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*May 2009*

Cahier n° 2009-54

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# Upstream Competition between Vertically Integrated Firms<sup>1</sup>

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## Abstract

We propose a model of two-tier competition between vertically integrated firms and unintegrated downstream firms. We show that, even when integrated firms compete in prices to offer a homogeneous input, the Bertrand result may not obtain, and the input may be priced above marginal cost in equilibrium, which is detrimental to consumers' surplus and social welfare. We obtain that these partial foreclosure equilibria are more likely to exist when downstream competition is fierce. We then use our model to assess the impact of several regulatory tools in the telecommunications industry.

*Journal of Economic Literature* Classification Number: L13, L51.

*Keywords:* Vertical foreclosure, vertically-related markets, telecommunications.

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<sup>1</sup>Intellectual and financial support by France Télécom and CEPREMAP is gratefully acknowledged. This paper has previously been circulated under the title "Wholesale Markets in Telecommunications". We also wish to thank Eric Avenel, Bernard Caillaud, Philippe Février, Steffen Hoernig, Marc Ivaldi, Bruno Julien, Tilman Klumpp, Laurent Lamy, Marc Lebourges, Laurent Linnemer, Patrick Rey, Katharine Rockett, Bernard Salanié, Jean Tirole, Thomas Trégouët, Timothy Van Zandt and seminar participants at University of Toulouse, PSE, CREST-LEI, University of Grenoble (GAEL) and ARCEP, and participants at the 2006 Telecom Paris Conference on the Economics of Information and Communication Technologies, the Conferência sobre Telecomunicações, the Joint Workshop HEC-INSEAD-PSE, the Eighth CEPR Conference on Applied Industrial Organization, and the Second European Conference in Competition and Regulation for helpful comments on an earlier draft. We are solely responsible for the analysis and conclusions.

# 1 Introduction

In several industries, production has a two-tier structure: firms need to obtain an intermediate input in order to serve final consumers. In this paper, we focus on industries in which the intermediate input is produced by vertically integrated firms only. Examples of such a market structure are numerous. In the broadband market, Digital Subscriber Line (DSL) operators and cable networks own a broadband infrastructure and compete at the retail level. They can also compete to provide wholesale broadband services to unintegrated downstream firms, which have not built their own network. Similarly, in the mobile telephony market, Mobile Virtual Network Operators (MVNOs) do not have a spectrum license nor a mobile network and therefore have to purchase a wholesale mobile service from Mobile Network Operators (MNOs). Other examples can be found in licensing contexts. For instance, Arora, Fosfuri, and Gambardella (2001) report that, at the end of the 1990s, Dow Chemicals and Exxon had developed rival metallocene technologies, which enabled them to produce polyethylenes. They also licensed their technologies to downstream polyethylenes producers.<sup>2</sup> In the video game industry, some firms (e.g., Epic Games, Valve Corporation) have designed their own 3D engines to develop 3D video games. They also license these engines to rival downstream firms (e.g., Electronic Arts).

This raises the following policy issue: does competition between vertically integrated firms on the upstream market level the playing field between the downstream rivals? More specifically, consider that an unintegrated downstream entrant has managed to get access to the intermediate input; should we expect competition on the upstream market to drive the input price down to marginal cost?

To answer this question, we propose a model, in which two vertically integrated firms and an unintegrated downstream firm compete in prices with differentiated products on a downstream market. The goods sold to end-users are derived from an intermediate input that the integrated firms can produce in-house. Integrated firms compete, first on the upstream market to provide the input to the unintegrated downstream firm, and second on the downstream market with the unintegrated downstream firm. The upstream market exhibits the usual ingredients of tough competition: integrated firms compete in (linear) prices, produce a perfectly homogeneous upstream good and incur the same constant marginal cost. Yet, we show that upstream competition may not drive the input price down to marginal cost, thereby giving rise to partial foreclosure equilibria. In particular, there can exist monopoly-like equilibria, in which one vertically integrated firm supplies the intermediate input at its monopoly upstream price, while its integrated rival decides rationally to make no upstream

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<sup>2</sup>See Arora (1997) for other examples in the chemical industry.

offer.

The intuition is the following. Assume that integrated firm  $i$  supplies the wholesale market at a strictly positive price-cost margin, and consider the incentives of its integrated rival  $j$  to corner that market. Notice first that, when firm  $i$  increases its downstream price, it recognizes that some of the final consumers it loses will eventually purchase from the unintegrated downstream firm, thereby increasing upstream demand and revenues. This implies that firm  $i$  charges a higher downstream price than its integrated rival  $j$  at the downstream equilibrium. This effect obviously benefits firm  $j$ , which faces a less aggressive competitor on the final market: this is the softening effect. Now, if firm  $j$  undercuts firm  $i$  on the upstream market and becomes the upstream supplier, the roles are reversed: firm  $i$  decreases its downstream price, while firm  $j$  increases it. To sum up, firm  $j$  faces the following trade-off when deciding whether to undercut. On the one hand, undercutting yields wholesale profits; on the other hand, it makes integrated firm  $i$  more aggressive on the downstream market. When the latter effect is strong enough, the incentives to undercut vanish and the Bertrand logic collapses.<sup>3</sup>

This implies that, when the softening effect is strong enough, the monopoly outcome on the upstream market may persist even under the threat of competition on that market. Other equilibria may exist, but monopoly-like equilibria are Pareto-dominant from the integrated firms' viewpoint. Besides, we show that partial foreclosure equilibria degrade both social welfare and consumers' surplus.

The degree of differentiation at the downstream level has an important impact on the strength of the softening effect, hence on the competitiveness of the upstream market. Intuitively, when final products are strongly differentiated, downstream demands are almost independent and the softening effect is consequently weak. As a result, undercutting on the upstream market is always profitable, and competition drives the wholesale price down to marginal cost. Conversely, when downstream products are strong substitutes, the softening effect is strong and the monopoly outcome is an equilibrium.

We obtain an even stronger result under two-part tariff competition. We show that partial foreclosure equilibria with strictly positive upstream profits always exist when firms compete in two-part tariffs on the upstream market.

Our framework is especially relevant to analyze competition on wholesale markets in the telecommunications industry. As we have just seen, wholesale competition in telecoms may fail to develop, and therefore, there is a scope for regulatory intervention in these markets. We

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<sup>3</sup>That an integrated firm changes its downstream behavior when it supplies a non-integrated rival has already been noted in the literature. See Chen (2001), Fauli-Oller and Sandonis (2002), Sappington (2005) and Chen and Riordan (2007) among others. The novelty of our paper is to analyze the implications of these upstream-downstream interactions on upstream competition between vertically integrated firms.

show that several regulatory tools, which have been considered or implemented by telecoms regulators, can destroy all partial foreclosure equilibria. First, the vertical separation of an integrated firm, or the entry of an unintegrated upstream competitor, can restore the competitiveness of the wholesale market. Intuitively, an unintegrated upstream firm is always willing to corner the upstream market, for it is its sole source of profit. Conversely, an integrated firm is always willing to undercut an unintegrated upstream competitor since, in this case, it does not lose a softening effect. Second, we derive conditions on the demand and cost functions, under which a wholesale price cap can restore the competitiveness of the upstream market.

Our paper is related to several strands of the literature. The literature on one-way access pricing deals with situations in which a service-based firm must gain access to the network of a historical incumbent (Laffont and Tirole 2001, Armstrong 2002, de Bijl and Peitz 2002). These works are, by definition, silent on the issue of wholesale competition.

The question addressed in this paper echoes the old antitrust debate on the anticompetitive effects of vertical integration. According to the traditional foreclosure doctrine, vertical integration can be anticompetitive, since vertically integrated firms have incentives to raise their rivals' costs. This theory was criticized by Chicago School authors (see Bork 1978 and Posner 1976), on the ground that firms cannot leverage market power from one market to another one. More recently, the literature on vertical mergers has revisited these issues by analyzing extensively wholesale competition between a vertically integrated firm and an unintegrated upstream firm. Ordober, Saloner and Salop (1990) argue that such a market structure is unlikely to yield tough competition on the wholesale market when the vertically integrated firm can commit *ex ante* to its upstream price. Choi and Yi (2000) provide foundations for this commitment power through the choice of input specification. With upstream cost asymmetries and upstream switching costs, Chen (2001) shows that the integrated firm partially forecloses its unintegrated downstream rival in equilibrium. Chen and Riordan (2007) argue that an exclusive dealing contract enables the integrated firm to implement partial foreclosure in equilibrium. Surprisingly, this strand of literature has not dealt with wholesale competition between vertically integrated structures, which, as we argued previously, is crucial in several industries. Our main result is that upstream competition between integrated firms may lead to equilibrium foreclosure even in the absence of upstream commitment power, input choice specification, upstream cost asymmetries or switching costs, and exclusive dealing contracts.<sup>4,5</sup>

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<sup>4</sup>Salinger (1988) and Nocke and White (2007) consider situations in which several integrated firms compete on the wholesale market. These papers, however, do not study tough price competition. The former assumes Cournot competition on both markets, while the latter focuses on tacit collusion on the input market.

<sup>5</sup>Hart and Tirole (1990) have initiated another strand of the vertical foreclosure literature, which analyzes

Two exceptions are Ordober and Shaffer (2007) and Brito and Pereira (2006), who present models with several vertically integrated firms and a downstream entrant. Both papers are mainly interested in whether a wholesale market will emerge at all, i.e., whether the entrant can be completely foreclosed in equilibrium. Though we think that complete foreclosure is an interesting question, we would like to point out that we do observe unintegrated downstream firms in the telecommunications industry, or in licensing contexts. For instance, there were 290 MVNOs in 15 European countries in July 2006, and around 50 MVNOs in the US in 2006.<sup>6</sup> This motivates our focus on partial foreclosure. Using particular demand and cost specifications, both Ordober and Shaffer (2007) and Brito and Pereira (2006) argue that, when the entrant is not completely foreclosed, wholesale competition drives the input price down to marginal cost. By contrast, we work out a more general model, and obtain that partial foreclosure is actually an issue.

Höfler and Schmidt (2008) take a complementary perspective and study the impact on consumers' surplus of the entry of unintegrated downstream firms. They show that service-based competition can be detrimental to consumers, due to the softening effect. However, they assume away any form of wholesale competition: an upstream supplier is exogenously chosen, and it is free to impose its monopoly wholesale price. Our results indicate that allowing competition on the upstream market may leave integrated firms with as much market power as when the upstream market structure is exogenously fixed.

The paper is organized as follows. Section 2 describes the model. Section 3 presents our main results. Section 4 analyzes the efficacy of several regulatory tools. Section 5 discusses several extensions and robustness checks of our basic framework. Section 6 concludes.

## 2 The Model

**Firms.** There are two vertically integrated firms, denoted by 1 and 2, and one unintegrated downstream firm, denoted by  $d$ . Integrated firms are composed of an upstream and a downstream unit, which produce the intermediate input and the final good, respectively. The unintegrated downstream competitor is composed of a downstream unit only. In order to be active on the final market, it must purchase the intermediate input from one of the integrated firms on the upstream market.

Both integrated firms produce the upstream good under constant returns to scale at unit

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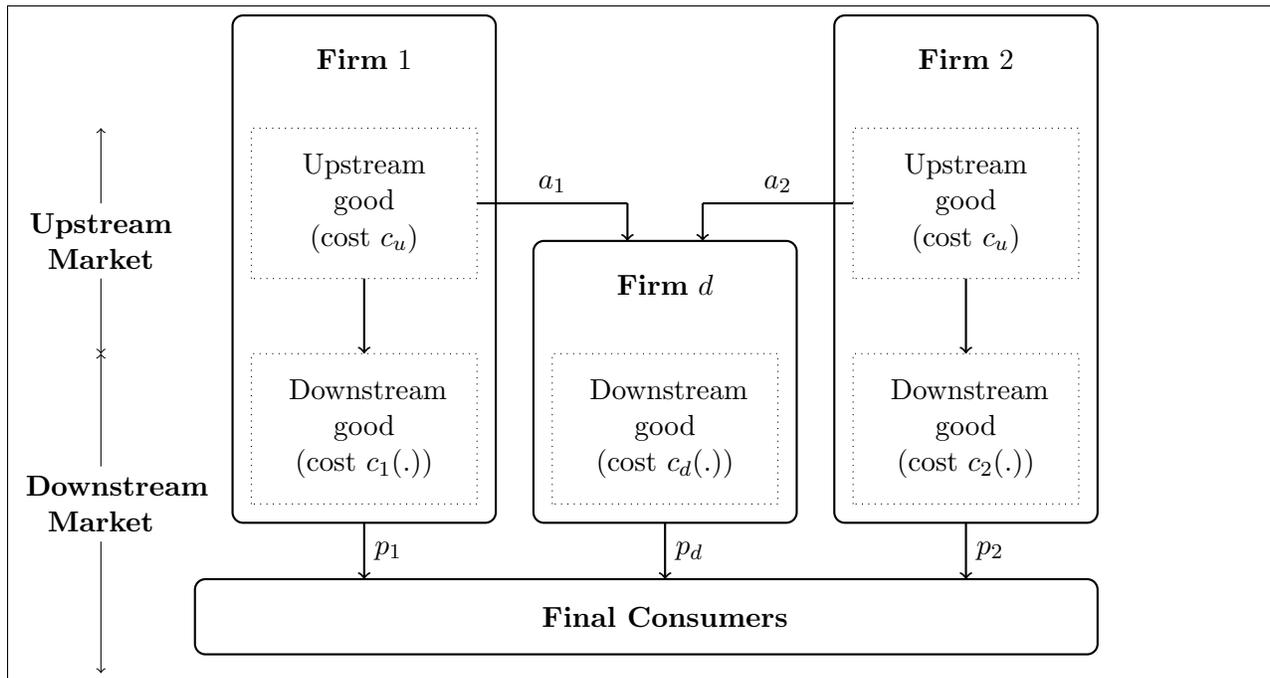
the consequences of upstream secret offers, and focuses mainly on the commitment problem faced by an upstream monopolist.

<sup>6</sup>See Federal Communications Commission (2008) and European Commission (2007). In the broadband market, there were 453 agreements between incumbent operators and new entrants and 634 agreements for resale in the EU25 member states, as of January 1st, 2008 (see European Commission 2008).

cost  $c_u$ . The downstream product is derived from the intermediate input on a one-to-one basis with the twice continuously differentiable cost function  $c_k(\cdot)$ , for firm  $k \in \{1, 2, d\}$ . We assume that integrated firms have the same downstream cost function:  $c_1(\cdot) = c_2(\cdot)$ .

**Markets.** All firms compete in prices on the downstream market and provide imperfect substitutes to final customers. Let  $p_k$  be the downstream price set by firm  $k \in \{1, 2, d\}$  and  $p \equiv (p_1, p_2, p_d)$  the vector of final prices. Firm  $k$ 's demand, denoted by  $D_k(p)$ , is twice continuously differentiable; it depends negatively on firm  $k$ 's price and positively on its competitors' prices:  $\partial D_k / \partial p_k \leq 0$  with a strict inequality whenever  $D_k > 0$ , and  $\partial D_k / \partial p_{k'} \geq 0$  with a strict inequality whenever  $D_k > 0$  and  $D_{k'} > 0$ , for  $k \neq k' \in \{1, 2, d\}$ . We also suppose that the total demand is non-increasing in each price: for all  $k' \in \{1, 2, d\}$ ,  $\sum_{k \in \{1, 2, d\}} \partial D_k / \partial p_{k'} \leq 0$ . Symmetry of the integrated firms is assumed again:  $D_1(p_1, p_2, p_d) = D_2(p_2, p_1, p_d)$  and  $D_d(p_1, p_2, p_d) = D_d(p_2, p_1, p_d)$  for all  $p$ .

On the upstream market, integrated firms compete in prices and offer perfectly homogeneous products. We denote by  $a_i$  the upstream price set by integrated firm  $i \in \{1, 2\}$ .<sup>7</sup> The structure of the model is summarized in Figure 1.



**Figure 1:** Structure of the model.

**Timing.** The sequence of decision-making is as follows:

<sup>7</sup>Throughout the paper, subscripts  $i$  and  $j$  refer to integrated firms only, whereas subscript  $k$  refers either to an integrated firm or to the unintegrated downstream firm.

*Stage 1 – Upstream competition:* Vertically integrated firms announce their prices on the upstream market. Then, the unintegrated downstream firm elects at most one upstream provider.

*Stage 2 – Downstream competition:* All firms set their prices on the downstream market.

We focus on pure strategy subgame-perfect equilibria and reason by backward induction.

**Profits.** Assume that the unintegrated downstream firm is active on the downstream market. The profit of integrated firm  $i \in \{1, 2\}$  which supplies the upstream market at price  $a_i$  is:<sup>8</sup>

$$\tilde{\pi}_i^{(i)}(p, a_i) = (p_i - c_u)D_i(p) - c_i(D_i(p)) + (a_i - c_u)D_d(p).$$

The profit of integrated firm  $j \neq i \in \{1, 2\}$  which does not supply the upstream market is given by:

$$\tilde{\pi}_j^{(i)}(p, a_i) = (p_j - c_u)D_j(p) - c_j(D_j(p)).$$

The profit of unintegrated downstream firm  $d$  is:

$$\tilde{\pi}_d^{(i)}(p, a_i) = (p_d - a_i)D_d(p) - c_d(D_d(p)).$$

Note that when the upstream price is equal to the upstream unit cost, i.e.,  $a_i = c_u$ , there is no upstream profit and all firms compete on a level playing field. This is the perfect competition outcome on the upstream market.

## 3 Main Results

### 3.1 Preliminaries

**Downstream market competition.** Consider that at least one integrated firm has made an *acceptable* upstream offer, i.e., an offer that allows firm  $d$  to be active and earn strictly positive profits on the downstream market. Denote by  $i \in \{1, 2\}$  the upstream supplier. For  $k \in \{1, 2\}$ , define  $BR_k^{(i)}(p_{-k}, a_i) = \arg \max_{p_k} \tilde{\pi}_k^{(i)}(p, a_i)$  the best-response function of integrated firm  $k$ .<sup>9</sup> We assume that  $BR_k^{(i)}(p_{-k}, a_i)$  is unique, bounded and well-defined by the corresponding first-order condition for any  $p_{-k} \in [0, \infty)^2$  and any acceptable  $a_i$ . In order

<sup>8</sup>Throughout the paper, the superscript in parenthesis indicates the identity of the upstream supplier.

<sup>9</sup>As usual,  $p_{-k}$  is the vector obtained by removing  $p_k$  from vector  $p$ .

to obtain well-behaved comparative statics, we make the following stability assumption:  $\left| \frac{\partial BR_j^{(i)}}{\partial p_{j'}}(p_{j'}, p_d, c_u) \right| < 1$ , for all  $j' \neq j$  in  $\{1, 2\}$ , for all  $p_{j'}, p_d \geq 0$ .<sup>10</sup>

We assume that there exists a unique Nash equilibrium on the downstream market, and we denote by  $p_k^{(i)}(a_i)$  the equilibrium price of firm  $k \in \{1, 2, d\}$ , and by  $p^{(i)}(a_i)$  the vector of these downstream prices. At the equilibrium of this subgame, firms' profits are given by functions  $\pi_k^{(i)}(a_i) \equiv \tilde{\pi}_k^{(i)}(p^{(i)}(a_i), a_i)$ , which are defined over the set of acceptable offers. Note that, when the upstream product is priced at marginal cost,  $p_i^{(i)}(c_u) = p_j^{(i)}(c_u)$  and  $\pi_i^{(i)}(c_u) = \pi_j^{(i)}(c_u)$ . We assume that  $\pi_i^{(i)}(c_u)$  and  $\pi_d^{(i)}(c_u)$  are strictly positive.

**Choice of upstream supplier.** If only one integrated firm has made an acceptable offer, then it is obviously chosen by the unintegrated downstream firm.

Consider now that both offers are acceptable. If  $\pi_d^{(i)}(a_i) > \pi_d^{(j)}(a_j)$ , then firm  $d$  chooses firm  $i$  as its upstream supplier. If both offers lead to the same profit, then firm  $d$  chooses any of them.<sup>11</sup> We now make the following economically meaningful assumption:

**Assumption 1.**  $\pi_d^{(i)}(\cdot)$  is strictly decreasing.<sup>12</sup>

If firm  $d$  preferred to choose the most expensive upstream provider, we would have another, somewhat trivial (and pathological), reason for the existence of partial foreclosure equilibria. Assumption 1 rules out these cases.

**Upstream monopoly benchmark.** Consider the hypothetical scenario in which the upstream market is monopolized by integrated firm  $i$ . We assume that, in this case, firm  $i$  makes an acceptable offer to firm  $d$ :

**Assumption 2.** When integrated firm  $j$  has not made any acceptable upstream offer, integrated firm  $i$  does not completely foreclose firm  $d$ .

<sup>10</sup>This corresponds to the stability condition in a hypothetical game, in which unintegrated downstream firm  $d$  would be supplied at price  $c_u$  and would set downstream price  $p_d$ , and both vertically integrated firms would set their downstream prices simultaneously. See Vives (1999, p. 51).

<sup>11</sup>Other tie-breaking rules would not change our results. For instance, in Section 5, we consider the possibility that firm  $d$  splits its upstream demand between integrated firms.

<sup>12</sup>An increase in firm  $d$ 's cost has typically two impacts on its profit. First, the price-cost margin is directly reduced, leading unambiguously to a lower profit. Second, the best-response (in downstream price) of firm  $d$  shifts upward, which affects the equilibrium of the final market. These two effects are standard in any IO models with price competition and product differentiation. In our context, there is also a third effect since the best-response of the upstream supplier also shifts upward (the softening effect that we explain later on). The overall impact on firm  $d$ 's profit is a priori ambiguous and depends typically on the strategic interaction on the downstream market. In line with most IO models, Assumption 1 posits that the direct effect outweighs the strategic ones.

As a consequence, under Assumption 2, complete foreclosure of firm  $d$  never arises in equilibrium. As discussed in Section 5, this assumption may hold because firm  $i$  actually wants to supply firm  $d$  in order to attract new consumers, or because of regulatory constraints, or because firm  $d$  can buy the input from an alternative and less efficient source. We also assume:

**Assumption 3.**  $\pi_i^{(i)}(\cdot)$  is strictly quasiconcave, and it has a unique maximum at  $a_m > c_u$ .

To summarize, if the upstream market were exogenously monopolized, the unintegrated downstream firm would not be completely foreclosed, and monopoly market power on the upstream market would lead to a strictly positive markup on the price of the intermediate input, i.e., to partial foreclosure. In the sequel,  $a_m$  is referred to as the monopoly upstream price.

At this stage, we can already notice that, under Assumptions 1 and 3, integrated firms offering  $a_1 = a_2 = c_u$  is an equilibrium outcome.

**Remark 1.** Under Assumptions 1 and 3, the perfect competition outcome on the upstream market is an equilibrium.

*Proof.* Consider that integrated firms offer  $a_1 = a_2 = c_u$ . Then, if firm  $d$  elects firm  $i$  as its upstream supplier,  $\pi_i^{(i)}(c_u) = \pi_j^{(i)}(c_u)$ . If an integrated firm deviates upwards, then, by Assumption 1, firm  $d$  still purchases the input at marginal cost from the other integrated firm, and the integrated firms' profits are not affected. If on the other hand, an integrated firm, say  $i$ , deviates downwards by setting  $a_i < c_u$ , its profit becomes  $\pi_i^{(i)}(a_i)$ . This is less than  $\pi_i^{(i)}(c_u)$ , since  $\pi_i^{(i)}$  is strictly quasi-concave and  $a_m > c_u$ .  $\square$

### 3.2 Persistence of the monopoly outcome

We now study the first stage of our game in which integrated firms compete on the upstream market, and establish the main result of the paper. We show that the usual mechanism of Bertrand competition may be flawed and that partial foreclosure equilibria may exist.

Assume that integrated firm  $i$  has made an acceptable upstream offer to firm  $d$ ,  $a_i > c_u$ , and let us see whether integrated firm  $j \neq i$  is willing to slightly undercut to corner the upstream market, as would be the case with standard (single-market) Bertrand competition. The analysis proceeds in two steps. First, we look at prices at the downstream equilibrium. Second, we compare the profits of firms  $i$  and  $j$ .

The integrated firms' best-responses on the downstream market are characterized by the following first-order conditions:

$$(1) \quad \frac{\partial \tilde{\pi}_i^{(i)}}{\partial p_i}(p, a_i) = D_i + (p_i - c'_i(D_i) - c_u) \frac{\partial D_i}{\partial p_i} + (a_i - c_u) \frac{\partial D_d}{\partial p_i} = 0,$$

$$(2) \quad \frac{\partial \tilde{\pi}_j^{(i)}}{\partial p_j}(p, a_i) = D_j + (p_j - c'_j(D_j) - c_u) \frac{\partial D_j}{\partial p_j} = 0.$$

The comparison between (1) and (2) indicates that the upstream supplier has more incentives to raise its downstream price than its integrated rival. Realizing that final customers lost on the downstream market may be recovered via the upstream market, the upstream supplier is less aggressive than its integrated rival on the downstream market. As formally shown in Appendix, this mechanism, together with our stability assumption, implies that the upstream supplier charges a higher downstream price than its integrated rival at the subgame equilibrium.

**Lemma 1.** *Let  $a_i > c_u$  be an acceptable offer. Then the upstream supplier charges a strictly higher downstream price than its integrated rival:*

$$p_i^{(i)}(a_i) > p_j^{(i)}(a_i).$$

*Proof.* See Appendix A.1. □

This soft behavior favors the other integrated firm which, by a revealed preference argument, earns more downstream profit than the upstream supplier. We shall refer to that mechanism as the ‘softening effect’.

**Lemma 2.** *Let  $a_i > c_u$  be an acceptable offer. Then, the upstream supplier earns strictly smaller downstream profits than its integrated rival:*

$$\left[ p_i^{(i)}(a_i) - c_u \right] D_i(p^{(i)}(a_i)) - c_i(D_i(p^{(i)}(a_i))) < \left[ p_j^{(i)}(a_i) - c_u \right] D_j(p^{(i)}(a_i)) - c_j(D_j(p^{(i)}(a_i))).$$

*Proof.* See Appendix A.2. □

A key consequence of that result is that we cannot tell unambiguously which of the integrated firms earns more total profits. On the one hand, the upstream supplier extracts revenues from the upstream market. On the other hand, its integrated rival benefits from larger downstream profits, owing to the softening effect.

We can now come back to our initial question. When an integrated firm undercuts the upstream market, it obtains the upstream profits at the cost of making its integrated rival

more aggressive on the downstream market. Therefore an integrated firm may not always want to undercut its integrated rival on the upstream market. Notice that we do not need any assumptions on the strategic interactions between downstream prices to obtain this tradeoff between capturing the upstream profits and benefiting from the softening effect. When the latter effect outweighs the former, the usual logic of Bertrand competition may not work anymore, and in particular, the monopoly outcome can be an equilibrium.

**Proposition 1.** *Under Assumptions 1-3, there exists an equilibrium in which the upstream market is supplied by an integrated firm at price  $a_m$  if and only if*

$$(3) \quad \pi_j^{(i)}(a_m) \geq \pi_i^{(i)}(a_m).$$

*These equilibria are referred to as monopoly-like equilibria.*<sup>13</sup>

*Proof.* Assume that condition (3) holds. Suppose firm  $i$  offers  $a_i = a_m$  and firm  $j$  makes an unacceptable offer. Then, firm  $j$  has no incentives to undercut firm  $i$  and, by Assumptions 2 and 3, firm  $i$  has no incentives to deviate. Conversely, assume that condition (3) does not hold. If firm  $i$  supplies the upstream market at price  $a_m$ , then it is strictly profitable for firm  $j$  to propose  $a_m - \varepsilon$  (with  $\varepsilon > 0$  sufficiently small), an offer which firm  $d$  would accept, by Assumption 1.  $\square$

Proposition 1 might sound somewhat tautological. Yet, our contribution is to show that, contrary to what the conventional wisdom and the existing literature state, condition (3) may well be satisfied. Because losers on the upstream market become winners on the downstream market, the usual competitive forces may collapse. This effect does not hinge on any commitment device for the integrated firms to exit the upstream market;<sup>14</sup> nor does this rely on any kind of overt or tacit collusion. To complete the analysis, it remains to show that condition (3) is indeed satisfied with several standard demand specifications. We provide such examples in Sections 3.5 and 5.

As compared to the existing literature, we do not specify the downstream demand functions. This allows us to stay at a higher level of generality. Ordober and Shaffer (2007) consider a linear specification over a particular range of parameters. In Brito and Pereira (2006), there is a third integrated firm and downstream competition takes place in the Hotelling circle. In both specifications, condition (3) turns out not to be satisfied.<sup>15</sup> By carefully picking

<sup>13</sup>Notice that different strategies can be used to support a monopoly-like equilibrium:  $a_i = a_m$  and  $a_j$  unacceptable, or  $a_i = a_m$  and  $a_j > a_m$  acceptable such that  $\pi_j^{(i)}(a_j) \leq \pi_i^{(i)}(a_m)$ .

<sup>14</sup>Firms cannot commit not to enter the upstream market; however, endogenously, the incentives to corner the upstream market may disappear.

<sup>15</sup>Brito and Pereira (2006) note that, in their framework, it could actually be satisfied if one of the three integrated firms could commit to exit the upstream market.

apart the main effects which govern firms' undercutting decisions, we show that their results have limitations, and that we cannot predict the outcome of the upstream price competition game without assessing the strength of the softening effect.

### 3.3 Other equilibria

In this section, we give a complete characterization of the subgame-perfect equilibria of our game.

**Proposition 2.** *Under Assumptions 1 and 3, there exists an equilibrium, in which  $a_1 = a_2 = a_*$  if and only if  $a_* \leq a_m$  and  $\pi_i^{(i)}(a_*) = \pi_j^{(i)}(a_*)$ . These equilibria are referred to as matching-like equilibria.*

*Proof.* See Appendix A.3. □

In a matching-like equilibrium, both integrated firms offer the same upstream price and are indifferent between supplying the upstream market and not supplying it. Notice that the perfect competition outcome  $a_1 = a_2 = c_u$  is always a (matching-like) equilibrium outcome, as already stated in Remark 1.

However, nothing precludes the existence of other matching-like equilibria featuring either a supra-competitive upstream market ( $a_* > c_u$ ) or a super-competitive upstream market ( $a_* < c_u$ ). The existence of these equilibria also hinges on the softening effect. For  $a_* > c_u$ , the integrated firm which does not supply the upstream market benefits from the softening effect and does not want to undercut. For  $a_* < c_u$ , the softening effect is reversed. The upstream supplier offers a low downstream price to reduce the upstream demand, which hurts its integrated rival. Even though the upstream supplier makes losses on the upstream market, it does not want to exit that market since it would then suffer from an adverse softening effect.

We conclude this paragraph with the following result:

**Proposition 3.** *Under Assumptions 1-3:*

- *Only monopoly-like and matching-like outcomes can arise in equilibrium.*
- *From the viewpoint of the integrated firms, any monopoly-like equilibrium Pareto-dominates any matching-like equilibrium.*

*Proof.* See Appendix A.4. □

Propositions 1, 2 and 3 provide a characterization of all the possible equilibria of our game. Moreover, the monopoly-like equilibria, when they exist, Pareto-dominate all other equilibria from the integrated firms' standpoint. Therefore there is a strong presumption that these equilibria will actually be played when they exist.

### 3.4 Welfare analysis

As the following proposition shows, partial foreclosure equilibria can significantly degrade consumers' surplus and social welfare:

**Proposition 4.** *Assume that downstream prices are strategic complements.*

- *Then, consumers strictly prefer the perfect competition outcome to a partial foreclosure equilibrium.*
- *Besides, if firms' downstream divisions are identical and downstream costs are weakly convex, then, social welfare is strictly higher in the perfect competition outcome than in a partial foreclosure equilibrium.*

*Proof.* See Appendix A.5. □

Strategic complementarity ensures that all prices increase when the industry shifts from the perfect competition outcome to a partial foreclosure equilibrium. In this case, a partial foreclosure equilibrium is clearly detrimental to all consumers.

Assessing the impact on social welfare requires more assumptions. If the assumptions made in Proposition 4 are satisfied, then, a shift from the perfect competition outcome to a partial foreclosure equilibrium has the following implications. First, since all prices go up due to strategic complementarity, the total quantity produced diminishes. This is clearly welfare-degrading, since the total demand is already too low at the perfect competition outcome, due to positive markups on the downstream market. Second, the outcome on the final market becomes more asymmetric: firms  $i$  and  $d$  have more incentives to increase their downstream prices than firm  $j$ . This merely shifts some demand from firms  $i$  and  $d$  to firm  $j$ , which is again detrimental to welfare if downstream costs and preferences are convex.

### 3.5 The dilemma between upstream and downstream competitiveness

A key determinant of the persistence of the monopoly outcome is the degree of differentiation of the unintegrated downstream firm. Suppose that the entrant is on a niche market, in the

sense that its demand does not depend on the prices set by the downstream rivals and vice-versa.<sup>16</sup> In that situation, the wholesale profit of the upstream supplier is fully disconnected from its retail behavior and the softening effect disappears. Hence, with an unintegrated downstream firm on a niche market, the perfect competition outcome always emerges in equilibrium.

In order to refine this intuition, consider the symmetric linear case. The demand that addresses to firm  $k \in \{1, 2, d\}$  is given by  $D_k(p) = D - p_k - \gamma(p_k - \frac{p_1+p_2+p_d}{3})$ , where  $\gamma \geq 0$  parameterizes the degree of differentiation between final products, which can be interpreted as the intensity of downstream competition. Perfect competition corresponds to  $\gamma$  approaching infinity and local monopolies to  $\gamma = 0$ . All firms incur the same linear downstream costs:  $c_k(q) = cq$ .<sup>17</sup> With that specification, the assumptions we have made on the second stage demands, payoff functions, best-responses, etc. are satisfied. Assumptions 1 and 3 are satisfied as well. Finally, assuming firm  $d$  can obtain the input from an alternative and less efficient source ensures, as discussed in Section 5, that Assumption 2 also holds.

Figure 2 offers a graphical representation of the profit functions  $\pi_i^{(i)}(\cdot)$ ,  $\pi_j^{(i)}(\cdot)$  and  $\pi_d^{(i)}(\cdot)$ . As discussed in Subsection 3.2, when  $a_i > c_u$ , two opposite effects are at work. On the one hand, the upstream supplier derives profit from the upstream market; on the other hand, its integrated rival benefits from the softening effect on the downstream market. When the upstream price is not too high, the upstream profit effect dominates and  $\pi_i^{(i)}(a_i) > \pi_j^{(i)}(a_i)$ . When the upstream price is high enough, upstream revenues shrink, the softening effect is strengthened and  $\pi_i^{(i)}(a_i) < \pi_j^{(i)}(a_i)$ .

We then obtain the following proposition.

**Proposition 5.** *Consider the symmetric linear case. There exists  $\bar{\gamma} > 0$  such that: If  $\gamma \geq \bar{\gamma}$ , then there exist four equilibrium outcomes on the upstream market.<sup>18</sup>*

- *the perfect competition outcome;*
- *a supra-competitive matching-like outcome;*
- *two monopoly-like outcomes.*

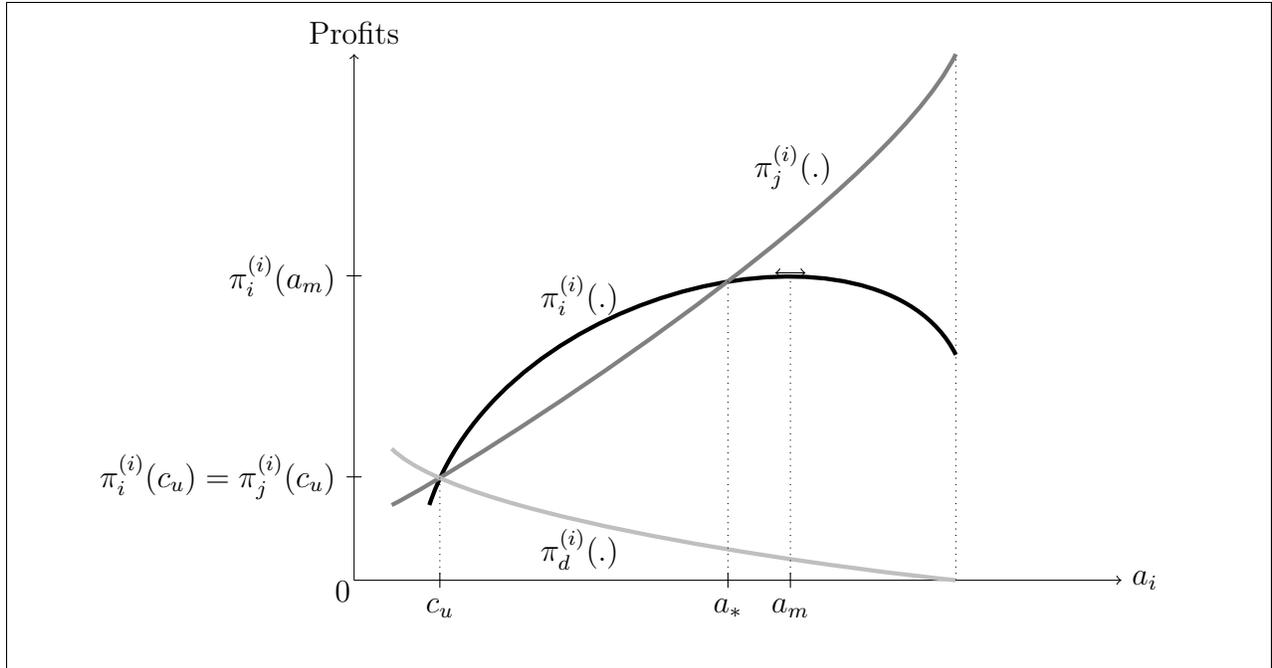
*Otherwise, the perfect competition outcome is the only equilibrium outcome.*

*Proof.* See Appendix A.6. □

<sup>16</sup>Formally,  $\partial D_d / \partial p_i = \partial D_i / \partial p_d = 0$  for  $i \in \{1, 2\}$ .

<sup>17</sup>We assume that the total unit cost  $c_u + c$  is strictly smaller than the intercept of the demand functions  $D$ , otherwise, it would not be profitable to be active on the final market.

<sup>18</sup>The perfect competition and monopoly-like equilibria are stable; the matching-like is unstable.



**Figure 2:** Profits in the symmetric linear case ( $\gamma \geq \bar{\gamma}$ ).

To grasp the intuition of the proposition, suppose that the upstream market is supplied at the monopoly upstream price. When the substitutability between final products is strong, the integrated firm which supplies the upstream market is reluctant to set too low a downstream price since this would strongly contract its upstream profit. The other integrated firm benefits from a substantial softening effect and, as a result, is not willing to corner the upstream market. There exists a monopoly-like equilibrium when downstream products are sufficient substitutes. By the reverse token, only the perfect competition outcome emerges when the competition on the downstream market is sufficiently weak. In other words, there is a tension between competitiveness on the downstream market and competitiveness on the upstream market. Intuitively, the same downstream interactions which strengthen the competitive pressure on the downstream market, are those which soften the competitive pressure on the upstream market.

This tension is revealed in downstream prices, which turn out to be non-monotonic in the substitutability parameter (provided that a monopoly-like equilibrium is selected when it exists). The level of downstream prices results indeed from two combined forces: the level of upstream prices on the one hand, and the intensity of downstream competition / substitutability on the other hand.

This suggests that strongly differentiated unintegrated downstream firms are more likely to enter in the market, not only because entrants have incentives to differentiate to avoid head-to-head competition, but also because they are more likely to benefit from attractive

wholesale offers by integrated firms. In the mobile market, the evidence seems to confirm this intuition. Indeed, many MVNOs target specific market segments, either by using their brand reputation,<sup>19</sup> or by investing in network elements to increase their differentiation possibilities.<sup>20</sup>

## 4 Regulation in the Telecommunications Industry

We have just seen that competition on wholesale markets may fail to develop, thereby giving rise to partial foreclosure equilibria, which degrade both social welfare and consumers' surplus. This provides a rationale for regulatory intervention. In this section, we show that several tools, which have been used or considered by telecoms regulators, such as a wholesale price cap, the entry of an unintegrated upstream competitor, or the vertical separation of an integrated operator, can restore the competitiveness of the upstream market.

### 4.1 Price cap

In several countries (e.g., France, Spain, Belgium, Italy), the telecoms regulator sets a price at which the broadband incumbent has to supply any service-based firm. This does not prevent the incumbent from negotiating lower tariffs with downstream firms. Therefore, the regulated price can be seen as a price cap on the incumbent's wholesale offer. In the following, we show that this kind of regulation can favor the development of tough wholesale competition, and remove all partial foreclosure equilibria, even if the price cap is strictly above marginal cost.

As a first step, let us inspect Figure 2, which depicts firms' profits in the symmetric linear case. Notice that for any  $a_i \in (c_u, a_*)$ ,  $\pi_i^{(i)}(a_i) > \pi_j^{(i)}(a_i)$ : in this range of upstream prices, it is always better to be the upstream supplier. Consequently, if the regulator sets any price cap between  $c_u$  and  $a_*$ , then, the only equilibrium is the perfect competition outcome.

Now we would like to extend this result to more general demand and cost systems. To do so, we have to compare  $\pi_i^{(i)}(a_i)$  and  $\pi_j^{(i)}(a_i)$  for  $a_i$  slightly above  $c_u$ . Put differently, we need to derive conditions under which  $\frac{d\pi_i^{(i)}}{da_i}(c_u) > \frac{d\pi_j^{(i)}}{da_i}(c_u)$ . We obtain the following proposition:

**Proposition 6.** *Assume that firms' downstream divisions are identical and downstream costs are weakly convex. Then, a low enough price cap, strictly above the upstream marginal cost, destroys all partial foreclosure equilibria if*

- *downstream prices are strategic complements and  $\frac{\partial^2 D_k}{\partial p_k^2} \leq 0$  for all  $k \in \{1, 2, d\}$ .*

<sup>19</sup>For instance, teenagers for Virgin Mobile in the UK or NRJ Mobile in France.

<sup>20</sup>For instance, Euskaltel and Budget Telecom are 'full' MVNOs, i.e., they own all the network elements of a traditional mobile operator except the radio equipments. They specifically offer cross-border services.

- or,  $\frac{\partial^2 D_k}{\partial p_k \partial p_{k'}} \geq 0$  for all  $k \neq k'$  in  $\{1, 2, d\}$ .

*Proof.* See Appendix A.7. □

A price cap strictly larger than the upstream marginal cost can restore the competitiveness of the wholesale market, provided that the upstream supplier earns more profits than its integrated rival when the upstream price is slightly above the marginal cost. Put differently, the upstream profit effect has to dominate the softening effect for  $a_i$  sufficiently close to  $c_u$ . A good proxy to assess the strength of the softening effect is the difference between the upstream supplier's and the integrated rival's downstream prices. This gap is small if the upstream supplier does not raise its downstream price by much when the upstream price increases, which is the case when a firm's demand is concave with respect to its own price, and downstream costs are convex. Besides, if prices are strategic complements, the integrated rival increases its price as well, which implies an even smaller gap between downstream prices, hence, a small softening effect. This is the first sufficient condition in Proposition 6.

Second, even if the upstream supplier does increase its price a lot, the gap may still be small if the integrated rival reacts by also increasing its price a lot, namely, if downstream prices are strongly strategic complements. A sufficient condition for this is  $\partial^2 D_k / \partial p_k \partial p_{k'} \geq 0$  and convex costs. This is the second condition in the proposition.<sup>21</sup>

We would like to emphasize that Proposition 6 does not come from a simple mechanical effect. Of course, imposing a price cap reduces the upstream price mechanically. But, more fundamentally, under the assumptions detailed in Proposition 6, a price cap initiates a process by which integrated firms will undercut each other, leading to tough competition in the wholesale market. Interestingly, a price cap can influence the outcome of the market even though the regulatory constraint does not bind (i.e., the upstream price is strictly smaller than the price cap) in equilibrium. Note also that it is sufficient to impose a price cap on one of the integrated firms only to fuel competition in the wholesale market.

Notice that the threat of investment by firm  $d$  can have the same impact as a price cap on the wholesale market. Consider the following alteration of our game: between stage 1 and stage 2, after having observed the integrated firms' upstream offers, the unintegrated downstream firm can pay a sunk investment cost to build its own network. If it does so, it becomes able to produce the intermediate input at marginal cost  $c_u$ . If the investment cost is not too large, there is a threshold  $\bar{a}$ , such that firm  $d$  invests if, and only if the cheapest wholesale offer is above  $\bar{a}$ . Since integrated firms prefer to face a relatively less efficient

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<sup>21</sup>It should be noticed that this reasoning, which derives conditions for the upstream profit effect to dominate the softening effect, is only valid in the neighborhood of  $c_u$ . Therefore, the sufficient conditions given in Proposition 6 do not imply that partial foreclosure equilibria do not exist. For instance, in the symmetric linear case, both sufficient conditions hold and monopoly-like equilibria exist when  $\gamma$  is high enough.

competitor, at least one integrated firm will make an offer below  $\bar{a}$  to prevent firm  $d$  from investing: firms behave exactly as if  $\bar{a}$  were a price cap. If the cost of bypass is low, then  $\bar{a}$  is low as well, and, under the assumptions of Proposition 6, only the perfect competition outcome emerges.

This result has interesting policy implications. In the mobile industry, it means that favorable terms for spectrum licences (e.g., terms for ungranted mobile licences, or for Wimax licences) can increase MNOs' incentives to set low wholesale prices for MVNOs. In the broadband market, it implies that favorable conditions for local loop unbundling investments (e.g., low rates for colocation in the historical operator's premises) might stimulate the development of the wholesale broadband market.

## 4.2 Entry of an unintegrated upstream competitor

Suppose that, in addition to integrated firms 1 and 2, an unintegrated upstream competitor, firm  $u$ , is able to produce the intermediate input at constant marginal cost  $c_u$ . There are two situations in which such an upstream unit can enter the wholesale market. First, local authorities can invest in broadband networks and offer wholesale services to service-based operators.<sup>22</sup> Second, some private companies can decide to enter as unintegrated upstream providers. For instance, in the broadband market, firms like Covad or Northpoint in the US, or Mangoosta in France, adopted this strategy. In the mobile market, so-called mobile virtual enablers (MVNEs) are also unintegrated upstream firms.<sup>23</sup> We argue that this can stimulate competition in wholesale markets more surely than entry of (and competition between) integrated firms:

**Proposition 7.** *Add an unintegrated upstream firm to the basic model. Under Assumption 1, if downstream prices are strategic complements, then in any equilibrium, the upstream market is supplied at or below marginal cost.*

*Proof.* See Appendix A.8. □

Let us briefly explain the intuition underlying that proposition. Assume first that integrated firm  $i \in \{1, 2\}$  supplies the upstream market at a price  $a_i > c_u$ . Then, firm  $u$  always

<sup>22</sup>In France, at the end of 2007, 2% of the population had no access to broadband services. In addition to these so-called 'white zones', the French regulator, ARCEP, keeps track of the 'grey zones', in which only the incumbent operator France Télécom has installed broadband equipments. ARCEP views investments by municipalities in both 'white' and 'grey' zones as legitimate; in the white zones, to offer broadband services; in the grey zones, to foster facility-based competition. In March 2008, there were 55 projects from municipalities for a total amount of 1.3 billion euros. Out of a total number of 2,674 main distribution frames in October 2007, there were 988 with a municipal network.

<sup>23</sup>Examples of MVNEs are Versent Mobile and Visage Mobile in the US, Transatel in France and Belgium, Effortel in Belgium.

wants to undercut, for its sole source of profit comes from the upstream market. Moreover, firm  $u$  is always able to attract firm  $d$ , provided that it offers a sufficiently low upstream price.<sup>24</sup>

Conversely, if firm  $u$  supplies the upstream market at price  $a_u > c_u$ , then, we claim that both firms 1 and  $d$  are strictly better off if firm 1 matches firm  $u$ 's price, and firm  $d$  elects firm 1 as its upstream supplier. If firm  $d$  agrees to purchase the input from firm 1 at price  $a_u$ , then, firm 1 becomes less aggressive on the downstream market, and, by strategic complementarity, firms 2 and  $d$  react by increasing their prices as well. By a revealed preference argument, these price increases benefit firms 1 and  $d$ . Moreover, when firm 1 matches firm  $u$ , it also earns upstream profits, which provides additional incentives to match.

Having said that, it becomes clear that the upstream market cannot be supplied above marginal cost in equilibrium. As noted earlier, in our basic setting, Remark 1 implies that the perfect competition outcome is always a subgame-perfect equilibrium. Obviously, adding an unintegrated upstream competitor does not affect this result.<sup>25</sup>

### 4.3 Vertical separation

Viviane Reding, Member of the European Commission responsible for Information Society and Media, has many times argued that structural separation of the dominant operator was a policy option.<sup>26</sup> Since the broadband industry has a three-tiered structure (local loop-wholesale products-retail services), two types of separation could be implemented. First, the local access unit of the ILEC could be separated from its wholesale-retail unit. Second, the Internet service provider unit of the ILEC could be separated from the local loop-wholesale unit.<sup>27</sup>

The typical argument in favor of vertical separation, which only applies when a firm has a monopoly position over a bottleneck, is that a vertically integrated firm has an incentive to use its upstream price to raise its rivals' costs; vertical separation then annihilates this incentive. Our argument is different. In the following, we claim that when several integrated firms compete on the upstream market, vertical separation of one integrated firm annihilates the softening effect and restores the firms' incentives to compete fiercely on the upstream

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<sup>24</sup>This is true, since  $\pi_d^{(i)}(\cdot)$  is strictly decreasing and  $\pi_d^{(u)}(c_u) = \pi_d^{(i)}(c_u)$ .

<sup>25</sup>However, nothing precludes a priori equilibria in which the input would be priced below marginal cost.

<sup>26</sup>In May 2007, in a speech, she declared: "I believe that functional separation (...) could indeed serve to make competition more effective in a service-based competition environment where infrastructure-based competition is not expected to develop in a reasonable period. It may be a useful remedy in specific cases. It is certainly not a panacea." (Viviane Reding, "How Europe can Bridge the Broadband Gap", Brussels, 14 May 2007).

<sup>27</sup>The latter situation has been observed in some countries. For instance, in France, the ILEC's Internet service provider, Wanadoo, was a subsidiary of its parent company, France Télécom, between 2000 and 2004.

market. To make this point, we assume that firm 2 has been vertically separated and call  $d1$  and  $d2$  the unintegrated downstream firms. We establish that, under an additional technical assumption, Proposition 7 extends to the vertical separation of an integrated firm.

**Assumption 4.** (i) *Upstream suppliers cannot discriminate between downstream firms.* (ii) *Downstream firms choose their upstream supplier after downstream prices have been set.*<sup>28</sup>

**Proposition 8.** *Consider the vertical separation of a vertically integrated firm in the basic model. If Assumption 4 holds and if downstream prices are strategic complements, then, the perfect competition outcome is the only equilibrium.*

*Proof.* See Appendix A.9. □

There is an important difference between vertical separation and the entry of a pure upstream competitor in our framework: after vertical separation has taken place, there are two unintegrated downstream firms. This implies that we need both (i) and (ii) in Assumption 4 to prove Proposition 8.

To see why, consider first that discrimination is allowed on the upstream market. Then, we cannot exclude that the following situation arises in equilibrium. Integrated firm 1 offers upstream price  $a_1$  to firm  $d1$ , and unintegrated upstream firm  $u$  offers price  $a_u$  to firm  $d2$ , where  $a_1$  and  $a_u$  are the monopoly prices of firms 1 and  $u$  respectively.<sup>29</sup> Firm  $u$  prefers not to make an acceptable offer to unintegrated downstream firm  $d1$ , since, if that offer were eventually accepted, firm 1 would become more aggressive on the downstream market, which would erode the profit earned by firm  $u$  on firm  $d2$ . Similarly, firm 1 prefers not to make an acceptable offer to firm  $d2$ , since if that offer were accepted, firm 1 would become less aggressive on the downstream market. By strategic complementarity, firm  $d1$  would increase its downstream price as well, which could lower its demand, and hence, the upstream profit that firm 1 makes on firm  $d1$ .

Consider now that firms cannot discriminate on the upstream market, and assume that unintegrated downstream firms elect their upstream suppliers before the downstream competition stage. Then, the following situation may be an equilibrium. Firms 1 and  $u$  set the monopoly upstream prices  $a_1$  and  $a_u$ , as defined in the previous paragraph. Consider that  $a_1 > a_u$ , which makes sense, since a vertically integrated firm has more incentives than an unintegrated upstream firm to charge a high upstream price. It may then be that firm  $d1$

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<sup>28</sup>Notice that this assumption would not change the results of the previous sections, since Assumption 1 already ensured that the unintegrated downstream firm chose the cheapest supplier in our basic framework.

<sup>29</sup>Formally,  $a_1 = \arg \max_a \pi_1^{(1,u)}(a, a_u)$  and  $a_u = \arg \max_a \pi_u^{(1,u)}(a_1, a)$ , where  $\pi_i^{(j,k)}(a_j, a_k)$  denotes the profit of firm  $i$  when firms  $d_1$  and  $d_2$  are supplied by firms  $j$  and  $k$ , respectively, at prices  $a_j$  and  $a_k$ .

purchases from firm 1 to make the integrated firm less aggressive on the final market, while firm  $d2$  chooses firm  $u$  to benefit from a lower upstream price.

The situations described in the above paragraphs seem rather unlikely, and we have not been able to exhibit them using standard demand specifications. Assumption 4 enables us to rule them out in our general framework.

## 5 Extensions and Discussions

We now discuss some extensions and robustness checks.<sup>30</sup>

**Two-part tariffs.** We now show that partial foreclosure equilibria with positive upstream profits always exist under two-part tariff competition on the upstream market. Denote by  $a_i$  (respectively,  $T_i$ ) the variable (respectively, the fixed) part of the tariff set by firm  $i$ . In a monopoly-like outcome, firm  $i$  sets the variable part which maximizes the sum of its profit and firm  $d$ 's profit, i.e.,  $a_{tp} = \arg \max_{a_i} \pi_i^{(i)}(a_i) + \pi_d^{(i)}(a_i)$ ,<sup>31</sup> while firm  $j$  makes no upstream offer. The fixed fee  $T_i$  captures firm  $d$ 's profit, i.e.,  $T_i = \pi_d^{(i)}(a_{tp})$ . This is an equilibrium provided that firm  $j$  does not want to undercut, or:  $\pi_i^{(i)}(a_{tp}) + \pi_d^{(i)}(a_{tp}) \leq \pi_j^{(i)}(a_{tp})$ .

If the above inequality is not satisfied, then there exists an equilibrium in which both integrated firms charge the variable part  $a_{tp}$  and a fixed fee equal to  $\pi_j^{(i)}(a_{tp}) - \pi_i^{(i)}(a_{tp})$ , which makes them indifferent between supplying the upstream demand or not. Under two-part tariff competition, this is a matching-like equilibrium.<sup>32</sup>

**Proposition 9.** *Under two-part tariff competition on the upstream market, there exist monopoly-like or matching-like equilibria.*

*Proof.* Immediate. □

Once again, upstream competition may not modify the outcome with respect to the monopoly benchmark. If the upstream equilibrium is monopoly-like, both the fixed and the variable parts of the tariff remain the same; obviously, downstream prices are not affected either. If the equilibrium is matching-like, competition modifies the fixed part only, without affecting any downstream prices. In other words, the only impact of competition is to redistribute some profits from the integrated firms to the unintegrated downstream firm. Besides, provided that  $a_{tp} > c_u$ ,<sup>33</sup> it is straightforward to show that the upstream profit is strictly

<sup>30</sup>All the computations in this section are available at <http://jerome.pouyet.googlepages.com/Mathematica.BHPS.Extensions.zip>.

<sup>31</sup>See Bonanno and Vickers (1988).

<sup>32</sup>For conciseness, we do not report other equilibria in which the upstream market is supplied by one firm, whose upstream offer is constrained by the offer of its integrated rival.

<sup>33</sup>This is the case if downstream prices are strategic complements, or not too strategic substitutes.

positive.<sup>34</sup> In this sense, competition on the upstream market may still be ineffective with two-part tariffs.

**Complete vs. partial foreclosure.** When Assumption 2 holds, we have identified conditions under which partial foreclosure arises in equilibrium.

This assumption can hold for exogenous reasons. First, the regulator may force one of the integrated firms to make an acceptable upstream offer, which is a common practice in the broadband industry.<sup>35</sup> Second, the unintegrated downstream firm may have the possibility to build its own network. More precisely, consider that firm  $d$  can invest in its own infrastructure between stages 1 and 2. If it does so, it pays some sunk cost and becomes able to compete on a level playing field with firms 1 and 2. Faced with this threat, integrated firms prefer to make acceptable upstream offers, rather than triggering the investment of the unintegrated downstream firm. A similar insight obtains if firm  $d$  can purchase the input from an alternative provider with marginal cost  $c'_u > c_u$ .

Assumption 2 can also hold for endogenous reasons. Cost and demand parameters may be such that an integrated firm prefers to supply the unintegrated downstream firm at the monopoly upstream price rather than completely foreclosing it. It is worth emphasizing that the determinants of complete foreclosure and of partial foreclosure are different. The crucial determinant of partial foreclosure is the softening effect, according to which an integrated firm gains from letting its integrated rival supply the upstream market. By contrast, the softening effect does not affect the incentives of an integrated firm to supply an entrant with no alternative upstream offer.<sup>36</sup> In the following, we exhibit two examples with commonly used demand and cost functions, in which Assumption 2 holds endogenously, and partial foreclosure equilibria exist:

- (a) Firms compete on the Salop (1979) circle, consumers have unit demands, transport costs are linear. When firm  $d$  is active, firms are symmetrically located. When firm  $d$

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<sup>34</sup>If the equilibrium is monopoly-like, this is obvious. If it is matching-like, then the upstream profit is equal to  $[a_{tp} - c_u] D_d(p^{(i)}(a_{tp})) + \pi_j^{(i)}(a_{tp}) - \pi_i^{(i)}(a_{tp}) = [p_j^{(i)}(a_{tp}) - c_u] D_j(p^{(i)}(a_{tp})) - c_j (D_j(p^{(i)}(a_{tp}))) - [p_i^{(i)}(a_{tp}) - c_u] D_i(p^{(i)}(a_{tp})) + c_i (D_i(p^{(i)}(a_{tp})))$ , which is strictly positive by Lemma 2.

<sup>35</sup>In France, bitstream access has been regulated since 1999. In 2007, the regulator, ARCEP, removed the obligation to provide bitstream access at the national level (See Decision n° 2007-0089, 30 January 2007), but an obligation to provide a cost-based bitstream access offer at the regional level remains (See Decision n°05-0280, 19 May 2005). In the UK, BT has an obligation to provide a wholesale broadband offer on a retail minus basis. The Italian and Spanish regulators recently introduced an obligation to offer a cost-based bitstream access offer. By contrast, in Germany, there is no obligation for the incumbent to provide a wholesale broadband (bitstream access) offer (See European Commission 2007).

<sup>36</sup>The tradeoff faced by an integrated firm when deciding whether to supply a new downstream entrant is analyzed in Hombert, Pouyet and Schutz (2009).

is completely foreclosed, it is straightforward to show that the locations of firms 1 and 2 do not affect their demand functions. Downstream costs are linear.

With this specification, the duopoly profit is strictly smaller than  $\pi_i^{(i)}(a_m)$ , therefore Assumption 2 holds. Besides, since  $\pi_i^{(i)}(a_m) < \pi_j^{(i)}(a_m)$ , monopoly-like equilibria exist.

- (b) The linear demand functions that we use in Section 3.5 comes from the maximization of a quadratic quasilinear utility function. When firm  $d$  is active on the downstream market, demand functions are as given in Section 3.5. When it is squeezed, demand functions are obtained by imposing the constraint that the quantity purchased from firm  $d$  is zero in the representative consumer's program.<sup>37</sup> Downstream costs are quadratic.

When downstream costs are slightly convex and the substitutability parameter is large enough, Assumption 2 holds together with the existence condition for monopoly-like equilibria.

**Input differentiation.** We claim that input differentiation would actually increase the scope for monopoly-like equilibria. Assume that the intermediate inputs produced by the vertically integrated firms are differentiated, and that this differentiation translates into the final good. As pointed out by Ordober and Shaffer (2007), this implies that the unintegrated downstream firm's product is a closer substitute to its wholesale supplier's product than to the other vertically integrated firm's product.

Once plugged into our setting, this assumption makes monopoly-like equilibria more likely. Indeed, as the upstream supplier's and the downstream entrant's products become closer substitutes, the softening effect strengthens. At the same time, the upstream profit effect weakens, as fewer additional final customers can be reached through the pure retailer. Both effects reduce the incentives to undercut the upstream market.<sup>38</sup>

**Quantity competition.** The softening effect exists if the upstream supplier can enhance its upstream profits by behaving softly on the downstream market. As discussed previously, this requires that it actually interacts with the unintegrated downstream firm. One may wonder whether the softening effect hinges on the assumption of price competition on the downstream market, for if the downstream strategic variables are quantities and all firms play simultaneously, then the upstream supplier can no longer impact its upstream profit through its downstream behavior. However, if for instance integrated firms are Stackelberg leaders on the downstream market, then the upstream supplier's quantity choice modifies its

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<sup>37</sup>See Höfler (2008).

<sup>38</sup>Using the specification of Ordober and Shaffer (2007), we checked that input differentiation expands the set of parameters sustaining monopoly-like equilibria.

upstream profit, and the softening effect is still at work. To summarize, the question is not whether firms compete in prices or in quantities, but whether the strategic choice of a firm can affect its rivals' quantities.<sup>39</sup>

**Upstream demand sharing.** Throughout the paper, we have assumed that the upstream market could be supplied by one integrated firm only. Consider now that, when upstream offers are identical, the upstream demand is split equally between integrated firms. This would be a reasonable assumption if there were several downstream firms. In that case, we can still think about the upstream market in terms of softening effect and upstream profit effect. When the integrated firms share the upstream market, they both obtain some upstream profits, and they both benefit from a softening effect, since they both have incentives to protect their upstream revenues. Behaviors on the upstream market still trade off the softening effect and the upstream profit effect.

It becomes clear that the possibility for equilibrium partial foreclosure remains. Monopoly-like equilibria feature the same outcome as in the basic model. Matching-like equilibria can also exist, in which the upstream market is shared between integrated firms.<sup>40</sup>

**Downstream strategic interaction.** In Bourreau, et al. (2007), we provide another example, in which downstream prices can be either strategic substitutes or strategic complements. We obtain two results of interest. First, as downstream prices become more strategic complements, the softening effect weakens and the incentives to undercut on the upstream market are reinforced; therefore, the nature of the strategic interaction on the downstream sector may give some hints on the potential competitiveness of the upstream market. Second, with high strategic substitutability and assuming that there exists an unintegrated upstream supplier, there is an equilibrium in which both integrated firms are inactive on the upstream market and the unintegrated upstream firm sets its monopoly price on that market; in that case, vertical separation is not the ideal remedy to a poorly competitive upstream market.

## 6 Conclusion

Our analysis has focused on the links between vertically-related markets, when the upstream good is an essential input to the downstream product, and when the competitors on the

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<sup>39</sup>With a linear demand function and quantity competition, if integrated firms are Stackelberg leaders on the downstream market, then a monopoly-like equilibrium always exists.

<sup>40</sup>For instance, solving the Hotelling-Salop case, and assuming that the unintegrated downstream firm splits equally its demand when upstream prices are identical, we obtain the following subgame-perfect equilibria: the two monopoly-like outcomes, the perfect competition outcome, and a continuum of equilibria in which both integrated firms set the same price above marginal cost, and share the upstream demand.

upstream market are also rivals on the downstream one. One of the main insights conveyed in the paper is that competition on these upstream markets may not be effective, and the monopoly outcome may persist even when competition in that market is possible.

Our results may rationalize the concerns expressed by the French Competition Authority when reviewing the proposed merger between two broadband providers, Cegetel and Neuf Telecom. Before the merger, these operators owned between 30 and 50 percent of the national wholesale broadband market, while the incumbent historical operator (France Télécom) served the rest. The merger received clearance in August 2005 under the condition that the joint entity continued to provide its wholesale services, highlighting the perceived fear that the competition between two integrated firms on the upstream market may de facto lead to a monopoly.<sup>41</sup>

However, our results do not shed light on why the competitive situation on the upstream market would deteriorate when the number of integrated firms decreases. Answering this question would require to study the impact of the market structure on the emergence of monopoly-like equilibria. In our general setting, the impact of the number of vertically integrated firms and of unintegrated downstream firms on the competitiveness of the wholesale market is ambiguous. For instance, an additional retailer strengthens both the softening effect and the wholesale profit effect, which may or may not strengthen the incentives to undercut the wholesale market. A complete analysis is left for future research.

## A Appendix

### A.1 Proof of Lemma 1

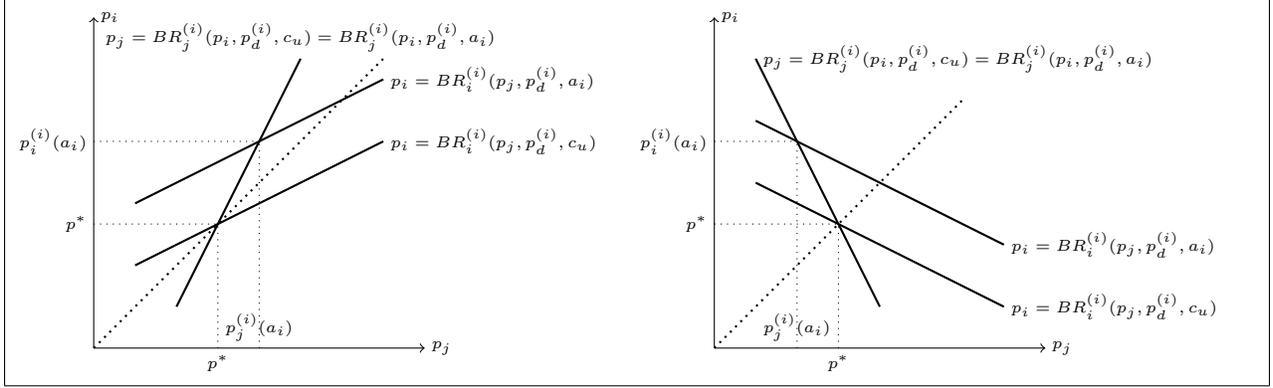
Assume that integrated firm  $i \in \{1, 2\}$  is the upstream supplier at price  $a_i > c_u$ , and let us show that  $p_i^{(i)}(a_i) > p_j^{(i)}(a_i)$ .

Let us first give a sketch of the proof. Start from a hypothetical situation, in which integrated firm  $i$  would supply the upstream market at marginal cost, and unintegrated downstream firm  $d$  would set  $p_d^{(i)}(a_i)$  on the downstream market, and analyze the outcome of the ensuing price competition game between the two integrated firms. There exists a symmetric Nash equilibrium in this reduced game. Then, assume that firm  $i$  increases its upstream price from  $c_u$  to  $a_i$ . Its downstream best-response shifts upwards, while firm  $j$ 's best-response remains unaffected. These curves intersect at least once, at coordinates  $(p_i^{(i)}(a_i), p_j^{(i)}(a_i))$ , since  $(p_i^{(i)}(a_i), p_j^{(i)}(a_i), p_d^{(i)}(a_i))$  is a Nash equilibrium of the three-player game. Since the slopes of the best-response functions are smaller than one, we obtain that  $p_i^{(i)}(a_i) > p_j^{(i)}(a_i)$ .

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<sup>41</sup>See DGCCRF, Decision C2005-44 related to the merger between Neuf Telecom and Cegetel.

This is depicted graphically in Figure 3 for the case of strategic complements (left) and strategic substitutes (right).



**Figure 3:** Graphical representation of the proof of Lemma 1

Let us now write down the formal proof. Assume that firm  $i$  supplies the upstream market at marginal cost, and firm  $d$  sets  $p_d^{(i)}(a_i)$  on the downstream market. Let  $\bar{p}$  an upper bound for the integrated firms' best-response functions.  $p \in [0, \bar{p}] \mapsto BR_i^{(i)}(p, p_d^{(i)}(a_i), c_u) \in [0, \bar{p}]$  is continuous.<sup>42</sup> By the Brouwer fixed point theorem, this function has a fixed point, which we denote by  $p^*$ . When the upstream price is set at marginal cost, integrated firms are identical, therefore, they have the same best-responses, and  $p^*$  is a fixed point for  $BR_j^{(i)}(\cdot, p_d^{(i)}(a_i), c_u)$  as well. As a result, there exists a Nash equilibrium of the two-firm game, in which both firms set  $p^*$ .  $p^*$  satisfies  $p^* = BR_i^{(i)}(p^*, p_d^{(i)}(a_i), c_u) = BR_j^{(i)}(p^*, p_d^{(i)}(a_i), c_u)$ .

Assume now that integrated firm  $i$  increases its upstream price from  $c_u$  to  $a_i$ . Clearly, firm  $j$ 's best-response is not affected (see equation (2)). The impact on firm  $i$ 's best-response is obtained by differentiating equation (1):  $\frac{\partial BR_i^{(i)}}{\partial a_i} = -\frac{\partial^2 \bar{\pi}_i^{(i)}}{\partial p_i \partial a_i} / \frac{\partial^2 \bar{\pi}_i^{(i)}}{\partial p_i^2} = -\frac{\partial D_d}{\partial p_i} / \frac{\partial^2 \bar{\pi}_i^{(i)}}{\partial p_i^2}$ , which is strictly positive since the second-order condition is satisfied. Therefore, firm  $i$ 's best-response shifts upwards when  $a_i$  increases. In particular,  $BR_i^{(i)}(\cdot, \cdot, a_i) > BR_i^{(i)}(\cdot, \cdot, c_u) = BR_j^{(i)}(\cdot, \cdot, a_i)$ .

Since  $(p_i^{(i)}(a_i), p_j^{(i)}(a_i), p_d^{(i)}(a_i))$  is a downstream equilibrium, we have

$$p_i^{(i)}(a_i) = BR_i^{(i)}\left(BR_j^{(i)}\left(p_i^{(i)}(a_i), p_d^{(i)}(a_i), a_i\right), p_d^{(i)}(a_i), a_i\right).$$

Therefore,

$$(4) \quad p_i^{(i)}(a_i) > BR_i^{(i)}\left(BR_j^{(i)}\left(p_i^{(i)}(a_i), p_d^{(i)}(a_i), c_u\right), p_d^{(i)}(a_i), c_u\right).$$

Define  $\Phi(p) = BR_i^{(i)}\left(BR_j^{(i)}\left(p, p_d^{(i)}(a_i), c_u\right), p_d^{(i)}(a_i), c_u\right) - p$ , and notice that  $\Phi(p^*) = 0$ .  $\Phi$

<sup>42</sup>As can be seen by applying the implicit function theorem to equation (1), this function is even continuously differentiable.

is continuously differentiable and strictly decreasing, since the slopes of the best-response functions are strictly smaller than 1. Therefore,  $\Phi(p) < 0$  if and only if  $p > p^*$ . Together with inequality (4), this implies that  $p_i^{(i)}(a_i) > p^*$ .

We conclude the proof by using the stability condition  $\left| \frac{\partial BR_j^{(i)}}{\partial p_i} \right| < 1$ :

$$\begin{aligned} p_j^{(i)}(a_i) &= BR_j^{(i)}\left(p_i^{(i)}(a_i), p_d^{(i)}(a_i), a_i\right) = BR_j^{(i)}\left(p_i^{(i)}(a_i), p_d^{(i)}(a_i), c_u\right) \\ &= p^* + \int_{p^*}^{p_i^{(i)}(a_i)} \frac{\partial BR_j^{(i)}}{\partial p_i}\left(p_i, p_d^{(i)}(a_i), c_u\right) dp_i < p_i^{(i)}(a_i). \end{aligned}$$

## A.2 Proof of Lemma 2

Let integrated firm  $i \in \{1, 2\}$  be the upstream supplier at price  $a_i > c_u$ . Its downstream profit is given by:

$$(5) \quad (p_i^{(i)}(a_i) - c_u)D_i(p_i^{(i)}(a_i), p_j^{(i)}(a_i), p_d^{(i)}(a_i)) - c_i \left( D_i(p_i^{(i)}(a_i), p_j^{(i)}(a_i), p_d^{(i)}(a_i)) \right).$$

Since  $p_i^{(i)}(a_i) > p_j^{(i)}(a_i)$  by Lemma 1, there exists  $\hat{p} > p_i^{(i)}(a_i)$  such that  $D_i(p_i^{(i)}(a_i), p_j^{(i)}(a_i), p_d^{(i)}(a_i)) = D_i(\hat{p}, p_i^{(i)}(a_i), p_d^{(i)}(a_i))$ . Downstream profit (5) is thus strictly smaller than:

$$(\hat{p} - c_u)D_i(\hat{p}, p_i^{(i)}(a_i), p_d^{(i)}(a_i)) - c_i \left( D_i(\hat{p}, p_i^{(i)}(a_i), p_d^{(i)}(a_i)) \right).$$

By symmetry between integrated firms, this expression is equal to the downstream profit that rival integrated firm  $j$  would earn if it charged the downstream price  $\hat{p}$  instead of its actual equilibrium price  $p_j^{(i)}(a_i)$ . This profit, by revealed preference, is strictly smaller than the actual downstream profit of firm  $j$ , which concludes the proof.

## A.3 Proof of Proposition 2

Suppose first that both integrated firms offer the same upstream price  $a_* \leq a_m$ , such that  $\pi_i^{(i)}(a_*) = \pi_j^{(i)}(a_*)$ . The unintegrated downstream firm chooses indifferently one of them as its upstream supplier, and both integrated firms earn the same profit.

Consider an upward deviation of integrated firm  $i$ , be it the upstream supplier or not. Now, by Assumption 1, firm  $d$  strictly prefers buying the upstream good from firm  $j$  at price  $a_*$ , and firm  $i$ 's profit is unchanged. Consider now a downward deviation:  $a_i < a_*$ . Firm  $d$  strictly prefers to buy from firm  $i$ , which then earns  $\pi_i^{(i)}(a_i)$ . By Assumption 3, since  $a_* \leq a_m$ , this profit is smaller than  $\pi_i^{(i)}(a_*) = \pi_j^{(i)}(a_*)$ . That situation is therefore an equilibrium.

Conversely, consider that both integrated firms offer the same upstream price  $a_*$ . Suppose

first that  $a_* > a_m$ . The upstream supplier then has a strictly profitable deviation: propose  $a_m$ .

If  $\pi_i^{(i)}(a_*) < \pi_j^{(i)}(a_*)$ , then the upstream supplier would rather set an upstream price above  $a_*$  to earn  $\pi_j^{(i)}(a_*)$ .

If  $\pi_i^{(i)}(a_*) > \pi_j^{(i)}(a_*)$ , then the integrated firm which does not supply the upstream market would rather set an upstream price slightly smaller than  $a_*$  to earn a profit almost equal to  $\pi_i^{(i)}(a_*)$ .

## A.4 Proof of Proposition 3

Consider by contradiction an equilibrium configuration in which  $a_i < a_j$  and  $a_i \neq a_m$ . By Assumption 1, the upstream supplier is firm  $i$ . If  $a_j > a_m$ , it is a strictly profitable deviation for firm  $i$  to offer  $a_m$ . If  $a_j \leq a_m$ , firm 1 would rather charge any upstream price in  $(a_i, a_j)$ , since  $\pi_i^{(i)}(\cdot)$  is increasing in this interval by Assumption 3.

Let us now show that a monopoly-like equilibrium Pareto-dominates any matching-like equilibrium, from the viewpoint of integrated firms. We have  $\pi_j^{(i)}(a_m) \geq \pi_i^{(i)}(a_m)$  by Proposition 1. Consider a matching-like equilibrium at upstream price  $a_*$ . By definition of  $a_m$ ,  $\pi_i^{(i)}(a_m) \geq \pi_i^{(i)}(a_*) = \pi_j^{(i)}(a_*)$ . This concludes the proof.

## A.5 Proof of Proposition 4

Consider a partial foreclosure equilibrium with upstream price  $\hat{a} > c_u$ . Assume that downstream prices are strategic complements, and let us show that downstream prices are higher in the partial foreclosure equilibria than in the perfect competition outcome, and that at least one of these prices is strictly higher.

Strategic complementarity writes as  $\partial^2 \tilde{\pi}_k^{(i)} / \partial p_k \partial p'_k \geq 0$  for  $k \neq k'$ , hence the game defined by payoff functions  $(p_k, p_{-k}) \in [0, +\infty)^3 \mapsto \tilde{\pi}_k^{(i)}(p_k, p_{-k}, a)$  is smooth supermodular, parameterized by  $a \in \{c_u, \hat{a}\}$ . For all  $k$ ,  $\partial \tilde{\pi}_k^{(i)}(\cdot, \cdot, \hat{a}) / \partial p_k \geq \partial \tilde{\pi}_k^{(i)}(\cdot, \cdot, c_u) / \partial p_k$ , therefore,  $\tilde{\pi}_k^{(i)}(p_k, p_{-k}, a)$  has increasing differences in  $(p_k, a)$ . Besides, the downstream equilibrium is, by assumption, unique. Supermodularity theory (see Vives 1999, Theorem 2.3) tells us that the equilibrium of this game is increasing in  $a$ . Therefore,  $p_k^{(i)}(c_u) \leq p_k^{(i)}(\hat{a})$  for all  $k$ .

Besides, by Lemma 1,  $p_i^{(i)}(\hat{a}) > p_j^{(i)}(\hat{a}) \geq p_j^{(i)}(c_u) = p_i^{(i)}(c_u)$ , therefore,  $p_i^{(i)}(\hat{a}) > p_i^{(i)}(c_u)$ . Therefore, consumers are worse off in the partial foreclosure equilibrium.

Assume also: a representative consumer with a quasi-linear, continuously differentiable and quasi-concave utility function exists; firms have symmetric and identical demands; firms have the same convex downstream costs functions. Let us show that partial foreclosure lowers the social welfare.

When the upstream price is set at marginal cost, the three firms are perfectly identical. Hence, since the downstream equilibrium is unique, it is symmetric, and  $p_k^{(i)}(c_u) = \hat{p}$  for all  $k$ . Let  $(\hat{p}_1, \hat{p}_2, \hat{p}_3)$  the permutation of the triple  $(p_k^{(i)}(\hat{a}))_{k=1,2,3}$  such that  $\hat{p}_1 \leq \hat{p}_2 \leq \hat{p}_3$ , and let us relabel firms so that firm  $k$  is the firm that charges  $\hat{p}_k$  when the upstream price is  $\hat{a}$ . Recall that  $\hat{p}_k \geq \hat{p}$  for all  $k$ , and that this inequality is strict for  $k = 3$ .

Keeping in mind that firms have been relabeled, let us denote by  $U(q_0, q_1, q_2, q_3) = q_0 + u(q_1, q_2, q_3)$  the utility function of the representative consumer, where  $q_0$  denotes consumption of the numeraire, and  $q_k$  denotes consumption of product  $k \in \{1, 2, 3\}$ . We can then write the social welfare as a function of the downstream price vector  $p$ .<sup>43</sup>

$$W(p) = u(D_1(p), D_2(p), D_3(p)) - \sum_{k=1}^3 \{c_u D_k(p) + c(D_k(p))\},$$

where  $c(\cdot)$  denotes the downstream cost function, which, by assumption, is the same for the three firms. This welfare function is continuously differentiable, since functions  $u$ ,  $c$  and  $D_k$  are continuously differentiable. To prove the result, we need to show that  $W(\hat{p}_1, \hat{p}_2, \hat{p}_3) - W(\hat{p}, \hat{p}, \hat{p})$ , the variation in social welfare when downstream prices increase from  $(\hat{p}, \hat{p}, \hat{p})$  to  $(\hat{p}_1, \hat{p}_2, \hat{p}_3)$ , is strictly negative. This variation can be written as:

$$\begin{aligned} & (W(\hat{p}_1, \hat{p}_2, \hat{p}_3) - W(\hat{p}_1, \hat{p}_2, \hat{p}_2)) + (W(\hat{p}_1, \hat{p}_2, \hat{p}_2) - W(\hat{p}_1, \hat{p}_1, \hat{p}_1)) + (W(\hat{p}_1, \hat{p}_1, \hat{p}_1) - W(\hat{p}, \hat{p}, \hat{p})) \\ &= \int_{\hat{p}_2}^{\hat{p}_3} \frac{\partial W}{\partial p_3}(\hat{p}_1, \hat{p}_2, r) dr + \int_{\hat{p}_1}^{\hat{p}_2} \sum_{k=2}^3 \frac{\partial W}{\partial p_k}(\hat{p}_1, r, r) dr + \int_{\hat{p}}^{\hat{p}_1} \sum_{k=1}^3 \frac{\partial W}{\partial p_k}(r, r, r) dr. \end{aligned}$$

We know that  $\hat{p}_1 < \hat{p}_2$  or  $\hat{p}_2 < \hat{p}_3$ . Assume first that  $\hat{p}_2 < \hat{p}_3$ . Then, we claim that the first integral in the right-hand side is strictly negative, while the two other ones are non-positive. Let us start with the first one. Let  $r \in (\hat{p}_2, \hat{p}_3]$ . Then,

$$\begin{aligned} \frac{\partial W}{\partial p_3}(\hat{p}_1, \hat{p}_2, r) &= \sum_{k=1}^3 \left( \frac{\partial u}{\partial q_k} \frac{\partial D_k}{\partial p_3} - (c_u + c'(D_k)) \frac{\partial D_k}{\partial p_3} \right), \\ &= \frac{\partial D_3}{\partial p_3} (r - c_u - c'(D_3)) + \sum_{k=1}^2 \frac{\partial D_k}{\partial p_3} (\hat{p}_k - c_u - c'(D_k)). \end{aligned}$$

Let  $k \in \{1, 2\}$ . Firm 3 has the highest markup:  $r - c_u - c'(D_3) > \hat{p}_k - c_u - c'(D_k)$ . Indeed, since  $r > \hat{p}_k$ ,  $D_3 < D_k$ , and, since costs are convex,  $c'(D_3) < c'(D_k)$ . Besides, firm 3's markup is strictly positive. Indeed, since  $D_3(\hat{p}_1, \hat{p}_2, r) < \frac{1}{3} \sum_{k=1}^3 D_k(\hat{p}_1, \hat{p}_2, r) \leq \frac{1}{3} \sum_{k=1}^3 D_k(\hat{p}, \hat{p}, \hat{p}) = D_3(\hat{p}, \hat{p}, \hat{p})$ , then  $r - c_u - c'(D_3(\hat{p}_1, \hat{p}_2, r)) > \hat{p} - c_u - c'(D_3(\hat{p}, \hat{p}, \hat{p}))$ , which is strictly positive,

<sup>43</sup>This function does not depend on the upstream price.

since  $\pi(c_u) > 0$ . As a result,

$$\frac{\partial W}{\partial p_3}(\hat{p}_1, \hat{p}_2, r) < (r - c_u - c'(D_3)) \sum_{k=1}^3 \frac{\partial D_k}{\partial p_3} < 0.$$

Therefore, the first integral is indeed strictly negative, since  $\hat{p}_2 < \hat{p}_3$ . A similar argument shows that the two other integrands are non-positive, and we can conclude that the social welfare is strictly lower in a partial foreclosure equilibrium.

If  $\hat{p}_1 < \hat{p}_2$ , we can make the same reasoning to show that the first and third integrals are non-positive, while the second one is strictly negative. This concludes the proof.

## A.6 Proof of Proposition 5

Assume that integrated firm  $i$  supplies the upstream market at price  $a_i$ , and denote its integrated rival by  $j$ . To begin with, it is straightforward to see that we can normalize the intercepts of the linear demands to  $D = 1$  and all upstream and downstream costs to  $c = c_u = 0$ , by redefining upstream prices as  $\frac{a_i - c_u}{D - c - c_u}$  and downstream prices as  $\frac{p_k - c - c_u}{D - c - c_u}$ .

Then, for all downstream and upstream prices, for  $k \in \{1, 2, d\}$ , we have  $\frac{\partial^2 \pi_k^{(i)}}{\partial p_k^2} = -2(1 + \frac{2}{3}\gamma) < 0$ . This ensures that the best-response functions are uniquely defined. The stability condition is satisfied, since, for all  $k \neq k'$ , we have  $\left| \frac{\partial BR_k^{(i)}}{\partial p_{k'}} \right| = \frac{\gamma}{6+4\gamma} < 1$ . There is a unique downstream equilibrium, which can be computed by solving the set of first-order conditions. The equilibrium quantity served by downstream firm  $d$  is positive if and only if  $a_i \leq a_{\max}(\gamma) \equiv \frac{6+5\gamma}{6+7\gamma+\gamma^2} > 0$ . Assumption 1 is satisfied, since  $\pi_d^{(i)}(a_i) = \frac{3(1+\gamma)^2(6+\gamma)^2(3+2\gamma)}{4(3+\gamma)^2(6+5\gamma)^2} [a_i - a_{\max}(\gamma)]^2$ , thus  $\pi_d^{(i)}(\cdot)$  is decreasing for  $a_i \leq a_{\max}(\gamma)$ .

The profit of the upstream supplier is strictly concave since  $\frac{d^2 \pi_i^{(i)}}{da_i^2} = \frac{-648+1944\gamma+2205\gamma^2+1158\gamma^3+269\gamma^4+20\gamma^5}{2(18+21\gamma+5\gamma^2)^2} < 0$ . Firm  $i$ 's maximum is reached for  $a_i = a_m(\gamma) \equiv \frac{324+594\gamma+360\gamma^2+75\gamma^3}{648+1296\gamma+909\gamma^2+249\gamma^3+20\gamma^4}$ . Since  $a_m \in (0, a_{\max}(\gamma))$ , Assumption 3 is satisfied.  $\pi_i^{(i)}(\cdot)$  and  $\pi_j^{(i)}(\cdot)$  are parabolas, they cross each other twice, in  $a_i = c_u$  and in  $a_i = a_*(\gamma) \equiv \frac{9(12+16\gamma+5\gamma^2)}{108+180\gamma+93\gamma^2+13\gamma^3}$ .  $\pi_i^{(i)}(\cdot)$  is strictly concave and  $\pi_j^{(i)}(\cdot)$  is convex since  $\frac{d^2 \pi_j^{(i)}}{da_i^2} = \frac{3(3+2\gamma)\gamma^2(1+\gamma)^2}{2(3+\gamma)^2(6+5\gamma)^2} \geq 0$ . Hence, we have:

$$(6) \quad \pi_i^{(i)}(a_i) \geq \pi_j^{(i)}(a_i) \Leftrightarrow a_i \in [0, a_*(\gamma)].$$

Let us now check whether  $a_m(\gamma) \in [0, a_*(\gamma)]$ :

$$a_m(\gamma) - a_*(\gamma) = \frac{3(3+\gamma)(6+5\gamma)(-648-1296\gamma-864\gamma^2-183\gamma^3+5\gamma^4)}{(108+180\gamma+93\gamma^2+13\gamma^3)(648+1296\gamma+909\gamma^2+249\gamma^3+20\gamma^4)}.$$

Analyzing the above function, we establish that there exists  $\bar{\gamma} > 0$ , such that  $a_m(\gamma) \geq a_*(\gamma)$  if, and only if,  $\gamma \geq \bar{\gamma}$ .

Since  $\pi_i^{(i)}(0) = \pi_j^{(i)}(0)$  and  $0 \leq a_m(\gamma)$ , Proposition 2 implies that the perfect competition outcome is always an equilibrium.

If  $\gamma < \bar{\gamma}$ , then  $0 < a_m(\gamma) < a_*(\gamma)$ . By Proposition 1, (6) implies that there is no monopoly-like equilibrium. Moreover  $a_*(\gamma) > a_m(\gamma)$  implies by Proposition 2 that there is no other matching-like equilibrium than the perfect competition outcome.

Similarly, if  $\gamma \geq \bar{\gamma}$ , then  $a_m(\gamma) \geq a_*(\gamma)$  and there exist monopoly-like equilibria. This is also a necessary and sufficient condition for the matching-like equilibrium with upstream price  $a_*$ .

## A.7 Proof of Proposition 6

Let  $i \neq j$  in  $\{1, 2\}$ . We show that the conditions stated in Proposition 6 are sufficient to have  $\frac{d\pi_i^{(i)}}{da_i}(c_u) > \frac{d\pi_j^{(i)}}{da_i}(c_u)$ . To simplify the exposition, we introduce the following notations.  $p$  denotes the equilibrium downstream price set by the three firms when the upstream market is supplied at marginal cost.  $D$  denotes the demand addressed to each firm when all downstream prices are equal to  $p$ . We also denote by  $c'$  and  $c''$  the first and second derivatives of the downstream cost function when the quantity produced is  $D$ . Last, we define  $\delta \equiv \frac{\partial D_k}{\partial p_k}$ ,  $\tilde{\delta} \equiv \frac{\partial D_k}{\partial p_{k'}}$ ,  $\gamma \equiv \frac{\partial^2 D_k}{\partial p_k^2}$  and  $\tilde{\gamma} \equiv \frac{\partial^2 D_k}{\partial p_k \partial p_{k'}}$ , where all the derivatives are taken at price vector  $(p, p, p)$ , and  $k \neq k'$  in  $\{1, 2, d\}$ .<sup>44</sup>

With these notations, when  $a_i = c_u$ , the first-order conditions on the downstream market can be rewritten as  $(p - c_u - c')\delta + D = 0$ . The second-order conditions are given by  $(2 - \delta c'')\delta + (p - c_u - c')\gamma < 0$ , and the stability condition is:

$$\left| \frac{(1 - \delta c'')\tilde{\delta} + (p - c_u - c')\tilde{\gamma}}{(2 - \delta c'')\delta + (p - c_u - c')\gamma} \right| < 1.$$

Notice also that, if downstream prices are strategic complements, then,  $(1 - \delta c'')\tilde{\delta} + (p - c_u - c')\tilde{\gamma} \geq 0$ . Besides, since the total demand is decreasing,  $\delta + 2\delta' \leq 0$ .

<sup>44</sup>Notice that, since the three firms are identical,  $p$ ,  $D$ ,  $c'$ ,  $c''$ ,  $\delta$ ,  $\tilde{\delta}$ ,  $\gamma$  and  $\tilde{\gamma}$  are well-defined, and do not depend on  $k$  or  $k'$ .

Differentiating the profit functions with respect to  $a_i$  for  $a_i = c_u$ , we obtain:

$$\begin{aligned}\frac{d\pi_i^{(i)}}{da_i}(c_u) &= (p - c_u - c')\tilde{\delta} \left( \frac{dp_j^{(i)}}{da_i}(c_u) + \frac{dp_d^{(i)}}{da_i}(c_u) \right) + D, \\ \frac{d\pi_j^{(i)}}{da_i}(c_u) &= (p - c_u - c')\tilde{\delta} \left( \frac{dp_i^{(i)}}{da_i}(c_u) + \frac{dp_d^{(i)}}{da_i}(c_u) \right).\end{aligned}$$

Therefore,

$$(7) \quad \frac{d\pi_i^{(i)}}{da_i}(c_u) - \frac{d\pi_j^{(i)}}{da_i}(c_u) = D - (p - c_u - c')\tilde{\delta} \left( \frac{dp_i^{(i)}}{da_i}(c_u) - \frac{dp_j^{(i)}}{da_i}(c_u) \right).$$

As usual, we obtain the expression of  $\frac{dp_i^{(i)}}{da_i}(c_u) - \frac{dp_j^{(i)}}{da_i}(c_u)$  by differentiating firms  $i$  and  $j$ 's first-order conditions with respect to  $a_i$ . We get:

$$\frac{dp_i^{(i)}}{da_i}(c_u) - \frac{dp_j^{(i)}}{da_i}(c_u) = \frac{\tilde{\delta}}{-\left((2 - \delta c'')\delta + (p - c_u - c')\gamma - ((1 - \delta c'')\tilde{\delta} + (p - c_u - c')\tilde{\gamma})\right)}.$$

Plugging this into equation (7), using the first-order conditions to get rid of the  $D$  term, and rearranging terms, we finally obtain:

$$\begin{aligned}\frac{d\pi_i^{(i)}}{da_i}(c_u) - \frac{d\pi_j^{(i)}}{da_i}(c_u) &= \frac{p - c_u - c'}{-\left((2 - \delta c'')\delta + (p - c_u - c')\gamma - ((1 - \delta c'')\tilde{\delta} + (p - c_u - c')\tilde{\gamma})\right)} \\ &\quad \times \left(-\tilde{\delta}^2 + \delta \left((2 - \delta c'')\delta + (p - c_u - c')\gamma - ((1 - \delta c'')\tilde{\delta} + (p - c_u - c')\tilde{\gamma})\right)\right).\end{aligned}$$

The stability and second-order conditions imply that the denominator in the right-hand side is positive. Therefore, the above expression is positive if, and only if

$$-\tilde{\delta}^2 + (-\delta) \left( (-\delta)(2 - \delta c'') - (p - c_u - c')\gamma + \tilde{\delta}(1 - \delta c'') + (p - c_u - c')\tilde{\gamma} \right) > 0.$$

Since  $\delta < 0$ ,  $-\delta > \tilde{\delta} > 0$ ,  $p - c_u - c' > 0$ ,  $c'' > 0$ , and  $(-\delta)(2 - \delta c'') - (p - c_u - c')\gamma > 0$  this is the case if:

- $\tilde{\delta}(1 - \delta c'') + (p - c_u - c')\tilde{\gamma} \geq 0$  (namely, if downstream prices are strategic complements at price vector  $(p, p, p)$ ), and  $\gamma \leq 0$ ,
- or if  $\tilde{\gamma} \geq 0$ .

## A.8 Proof of Proposition 7

First, this is obvious that the unintegrated downstream firm cannot be completely foreclosed in equilibrium, since the unintegrated upstream firm would then prefer to make any acceptable offer above the marginal cost. Similarly, suppose that integrated firm 1 supplies the upstream market at  $a_1 > c_u$ . Then, by Assumption 1,  $\pi_d^{(1)}(a_1) < \pi_d^{(1)}(c_u) = \pi_d^{(u)}(c_u)$ . By continuity, there exists  $a_u > c_u$ , such that  $\pi_d^{(u)}(a_u) > \pi_d^{(1)}(a_1)$ : a strictly profitable deviation for firm  $u$ .

Suppose now that the upstream market is supplied by unintegrated upstream firm  $u$  at price  $a > c_u$ . Let us show that integrated firm 1 can corner the upstream market and enhance its profit by matching firm  $u$ 's offer, i.e., by proposing  $a_1 = a$ . Assume that firms 1 and  $u$  propose the same upstream price  $a > c_u$ , while firm 2 makes no offer.<sup>45</sup> Since  $\partial^2 \tilde{\pi}_k^{(i)} / \partial p_k \partial p_{k'} \geq 0$  for all  $k \neq k' \in \{1, 2, d\}$ , the game defined by the payoff functions  $(p_k, p_{-k}) \in \mathbb{R}_+^3 \mapsto \tilde{\pi}_k^{(i)}(p_k, p_{-k}, a)$  is smooth supermodular, parameterized by  $i \in \{1, u\}$ . For all  $k$ ,  $\partial \tilde{\pi}_k^{(u)} / \partial p_k \leq \partial \tilde{\pi}_k^{(1)} / \partial p_k$ , therefore,  $\tilde{\pi}_k^{(i)}(p_k, p_{-k}, a)$  has increasing differences in  $(p_k, i)$  with the order relation  $u < 1$ . Besides, the downstream equilibrium is, by assumption, unique. Supermodularity theory (see Vives 1999, Theorem 2.3) tells us that the equilibrium of that game is increasing in  $i$ , i.e.,  $p_k^{(u)}(a) \leq p_k^{(1)}(a)$  for  $k = 1, 2, d$ . Moreover, the inequality is strict for  $k = 1$ . Indeed, since  $\tilde{\pi}_1^{(i)}(p_1, p_{-1}, a)$  has strictly increasing differences in  $(p_1, i)$ , we can write the following inequalities:

$$\begin{aligned} 0 &= \frac{\partial \tilde{\pi}_1^{(u)}}{\partial p_1}(p_1^{(u)}(a), p_2^{(u)}(a), p_d^{(u)}(a), a) \leq \frac{\partial \tilde{\pi}_1^{(u)}}{\partial p_1}(p_1^{(u)}(a), p_2^{(1)}(a), p_d^{(1)}(a), a) \\ &< \frac{\partial \tilde{\pi}_1^{(1)}}{\partial p_1}(p_1^{(u)}(a), p_2^{(1)}(a), p_d^{(1)}(a), a). \end{aligned}$$

A revealed preference along the line of the proof of Lemma 2 implies that firm  $d$  strictly prefers to purchase from firm 1 than from firm  $u$ .

By the same revealed preference argument, firm 1's downstream profit is larger when it supplies the upstream market than when firm  $u$  does. In addition, it gets strictly positive upstream profit. Therefore, matching firm  $u$ 's offer is a strictly profitable deviation, which concludes the proof.

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<sup>45</sup>The new notations are similar to the previous ones:  $\tilde{\pi}_k^{(u)}(.,., a)$  denotes the profit of firm  $k$  when the upstream market is supplied by firm  $u$  at price  $a$ , while  $p_k^{(u)}(a)$  denotes its downstream price at downstream equilibrium.

## A.9 Proof of Proposition 8

The proof that the upstream market cannot be supplied above marginal cost in equilibrium proceeds as in the proof of Proposition 7. To streamline the analysis, we only provide the main steps. To begin with, notice that, if a firm wants to corner the upstream market, it just needs to slightly undercut its rival. Indeed, because of Assumption 4, unintegrated downstream firms cannot commit to purchasing from a more expensive supplier. Consider that the upstream market is supplied by firm 1 at price  $a_1 > c_u$ . Then, firm  $u$  can set  $a_u = a_1 - \varepsilon$ , and corner the upstream market. Conversely, if firm  $u$  supplies the upstream market at price  $a_u > c_u$ , then firm 1 sets  $a_1 = a_u - \varepsilon$ , since, as in the proof of Proposition 7, this enables it to capture the upstream profits, and to relax downstream competition. Therefore, the upstream market cannot be supplied above marginal cost.

Let us now demonstrate that the perfect competition outcome is the only equilibrium. Assume that firms 1 and  $u$  set  $a_1 = a_u = c_u$ . Obviously, firm  $u$  does not want to undercut. Firm 1 does not want to undercut either, since it would then make upstream losses. Besides, with the supermodularity argument used in the proof of Proposition 4, all final prices would decrease, which gives even less incentives to undercut. Therefore, the perfect competition outcome is an equilibrium.

Last, let us check that the upstream market cannot be supplied below cost in equilibrium. First, if firm  $u$  supplies the upstream market below  $c_u$ , it has a strictly profitable deviation: set  $a_u = c_u$ . Conversely, if firm 1 supplies the upstream market at  $a_1 < c_u$ , then, we claim that it can also strictly increase its payoff by setting  $c_u$ . To make this point, we need to distinguish two cases, depending on whether  $a_u \leq c_u$ . First, if  $a_u \geq c_u$ , then, firm 1's payoff increases, since the integrated firm gets rid of its upstream losses, and, using again the supermodularity argument from the proof of Proposition 4, all downstream prices increase. If, on the other hand,  $a_u < c_u$ , then, firm  $u$  becomes the upstream supplier. Again, firm 1 gets rid of its upstream losses, and, using this time the supermodularity argument from the proof of Proposition 7, all prices increase. This concludes the proof.

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