CONCOURS D’ADMISSION FUI 2019 – SESSION AUTOMNE
FILIÈRE UNIVERSITAIRE INTERNATIONALE

PHYSICS TEST

(Duration : 2 hours)

The two parts of the subject are independant.
The use of calculators is not allowed.

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This topic deals with different aspects of seismic waves and their analysis which allows geophysicists to obtain information on the elasticity, the hardness of rocks, the presence of faults or geological anomalies. The seismic waves that travel the Earth may be surface waves or volume waves, transverse or longitudinal.
The subject is composed of a problem and an exercise. The problem proposes some simple modelizations of the seismic waves by studying equivalent mechanical systems. The exercise analyzes a seismic wave detection system.

Mathematical form :

- $\nabla f(x, y, z) = \left( \frac{\partial f}{\partial x} \right) \boldsymbol{\varepsilon}_x + \left( \frac{\partial f}{\partial y} \right) \boldsymbol{\varepsilon}_y + \left( \frac{\partial f}{\partial z} \right) \boldsymbol{\varepsilon}_z$
- $\Delta f(x, y, z) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$
Problem: modeling P and S seismic waves

1. The scheme below illustrates the characteristics of different types of seismic waves.

What is a transverse wave? Longitudinal wave? What type of waves do the so-called P and S seismic waves belong to in the previous scheme?

2. Modeling a longitudinal wave.

The system shown on the next figure is considered to be an infinite succession of masses $m$, distant from each other by a distance $a$, and coupled by non-contiguous massless springs, of spring constant $k$, of rest length equal to the distance $a$.

One assumes that the system is constrained to move along the horizontal axis by a support exerting a reaction force without friction. We note the difference of the position of the $n$th mass with respect to its equilibrium position. Make an assessment of the forces exerted by the springs on the $n$th mass and deduce that the differential equation of its movement is written:

$$m \frac{d^2 x_n}{dt^2} + k (2x_n - x_{n-1} - x_{n+1}) = 0 \quad (1)$$

3. Check that the previous differential equation allows solutions of the form:

$$x_n = A \exp i(\omega t - \beta n a) + B \exp i(\omega t + \beta n a)$$

provided that the temporal pulsation $\omega$ and the spatial pulsation $\beta$ are linked by an equation to be written. Plot the curve representing the variations of $\omega$ according to $\beta$. Deduce that, in this chain of springs, the waves can propagate only with temporal pulsations less than a cut-off value to be shown on the curve.
4. Longitudinal wave in a solid.

We consider a deformable solid of parallelepipedal shape whose 3 edges define the $0xyz$ trihedron. As shown in the figure, a force per unit area $N_z$ is applied on the faces orthogonal to $O_z$. In the following will be designated by the generic term "constraints" these forces per unit area.

If the solid is isotropic, it remains of parallelepipedal shape, but its length $l$ along $O_z$ undergoes an elongation $\Delta l$. Its relative elongation $\delta_z$ is given by Hooke's law:

$$\delta_z = \frac{\Delta l}{l} = \frac{1}{E} N_z$$

where $E$ is the solid’s Young’s modulus.

The other faces of the solid undergo symmetrical contractions, defined by:

$$\delta_x = \delta_y = -\sigma \delta_z = -\sigma E N_z$$

where we introduced the Poisson’s coefficient $\sigma$.

Give the dimensions of $E$ and $\sigma$. What is the value of $\sigma$ in the ideal case of a perfectly symmetrical and isotropic cube?

5. For a solid with constraints $N_x$, $N_y$ and $N_z$ on each of its face, show that we have a simple linear equation:

$$\begin{pmatrix} \delta_x \\ \delta_y \\ \delta_z \end{pmatrix} = \mathcal{M} \cdot \begin{pmatrix} N_x \\ N_y \\ N_z \end{pmatrix}$$

where $\mathcal{M}$ is a symmetrical matrix that one will write as a function of $\sigma$ and $E$.

6. One considers a slice of a solid medium, infinite along $O_y$ and $O_z$ and of uniform width along $O_x$. This plate is stretched along $O_x$ with a constraint $\tilde{N}_x$. Show in that particular case that:

$$\delta_x = \frac{N_x}{\alpha}$$

with

$$\alpha = E \cdot \frac{1 - \sigma}{(1 + \sigma)(1 - 2\sigma)}$$

7. We now consider an unlimited homogeneous isotropic medium in which a compression/dilatation disruption propagates, modeled as a propagating constraint, created by a source such as the epicenter of an earthquake, for example. On the next figure one shows a slice of a solid medium stretched at time $t$ by a constraint $N_x$. Initially the medium is between the planes $x$ and $x + dx$. 
Under the constraint’s action, the first plane moves by a quantity \( u(x, t) \). At the same time \( t \),
the elongation of the plane of \( x + dx \) abscissa will write:
\[ u + \frac{\partial u}{\partial x} dx. \]

By adopting the notations of the diagram above, justify that the plane of abscissa \( x + dx \) undergoes the constraint:
\[ N_x + \frac{\partial N_x}{\partial x} dx = N_x + \alpha \frac{\partial^2 u}{\partial x^2} dx. \]

8. Applying the fundamental principle of dynamics to the solid’s slice, whose density \( \rho \) is considered as a constant, show that one leads to the equation:
\[ \frac{\partial^2 u}{\partial x^2} - \frac{1}{V_t^2} \frac{\partial^2 u}{\partial t^2} = 0 \quad (2), \]
where we will express the parameter \( V_t \) as a fonction of \( \rho \) and \( \alpha \). What is the dimension of this parameter? What does it represent in relation to the propagation of the wave?
Considering that the elongations of the springs of question 2. are infinitely small, study the analogy between equations (1) and (2).

9. **Modeling a transverse wave.**

We now consider a mass weight \( m \), connected by two identical massless wires without elasticity at two fixed points \( A \) and \( B \) \((OA = OB = a)\) as shown in the figure.
The axis \( Oy \) is vertical ascending. The horizontal axis is noted \( Ox \).

The wires exert tension forces on the masses whose direction remains tangent to the wires. We move the mass of an ordinate \( y \) perpendicular to the axis \( AB \). Show that both tensions have the same modulus and establish the differential equation in \( y \) assuming that the angle of inclination \( \alpha \) of each of the wires with respect to the \( Ox \) axis is small. How would this result be modified if the gravity field was taken into account?

10. We consider an infinite succession of masses \( m \), distant from each other of the same distance \( a \), and coupled by a wire without mass and elasticity. Each wire, of length \( a \), exerts a tension of the same modulus \( T \) constant at both ends.

The system is shown in the figure below.

The gravitational forces are neglected with respect to the forces exerted by the wires on the masses. In this question we study strictly transverse movements. We note \( y_n \), the ordinate of the displacement of the mass \( n \).
Write for the nth mass, the differential equation of the movement according to the ordinates \( y_{n-1}, y_n \) and \( y_{n+1} \). This equation will be noted (3).

11. Check then that one can write a solution of the form:

\[
y_n = A' \exp i(\omega't - \beta' n a) + B' \exp i(\omega't + \beta' n a)
\]

provided the temporal pulsation \( \omega' \) and the spatial pulsation \( \beta' \) are linked by an equation that one will write. Compare to the solution found in 4.

12. A reasoning similar to that which was conducted in questions 4. to 8. applies when one is interested in shear stresses (parallel to the faces of a solid) rather than compression stresses (normal to faces of a solid).

Denoting \( v \) the transverse elongation one finds the propagation equation:

\[
\frac{\partial^2 v}{\partial x^2} - \frac{1}{V_i^2} \frac{\partial^2 v}{\partial t^2} = 0 \quad (4).
\]

Knowing that the S waves get their name from the Latin Secondae (which come in second position) and that the P waves draw theirs from the Latin Primae (which come in first position), propose an equation of order between the parameters and in connection with the figure in question 1.

13. Comment on the similarities between equations (1) and (3) obtained in "discrete" models on the one hand, and equations (2) and (4) obtained in "continuous" models on the other hand.

**Exercise: Recording of seismic movements using a seismograph.**

1. A mass \( M \) is suspended, by means of a massless spring of constant \( k \), to a frame relative. With respect to this frame the mass undergoes a vertical movement without friction, in the gravity field, assumed uniform, \( g \). Let \( O \) be the spring suspension point on the chassis. We note \( z \) the ordinate of the mass with respect to the point \( O \), \( z_c \) the rest value of \( z \) and \( Z = z - z_c \). It is assumed that the chassis is subject to the vertical ground vibration: \( Z_{sol} = Z_0 \cos \omega t \). The system is shown below. We can associate to any physical quantity \( Z \) the complex quantity \( Z \) which real part is \( Z \). Hence \( Z_{sol} = Z_0 \exp j \omega t \) where \( j^2 = -1 \).

Write the differential equation governing the variations of \( Z \) in the frame supposed Galilean. What is the forced sinusoidal response of the system to ground vibration, in the form of a relationship \( Z = A(\omega)Z_{sol} \), where the factor \( A \) is expressed according to the characteristics of the device?
2. The mass is more firmly attached to a negligible mass coil, with electrical resistance $r$ (we will neglect the coefficient of self-induction $L$ of the coil). The coil comprises $N$ turns, each of length $l$ and it has the same vertical movement as the mass, while remaining entirely in the gap of a magnet creating in the coil a radial magnetic field having the same constant standard in all directions. The terminals of the coil are closed on a resistor $r'$ and the total resistance of the circuit thus constituted is noted $R = r + r'$.

Write the equation of mechanical balance of the mass movement.

3. Write the electrical equation of the loop formed by the coil and the resistor on which the coil is closed.

Deduce the response of the system to a ground vibration, on the form of an equation $Z = A'(\omega) Z_{sol}$, where the factor $A'$ will be expressed as a function of the characteristics of the device.

4. Which conditions should be satisfied by the ground vibration pulsation $\omega$ in order that $Z$ is proportional to $Z_{sol}$? Is the voltage drop at the terminals of resistor $r'$ proportional to the ground vibration? By what type of electrical circuit should one replace resistor $r'$ to get a signal proportional to the ground vibration? Propose a scheme of the circuit.