Uniqueness for a system of SDE, in the context of Scaling limits of G-W with sex

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The following question was raised by Vincent last year in his presentation at Berlin-Paris Young Researchers Workshop, second edition Stochastic Analysis with applications in Biology and Finance (May 2-4 2018):

Does the following system of SDEs

\[ X_t = x_0 + \alpha_1 \int_0^t \xi_s ds + \sigma_1 \int_0^t \sqrt{\xi_s} dB^1_s + \int_0^t 1_{\theta \leq e_{s-}} z 1_{s \leq 1} d\tilde{N}^1 + \int_0^t 1_{\theta \leq e_{s-}} z 1_{s \geq 1} dN^1 \]

\[ Y_t = y_0 + \alpha_2 \int_0^t \xi_s ds + \sigma_2 \int_0^t \sqrt{\xi_s} dB^2_s + \int_0^t 1_{\theta \leq e_{s-}} z 1_{s \leq 1} d\tilde{N}^2 + \int_0^t 1_{\theta \leq e_{s-}} z 1_{s \geq 1} dN^2 \]

has a unique solution? Here \( \xi_t = X_t \wedge Y_t \), \( B^i, i = 1, 2 \) are BM and \( N^i, i = 1, 2 \) are Poisson processes (independent??) with intensities \( d\nu^i(ds,d\theta,dz) = ds d\theta \lambda_i(dz) \).

Note that this system appears as scaling limits in the context of G-W with sex (work in progress V. Bansaye, M.E. Caballero and S. Méléard).

We shall prove uniqueness of this system under the hypotheses

\[ \int_{\mathbb{R}_+} (z^2 \wedge 1) \lambda_i(dz) < \infty. \]

plus a technical condition to be explained.