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Skill Supply and Biased technical change

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Résumé: Cet article contribue au débat sur le progrès technique biaisé en analysant la dynamique de l'offre de travail qualifié et l'inégalité salariale dans un modèle de croissance endogène avec progrès technique biaisé en faveur des capacités. En raison d'un effet de découragement, l'augmentation des inégalités intra groupes réduit l'incitation à s'éduquer pour ceux qui ont des capacités ordinaires. Ce mécanisme induit une relation non monotone entre le taux de croissance de l'économie et l'offre de travail qualifié, phénomène qui s'est manifesté dans certains grands pays de l'OCDE au cours des années 1970 et 1980.

Abstract: This article contributes to the debate on skill-biased technical change by studying the dynamics of skill supply and wage inequality in an endogenous growth model with ability-biased technical progress. Due to a discouragement effect, rising within groups inequality reduces incentives to become educated for ordinary-ability workers. This mechanism generates a non-monotonic relationship between the growth rate and skill supply driving wage inequality upward during periods of accelerating technical change. This theoretical explanation is consistent with the apparent negative relationship between the relative skill supply and premium in the 1970s and 1980s in major OECD countries.

Mots clés : Inégalités de salaires, Progrès technique biaisé, Offre de travail qualifié, Croissance fondée sur l'innovation

Key Words : Wage Inequality, Biased Technical Change, Skill Supply, Innovation-Driven Growth

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Skill Supply and Biased Technical Change ^{*}

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1 Introduction

This paper provides a theoretical analysis of the dynamics of skill supply and wage inequality in an endogenous growth model with ability-biased technical progress. Combined with the signalling role of education, such an ability-bias may imply a discouragement effect to become educated for workers with ordinary ability. In turn, a negative relationship between skill supply and growth could characterize periods of widening within-groups inequality like the new information and communication technologies revolution.

This theoretical explanation is consistent with the behaviour of skill supply, and the wage premium during the 1970s and 1980s in the United States and Great Britain, and with the evolution of the growth rate of the GDP and the growth rate of labour supply since the 1970s in major OECD countries, as documented in details in section 2. The model proposed formalizes ability-biased technical change in a dynamic framework where workers' human capital is determined both by technological progress and ability, and the efficiency units supplied by the most able are not influenced by the technological level but by its growth rate, in the spirit of Galor and Moav's (2000) basic argument. But our approach then focuses on the negative relationship between technical change and skilled labour supply induced by ability-biased technical change. Indeed, ability-biased innovations cause a smaller fraction of workers with ordinary ability to choose to become educated by reducing the relative returns to skills for such workers. We propose a dynamic general equilibrium framework with endogenous technical change to analyze the non-monotonic response of skill-supply to wage inequality both between and within groups.

The argument that skill supply do matter in the evolution of biased technical change and wage inequality has echoes in two kinds of models skill-biased technical change. A first category of models considers the impact of skill supply on wage inequality. In

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Acemoglu (1998), the rise in the supply of skills is the source of skill-biased technical change, but supply itself is not influenced by the rate of technological progress. In Eicher & García-Peñalosa (2001), a direct effect of an increase in the relative supply is to reduce the skill premium, but an indirect effect is to generate more innovations and therefore a higher demand for skilled workers to absorb new technologies. Supply hence is crucial, but its effects are ambiguous. In Galor & Tsiddon (1997), the supply of skilled labour is likely to decline along the lifecycle of technology. During periods of major technological change (inventions), the returns to ability increase, which increases inequality and relative supply. Yet, once technologies become more accessible (with innovations), the returns to ability decline, thereby reducing inequality and relative supply. However, a low skill supply response is not contemporaneous to periods of major technological breakthroughs.

A second category of models analyzes the inverse and equivocal effect of biased technical change on skill supply. In this retrospect, the present paper is closer to Eicher (1996) who explains the positive links between technological progress and the relative wage on the one hand, and the long-term inverse fluctuations in the relative supply and wage of skilled labour on the other hand. In his model, the non monotonic relationships between supply, demand and relative wages are driven by an absorption effect. The absorption of bursts in technological progress requires the withdrawal of skilled labour from research and education, which subsequently increases the costs of both human capital investment and innovation¹. The incentives to accumulate human capital and the supply of skilled labour is reduced, which depresses growth. A higher rate of technological change can therefore reduce the stock of human capital, and higher relative wages can lead to a decline in the supply of skilled labour. The absorption cost of new technologies hence allows for inverse movements in the relative wage and supply of skilled labour in response to accelerations in technological change.

This article proposes a model that analyzes the skilled labour supply response to ability-biased technological change. Following Rubinstein & Tsiddon (1997), inequality both within and between education groups comes from the same source: the rise in the returns to ability. When it increases the returns to education, technological progress rises the productivity of skilled labour, independently individual ability. This mechanism explains the contemporaneous rise in both wage inequality between groups and relative skilled labour supply (since the incentives to become educated is higher). But when technological progress increases the returns to ability, it reduces, among those who choose to become educated, the productivity of the least able. The positive relationship between technical change and wage inequality within groups then reduces the relative supply of educated labour, thereby driving wage inequality upward.

This paper is organized as follows. Section 2 reports empirical evidence available for major OECD countries regarding the evolution of the growth rates of GDP and labour supply, and the relative percentile distribution of earnings since the 1970s. Section 3 presents the basic structure of the model. Section 4 defines the general competitive equilibrium and section 5 analyzes the impact of ability-biased technical change on skill supply and wage inequality. Section 6 concludes the paper.

¹Using a similar argument, Lloyd-Ellis (1999) shows that this absorption effect reduces innovation and growth.

2 Empirical evidence in major OECD countries

In this section, I provide some evidence indicating that in most OECD countries, periods of high growth rates of the GDP seem to go along with periods of slow and even decline of the growth rate of labour supply, suggesting a non-monotonic relationship between skill supply and biased-technical change, as developed in this paper.

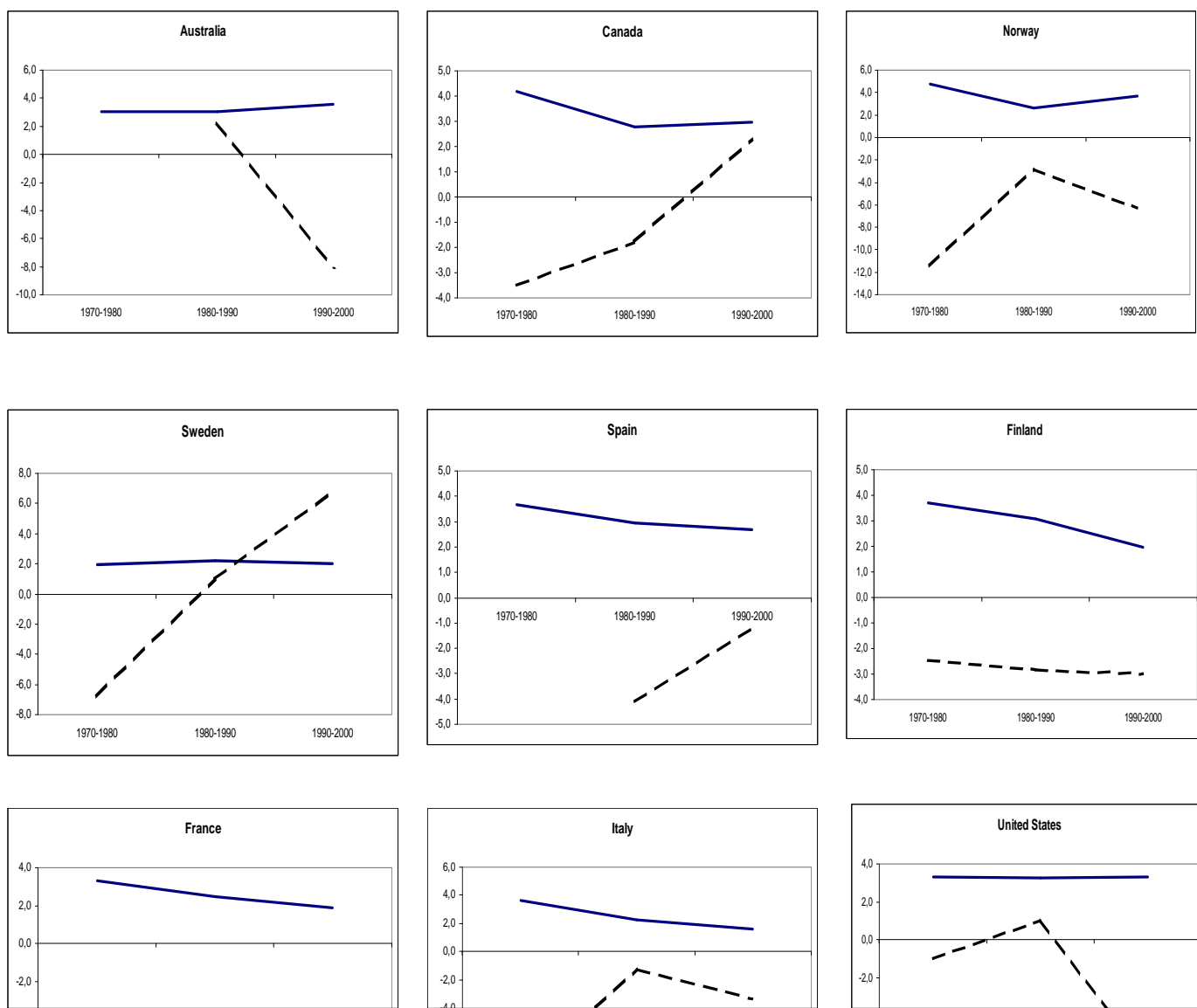
The growth rates of the Gross Domestic Product and labour supply are reported for the major OECD countries in the following figures (drawn from Tables I and II in appendix 7.1). Only comparable data have been reported, in particular regarding the available years for each type of data. This in turn reduced the number of OECD countries to be taken into account. Nevertheless, data related to the major industrial countries (the G-7 countries) since the 1970s were available and are reported in this section.

Following Prescott (2004), the labour supply measure considered is the number of hours worked per person aged 15-64 in the taxed market sector. However, as pointed out in the OECD employment outlook, the absolute values of such a variable may not be comparable between countries, hence it is the evolution of such a variable that matters. This precisely fits the goal of this paper which focuses on the links between the growth rates of labour supply and GDP during different phases of the growth process.

Figure 1: Average growth rates of GDP and labour supply in major industrialized countries, 1970-90

Source: OECD Productivity Database (2004), and Employment Outlook (1996, 2003)

— GDP
- - - Labour Supply



These figures emphasize the fact that the growth rate of GDP and labour supply seem to be negatively correlated for most of the major industrialized countries. More precisely, two groups of countries can be distinguished.

- The first group is composed of Australia, Canada, Norway, Spain and Sweden.

These countries are characterized by a complete negative relationship between the growth rate of GDP and labour supply since the 1970s. Between the 1970s and the 1980s, Canada, Norway, Spain and Sweden experienced a decline in the growth rate of the GDP, together with an increase in the growth rate of labour supply, a phenomenon which reversed between the 1980s and the 1990s. Australia was characterized by the same negative relationship but not during the same decades.

- The second group of countries is composed of Finland, France, United States, United Kingdom, Germany and Japan.

These countries have faced an apparent similar evolution of the growth rate of both the GDP and labour supply. However, since the 1970s, labour supply has been declining (its growth rate being permanently negative), while the growth rate of the GDP has remained positive. Hence, if there is any correlation between both variables, it is not a positive one. Besides, when taking a closer look at the figures, on average, the lowest decline in the growth rate of labour supply comes during a period in which the growth rate of the GDP was the highest (1970s) and vice versa during the 1990s which was a period characterized by the highest (lowest) decline in the growth rate of labour supply (GDP).

Regarding the behaviour of skill supply and the wage premium during the 1970s and 1980s, Katz and Murphy (1992) and Murphy, Riddell and Romer (1998) observe that in the United States, the largest increase in the supply of skilled workers comes during the 1971-1979 period in which the skill premium declined. Inversely, the smallest growth of the supply comes during the 1979-1987 period in which the skill premium expanded sharply. Gregg & Manning (1997) observe a similar pattern in Great Britain between 1975 and 1993. This apparent inverse relationship between relative supplies and demand for skills is reproduced in Figure 2 for nine OECD countries (drawn from Tables II and III in appendix 7.1).

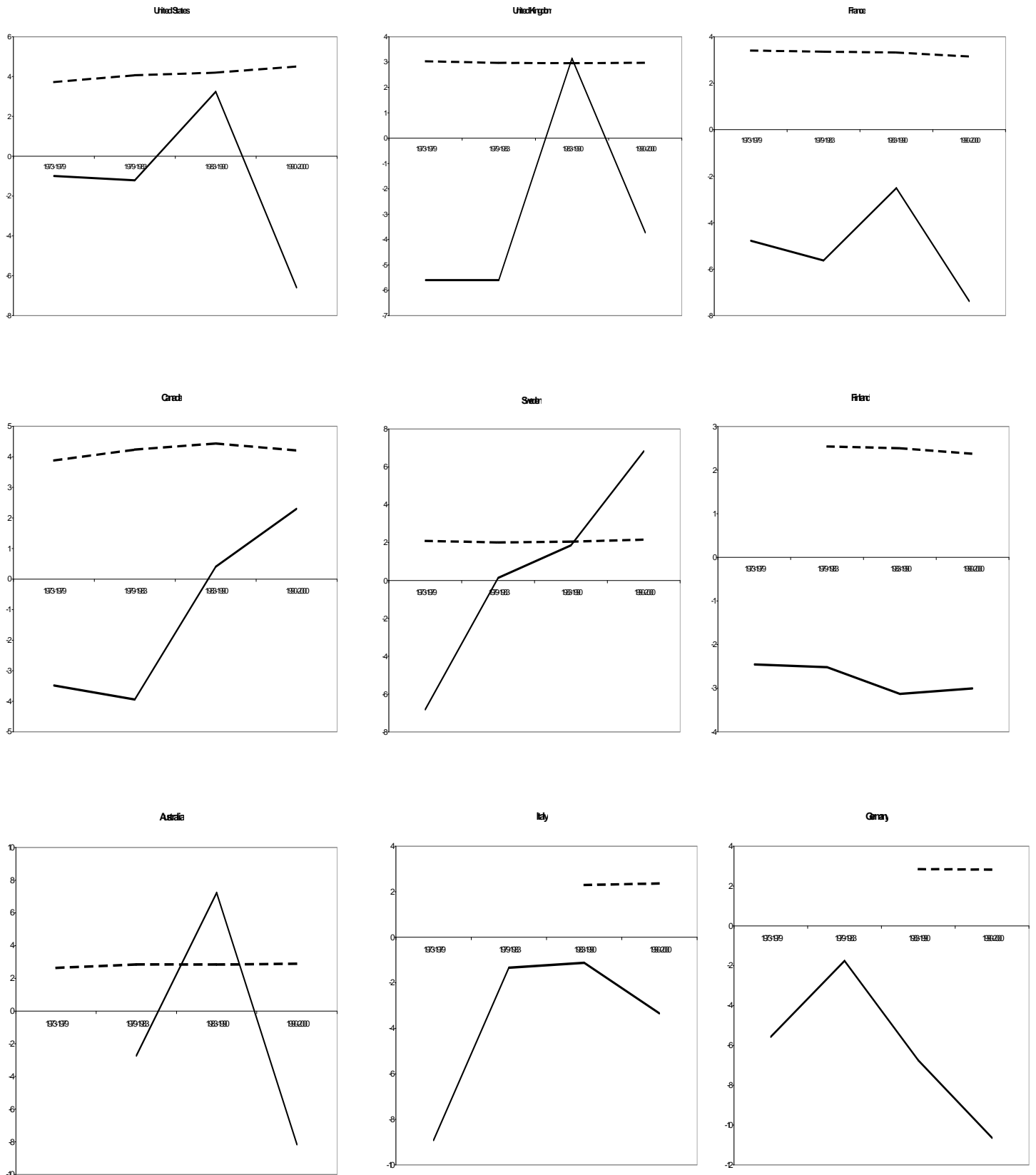
For Murphy, Riddell and Romer (1998), in the absence of policies facilitating substantial growth in post-secondary education at both the college and university levels, Canada would have experienced an increase in wage inequality between groups similar to that observed in the United States. Similarly, Nickell and Bell (1996) report that the relative wages of the unskilled have not fallen in Germany but have fallen substantially in Britain and the United States. They conclude that it would be a far higher education and training level in the bottom half of the ability range that would have enabled the German economy to respond to demand shifts toward the skilled in a more robust fashion. If the relative earnings of more- and less-educated workers do respond to changes in their relative supply, the acceleration in skill-biased technical associated with the “NICT revolution” may not have induced a sufficient response in supply to compensate the rise in relative wages.

Hence, for major OECD countries, the data available seem to highlight that the highest growth rate of the GDP on average comes during periods in which the growth of labour supply is the lowest, and the wage premium is declining, and vice versa. In other words, this evidence suggests a negative correlation between growth and skill supply on average since the 1970s in major OECD countries.

Figure 2: Average annual growth rates of Labour Supply and Relative Percentile (P90/P10) of gross earnings.

Source: OECD Labour Market Statistics (July 2004) and Employment Outlooks (1996, 2003)

— Labour Supply - - - P90/P10



3 The model

The framework builds upon the basic Romer (1990)'s model in an overlapping generations framework where individuals live for two periods. In the first period, individuals choose to become educated or not and are then employed as unskilled or skilled workers. In the second period, individuals retire consuming their savings. The economy is composed of a final good sector, an intermediate goods sector and a research sector. The final good can be used for consumption (of households), investment (research) and intermediate goods production. The research sector produces innovations which are commercialized and sold by intermediate good producers to the final good sector. Labour is employed only in final good manufacturing.

3.1 The final good sector

The final good is the numeraire. It is produced in a competitive environment using skilled workers, H_t , unskilled workers, L_t and intermediate goods $x_t(i)$ according to the following technology:

$$Y_t = (H_t)^{\alpha\beta} (L_t)^{(1-\alpha)\beta} \int_0^{n_t} x_t(i)^{1-\beta} di \quad (1)$$

where $0 < \alpha < 1$, $0 < \beta < 1$ and n_t is the number of intermediate goods produced in every period. Profit maximization by a representative firm in this sector leads to the following inverse demand for factors of production:

$$w_t^s = \frac{\partial Y_t}{\partial H_t}, \quad w_t^u = \frac{\partial Y_t}{\partial L_t}, \quad p_t(i) = \frac{\partial Y_t}{\partial x_t(i)} \quad (2)$$

with w_t^s the wage rate per efficiency unit of skilled labour, w_t^u the wage rate per efficiency unit of unskilled labour and $p_t(i)$ the price of intermediate good i .

3.2 The intermediate goods sector

The intermediate goods sector is monopolistic: once a good is invented in the research sector, an intermediate goods firm can produce it provided it buys the corresponding patent (or licence) to the innovator. We assume that intermediate goods are produced using final good, according to a one-for-one technology ($x_t(i)$ units of intermediate good i requires $x_t(i)$ units of final good). Given the inverse demand for intermediate goods in the final good sector, the optimization program for firm $i \in [0, n_t]$ writes:

$$\max_{x_t(i)} \pi_t(i) = p_t(i)x_t(i) - x_t(i) = (1-\beta)(H_t)^{\alpha\beta} (L_t)^{(1-\alpha)\beta} x_t(i)^{1-\beta} - x_t(i).$$

The first-order condition of this program yields:

$$p_t(i) = p_t = \frac{1}{1-\beta}, \quad x_t(i) = x_t = (1-\beta)^{2/\beta} (H_t)^\alpha (L_t)^{1-\alpha}, \quad \pi_t(i) = \pi_t = \frac{\beta}{1-\beta} x_t. \quad (3)$$

Hence intermediate goods producers are symmetric in equilibrium.

3.3 The research sector

We consider that firms' lifetime is finite. In standard models of growth with expanding product variety, patents are infinitely-lived. In our framework, this assumption implies that patents can be sold at the end of each period. There are two ways to produce an intermediate good: either buy a patent on a pre-existing product (n_t firms), or buy a patent on a new variety ($n_{t+1} - n_t$ firms). Firms willing to enter the market by purchasing an existing patent have to pay an entry cost equal to the patent price. Firms willing to enter by inventing a new variety have to devote resources to deliberate R&D. The R&D process is deterministic and requires to spend F units of final good to introduce a new variety².

Entry decisions in period t are made in period $t-1$. The price that a potential entrant will be willing to pay for an existing patent, $v_{t-1}^e(1+r_t)$, with r_t the interest rate, must equal the present value of profits earned by an intermediate good producer in period t , π_t augmented with the market price of the patent at the end of the period, v_t^e . The value of a patent therefore writes:

$$v_{t-1}^e = \frac{\pi_t + v_t^e}{1+r_t} \quad (4)$$

Free-entry in the research sector implies:

$$v_t^e(i) = F = v_t^e \quad (5)$$

Equations (4) and (5) together determine the interest rate of the economy: $r_t = (\pi_t + F)/F$. The overall investment level in period t is composed of the amount of resources dedicated to the acquisition of existing patents, $n_t v_t^e$ and the amount of resources devoted to creating new varieties, $(n_{t+1} - n_t)F$. Hence, $I_t = n_t v_t^e + (n_{t+1} - n_t)F$. Using the free-entry condition then implies the following investment level:

$$I_t = n_{t+1}F \quad (6)$$

² The assumption that R&D requires resources expressed in terms of final good is common to models based on the standard framework of Grossman and Helpman (1991). This assumption makes sense in the present model because we model labour supply decision in terms of educational choices and not occupational choices. Relaxing this assumption and assuming instead that educated labour is the input of the R&D process would induce a mobility condition for skilled workers that is, in equilibrium, an indifference condition between working in the final good or in the research sector. This would not change the endogenous allocation of the workforce between skilled and unskilled labour, but would only make the skilled wage dependent on the patent price, that is in fine, on the price and quantity of intermediate goods. Since the research process remains deterministic in such kind of models, given the equilibrium conditions developed in section 4 (see equation 11), this would not change the qualitative results of the model. In a different framework, based on the model proposed by Aghion and Howitt (1992), Crifo-Tillet and Lehmann (2004) analyze such a mobility condition, considering an endogenous labour supply based on an occupational choice. In this framework, the model becomes more complex but the qualitative results still hold.

3.4 Labour market and resources constraints

The size of the population is normalized to one. Individuals differ in their ability: they can have either high ability or ordinary ability. The distribution of ability is fixed and exogenous. However, individuals can acquire education, so that the allocation of workers between skilled and unskilled labour is endogenous. Let M denote the proportion of workers that have high ability. We denote by E_t (respectively O_t) the fraction of workers with high (respectively ordinary) ability who choose to become educated. Resources constraints on the labour market then write:

$$N_t^{se} + N_t^{so} + N_t^u \equiv 1 \quad \text{with} \quad \begin{cases} N_t^{se} = ME_t \\ N_t^{so} = (1-M)O_t \\ N_t^u = M(1-E_t) + (1-M)(1-O_t) \end{cases} \quad (7)$$

where N_t^{se} (respectively N_t^{so}) is the number of skilled workers with high (respectively ordinary) ability, and N_t^u is the number of unskilled workers.

3.5 Households' decisions

Preferences of an individual j in generation t are represented by the following utility function: $u_t^j = (c_{1t}^j)^{1-\phi} (c_{2t+1}^j)^\phi$, $0 < \phi < 1$ where c_{1t}^j is the first period consumption, c_{2t+1}^j is the second period consumption and ϕ is the expenditure share for second period consumption. In the first period, individuals make two decisions: becoming educated or not and saving for the second period consumption. Savings decisions are made according to the following program:

$$\max_{c_{1t}^j, c_{2t+1}^j} u_t^j = (c_{1t}^j)^{1-\phi} (c_{2t+1}^j)^\phi \quad \text{s.t.} \begin{cases} \Omega_t^j = c_{1t}^j + s_t^j \\ c_{2t+1}^j = (1+r_{t+1})s_t^j \end{cases} \quad (8)$$

where s_t^j denotes saving, r_{t+1} is the interest rate and Ω_t^j is the income net of education costs.

Given the distribution of ability, the choice of becoming educated allows workers with high (respectively ordinary) ability to supply k_t efficiency units of labour (respectively l_t efficiency units of labour). Individuals who choose not to acquire skills supply m_t efficiency units of labour. We assume that the number of efficiency units of unskilled labour is independent of ability to capture the idea that unskilled workers are employed in simple, routine jobs which do not allow them to take advantage of their intrinsic competencies. In contrast, skilled workers are employed in complex jobs which enable those with high ability to exploit their comparative advantage and supply higher efficiency units of labour. Parameters k , l and m are such that

$$k_t > l_t \geq m_t \quad (\text{A1})$$

The inequality $k_t > l_t$ captures returns to ability. It means that the productivity (efficiency units of labour) of individuals with high ability who choose to become

educated is higher than the productivity of individuals with ordinary ability who choose to become educated. The inequality $l_t \geq m_t$ captures returns to skills as it implies that unskilled workers supply lower efficiency units of labour than skilled workers. Ability and education hence are complement inputs in the formation of human capital as in Becker (1975)'s theory of human capital. Given both kinds of returns, the earnings of individuals, based on wage rates per efficiency unit of labour, are given by:

$$W_t^{se} = k_t w_t^s, \quad W_t^{so} = l_t w_t^s, \quad W_t^u = m_t w_t^u \quad (9)$$

The cost of education is such that individuals who choose to become skilled workers devote a fraction $0 < d < 1$ of their unit-time endowment to the formation of human capital³. Individuals who choose to become skilled supply a fraction $(1-d)$ of their potential efficiency units of skilled labour. Expected incomes of each category of workers then write: $\Omega_t^u = W_t^u$, $\Omega_t^{se} = (1-d)W_t^{se}$, $\Omega_t^{so} = (1-d)W_t^{so}$.

The assumption that educated workers with ordinary ability are less efficient than educated workers with high ability models the signaling role of education first explored by Spence (1973). Stated differently, education act as a signal of ability and differences in educational attainment arise as a consequence of heterogeneity in ability. Willen, Hendel and Shapiro (2004) consider a model where, when households face credit constraints, lack of education could mean either low ability, or high ability and low financial resources. Despite the difference in their modelling strategy, their results are not in contradiction with the present model. In their model indeed, the wage of uneducated workers reflect the mix of abilities: the smaller the proportion of high-ability persons in the uneducated pool, the lower the wage for unskilled labour. In turn, improving opportunities for higher education, either by providing direct grants for tuition or by reducing the interest rate that households pay to borrow for an education, more high-ability workers get an education and the quality of the unskilled pool drops, lowering the unskilled wage. Besides, they show that considering an additional productivity-enhancing aspect to education do not change their basic results.

Individuals' utility function is defined over first and second period consumption and is strictly increasing in both variables. Given that individuals work only in the first period, maximizing first period income is a necessary condition for maximizing utility. In other words, individuals choose to become skilled workers if the skilled income is higher than the unskilled one. In the present model, individuals with ordinary ability then choose to become skilled workers as long as their expected income as skilled workers, Ω_t^{so} is higher than that of unskilled workers, Ω_t^u . In equilibrium, this condition is binding, implying that the number of workers with ordinary ability who choose to become educated satisfies the following indifference condition: $\Omega_t^{so} = \Omega_t^u$. Regarding individuals with high ability, the assumption that $k_t > l_t$ implies that $W_t^{se} > W_t^{so}$, therefore $\Omega_t^{se} > \Omega_t^{so}$. Education decisions in turn satisfy the following rule:

³This fraction is identical for all individuals, whatever their ability, differences between workers being captured by the returns to skills and ability. d represents the cost of education, expressed in terms of units of time necessary for the formation of human capital.

$\Omega_t^{se} > \Omega_t^{so} = \Omega_t^u$ which implies that, all individuals with high ability choose to become educated:
 $E_t = 1$ (10)

4 Competitive general equilibrium

A competitive general equilibrium for this economy in every period t is characterized by the following conditions.

- Firms in the final good sector determine the quantity of inputs (skilled labour, unskilled labour and intermediate goods) that maximize profits. This yields the inverse demand functions (2):

$$w_t^s = \alpha\beta \frac{Y_t}{H_t}, \quad w_t^u = (1-\alpha)\beta \frac{Y_t}{L_t}, \quad p_t = (1-\beta) \frac{Y_t}{n_t x_t} \quad (11)$$

- Firms in the intermediate goods sector are symmetric. Price and quantities produced in equilibrium are given by equations (3).

- Firms in the research sector enter the market either by purchasing a patent on previous products (n_t) or by developing new products ($n_{t+1} - n_t$). All profit opportunities are exploited in equilibrium and the investment level is given by equation (6).

- Education decisions are such that all individuals with high ability choose to become educated: $E_t = 1$, and on the other hand, given the rule $\Omega_t^{se} > \Omega_t^{so} = \Omega_t^u$, workers with ordinary ability are indifferent between becoming educated or not. The ratio Ω_t^{so}/Ω_t^u therefore satisfies the following condition:

$$\frac{\Omega_t^{so}}{\Omega_t^u} = 1 \Leftrightarrow \frac{l_t w_t^s}{m_t w_t^u} = \frac{1}{1-d} \quad (12)$$

- Equilibrium on the labour market implies equality between demand and supply in efficiency units:

$$H_t = k_t N_t^{se} + l_t N_t^{so}, \quad L_t = m_t N_t^u \quad (13)$$

The number O_t of workers with ordinary ability that choose to become educated is

determined using equations (12) $\frac{l_t w_t^s}{m_t w_t^u} = \frac{1}{1-d}$ where the skill premium $\frac{w_t^s}{w_t^u}$ is obtained

using equations (2), (7) and (13): $\frac{w_t^s}{w_t^u} = \frac{\alpha}{1-\alpha} \frac{L_t}{H_t} = \frac{\alpha}{1-\alpha} \frac{m_t(1-M)(1-O_t)}{k_t M + l_t(1-M)O_t}$ (14)

$$\frac{\alpha}{1-\alpha} \frac{m_t(1-M)(1-O_t)}{k_t M + l_t(1-M)O_t} = \frac{1}{1-d} \Leftrightarrow O_t = \frac{\frac{\alpha}{1-\alpha}(1-d) - \frac{k_t}{l_t} \frac{M}{1-M}}{\frac{\alpha}{1-\alpha}(1-d) + 1} \quad (15)$$

• Individuals determine the level of consumption and savings that maximize their intertemporal utility, subject to the budget constraints according to (8). This leads to the following individual saving function: $s_t^i = \phi \Omega_t^i$, with ϕ the marginal propensity to save (around 10% on average in OECD countries).

Aggregate savings is then given by :

$$S_t = \phi \left(\Omega_t^u N_t^u + \Omega_t^{se} N_t^{se} + \Omega_t^{so} N_t^{so} \right) = \phi \left((1-d)k_t w_t^s N_t^{se} + (1-d)l_t w_t^s N_t^{so} + w_t^u m_t N_t^u \right)$$

Using the labour market clearing condition (equation 13), one gets:

$$S_t = \phi \left(w_t^u L_t + (1-d)w_t^s H_t \right) = \phi w_t^u L_t \left(1 + (1-d) \frac{w_t^s H_t}{w_t^u L_t} \right)$$

Substituting for the skill premium $\frac{w_t^s}{w_t^u} = \frac{\alpha}{1-\alpha} \frac{L_t}{H_t}$ (equation 14), we get:

$S_t = \phi w_t^u L_t \left(1 + (1-d) \frac{\alpha}{1-\alpha} \right)$. Replacing w_t^u by its value in (11), one has then:

$S_t = \phi(1-\alpha)\beta \left(1 + (1-d) \frac{\alpha}{1-\alpha} \right) Y_t$, and using the fact that $Y_t = \frac{n_t p_t x_t}{1-\beta}$ (see 11),

aggregate savings is such that: $S_t = \phi \frac{(1-\alpha)\beta}{1-\beta} \left(1 + (1-d) \frac{\alpha}{1-\alpha} \right) n_t p_t x_t$.

Finally, using equations (3): $p_t x_t = \frac{1}{1-\beta} (1-\beta)^{\frac{2}{\beta}} (H_t)^\alpha (L_t)^{1-\alpha} = (1-\beta)^{\frac{2-\beta}{\beta}} (H_t)^\alpha (L_t)^{1-\alpha}$,

savings write:

$$S_t = \phi(1-\beta)^{\frac{2(1-\beta)}{\beta}} (1-\alpha)\beta \left(1 + (1-d) \frac{\alpha}{1-\alpha} \right) n_t H_t^\alpha L_t^{1-\alpha} \quad (16)$$

• Equilibrium on the goods market implies equality between aggregate saving and investment: $S_t = I_t$.

Equalizing (16) and (6), implies:

$$n_{t+1} F = \phi(1-\beta)^{\frac{2(1-\beta)}{\beta}} (1-\alpha)\beta \left(1 + \frac{\alpha}{1-\alpha} (1-d) \right) n_t H_t^\alpha L_t^{1-\alpha}$$

The growth rate is defined as follows:

$$1 + g_{t+1} = \frac{n_{t+1}}{n_t} = \frac{\phi}{F} (1-\beta)^{\frac{2(1-\beta)}{\beta}} (1-\alpha)\beta \left(1 + \frac{\alpha}{1-\alpha} (1-d) \right) H_t^\alpha L_t^{1-\alpha}$$

Substituting for H_t and L_t by their values defined in equations (7) and (13), one gets:

$$1 + g_{t+1} = \frac{\phi}{F} \beta(1-\beta)^{2(1-\beta)} (1-\alpha)^{1-\alpha} (1-d)^{1-\alpha} \left(\frac{\alpha(1-d)}{1-\alpha} + 1 \right) (k_t M + l_t(1-M)O_t)^\alpha (m_t(1-M)(1-O_t))^{1-\alpha}$$

Replacing O_t by its value given in equation (15) we get after simplifications:

$$1 + g_{t+1} = \frac{\phi}{F} \beta(1-\beta)^{2(1-\beta)} (1-\alpha)^{1-\alpha} (1-d)^{1-\alpha} \left(\frac{\alpha(1-d)}{1-\alpha} + 1 \right) \left(\frac{m_t}{l_t} \right)^{1-\alpha} (k_t M + l_t(1-M)) \quad (17)$$

5 Skill supply and ability-biased technical change

To analyze the impact of ability-biased technical change on skill supply and relative wages, we first consider a stationary environment where parameters k , l and m are constant and we determine the impact of a shock increasing the returns to ability k/l . Two inequality indexes can be defined: wage inequality within skilled workers, denoted by $\Gamma^{s/u}$, and wage inequality between skilled and unskilled workers, denoted by Γ^s .

The average income of skilled workers is defined by the income (in efficiency units) of skilled workers, divided by the size of the skilled work force: $\bar{W}_t^s = \frac{k_t N_t^{se} + l_t N_t^{so}}{N_t^{se} + N_t^{so}} w_t^s$.

Similarly, the average income of unskilled workers is given by: $\bar{W}_t^u = \frac{m_t N_t^u}{N_t^u} w_t^u = m_t w_t^u$

Wage inequality within groups is defined by:

$$\Gamma^s = \frac{N_t^{se} W_t^{se}}{N_t^{so} W_t^{so}} = \frac{k_t}{l_t} \frac{M}{1-M} \frac{1}{O_t} = \frac{k_t}{l_t} \frac{M}{1-M} \frac{\frac{\alpha(1-d)}{1-\alpha} + 1}{\frac{\alpha(1-d)}{1-\alpha} - \frac{k_t}{l_t} \frac{M}{1-M}} \quad (18)$$

Between groups inequality writes: $\Gamma^{s/u} = \frac{\bar{W}_t^s}{\bar{W}_t^u} = \frac{\frac{k_t N_t^{se} + l_t N_t^{so}}{N_t^{se} + N_t^{so}}}{m_t w_t^u}$

Hence, given the resources constraints (7): $\Gamma^{s/u} = \frac{k_t M + l_t(1-M)O_t}{M + (1-M)O_t} \frac{w_t^s}{m_t w_t^u}$

Substituting for $\frac{w_t^s}{w_t^u}$ (equation 14) and O_t (equation 15) we have after simplifications:

$$\Gamma^{s/u} = \frac{\alpha (1-M)(1-O)}{1-\alpha M + (1-M)O} = \frac{\frac{\alpha}{1-\alpha} \left[M \left(\frac{k}{l} - 1 \right) + 1 \right]}{\frac{\alpha}{1-\alpha} (1-d) - M \left(\frac{k}{l} - 1 \right)} \quad (19)$$

Proposition 1. *A shock increasing the returns to ability k/l induces a decline in the number of workers with ordinary ability who choose to become educated and an increase in wage inequality both between groups and within skilled workers.*

Proof: immediate from equations (15), (18) and (19). \square

k/l measures the returns to ability as it is the ratio of the income of skilled workers with high ability to the income of skilled workers with ordinary ability (Ω^{se}/Ω^{so}). An increase in the returns to ability reduces the number of workers with ordinary ability who choose to become educated via a productivity effect. When the returns to skills for workers with high ability increases, this is equivalent in this framework to a reduction in the returns to skills for workers with ordinary ability. Firms are incited to save workers relatively less productive and increase their demand for the most productive workers. The supply of skilled labour by workers with ordinary ability adjusts downward. This effect is responsible for the increase in wage inequality between groups (see 19). Intuitively, a higher k/l implies that high-ability workers provide lots of efficiency units of skilled labour, which tends to depress the wage and hence the income of ordinary-ability agents if they become skilled.

From (18), we see that an increase in the returns to ability increases wage inequality within skilled workers through two effects. On the one hand, there is a direct income effect through the increase in the efficiency of high-ability relatively to ordinary-ability workers (k/l). On the other hand, there is an indirect supply effect ($1/O$). The reduction in the number of workers with ordinary ability who choose to become educated reinforces the upward pressure on the relative income of high-ability workers.

In Eicher (1996)'s model, skill-biased technical change exerts a negative impact on the cost of accumulation of human capital. The relative wage of skilled workers is determinant both in the cost of absorbing new technologies in production and in the formation of human capital. A higher rate of technological progress rises the relative wage of skilled workers (and hence inequality between groups) and reduces the number of skilled workers in the education sector. This reduces the incentives to become educated. In the present model, the rise in wage inequality within groups reduces the marginal benefit of education for individuals with ordinary ability. It is as if the opportunity cost of education for these individuals was rising in the presence of within groups inequality, inducing a discouragement effect. A rise in the relative wage of skilled workers is compatible with a decrease in the relative supply of skilled labour, and the inverse relationship between supply and demand stems from within groups inequality which reduces the incentives to become educated for ordinary-ability workers.

We turn now to the analysis of ability-biased technical change. We consider that technological change exerts an erosion effect on the efficiency units of labour supplied by unskilled and ordinary-ability workers. Intuitively, the faster the rate of technological change, the more one has to cope with tasks and situations not previously encountered. Such an instable environment demands ability to adapt and learn. Even for more educated individuals, the lower the ability, the lower the productive efficiency in an environment that changes rapidly. To formalize this erosion effect, we consider that the

number of efficiency units of labour supplied by unskilled and ordinary-ability workers are decreasing in the rate of technological progress. In contrast, the number of efficiency units of labour supplied by high-ability workers is not depreciated by technological progress. The returns to skills are therefore such that:

$$\begin{aligned} k_t &= k, \quad l_t = \lambda(1 + g_t), \quad m_t = \mu(1 + g_t) \\ \text{with } \lambda(\cdot) &> \mu(\cdot), \quad \lambda'(\cdot) \leq \mu'(\cdot) < 0 \end{aligned} \quad (20)$$

The erosion effect is such that l_t and m_t are decreasing with the rate of technical change. Technical change is ability-biased because the efficiency units of labour supplied by the most able workers are constant both in a stationary and a non-stationary environment. Technical change is also such that the efficiency units of skilled workers with ordinary-ability workers are decreasing at a higher pace compared to unskilled workers. This assumption captures the fact that in a rapidly changing environment, skilled jobs based on non-repetitive, interactive tasks impose tighter constraints on cognitive ability, thereby depreciating more rapidly the returns to a signalling education system for ordinary ability workers, while unskilled jobs, based on repetitive routine tasks, though being costly in terms of adaptation to new technological environments, imply a flatter depreciation rate. This argument is consistent with the observation on OECD data that workplaces where few or no qualifications are required are less likely to have adapted to new technological and organizational environments (see OECD Employment Outlook, 1999).

Given (20), the economy's growth rate writes:

$$1 + g_{t+1} = \gamma \left[\frac{\mu(1 + g_t)}{\lambda(1 + g_t)} \right]^{1-\alpha} [kM + (1-M)\lambda(1 + g_t)] \quad (21)$$

where $\gamma = \frac{\phi}{F} \beta(1-\beta)^{2(1-\beta)} (1-\alpha)^{1-\alpha} (1-d)^{1-\alpha} \left(\frac{\alpha}{1-\alpha} (1-d) + 1 \right)$ is such that

$$\frac{(\lambda(1)/\mu(1))^{1-\alpha}}{kM + (1-M)\lambda(1)} < \gamma < \frac{(\lambda(2)/\mu(2))^{1-\alpha}}{kM + (1-M)\lambda(2)} \quad (A2)$$

This assumption guarantees that the steady-state growth rate belongs to the unit interval.

Lemma 1 *Under assumption (A2) and given (19), there exists a unique steady-state rate of technical change $\bar{g} \in (0,1)$. The economy converges with oscillations to \bar{g} for all $g_0 \in (0,1)$.*

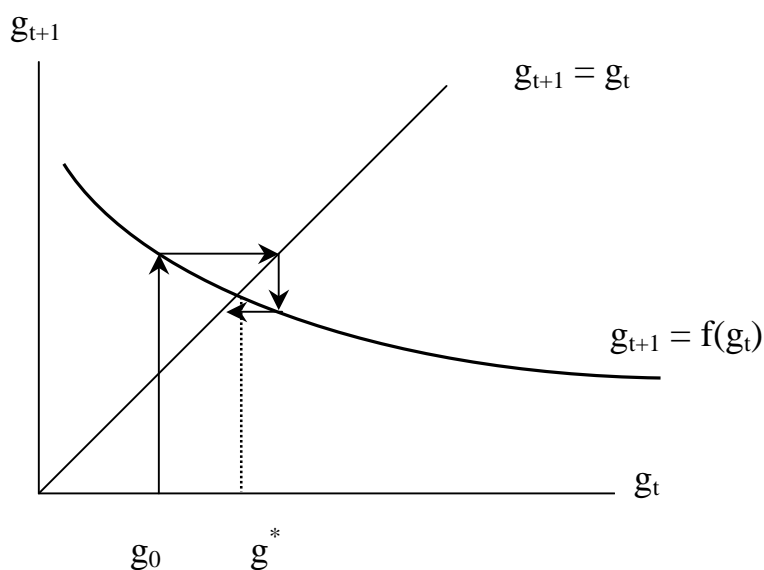
Proof. see appendix 7.2. \square

Figure 3 reproduces the dynamics of the economy's growth rate. The evolution of inequality and skill supply along the transition toward the steady-state are described in the following proposition.

Proposition 2 *Along the transition to the steady-state, ability-biased technical change exerts a non monotonic pressure on wage inequality both between and within groups, and on the number of individuals with ordinary ability who choose to become educated.*

Proof. After substituting (21) into (17), (19) and (20), these results are corollaries of Proposition 1 and Lemma 1. \square

Figure 3: Dynamics of the growth rate with ability-biased technical change



The non monotonic relationship between wage inequality within groups and skill supply stems from the erosion and discouragement effects due to ability-biased technical change. For low levels of the growth rate (below its stationary value), the erosion effect on the efficiency units of labour supplied by ordinary-ability workers is low. Since their relative income is relatively high, there are enough incentives to become educated for ordinary-ability individuals, and the growth rate is enhanced. Yet, this increase in the growth rate in turn erodes their relative efficiency units of labour. Due to a discouragement effect, the number of skilled workers with ordinary ability then decreases, which depresses the growth rate. The technological transition therefore is oscillatory, implying a non monotonic relationship between skill supply wage inequality and the rate of technical change.

This model can contribute to explain the observation that in the United States and Great Britain, the skill premium declined when supply grew at its highest pace (1970s) and inversely, the highest growth in the supply of skilled labour occurred when the skill premium was the lowest (1980s). In the model, periods of high (respectively low) growth increase (respectively reduce) wage inequality between and within groups. The rise (respectively reduction) in within groups inequality reduces (respectively increases) the incentives to become educated for workers with ordinary ability, thereby exerting an upward (respectively downward) pressure on between groups inequality. Ability-biased

technical change hence induces a non monotonic relationship between relative demand, relative supply and growth. Bursts of technical change widens the gap between the returns to skills of high-ability workers and that of ordinary-ability workers. The erosion effect reduces the number of skilled workers with ordinary ability and rises inequality between groups. The increase in wage inequality between and within groups comes from the same source: the increase in the demand for ability in periods of technological acceleration.

6 Conclusion

The theoretical framework proposed in this article enables to characterize the links between technical change and inequality between and within groups via two relationships. First, any increase in the growth rate, if it increases the returns to ability, increases inequality both between and within groups. Second, an increase in within groups inequality reduces the incentives to become educated for workers with ordinary ability. This model reproduces a stylized version of the evolution of the relative demand and supply of skills major OECD countries since the 1970s. Slow growth goes along with low inequality and relatively high supply of skilled labour. Inversely, an acceleration in ability-biased technical change induces high inequality and lower supply of skilled labour.

The results highlight that, when education acts as a signalling device of ability, the supply response to skill-biased technical change is conditioned by its impact on within groups inequality. There is no opposition between the forces that determine supply and the forces that determine demand, but rather interaction between both. Periods of high within groups inequality reduce the incentives to acquire skills and increase inequality between groups. The existence of (and the rise in) within groups inequality reduces the marginal benefit of education for individuals with ordinary ability, thereby exerting a discouragement effect to become educated for these workers.

This model however does not explain why, with an apparent similar technological bias, European economies witnessed a far lower of inequality than the United States in the 1980s. A possible argument is to consider that it is unemployment rather than inequality that increased in Europe. To explain this phenomenon, one should incorporate imperfect wage setting in order to account for the institutional environment and endogenize the unemployment rate. This constitutes an area for future research.

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7 Appendix

7.1 Growth rates of GDP and Labour Supply

Table I reports the annual average growth rates of the Gross Domestic Product in per cent and are drawn from the OECD Productivity Database (September 2004).

Table I: Annual average growth rates of the Gross Domestic Product in per cent

	Canada	Sweden	Spain	Finland	France	Australia	United Kingdom	United States	Germany	Italy	Japan	Norway
1970-80	4,2	2,0	3,7	3,7	3,3	3,0	2,0	3,3	2,8	3,6	4,5	4,8
1980-90	2,8	2,2	2,9	3,1	2,5	3,1	2,7	3,3	2,3	2,3	4,0	2,6
1990-00	2,9	2,0	2,7	2,0	1,9	3,6	2,4	3,3	1,9	1,6	1,5	3,7
1990-01	2,8	1,9	2,7	1,9	1,9	3,6	2,4	3,1	1,8	1,6	1,4	3,6
1990-02	2,9	1,9	2,6	1,9	1,8	3,5	2,3	3,0	1,7	1,5	1,2	3,5
1990-03	2,8	1,9	2,6	1,9	1,7		2,3		1,5	1,4	1,3	3,2
1990-95	1,7	0,8	1,5	-0,8	1,1	3,3	1,7	2,5	2,1	1,3	1,5	3,9
1995-00	4,2	3,3	3,8	4,7	2,7	3,9	3,1	4,1	1,8	1,9	1,4	3,6
1995-01	3,8	2,9	3,7	4,1	2,6	3,9	3,0	3,5	1,6	1,9	1,2	3,5
1995-02	3,7	2,8	3,4	3,9	2,4	3,7	2,8	3,3	1,4	1,7	1,0	3,2
1995-03	3,5	2,6	3,3	3,6	2,1		2,7		1,2	1,5	1,2	2,8

Source: OECD Productivity Database (2004)

Table II reports the annual growth rates of labour supply, the latter being defined by OECD as the total hours worked of all persons employed. These data are computed from the average hours worked from the OECD Employment Outlook (1996 and 2003).

Table II: Annual average growth rates of hours worked of all persons employed in percent

	Canada	Sweden	Spain	Finland	France	Australia	United Kingdom	United States	Germany	Italy	Japan	Norway
1973-79	-3,5	-6,8		-2,5	-4,8		-5,6	-1,0	-5,6	-8,9	-3,4	-11,4
1979-83	-3,9	0,1	-3,9	-2,5	-5,6	-2,7	-5,6	-1,2	-1,8	-1,3	-1,5	-1,9
1983-90	0,4	1,9	-4,3	-3,1	-2,5	7,2	3,1	3,2	-6,8	-1,1	-3,1	-3,7
1990-00	2,3	6,8	-1,2	-3,0	-7,4	-8,2	-3,7	-6,6	-10,6	-3,3	-10,9	-6,3

Source: OECD Employment Outlook (1996 and 2003)

Table III documents the ratio of the relative percentile distribution of gross earnings P90/P10 in nine OECD countries.

Table III: Relative percentile of gross earnings P90/P10

	United States	United Kingdom	France (73-98)	Japan (75-99)	Sweden (75-98)	Canada (73-94)	Finland (77-99)	Germany (84-98)	Italy (86-96)
1973-1979	3,72	3,03	3,41	3,00	2,10	3,88			
1979-1983	4,07	2,97	3,36	3,05	2,01	4,24	2,54		
1983-1990	4,20	2,96	3,32	3,15	2,05	4,44	2,50	2,85	2,30
1990-2000	4,51	2,97	3,15	3,03	2,16	4,21	2,37	2,83	2,36

Source: OECD Labour Market Statistics (July 2004)

7.2 Dynamics and steady-state of the growth rate

Proof of Lemma 1.

Let $G_t = 1 + g_t$. Given (21), we have:

$$G_{t+1} = \gamma \left(\frac{\mu(G_t)}{\lambda(G_t)} \right) [kM + (1-M)\lambda(G_t)]$$

Deriving with respect to G_t and rearranging yields:

$$\begin{aligned} \frac{\partial G_{t+1}}{\partial G_t} &= \gamma \left(\frac{\mu(G_t)}{\lambda(G_t)} \right)^{1-\alpha} \square \\ &\left\{ [kM + (1-M)\lambda(G_t)] \square \left[\frac{\mu'(G_t)}{\mu(G_t)} - \alpha \left(\frac{\mu'(G_t)}{\mu(G_t)} - \frac{\lambda'(G_t)}{\lambda(G_t)} \right) \right] - [kM] \frac{\lambda'(G_t)}{\lambda(G_t)} \right\} \end{aligned} \quad (24)$$

The properties of $\lambda(\cdot)$ and $\mu(\cdot)$ in (20) imply that $0 \geq \frac{\mu'(G_t)}{\mu(G_t)} \geq \frac{\lambda'(G_t)}{\lambda(G_t)}$ and

$0 \geq \frac{\mu'(G_t)}{\mu(G_t)} - \alpha \left(\frac{\mu'(G_t)}{\mu(G_t)} - \frac{\lambda'(G_t)}{\lambda(G_t)} \right) \geq \frac{\lambda'(G_t)}{\lambda(G_t)}$ We therefore have:

$$\frac{\partial G_{t+1}}{\partial G_t} < 0 \quad (25)$$

The dynamics of the growth rate hence is oscillatory. To determine whether there is convergence, we have to show that $\left| \frac{\partial G_{t+1}}{\partial G_t} \right| < 1$ Given (25), this is equivalent to show that

$\frac{\partial G_{t+1}}{\partial G_t} > -1$ Rearranging (24), this condition writes

$$\begin{aligned} &\gamma \left(\frac{\mu(G_t)}{\lambda(G_t)} \right)^{1-\alpha} \square \left[kM \frac{\lambda'(G_t)}{\lambda(G_t)} - [kM + (1-M)\lambda(G_t)] \right] \\ &\square \left[\frac{\mu'(G_t)}{\mu(G_t)} (1-\alpha) - \frac{\lambda'(G_t)}{\lambda(G_t)} \right] < 1 \\ &\Leftrightarrow \frac{1}{\gamma} \left(\frac{\lambda(G_t)}{\mu(G_t)} \right)^{1-\alpha} \square [kM + (1-M)\lambda(G_t)] \square (1-\alpha) \left[\frac{\mu'(G_t)}{\mu(G_t)} - \frac{\lambda'(G_t)}{\lambda(G_t)} \right] \\ &+ \left[\frac{\lambda'(G_t)}{\lambda(G_t)} (1-M)\lambda(G_t) \right] > 0 \end{aligned}$$

The properties of $\lambda(\cdot)$ and $\mu(\cdot)$ in (20) imply that $0 \geq (1-\alpha) \frac{\mu'(G_t)}{\mu(G_t)} \geq (1-\alpha) \frac{\lambda'(G_t)}{\lambda(G_t)} > (1-\alpha) \frac{\lambda'(G_t)}{\lambda(G_t)} + \frac{\lambda'(G_t)}{\lambda(G_t)}$. We therefore have $(1-\alpha) \left[\frac{\mu'(G_t)}{\mu(G_t)} - \frac{\lambda'(G_t)}{\lambda(G_t)} \right] > \frac{\lambda'(G_t)}{\lambda(G_t)}$ that is:

$$\frac{\partial G_{t+1}}{\partial G_t} > -1.$$

The growth rate hence converges with oscillations to its steady state value. Given that G_{t+1} is monotonically decreasing in G_t , its curve will cross the increasing 45° line (for which $G_{t+1} = G_t$), and hence the steady-state exists. It remains to show that the stationary value of the growth rate $g_{t+1} = g_t = \bar{g}$ belongs to the unit interval, (or equivalently that $G_{t+1} = G_t = \bar{G} \in [1, 2]$). \bar{G} is defined implicitly by

$$\bar{G} = \gamma \left(\frac{\mu(\bar{G})}{\lambda(\bar{G})} \right) [kM + (1-M)\lambda(\bar{G})]$$

Let $H(x) = x - F(x)$, where $F(x) = \gamma \left(\frac{\mu(x)}{\lambda(x)} \right) [kM + (1-M)\lambda(x)]$. We have shown that $-1 < F(x) < 0$ hence

$$H'(x) = 1 - F'(x) > 0 \tag{26}$$

Given (2), the steady-state growth rate $\bar{G} \in [1, 2]$ if $\lim_{x \rightarrow 1} F(x) > 1$ and $\lim_{x \rightarrow 2} F(x) < 2$. Both conditions are verified under assumption (A2), a condition which, as a by-product, also rules out the possibility of a negative growth rate even in the short run. Note that both conditions are equivalent to $\lim_{x \rightarrow 1} H(x) < 0$ and $\lim_{x \rightarrow 2} H(x) < 0$, which guarantees the existence and uniqueness of \bar{G} .