Characterization of the spectral phase of ultrashort light pulses

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(Réçu le 31 juillet 2001, accepté le 13 septembre 2001)

Abstract.
The complete characterization of a short light pulse requires the measurement of the value of the electric field as a function of time, or conversely, as a function of the optical frequency. Many different techniques have been demonstrated for this purpose. They fall in two main categories, whether a known reference pulse whose spectrum encompasses that of the unknown pulse is available or not. In the first case, linear techniques such as time-domain or frequency-domain interferometry can be directly used, with the advantage of a high sensitivity. In the latter case, non-stationary filters can be implemented using optical non-linearities, in techniques such as Frequency Resolved Optical Gating (FROG) or Spectral Phase Interferometry for Direct Electric-field Reconstruction (SPIDER).

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ultrashort pulse / characterization / phase measurement / spectral interferometry / SPIDER / FROG / sonogram / spectrogram / tomography / dispersive Fourier-transform spectroscopy

Caractérisation de la phase spectrale d’une impulsion ultra-brève

Résumé. La caractérisation complète d’une impulsion courte requiert la mesure du champ électrique en fonction du temps, ou, de manière équivalente, en fonction de la fréquence. De nombreuses techniques ont été développées dans ce but. Celles-ci peuvent être classées en deux catégories, selon qu’une impulsion de référence de spectre approprié est disponible ou non. Dans le premier cas, des méthodes linéaires comme l’interférométrie dans le domaine temporel ou spectral peuvent être utilisées, ce qui procure l’avantage d’une grande sensibilité. Dans le second cas, des filtres non stationnaires peuvent être mis en place à l’aide de non-linéarités optiques, comme par exemple dans le cas du FROG (Frequency Resolved Optical Gating) ou du SPIDER (Spectral Phase Interferometry for Direct Electric-field Reconstruction). \textsuperscript{©} 2001 Académie des sciences/Éditions scientifiques et médicales Elsevier SAS

impulsion ultra-courte / caractérisation / mesure de phase / interférométrie spectrale / SPIDER / FROG / sonogramme / spectrogramme / tomographie / spectrométrie dispersive par transformée de Fourier
1. Introduction

The complete characterization of ultrashort pulses is of great importance in many femtosecond experiments. Indeed, the outcome of an experiment is strongly influenced by the pulse shape. Proper interpretation and simulation of actual experiments also requires the complete knowledge of the exciting pulse. In coherent and optimal control experiments, complicated pulses must be generated using elaborate pulse-shaping techniques. The complete characterization of the shaped pulse then provides the feedback mechanism needed for a proper operation of such shaping devices. In all these examples, reliable characterization techniques of the electric field are clearly desirable.

Let us consider an ultrashort pulse associated with the real electric field $\varepsilon(t)$. The Fourier transform of the electric field reads $\varepsilon(\omega) = 1/2(\tilde{E}(\omega) + \tilde{E}^*(-\omega))$ where $\tilde{E}(\omega)$ is a complex quantity taking non-zero values only for positive values of the frequency $\omega$. The electric field in the time domain then reads $\varepsilon(t) = \text{Re}\ E(t)$, where the complex quantity $E(t)$ is the inverse Fourier transform of $\tilde{E}(\omega)$. The characterization of the pulse is understood as the measurement of its analytic signal $E(t)$ or $\tilde{E}(\omega)$. In the frequency domain, the amplitude of the field, $|\tilde{E}(\omega)|$, can be easily determined by taking the square root of the power spectrum $|\tilde{E}(\omega)|^2$ obtained experimentally using a spectrometer. The main challenge in the characterization of an ultrashort pulse therefore lies in determining the spectral phase $\varphi(\omega) = \text{arg}\ \tilde{E}(\omega)$, hence the title of this article, although some techniques operate directly in the time domain.

We first consider the information that can be gained from an experimental setup such as the one sketched in figure 1 where only linear stationary optical components (mirrors, all elements based on refraction, beam splitters, density filters, delay lines, diffraction gratings, etc.) are allowed. Furthermore we assume that the detector response time is much longer than the pulse duration, which is usually the case for femtosecond pulses, so that it only measures the energy of the incident radiation. The entire set of linear stationary elements used in the experiment can be characterized by its linear transfer function $R(t,t') = R(t-t')$, the transmitted field $E'(t)$ being related to the input field $E(t)$ by:

$$E'(t) = \int_{-\infty}^{+\infty} R(t,t')E(t')\,dt' = \int_{-\infty}^{+\infty} R(t-t')E(t')\,dt' = \int_{-\infty}^{+\infty} \tilde{R}(\omega)\tilde{E}(\omega)\exp(-i\omega t)\,\frac{d\omega}{2\pi}$$

The detected signal then reads:

$$S = \int_{-\infty}^{+\infty} |E'(t)|^2\,dt = \int_{-\infty}^{+\infty} |\tilde{E}'(\omega)|^2\,\frac{d\omega}{2\pi} = \int_{-\infty}^{+\infty} |\tilde{R}(\omega)|^2|\tilde{E}(\omega)|^2\,\frac{d\omega}{2\pi}$$

This signal is thus clearly independent of the spectral phase of the ultrashort pulse, which makes the device inappropriate to a measurement of this quantity. At best, the power spectrum $|\tilde{E}(\omega)|^2$ of the pulse can be obtained. This is indeed achieved for example using a monochromator whose ideal transfer function is $\tilde{R}(\omega) \approx \delta(\omega - \omega_0)$ where the frequency $\omega_0$ is tunable. Another possible apparatus for measuring the spectrum is the Michelson interferometer, which delivers a sequence of two pulses separated by a variable time delay $\tau$. In this case the transfer function is $R(t) = 1/2(\delta(t) + \delta(t-\tau))$. As will be shown below, the energy of the output field recorded as a function of the delay $\tau$ can be Fourier transformed with respect to this variable to yield the power spectrum of the input radiation, technique known as Fourier-transform spectroscopy.

**Figure 1.** General sketch for an experimental setup making use of linear stationary optical components and time-integrating detection.
Therefore, no such device can provide a spectral phase measurement of an isolated pulse. However, the problem is entirely different when a well-characterized reference pulse is available. The present article is divided in two parts: in the first one, we shall review measurement techniques making use of a reference pulse, in which case techniques based on stationary filters can be used, and in the second part we shall discuss the techniques required for the characterization of an isolated pulse.

2. Spectral phase measurement making use of a reference pulse

We consider here the case where a well characterized reference pulse, \( E_0(t) \), is available, and detail the techniques that make use of this pulse to measure the electric field \( E(t) \) of the unknown pulse.

2.1. Linear measurement techniques

We assume here that the beams corresponding to the reference and unknown pulses are recombined collinearly using a beam splitter and sent to the detection apparatus. As mentioned above, such a device is only sensitive to the power spectrum of the electric field. However, this field is now the linear superposition of the reference and the unknown pulse. Therefore, the power spectrum \( |\tilde{E}(\omega) + \tilde{E}_0(\omega)|^2 \) includes interference terms such as \( f(\omega) = \tilde{E}(\omega)\tilde{E}_0^*(\omega) \), a feature that makes possible the recovery of the quantity \( \Delta \varphi(\omega) = \varphi(\omega) - \varphi_0(\omega) = \text{arg} f(\omega) \), the spectral phase difference between the two pulses. Since the spectral phase of the reference pulse, \( \varphi_0(\omega) \), is assumed to be known, this allows the determination of the spectral phase of the unknown pulse, \( \varphi(\omega) = \varphi_0(\omega) + \Delta \varphi(\omega) \), for any frequency where \( \tilde{E}_0(\omega) \) takes non-zero values. This interferometric approach is therefore valid as soon as the spectrum of the reference pulse encompasses that of the unknown pulse, if one knows how to extract the complex quantity \( f(\omega) \) from the interference pattern.

Note that in some experiments the unknown pulse is obtained from the reference pulse using a linear device of complex transfer function \( \tilde{R}(\omega) \), so that \( f(\omega) = \tilde{E}(\omega)\tilde{E}_0^*(\omega) = \tilde{R}(\omega)|\tilde{E}_0(\omega)|^2 \). The measurement yields the transfer function \( \tilde{R}(\omega) \) of the device in both amplitude and phase, from which the dispersion of optical elements can be determined [1]. Interestingly enough, the result depends only on the power spectrum of the reference pulse, whose phase therefore does not need to be measured in this case. Furthermore, even an incoherent light source can be used instead of a femtosecond laser in this kind of measurements. Additionally, it must be kept in mind that techniques previously developed using white-light sources such as time-domain or frequency-domain interferometry will be relevant to the measurement of the spectral phase of ultrashort pulses when a reference pulse is available.

2.1.1. Time-domain interferometry

As shown in figure 2a, time-domain interferometry simply consists in recording the energy of the superposition of the two fields as a function of the time delay \( \tau \). The measured signal reads:

\[
S(\tau) = E(t)\tilde{E}(\tau) + \tilde{E}(t)E^*(\tau) = |\tilde{E}(\tau)|^2 + |E(t)|^2 - 2 |\tilde{E}(\tau)||E(t)|\cos(\Delta \varphi(\omega) - \varphi_0(\omega) + \text{phase}(\tau))
\]

\( \text{phase}(\tau) \) is the phase of the interference pattern. It is given by

\[
\text{phase}(\tau) = \frac{1}{2} \varphi_0(\omega) + \frac{\Delta \varphi(\omega)}{2} + \frac{\varphi(\omega)}{2} + \frac{\tilde{R}(\omega)}{2}
\]

\( \text{arg} f(\omega) = \Delta \varphi(\omega) \).
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\[ S(\tau) = \int_{-\infty}^{+\infty} |E_0(t - \tau) + E(t)|^2 \, dt \]

\[ = \int_{-\infty}^{+\infty} |E_0(t - \tau)|^2 \, dt + \int_{-\infty}^{+\infty} |E(t)|^2 \, dt + \int_{-\infty}^{+\infty} E_0^*(t - \tau)E(t) \, dt + c.c. \]

The first two terms yield a constant background signal while the last two terms give interference fringes whose frequency is the center frequency of the measured pulse. These terms correspond directly to the correlation product between the two pulses. If the input light source is spectrally filtered around \(\omega_0\), the temporal displacement of the interferometric component directly yields the group delay difference between the two arms of the interferometer at this frequency [2]. More generally, the direct retrieval of the phase difference can be performed using a Fourier transform of the measured signal:

\[ \text{F.T.}[S](\omega) = A\delta(\omega) + \tilde{E}_0^*(\omega)\tilde{E}(\omega) + \tilde{E}_0(\omega)\tilde{E}^*(\omega) \]

where \(A\) is a constant value proportional to the total detected energy. If the two channels of the interferometer are identical \((E = E_0)\), this yields the power spectrum of the incident light. This is Fourier-transform spectroscopy, a technique widely used in the infrared domain [3]. In the case of interest here, where the two channels are different, one usually refers to this implementation as dispersive Fourier-transform spectroscopy. In the above equation, one can recognize the aforementioned function \(f(\omega)\) in the second term. Note that extracting \(f(\omega)\) from F.T.\([S]\) is straightforward since it is the only non-vanishing term for positive values of the frequency. Finally, the knowledge of \(f(\omega)\) immediately yields the spectral phase of the unknown pulse.

Time-domain interferometry is therefore a simple technique for retrieving the spectral phase. Its main drawback is that it requires the scanning of the time delay with an interferometric accuracy (i.e. a small fraction of the wavelength of the source), which makes it less practical in the visible than in the infrared domain.

Apart from the vast domain of Fourier-transform infrared spectroscopy [3] where a black-body light source is most often used, time-domain interferometry has also been used in the context of femtosecond lasers for measuring the group delay of optical components [2] or for measuring the wavelength dispersion of femtosecond laser cavities [4]. Programmable pulse-shaping devices have also been characterized [5]. It is noteworthy that due to the practical difficulties of interferometry in the visible domain, the interferogram is not always recorded with the required accuracy, allowing only the acquisition of its envelope providing an incomplete characterization of the field [2,5]. In the case of Naganuma and Sakai, a dual interferometer was designed in order to compensate for length fluctuation, thus providing a complete measurement of the cavity dispersion [4].

These requirements on interferometric accuracy are not so severe in the infrared domain, where time-domain interferometry has been used for example in recording the coherent infrared emission resulting from wavepacket oscillations in an asymmetric quantum well structure [6]. More recently, the complete measurement of the infrared field using time-domain interferometry made possible the determination of the two-dimensional vibrational spectra of peptides [7].

2.1.2. Frequency-domain interferometry

Pioneered by Froehly and coworkers [8], frequency-domain interferometry, or spectral interferometry, uses the very same setup as time-domain interferometry except that the detection apparatus now consists of a spectrometer while the time delay between the two pulses is kept constant (see figure 2b). The measured spectrum then reads:

\[ S(\omega) = |\tilde{E}_0(\omega) + \tilde{E}(\omega)e^{i\omega\tau}|^2 = |\tilde{E}_0(\omega)|^2 + |\tilde{E}(\omega)|^2 + \tilde{E}_0^*(\omega)\tilde{E}(\omega)e^{i\omega\tau} + c.c. \]

\[ = |\tilde{E}_0(\omega)|^2 + |\tilde{E}(\omega)|^2 + f(\omega)e^{i\omega\tau} + c.c. \]
The last two terms in this expression can be rewritten as $2|f(\omega)| \cos(\Delta \varphi(\omega) + \omega \tau)$ and result in interference fringes in the power spectrum. The average fringe spacing is inversely proportional to the time delay while the phase of the fringe pattern directly yields the constant term $\Delta \varphi(\omega_0)$ in $f(\omega)$ at the center frequency $\omega_0$ [9]. Furthermore, frequency-resolved information on $\Delta \varphi(\omega)$ can be obtained since the spectral fringes are more narrowly spaced when the phase varies more rapidly with frequency. This allows the determination of the spectral phase for example by pointing the maxima of the fringe pattern [10, 11]. However, more elaborate techniques have been developed in order to extract the determination of the spectral phase, for example by pointing the maxima of the fringe pattern [10, 11]. A direct measurement of the group delay can also be performed by precisely scanning the delay between the two arms of the interferometer [14]. We discuss here the particular case of Fourier-Transform Spectral Interferometry (FTSI), which allows a direct determination of $\Delta \varphi(\omega)$ with no change in the experimental setup [13].

FTSI starts by taking the inverse Fourier transform of the measured spectrum:

$$F.T.^{-1}[S](t) = E_0^*(-t) \otimes E_0(t) + E^*(-t) \otimes E(t) + f(t - \tau) + f(-t - \tau)^*$$

The first two terms correspond to the field autocorrelation functions of each pulse and are thus centered around $t = 0$. The third term is the correlation function shifted by $\tau$ and is centered around $t = \tau$, while the last term is centered around $t = -\tau$. If we choose a large enough value of $\tau$ these terms do not overlap and $f(t - \tau)$ can be extracted by numerically filtering out the other terms. At this point, we accessed the same information as in time-domain interferometry, and a Fourier transform back into the frequency domain yields $f(\omega)e^{i\omega\tau}$.

FTSI is therefore a straightforward technique for retrieving the spectral phase difference $\Delta \varphi(\omega)$. As is time-domain interferometry, FTSI is a linear method and is therefore extremely sensitive, which makes it appropriate for measuring very weak pulses [12]. As with any homodyne technique, the sensitivity can be further improved by increasing the power of the reference pulse. Compared with time-domain interferometry, FTSI has the important advantage that a single measurement, e.g. using an array detector at the Fourier plane of a spectrometer, allows the measurement of the whole interferogram. However, the use of a spectrometer brings some experimental difficulties, among which the most striking effect is the requirement for a very accurate frequency calibration [15]. The spectrometer finite resolution and the fact that the experimental data points are sampled unevenly with respect to frequency must also be properly accounted for [16,17].

Spectral interferometry has been widely used in the recent years. With respect to the measurement of the linear response of materials, one can mention the refractive indices of transparent dispersive materials [18] and of semiconductor nanostructures [19], the spectral phase modulation induced by a pulse shaper [20], and the group delay dispersion resulting from reflection on mirrors [21]. Because it is an interferometric technique, FTSI has also been proven a powerful tool for discriminating between coherent and incoherent radiation in secondary emission from semiconductor quantum wells [22,23]. With respect to nonlinear femtosecond spectroscopy, spectral interferometry has also been used in phase-resolved pump-probe experiments [9,11], in two-dimensional nonlinear optics [24,25], and in photon-echo or four-wave mixing experiments [26–28]. Finally, spectral interferometry is a key ingredient of SPIDER [29], as will be discussed in the second part of this article.

### 2.2. Nonlinear measurement techniques making use of a reference pulse

When the reference pulse spectrum does not overlap that of the unknown pulse, linear optical techniques become useless since $f(\omega)$ vanishes. Techniques implementing non-stationary filters through non-linear optics must be used. In most cases, the techniques that will be discussed in the second part of this article can be adapted to the case where a reference pulse is available. For example, the XFROG technique implements a frequency resolved cross-correlation of the unknown pulse with the reference pulse, while a frequency resolved autocorrelation is used in conventional FROG [30].
The case of THz pulses is of particular interest due to the low value of the oscillation frequency, which makes possible a direct determination of the electric field in time domain using an ultrashort visible pulse as a reference. Such a measurement can be achieved using either a photoconductive switch [31] or the electro-optic effect [32]. Note that the latter technique has been recently extended to higher frequencies, up to 70 THz, well into the mid-infrared domain [33].

3. Self-referencing techniques

The development of self-referencing techniques has closely followed the birth of ultrashort optical pulses [34,35]. The characterization of an isolated ultrashort light pulse, i.e. without the use of a well-characterized reference, has been formalized by Wong and Walmsley [36]. A successful technique for such a task must at least make use of a time-stationary and a time-non-stationary filter. This contrasts with the previous techniques, which are not sensitive to the time evolution of the electric field since they only rely on time-stationary filters.

Making use of a time–frequency representation of the pulse such as the Wigner function [37], i.e. a mathematical representation of the distribution of the energy of the pulse as a function of time and frequency, three main classes of techniques have been identified. Spectrographic techniques are based on the measurement of filtered time–frequency representations of the pulse, in an attempt to directly locate the energy of the pulse in the time–frequency space. Because the filters are usually unknown, the retrieval of the pulse from the experimental trace is obtained through iterative blind deconvolution algorithms. Tomographic techniques are based on the measurement of projections of the time–frequency representation of the pulse. They can in principle lead to direct reconstruction through Radon transforms, although this has never been experimentally demonstrated. Interferometric techniques make use of interferences between different frequencies of the pulse. Because they measure a single slice of the two-frequency correlation function, which totally defines the coherent field of an ultrashort light pulse, they rely on a smaller experimental trace and lead to direct algebraic inversion algorithms.

Nearly arbitrary time-stationary filters (i.e. a filter such that the output field is related to the input field by \( \tilde{E}_{\text{output}}(\omega) = \tilde{R}(\omega)\tilde{E}_{\text{input}}(\omega) \)) can be synthesized, but the most widely used are created by a zero-dispersion line or dispersion. Useful time-non-stationary filters (i.e. filters for which the output field is related to the input field by \( E_{\text{output}}(t) = R(t)E_{\text{input}}(t) \)) must have a temporal response of the order of the duration of the pulse. They have been synthesized by fast photodiodes, fast electronic modulators or streak cameras for picosecond response times, but for pulses in the femtosecond range, non-linear interaction of the unknown pulse with replicas or filtered replicas of itself in instantaneous nonlinear materials is most often used. It must be kept in mind that the requirement for a successful characterization is that the filter is non-stationary, and not that it is non-linear. In practice, using non-linear interactions to implement non-stationary filters is quite penalizing in terms of sensitivity compared with the previously exposed linear techniques.

In the following, we detail three techniques for the complete characterization of an ultrashort light pulse belonging to each of these classes: Frequency Resolved Optical Gating, Spectral Phase Interferometry for Direct Electric-field Reconstruction and chronocyclic tomography. We start by an overview of autocorrelation techniques. Although they do not provide a complete characterization of the electric field, they represent the first historical attempt to characterize short optical pulses, and are still widely used.

3.1. Autocorrelation techniques

Autocorrelations have been proposed very early as a tool to capture the time evolution of short pulses of light [38], and were soon after the first techniques to directly reveal picosecond structures in laser pulses. Following the geometry exposed in figure 3, the interaction of the short pulse \( E(t) \) with \( E(t - \tau) \) in an instantaneous nonlinear element with infinite phase matching leads to the intensimetric autocorrelation

\[
A(\tau) = \int_{-\infty}^{+\infty} I(t)I(t - \tau) \, dt
\]

measured on an integrating detector (for example a slow photodiode). This
evaluation of the time concentration of the energy of the pulse does not allow the retrieval of the electric field but leads to an estimate of the duration through a decorrelation factor if a temporal intensity profile is assumed for the pulse. Also, the interferometric autocorrelation $B(\tau) = \int_{-\infty}^{+\infty} |E(t) + E(t - \tau)|^4 dt$ can be measured, with the advantage that fringes give a calibration of the time axis (which is particularly useful for very short pulses) but with the drawback of a constant background precluding any high-dynamic range measurements. Second-order autocorrelations are symmetric in time and do not allow the evaluation of the asymmetry of the pulse. A third-order autocorrelation can solve this issue, and can lead to an increase of the dynamic range of the measurement. Although the intensity autocorrelation and the power spectrum do not allow the unambiguous determination of the pulse shape [39], an early development by Naganuma demonstrated that the additional knowledge of the second-harmonic power spectrum allows the unique determination of the electric field. Interestingly, the latter quantity is available in the Fourier transform of the interferometric second-order autocorrelation, so that an iterative technique retrieving the spectral phase from the interferometric autocorrelation and the power spectrum can be used [40]. The retrieval of the field from the measurement of several autocorrelations after known spectral dispersion has also been tested [41], as well as the measurement of cross-correlations [42]. All these attempts aim at providing simple and sensitive characterization techniques, but no complete theory of ambiguities is available up to now. The measurement of the bispectrum by mixing three replicas of the pulse has also been used, following developments made in astronomical imaging [43]. A practical simplification of second order autocorrelations now consists of the replacement of the non-linear crystal and one-photon detector by an adequate two-photon detector.

3.2. Spectrographic techniques

Spectrographic techniques aim at locating the energy of the pulse in the time–frequency space. We take here the example of Frequency Resolved Optical Gating (FROG), which is based on the measurement of the spectrum (the stationary filter) of the pulse after gating through a correlation process (the non-stationary filter) [44,45]. Mathematically, the two-dimensional experimental trace can be written as

$$F(\omega, \tau) = \left| \int_{-\infty}^{+\infty} E(t) g(t - \tau) \exp(i\omega t) dt \right|^2$$

The gate $g$ is often a replica or a product of replicas of the unknown field or its complex conjugate. For example, as sketched in figure 4, one simply frequency resolves the upconverted pulse in an intensimetric autocorrelator in Second Harmonic Generation (SHG) FROG, so that $g = E$. The FROG trace corresponds to a spectrogram of the pulse, i.e. the measurement of the frequency content of the temporal slices of the pulse.
signal. This is only an image because of the time–frequency duality. Indeed the frequency content of the
filter modifies the frequency content of the gated pulse. In practice, filters with duration of the order of the
duration of the pulse must be used.

Single-shot measurements [46] can be implemented using the time-to-space mapping in a non-collinear
interaction geometry identical to the one used in single-shot autocorrelators and a two-dimensional detector,
but can suffer from beam quality issues. Recently, a version of FROG using the angular dependence of the
narrow phase matching function associated with a thick non-linear crystal as the spectral filtering element
was demonstrated [47].

Retrieval of the field consists in finding the phase associated with the FROG trace. The solution must obey
two constraints: the experimental FROG trace and the mathematical expression of the gate as a function
of the unknown pulse. Because the gate is unknown, it is compulsory to use blind deconvolution iterative
algorithms, among which the most robust so far is based on principal component generalized projections,
making use of the fact that the FROG trace is the squared modulus of the Fourier transform of the outer
product of the gate and probe pulses. This is also the fastest deconvolution algorithm, and thanks to the
progress of personal computing, is now capable of update rates of the order of 1 Hz [48].

Some general drawbacks of iterative retrievals include problems of convergence due to local minima and
uniqueness of the solution. SHG FROG, the most widely used and most sensitive implementation, suffers
from the time-ambiguity of SHG autocorrelators, i.e. it is not possible to distinguish \( E(t) \) from \( E(-t) \). An
advantage of the FROG technique is that it makes use of the pump-probe setup used in many experiments
in physics and chemistry, making possible the characterization of the pulse at the exact location where it
will be used (for example, at the focus of a microscope objective [49]).

A dual approach, demonstrated earlier, consists of the measurement of a sonogram of the pulse, i.e.
a representation of the time of arrival of the different spectral components of the pulse, by inverting the order
of the time-stationary and time-non-stationary filters [37,50,51]. It is less popular, due to some practical
reasons (e.g., difficulty to run in single-shot) but could have some advantages (no time-ambiguity even when
a second-harmonic-generation process is used, use of a two-photon photodiode for the temporal gating
process). Retrieval of the field can be performed using iterative algorithms [52,53], although simplified
procedures have also been used (and were initially used).

3.3. Tomographic techniques

Tomography consists of the retrieval of a two-dimensional real-valued function using a set of its one-
dimensional projections. For example, an object is rotated and its spatial transmission is mapped as a
function of the rotation angle. A Radon transform can be used to retrieve the local transmission at each
point of the object from the set of measured transmissions. These techniques are widely used in medical
imaging and in quantum-state measurement. A spectrometer naturally projects the Wigner function of a
pulse onto the frequency axis, and can then be used as the projection element if a technique to arbitrarily
rotate the Wigner representation of the pulse in the time–frequency space is available. Such a rotation can
be performed by an adequate choice of a parabolic spectral phase modulation (the stationary filter) followed
by a parabolic temporal phase modulation (the non-stationary filter), as plotted in figure 5 [54]. Although

![Figure 5. Principle of topographic techniques. Arbitrary rotations of the time–frequency representation of the pulse can be obtained through spectral and temporal quadratic phase modulations. On this figure, a 90° rotation is implemented so that a projection of the rotated function onto the frequency axis leads to the temporal intensity of the input pulse (time-to-frequency converter).](image-url)
the latter could in principle be obtained through electronic modulation, in practice it would once again be required to use nonlinear optics.

A simplified version of this technique, the time-to-frequency converter, has been known for a long time. Indeed, if one rotates the Wigner function of the unknown pulse by $90^\circ$, the measured spectrum is a scaled representation of the temporal intensity of the input pulse, thus allowing the real-time monitoring of the temporal intensity of the pulse using a spectrometer. Such an operation is identical to the spatial Fourier transform operated by a lens. Correspondences between the time–frequency phase space of ultrashort light pulses with the near-field–far-field phase space of one-dimensional monochromatic spatial beams have been known for a long time [55,56], and have for example lead to experimental realizations of a time microscope [57,58]. The difficulty to manipulate short pulses directly in the time domain has limited their implementations.

The use of cross-phase modulation induced by the unknown pulse itself has also been used, but requires an iterative deconvolution of the experimental trace [59].

### 3.4. Self-referencing interferometric techniques

Interferometry is a very successful approach for the spatial characterization of optical fields [60]. Some concepts are now transferred to the time–frequency space. The problem in the characterization of ultrashort optical pulses is once again the difficulty of time-resolving structures and manipulating the field in the time domain.

We detail here Spectral-Phase Interferometry for Direct Electric-field Reconstruction (SPIDER, figure 6), which is a transposition of shearing interferometry to the frequency domain [29]. In spatial shearing interferometry, two replicas of the beam that must be characterized are relatively sheared by a quantity $X$. Extraction of the phase difference at point $x$ leads to $\varphi(x + X) - \varphi(x)$, function close to the derivative of the spatial phase of the distribution for a sufficiently small shear, which can be integrated to get the spatial phase of the input field. The bandwidth of electronic modulators being insufficient to directly generate a spectral shear (such an operation is a linear temporal phase modulation), one uses a nonlinear conversion with a highly chirped pulse. Indeed, in such a pulse, there is a linear relation between time and instantaneous frequency $t = \varphi^{(2)}(\omega)$. For a sufficient chirp, the instantaneous frequency is constant during the interaction with the short pulse, so that the upconverted pulse is a replica of the short pulse shifted along the frequency axis by this instantaneous frequency. Getting a relative shear between two replicas of the unknown pulse is then simply obtained by upconverting two replicas of this pulse delayed by $\tau$ with the same highly chirped pulse. This leads to a shear $\Omega = \tau/\varphi^{(2)}$. The phase difference $\varphi(\omega + \Omega) - \varphi(\omega)$ between these two replicas can be directly measured with FTSI in a single shot, and the spectral phase of the initial pulse is reconstructed algebraically by integration. As the experimental trace is monodimensional and the inversion is algebraic, single-shot real-time operation is straightforward [61]. This technique can be adapted to arbitrary pulses [62,63], and has recently been combined with conventional spatial shearing interferometry to completely characterize the field as a function of spectral and spatial coordinates [64].

An implementation of reversal shearing interferometry has been done along the same lines, but it suffers from the usual insensitivity to even-order phase terms as the spatial reversal shearing interferometers [65].

A transposition of the Young’s slits experiment, where the beating between various frequencies of the pulse spectrum is measured, was also demonstrated earlier [66,67]. Train of pulses can also be characterized using a modified version of spectral shearing interferometry [68].

**Figure 6.** Principle of spectral-phase interferometry for direct electric-field reconstruction. The interferogram of the two upconverted pulses is measured for a fixed delay $\tau$ between the two pulses from an interferometer and directly yields the spectral phase of the pulse.
4. Conclusions and perspectives

The characterization of ultrashort light pulses is a very active domain. Time-resolution on the femtosecond scale can be achieved through very different means. Linear, very sensitive, techniques can be used if an appropriate reference field is available. More general non-linear techniques are now well understood, and lead to the characterization of arbitrary isolated pulses. Although the characterization is now reasonably established for pulses with central frequencies close to the visible range, exciting developments concern the extension of these concepts to the pulses obtained by non-linear frequency conversion, from the X-ray to the far IR region. Another interesting direction consists in the characterization of extremely short pulses, with durations of the order of one femtosecond or smaller. Other developments will most likely include the characterization of pulses with complicated pulse shapes, as well as the space-time characterization of ultrashort optical pulses. Also, it must be pointed out that all the developments presented here deal with the characterization of an isolated pulse (or a train of identical pulses), i.e. a source with a degree of coherence equal to one. There could be some interests in characterizing partially coherent sources, as can be done in the spatial domain.

References

TRENDS IN FEMTOSECOND LASERS AND SPECTROSCOPY

Characterization of the spectral phase


