ECOLE POLYTECHNIQUE

CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

PRACTICES

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June 2008

Cahier n° 2008-33

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PRACTICES[1]

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Abstract: We examine an economy where professionals provide services to clients and where a professional can sell his practice to another. Professionals vary in quality, and clients in their need (or willingness-to-pay) for high-quality service. efficiency is measured as the number of matches between high-quality professionals and high-need clients. However, agent types are unobservable a priori. We find that trade in practices can facilitate the transmission of information about agent types; sometimes full efficiency is achieved. In cases where it is not, a tax on the sale of practices (based on the seller's age) can be used to achieve full efficiency. In addition, a ceiling on the price of services can be used to adjust the distribution of surplus between clients and professionals, while preserving efficiency.

Key Words : signaling, professional services, practices, goodwill

Classification JEL: C73, D82.

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[1] This is a preliminary draft. We thank Arianna Degan, Claude Fluet, Chris Green and Roberto Serrano for useful comments. Jean-Marc Bourgeon gratefully acknowledges financial support from the "Chaire AGF Risques Santé." Max Blouin gratefully acknowledges financial support from SSHRC (Canada) and FQRSC (Quebec).

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> At the time of which I speak, Holmes had been back for some months, and I at his request had sold my practice and returned to share the old quarters in Baker Street. A young doctor, named Verner, had purchased my small Kensington practice (. . .).
>
> — Dr. Watson in
> *The Return of Sherlock Holmes*

1 Introduction

We examine a market for professional (legal, medical, dental, etc.) services. We are interested not only in the interactions between professionals and their clients, but also in the phenomenon of "selling one's practice," which happens when one professional refers his clients to another professional, who pays a sum of money in exchange.

Such a transaction presents an interesting information problem. When a practice changes hands, the clientele is aware of the change and is free to take its business elsewhere. Clearly a practice only has value if the new owner is confident that the clientele will remain. But why should it? This is a key issue if a professional's quality is not observable at the time one pays for his services.

When we speak of practices in this paper, we abstract from such things as physical capital, supplies, payroll, advantages attributable to location, etc. We wish to focus exclusively on the truly intangible component, which has to do with clients' expectations regarding the quality of services they will receive; this is commonly called *goodwill*.

We will explore whether buying a practice may act as a signal of a professional's quality. We model professional services as experience goods. The first time a client and professional meet, the client does not know the professional's quality. If she (the client) hires him (the professional), then she finds out his quality and on that basis decides whether or not to hire him a second time. Needless to say, high-quality professionals have an easier time retaining clients than low-quality ones. For this reason, we might expect high-quality professionals to have a greater incentive to buy practices. When they do, they feel confident that they can keep the clientele for two periods. Low-quality professionals, on the other hand, can only expect to keep them for one. Is this enough to ensure separation of the two types? That is to say, is it the case in equilibrium that only high-quality professionals buy and sell practices?

A closely related issue is the efficiency of client-professional pairings. Clients are heterogeneous in our model: some (type-A clients) value high-quality services more than others (type-B clients). This willingness-to-pay is unobservable. Naturally professionals seek out type-A clients at the same time that clients seek out high-quality professionals. An efficient outcome would be one where the number of matches between high-quality professionals and type-A clients is maximized.

We find that efficient equilibria can exist, but only if there is a relative abundance of type-A clients: in these equilibria, all high-quality professionals buy practices, but some low-quality ones buy them too. When this abundance criterion is not met, equilibrium is inefficient. The problem is that the incentive for a low-quality professional to pass himself off as a high-quality one, even for a single period, is too strong. As a result, low-quality professionals buy practices. The fact that they do so dilutes the signal that the purchase of a practice is supposed to represent, inciting some high-quality professionals *not* to buy them.

This inefficiency can be remedied. The proper intervention is one which would block early resales of practices. To us the most obvious one is a tax on the sale of practices based on the age of the seller, an observable characteristic. Professionals who sell their practice before retirement pay the tax, whereas those who sell when they retire are exempt. With such a policy in place, one gets a separating equilibrium and efficient outcome.

However, taxation alone may not necessarily be desirable. In an inefficient equilibrium, some rents are captured by professionals, some by clients. Taxation can make the situation fully efficient, which it does by increasing total rents (or surplus); but in the process it can redistribute them and make clients worse off. We show that a social planner can achieve full efficiency *and* make clients better off than before by combining taxation with a second policy instrument, namely a ceiling on the price of professional services. The latter allows him to make a direct transfer of utility from professionals to clients, without any deadweight loss.

The rest of this section deals with the relevant literature. Section 2 presents a model of the market for professional services without any trade in practices; it introduces the reader to the way interactions between clients and professionals (pros for short) are treated in the paper. Section 3 then presents the model augmented by a trade in practices. Section 4 shows how taxation can be used to promote efficiency. Then Section 5 demonstrates how a price ceiling can be used to transfer surplus from professionals to clients. A brief summary and discussion follow in Section 6. All proofs are to be found in an appendix.

1.1 Related Literature

Most of the literature on reputation focuses on long-lived agents and does not consider reputation as an asset that can be traded between firms (see Fudenberg and Tirole, chap. 9, for a presentation of this literature). Kreps (1990) is the first investigation of reputation trading and shows that there exists an equilibrium where reputation is a valuable asset that provides incentives to short-lived firms to exert more effort. However, this reputation equilibrium is one among many and is as likely to happen as less favorable ones.

Mailath and Samuelson (2001) and Tadelis (2002) extend Kreps' setup to investigate the incentive aspect of reputation trading. They consider a heterogeneous set of firms, composed of inept (low-type) and competent (high-type) firms. The latter can choose to exert an effort to increase the probability of providing a high-quality service (a "success;" as opposed to a low-quality service, a "failure"). They show that reputation trading gives incentives for competent firms to exert effort. As in Tadelis (1999, 2003), they also investigate the ability of reputation trading to operate as a screening device between low-type (inept) and high-type firms.

In all these studies there is a clear distinction between a firm's name, or entity, and its owner's identity. Also it is assumed that customers cannot observe changes in firm ownership. The name's reputation is publicly known, i.e. customers and entrepreneurs know if the services provided by each name in the past were successes, failures, or any sequence of those events. This name's record determines the price that the owner of the name can charge customers, and thereby, the price of the name on the name market. They show that names' good reputations cannot serve as sorting devices that separate high-type from low-type entrepreneurs: there is no equilibrium in which only high-type entrepreneurs buy good reputation names. Indeed, if good reputation names were only bought by high-type entrepreneurs, customers' expectations about the quality of the firm's services would not be violently shaken by the occurrence of a failure, making a good reputation name very valuable for low-type firms. As high-type entrepreneurs are more likely than low-type ones to be successful in building their own name, they would value less a good name than their low-type counterparts, and customers' expectations would not be rational. In equilibrium on the name market, both types of entrepreneurs buy good reputation names.

Contrary to this approach, we assume that a professional's record is not public information: only clients who have benefited from his services in the past know the professional's type. Moreover, we assume that a change in practice ownership is observable. We consider clients with heterogeneous valuations for high-quality service, which also impedes the assessment of the practice by other professionals. However, a professional who desires to sell his practice can evidence its value by the practice's "books" (the record of the price paid by his recent clientele) to buyers who approach him.

Developing an approach somewhat close to ours, Hakenes and Peitz (HP, 2007) obtain completely different results. They find that reputation is tradeable, i.e. that a competitive market allows high-value practices to be sold to good professionals over time. The reason for this striking difference comes from two opposite assumptions on the information available to professionals and clients. First, HP assume that when a professional buys a practice, he can identify immediately the seller's type.

In our model, however, the only information a practice-buyer may rely upon is the last price paid by the client in this practice. Second, although clients in HP's model observe that the ownership of their practice has changed, their beliefs as to the type of the new practice-owner are based only on the services provided by the previous professional. In our model these beliefs are based on the age of the new owner. As a result, in HP's model, if a low-type professional buys a high-value practice, he pockets a price premium from the high-type clients the first period, but these good customers run away afterward and the practice becomes worthless. The professional cannot resell it for a good price after one period because potential buyers can identify his type and infer that since he has offered bad services in the previous period, the high-value clients will not return to this practice anymore. The high-type clients' equilibrium strategy, which is "go back to the practice if the service was good the previous period, otherwise switch to another professional," is supported by the belief that the quality of the next service will be the same as the previous one, whoever is the owner of the practice. In our model, clients can infer the type of the new practice owner from his age (either "young" or "old"), and from professionals' equilibrium strategies. Hence, even if they had a bad experience with the former owner, clients will first check who the new professional is (his age) and give him a try if they believe that he is a provider of good services. As a consequence, a low-type professional can buy a high-value practice and resell it at a high price if he runs it only for one period: he can prove from the books to a potential buyer that the practice is a high-value one, and the buyer knows that he will have his chance with the practice's client whatever the seller's type. Compared to HP, then, the conditions for efficiency and for separating equilibrium are more demanding in our setup, and the possibility of a high reputation being tradeable is very slim without government intervention. In a way, because clients obtain some information from the observation of the new practice owner, the resulting market equilibria are worse than if they had the "naive" (uninformed) beliefs that a service has the same quality from one period to the other no matter who runs the practice.

2 Benchmark: The Model Without Practices

To give a sense of how the economy functions in our model, we first present a version in which clients hire professionals, but professionals do not buy or sell practices. Much of the reasoning presented here will be used in later sections. For clarity, when a professional and client meet, we will refer to the professional as "him" and to the client as "her."

2.1 Setup

Time elapses discretely without beginning or end. We are interested in steady-state equilibria. The market is for services, which are provided by *professionals* (or pros) and purchased by *clients*. We shall not call them buyers and sellers, since we reserve these terms for the next section, where professionals buy and sell practices. All agents are risk-neutral and have a discount factor $\delta \in (0, 1)$.

Professionals work for two periods, first as *young* professionals, then as *old* ones; then they retire. At the start of each period, a measure 1 of young professionals enter the economy; those who were young in the previous period (also a measure 1) now become old; and those who were old in the previous period (again, a measure 1) now retire. Hence there is always a measure 2 of working professionals in the economy, half of them young, half of them old. Each generation of professionals is composed of a measure q of *high-quality* (type-H) professionals and a measure $1 - q$ of *low-quality* (type-L) ones. These qualities are exogenous and fixed: a professional is born with a certain type, and stays that way. Each professional has one indivisible unit of service for sale in each period; that is to say, he may see only one client per period.

Clients are infinitely-lived. They are of two types, A and B. Type-A clients value high-quality services more than type-B clients do, in a way which will be specified shortly. There is a measure ψ_A of type-A agents and a measure ψ_B of type-B agents present in the economy at all times. We assume that $\psi_A < 1 + q$. We also assume that $\psi_A + \psi_B > 2$: clients outnumber working professionals. [A professional is *working* if he is selling his services, i.e. if he is young or old; in the following section we will also have *retiring* professionals, who sell practices but not services.]

Types (A, B, L and H) are private information, as are past histories. When a professional and client transact, they learn one another's types. A professional's age is publicly observable.

A central question in this paper is how professionals and clients are matched. There will be no frictions as far as matching is concerned. Since clients outnumber working professionals, this means that in each period, every working professional will be matched with a client.

In some cases, a professional will serve the same client when young and old, while in other cases he will not. This is determined as follows. As soon as a professional becomes old, i.e. at the beginning of his second period, he negotiates with the client he served when young. This is an opportunity for the professional and client to renew or break off their business relationship. Note that at this point the two know each other's types. The professional makes a take-it-or-leave-it offer to the client. The offer is a price for his services in the period which has just begun. If the client accepts, they transact and are considered matched for the rest of that period; we say that their previous match has been renewed. If the client rejects the offer, the match is dissolved: the two part ways and must seek new matches. These negotiations take place at the beginning of a period, in what we call the *negotiation phase*.

The negotiation phase is immediately followed by an *open market phase*. This is a centralized market for professional services, and involves all unmatched agents. This means (i) all agents whose matches were broken during the negotiation phase, and (ii) all those who did not participate in the negotiation phase. The latter group is composed of young professionals (they have just entered the economy) and clients who in the preceding period were served by old professionals (now retired). Market-clearing prices are established as if by an auctioneer. Professionals' ages are observable, so there will be two prices in this phase: one for the services of young professionals, which we denote p_Y, and one for those of old professionals, which we denote p_O. In either case, the price is for one period only of service.

Per-period utilities are as follows. A professional, whatever his type, has no costs: his utility is simply the price he receives for his unit of service. A type-i client who pays p for a unit of service gets utility $\theta_{iH} - p$ if the professional he deals with is high-quality, and $\theta_L - p$ if low-quality, where

$$0 < \theta_L < \theta_{BH} < \theta_{AH} \quad . \tag{1}$$

Anyone who does not transact gets zero utility for that period.[1] All agents maximize lifetime discounted utility. Since clients outnumber working professionals, and since all professional services provide positive utility, in equilibrium all working professionals will be employed.

An equilibrium consists of:

- a market-clearing price for young professionals on the market, p_Y;
- a market-clearing price for old professionals on the market, p_O;
- for each possible client-professional pairing (A-H, A-L, B-H and B-L), an optimal price to be offered by the professional when the partnership is up for renewal (i.e. during the negotiation phase), and an optimal acceptance-rejection rule to be implemented by the client.

These values and rules must be the same from period to period.

2.2 Efficiency

Since type-A clients value high-quality service more than type-B ones do (relative to low-quality service and no service), social efficiency can be measured by the number of A-H matches. A socially optimal outcome is one where either all type-H professionals in a given period (young and old) are matched with type-A clients, or all type-A clients are matched with type-H professionals. That is to say, the

[1]Properly speaking, these parameters measure willingness to pay rather than utility. And so type-A clients may or may not be the wealthier ones. Suppose for instance that both types get the *same* utility from high-quality service (say, perfect health), but that type-A clients get *lower* utility from receiving low-quality service or no service at all. It may be, then, that both types have equal or comparable wealth, but that type-A clients *need* high-quality service more than type-B clients do. Then θ_{AH}, θ_{BH} and θ_L may still measure the appropriate willingnesses to pay, and (1) may still reflects the relation between them.

measure of A-H matches in a given period must be equal to $\min\{\psi_A, 2q\}$, since ψ_A and $2q$ are, respectively, the measures of type-A and type-H agents in the economy.

The assumption that $\psi_A < 1 + q$ will play a large part in our proofs, and it is not hard to see why. If we allowed $\psi_A \geq 1 + q$, then in equilibrium type-A clients would hire all young professionals and renew their relationships with all the type-H ones. The social optimum would be achieved, but only because type-A clients, by their sheer numbers, prevented type-B clients from ever meeting high-quality professionals. This could almost be called brute force. Even allowing trade in practices, something we do in the next section, does not affect this.

Things are more interesting when $\psi_A < 1+q$. In this case something rather more complex is required to obtain a socially efficient outcome. Whether the necessary mechanism occurs naturally in the marketplace or must be imposed by a governing authority is one of the key issues of this paper.

2.3 Equilibrium

Let V_i be the value to a type-i client of being on the open market, i.e. without a match. And let V_{ij} be the value to a type-i client of being in the negotiation phase with a type-j professional.

Let us look at the negotiation phase first. Consider a client-professional relationship which is up for renewal. By this time the client and professional have learned each other's types. The professional makes a take-it-or-leave-it offer, consisting of a price p for his services for one period. First, let us look at this from the client's point of view. Suppose, to illustrate, that the client's type is A and the professional's is H. If the client accepts the offer, her payoff this period is $\theta_{AH} - p$, and she returns to the open market next period (when the professional retires), which makes her total discounted payoff $\theta_{AH} - p + \delta V_A$. If she rejects the offer, she returns to the market this period and gets V_A. The highest price she will consider is that which makes these two values equal. In general if we denote by p_{ij} the highest price that a type-i client is willing to accept from a type-j professional, we have

$$p_{iH} = \theta_{iH} - (1-\delta)V_i \quad ; \tag{2}$$
$$p_{iL} = \theta_L - (1-\delta)V_i \quad . \tag{3}$$

In equilibrium the client will accept any offer $p \leq p_{ij}$ and reject any offer $p > p_{ij}$.

From the professional's perspective, the situation is quite simple. If his offer of p is accepted, he gets that price; if it is rejected, he goes on the market this period and gets p_O, the market price for old professionals' services. And no matter what happens this period, he will retire next period. Clearly the best thing for him to do if $p_{ij} > p_O$ is to offer p_{ij}, and this will be accepted. If $p_{ij} < p_O$ then no agreement is possible: the professional makes an unacceptable offer and both he and the client end up on the market. If $p_{ij} = p_O$ he is indifferent between the two outcomes.

We see that the client will never be offered a price below the maximum she is willing to pay. In equilibrium there are two outcomes for her: either she will accept an offer of p_{ij}, which will leave her no better and no worse off than being on the market; or she will reject an offer of $p > p_{ij}$, and she will end up on the market. Either way the result is the same:

$$V_{ij} = V_i \quad . \tag{4}$$

On the open market, a client can purchase the services of a young professional at price p_Y, those of an old professional at price p_O, or she can purchase nothing. If she hires a young professional, she has a probability q of getting high-quality service. Let q_O denote the probability of receiving high-quality service when hiring an old professional: this is the proportion of high-quality professionals among all old professionals on the open market. Thus a client gets expected utility $q\theta_{iH} + (1-q)\theta_L - p_Y$ for this period if she hires a young professional, and $q_O\theta_{iH} + (1-q_O)\theta_L - p_O$ if she hires an old one; she gets 0 if she hires no one. In all three cases she expects δV_i from next period onward, whether on the market or in negotiations, since $V_{ij} = V_i$ as we have shown. Thus the client's prospects can be written as

$$\begin{aligned}(1-\delta)V_i = \max\{&q\ \theta_{iH} + (1-q)\ \theta_L - p_Y\ , \\ &q_O\ \theta_{iH} + (1-q_O)\ \theta_L - p_O\ ,\ 0\} \quad .\end{aligned} \tag{5}$$

We may make two observations at this point. The first concerns q_O, the proportion of high quality among old professionals on the market. This fraction depends on which matches are renewed during the negotiation phase and which are not. For instance, if all A-H and B-H matches are renewed but some or all of the A-L and B-L matches are dissolved, then all the old professionals on the market will be type-L, so that $q_O = 0$. As we have seen, the renewal or dissolution of matches during negotiations depends on how p_O compares to the different p_{ij}'s. If $p_O < p_{ij}$ the match is renewed; if $p_O > p_{ij}$ it is dissolved. There is one thing we can be certain of: since $p_{AH} > p_{AL}$ and $p_{BH} > p_{BL}$, type-H professionals are at least as likely to renew their matches as type-L ones. Consequently $q_O \leq q$.

Our second observation concerns V_B. At all times there is a measure 2 of professionals in the economy, half of them young, half of them old. We have assumed $\psi_A + \psi_B > 2$, so there are more clients than professionals. For markets to clear, some clients have to demand no services; in other words, we must have either $V_A = 0$ or $V_B = 0$ or both. Inspection of (1) and (5) shows that $V_A = 0$ implies $V_B = 0$. So it is certainly the case that $V_B = 0$ in equilibrium.

We may now determine equilibrium values of p_Y, p_O and q_O.

Lemma 1. *In equilibrium some type-B clients hire young professionals, and* $p_Y = q\theta_{BH} + (1-q)\theta_L$.

Since $V_B = 0$, we must have $p_O \geq q_O\theta_{BH} + (1-q_O)\theta_L$. Using this, the price p_Y found in Lemma 1, and the fact that $q_O \leq q$, we can deduce through (5) that, for a type-A client, the value of hiring a young professional is positive and at least as high as the value of hiring an old one. Hence $(1-\delta)V_A = q\theta_{AH} + (1-q)\theta_L - p_Y = q(\theta_{AH} - \theta_{BH})$. Knowing V_A and V_B, we can use (2) and (3) to calculate threshold prices for the negotiation phase:

$$p_{AH} = q\theta_{BH} + (1-q)\theta_{AH} \quad ; \tag{6}$$
$$p_{BH} = \theta_{BH} \quad ; \tag{7}$$
$$p_{AL} = \theta_L - q(\theta_{AH} - \theta_{BH}) \quad ; \tag{8}$$
$$p_{BL} = \theta_L \quad . \tag{9}$$

Note that p_{AL} may be negative. This would simply mean that in an A-L match no positive price could induce the client to renew the relationship.

LEMMA 2. *In equilibrium $q_O = 0$, i.e. there are no old type-H professionals on the market.*

LEMMA 3. *In equilibrium $p_O = \theta_L$.*

From equations (6) through (9), we can easily ascertain that

$$p_{AL} < p_{BL} = p_O < p_{BH} < p_{AH} \quad . \tag{10}$$

As a result, all A-H and B-H matches are renewed; A-L matches are dissolved; and B-L matches can be either renewed or dissolved.

Looking at V_A once more, we see that a type-A client, when on the market, always hires a young professional: she strictly prefers this option to the others. If the professional turns out to be high-quality, the match is renewed at price p_{AH}: the professional serves the client one more period, then retires, and the client is back on the market. If the professional is low-quality, an unacceptable offer is made, and both go straight to the market.

Type-B clients on the market are indifferent among their three options — hiring a young professional, hiring an old one, and hiring no one — and indeed for markets to clear each option must be chosen by at least some of them. When a type-B client is matched with a young type-H professional, the match is renewed at price p_{BH}. A match between a type-B client and a young type-L professional may or may not be renewed, since $p_O = p_{BL} = \theta_L$. Whether or not it is does not affect anyone's payoff.

For completeness, let us now determine exactly how many type-A clients hire young professionals in a given period. Let α denote the proportion of type-A clients among those clients hiring young professionals. In this benchmark case, the measure of clients of either type who hire young professionals is 1, so the measure of type-A clients who hire young professionals is just α. Since all A-H matches are renewed, there must in each period be a measure αq of type-A clients served by old type-H professionals, with whom they have renewed their relationship from the previous period. Since A-L matches are not renewed and type-A clients do not hire old professionals, this accounts for all type-A clients. Therefore $\alpha + \alpha q = \psi_A$, which is to say

$$\alpha = \frac{\psi_A}{1+q} \quad . \tag{11}$$

Client behavior in turn tells us what may happen to a professional during his career. This is depicted in Figure 1. Time is represented horizontally and increases towards the right. Two generations of professionals are shown: one of them is young at time $t-1$ and old at t; the other is young at t and old at $t+1$. Each horizontal stratum represents a possible history for a professional. First he goes on the market when young, and is matched with either a type-A or type-B client; then his match can be renewed (indicated by an arrow) or not; if not, he again goes on the market and is matched again. As the figure shows, (i) all A-H and B-H matches are renewed, and (ii) old professionals on the market are matched with type-B clients, as type-A clients will not hire them. [B-L matches may or may not be renewed; in the diagram they are.] To the right of each stratum is written the measure of such histories in a generation. The column labeled t gives us an idea of the state of the economy in steady state.

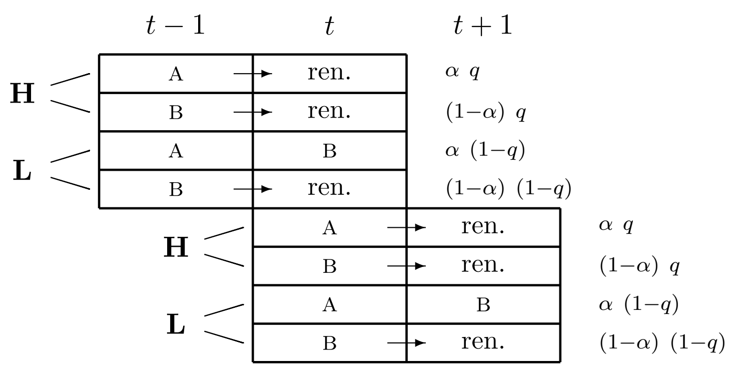

FIGURE 1. Professionals' histories in equilibrium. The letters A and B mean being hired by that type of client during the market phase. Arrows indicate matches renewed during the negotiation phase.

In equilibrium all A-H matches are renewed. In any period there are thus αq type-A clients matched with young type-H professionals, and the same number matched with old type-H professionals. The total, $2\alpha q$, is less than ψ_A, the total measure of type-A clients in the economy. It is also less than $2q$, the total measure of type-H professionals present at any one time. This is because A-L and B-H pairings are inevitably formed on the market, where young professionals have no way of revealing their type. As a result, we have

PROPOSITION 1. *An equilibrium without trade in practices is not socially efficient.*

3 The Model With Practices

We now add a market for practices. Practices in our model are essentially options to negotiate with particular clients. The market for practices is therefore distinct from the market for services. Services are provided by professionals and purchased by clients, whereas practices are traded among professionals only. Professionals who have just reached retirement age play a role in this section, so it is useful to keep in mind that there are *three* ages for professionals: young, old, and retiring.

Each period now has three phases: a *practice phase*, a negotiation phase, and an open market phase, in that order.

At the beginning of the period, professionals who served a client in the previous period (i.e. those who are now old or retiring) may sell this match for a sum k. When they do so, they give up the right to negotiate with that client in the upcoming negotiation phase: they are no longer matched. This is called selling a practice.

The professional who buys the practice acquires the right to negotiate with the client in question. He may be a young professional who has just entered the economy. Or he may be a professional who has just become old. In the latter case, since he can only negotiate with a single client, he must break up the match with his previous client, either by selling his old practice at the same time that he is buying the new one, or simply by abandoning his previous client.

When a professional sells a practice, he makes known to potential buyers the last price paid by the client for his services. This assumption has some basis in reality: a professional buying a practice often has access to "the books," i.e. records of past transactions between the seller of the practice and his clientele. In our model this information allows the buyer to ascertain the client's type.

When a practice is sold, the client is aware of the transaction (she sees that she is matched with someone new) but does not participate in it. Naturally we want to give her a say in the matter, i.e. a choice whether to accept this new match or take her business elsewhere. Formally we delay this choice until the negotiation phase: when the new owner of the practice makes the client an offer, then the client can decide whether to accept the offer or go on the market.

We wish to emphasize that we use the term *practice* only in the narrow sense of a clientele that is sold from one professional to another. In our model matches formed on the open market (during the market phase) are not called practices.

The buying and selling of practices takes place at the beginning of the period, during what we call the practice phase. Once the practice phase is over, retiring professionals formally retire and have no further role in the economy.

The negotiation phase comes next. As before, all professionals who are matched with clients make take-it-or-leave-it offers to those clients. This time the professionals involved are: (i) all those (young or old) who have just bought practices, and (ii) all old professionals who have not bought or sold practices. When a client accepts, the transaction proceeds at the proposed price and the two parties are considered matched for the remainder of the period. When she refuses, both parties go on the open market.

The one difference between negotiations here and negotiations in the previous section is that here information is not always perfect. If the professional has just acquired the practice, he and the client have not had an opportunity to learn each other's types. The professional will try to infer the client's type based on the information supplied him by the previous owner, i.e. the last price paid by the client. The client will form a belief as to the professional's type based on the one thing about him which she can observe: his age. This will be explained later.

The open market phase is, as before, a centralized market for professional services. All unmatched agents participate. These are:

- all agents whose matches were broken up during the negotiation phase;
- young professionals who did not buy practices;
- old professionals who have just sold practices and did not buy new ones;
- clients who in the previous period were served by old professionals (now retired), if these professionals did not sell their practices;
- clients who in the previous period were served by young professionals, if these professionals bought new practices without selling their old ones.

Figure 2 summarizes the timing.

3.1 Negotiation Strategies

As with the benchmark model, we begin our analysis with the negotiation phase. There are now two distinct situations to look at. The professional and client may have been matched in the previous period, in which case they now know each other's types. Or it may be that the professional has just bought a practice, in which case neither agent knows the other's type. In the latter situation, the professional knows the last price paid by the client. We assume for now that this price fully reveals the client's type. Afterward we will show that this must be the case in equilibrium.

In our model only practices consisting of type-A clients are exchanged. The reason why B-practices cannot be traded in equilibrium will be explained later.

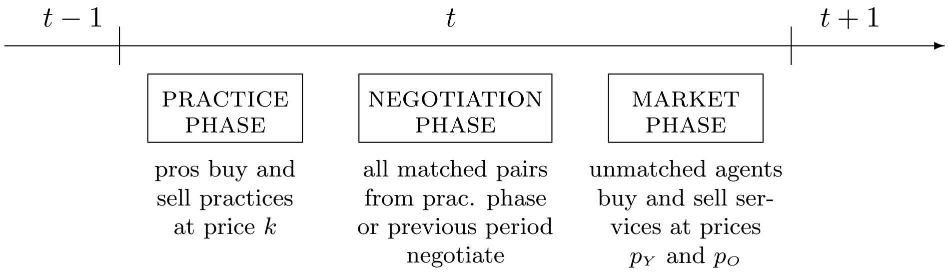

FIGURE 2. Timing.

The client's strategy during negotiations is to have a threshold price: she accepts any price equal to or less than this threshold and rejects any price above it. This threshold is based on her information and on her reservation payoff.

Let V_i be the value to a type-i client of being on the open market, just as in Section 2. The value to her of being in negotiations with *any* professional must be equal to V_i in equilibrium. The reasoning is basically the same as in Section 2. During negotiations, the professional knows the client's type — or infers it correctly from the last price paid. He knows, then, the client's threshold. This is the price which makes the client indifferent between accepting and rejecting. His optimal strategy is to offer this price (if he wants the offer to be accepted) or a higher one (if he wants the offer to be rejected). In either case, the client gets a payoff equal to her reservation payoff. And since a client goes directly to the open market upon rejecting an offer, this reservation payoff is V_i.

When the professional and client know each other from before, the maximum price the client is willing to accept is found in the same way as in Section 2. We again call this price p_{ij}, where i is the client's type and j the professional's. The four possibilities are given by equations (2) and (3).

If the professional is the new owner of a practice, however, then he is someone the client has never seen before. We call this the *new-owner situation.* Here the client, whom we suppose to be type-A, must form a belief as to the professional's type in order to decide how to proceed. That is, she ascribes a probability μ to the professional being type-H. If she accepts an offer of p, her expected payoff for this period is $\mu\theta_{AH} + (1-\mu)\theta_L - p$. This is followed next period by V_A (on the market or via negotiations), making the total $\mu\theta_{AH} + (1-\mu)\theta_L - p + \delta V_A$. If she rejects the offer, she goes on the market this period and gets V_A. The highest price she will consider is that which makes these two values equal. This belief μ can be contingent on the new owner's age, his only observable characteristic.[2] Let us denote this belief

[2]We do not allow the client to condition her belief on the *previous* owner's characteristics (age, type), since this would not be in the spirit of perfect Bayesian equilibrium. If for instance the client

by μ_Y when the new owner is young and by μ_O when he is old. The threshold prices are

$$p_{AY} = \mu_Y \theta_{AH} + (1-\mu_Y)\theta_L - (1-\delta)V_A \quad ; \tag{12}$$
$$p_{AO} = \mu_O \theta_{AH} + (1-\mu_O)\theta_L - (1-\delta)V_A \quad . \tag{13}$$

In equilibrium, as mentioned, the professional knows his client's threshold price. He compares this to what he can get on the market and makes his offer accordingly. For instance, a young professional who has just bought a practice compares p_{AY} to p_Y. If $p_{AY} > p_Y$ he wants to keep this client: he offers $p = p_{AY}$ and the offer is accepted. If $p_{AY} < p_Y$ he prefers to go on the market: he offers $p > p_{AY}$ and the offer is rejected. If $p_{AY} = p_Y$ he will do one or the other.

3.2 Market Values

Since young professionals may buy practices, the proportion of type H among the young ones on the market is not necessarily q anymore. Let q_Y denote this proportion. Define q_O as before: it is the analog of q_Y for old professionals. Clients on the market face the same choices, yielding this time

$$(1-\delta)V_i = \max\{q_Y\ \theta_{iH} + (1-q_Y)\ \theta_L - p_Y\ , \\ q_O\ \theta_{iH} + (1-q_O)\ \theta_L - p_O\ ,\ 0\} \quad . \tag{14}$$

And as before, clients outnumber working professionals, implying that some clients on the market have zero value. Since $V_A = 0$ implies $V_B = 0$, we must certainly have $V_B = 0$. As a consequence,

$$p_Y \geq q_Y \theta_{BH} + (1-q_Y)\theta_L \quad ; \tag{15}$$
$$p_O \geq q_O \theta_{BH} + (1-q_O)\theta_L \quad . \tag{16}$$

3.3 Professional strategies

A type-L professional who never buys practices obtains a career payoff of

$$V_L^{NN} = p_Y + \delta p_O \quad ; \tag{17}$$

labelled NN for "no practice, no practice." He gets p_O when old because he can secure this by going on the market but cannot get more than this from his previous client, since $p_O \geq \theta_L \geq \max\{p_{AL}, p_{BL}\}$. Another strategy is to go on the market when young and buy a practice when old; the payoff is

believed that a new practice owner was type-L whenever the previous owner was type-L, she would be performing non-Bayesian updating; her belief would not be consistent with how professionals actually behaved in equilibrium.

$$V_L^{NP} = p_Y - \delta k + \delta p_{AO} + \delta^2 k \quad . \tag{18}$$

labelled NP for "no practice, practice". Here k is the price of a practice. The professional pays it out when he buys the practice and gets it back when he sells the practice. Since he was old when he bought the practice, the client believes with probability μ_O that he is type-H. Hence he can obtain price p_{AO} for his services in that period.

If a type-L professional were to buy a practice when young, his client would believe with probability μ_Y that he is type-H, hence he could charge her p_{AY}. The following period, however, he could get no more than p_{AL} from this same client, since by then she would know his type. He would be better off selling the practice after a single period and looking for a new client. This gives rise to two more strategies. One of these consists of buying a practice when young, selling it after one period and going on the market ("practice, no practice"), which yields

$$V_L^{PN} = -k + p_{AY} + \delta k + \delta p_O \quad , \tag{19}$$

The other is to buy a practice when young, sell it after one period and immediately (i.e. during the same practice phase) buy another:

$$V_L^{PP} = -k + p_{AY} + \delta p_{AO} + \delta^2 k \quad . \tag{20}$$

Let us turn now to type-H professionals. Never buying a practice results in an expected payoff of

$$V_H^{NN} = p_Y + \alpha\ \delta \max\{p_O, p_{AH} + \delta k\} + (1-\alpha)\ \delta \max\{p_O, p_{BH}\} \quad . \tag{21}$$

Here α is the probability for a young professional on the market to be matched with a type-A client; we need not calculate it at this point. If he does meet a type-A client on the market when young, then the following period he can charge p_{AH}. Having charged p_{AH}, a price high enough to prove that his client is type-A, he can then sell the match when he retires. This cannot actually happen in equilibrium: since all practices which are bought are eventually sold, and since the number of practices being held must remain constant from one period to the next, we cannot allow new practices to be created in this way.

A type-H professional can also buy a practice when young and sell it after only one period, obtaining

$$V_H^{PN} = -k + p_{AY} + \delta k + \delta p_O \quad ; \tag{22}$$

or buy a practice and hold it for two periods, by renewing the match at price p_{AH}, obtaining

$$V_H^{PP} = -k + p_{AY} + \delta p_{AH} + \delta^2 k \quad . \tag{23}$$

Finally, he can wait until he is old to buy a practice. For a type-H pro, however, buying a practice when old only makes sense if he is not already matched with a type-A client from the previous period. If he is already matched with a type-A client, it would be preferable to renew the match, then sell it as a practice. [Again, this cannot happen in equilibrium.] The expected payoff is therefore

$$V_H^{NP} = p_Y + \alpha \ \delta \max\{p_O, p_{AH} + \delta k\} + (1-\alpha) \ \delta \ (-k + p_{AO} + \delta k) \quad . \tag{24}$$

3.4 Equilibrium Without Government Intervention

In this section we are interested in identifying properties of equilibrium, and in particular the conditions under which an efficient equilibrium exists. As in the benchmark model, all young and old professionals will be employed in equilibrium, since they are the short side of the market. Let us begin with some preliminary results.

Lemma 4. *In equilibrium $q_Y \leq q$ and $\mu_Y \geq q$.*

Lemma 5. *In equilibrium $q_O \leq q$.*

As mentioned, professionals selling practices show prospective buyers the last price paid by their client, as proof that these clients are type-A. In equilibrium, then, these prices must exceed the most that type-B clients would be willing to pay under the same circumstances. This is the object of the following result.

Lemma 6. *The prices p_{AY} and p_{AH} paid by clients in practices fully reveal that these clients are type-A.*

The lemma makes no mention of p_{AO}. In the equilibria we will consider, however, this price is never actually paid.

We now address the question of efficiency specifically.

Lemma 7. *In an efficient equilibrium, some type-B clients hire young professionals, some hire old ones, and as a result market prices are $p_Y = q_Y\theta_{BH} + (1-q_Y)\theta_L$ and $p_O = q_O\theta_{BH} + (1-q_O)\theta_L$.*

An equilibrium is deemed efficient when the number of A-H matches in each period is at its highest possible value, namely $\min\{\psi_A, 2q\}$. If $\psi_A \leq 2q$, this means that all type-A clients must be matched with type-H pros all the time: there can be no A-L matches. Such an equilibrium is not possible, however, as the following shows.

Lemma 8. *There can be no efficient equilibrium if $\psi_A \leq 2q$.*

For the opposite case, i.e. $\psi_A > 2q$, efficiency means type-H pros must be matched with type-A clients when young and old: there can be no B-H matches. In this case an efficient equilibrium is possible, but actually a stronger condition than $\psi_A > 2q$ is required. Type-A clients have to be abundant enough to employ all type-H pros *and* some of the type-L ones as well.

Proposition 2. *An efficient equilibrium exists if and only if*

$$\psi_A \quad \geq \quad 2q \quad + \quad \left[\frac{\theta_{BH} - \theta_L}{\theta_{AH} - \theta_{BH}}\right] \; q \quad . \tag{25}$$

Example 1. Let $\theta_L = 1$, $\theta_{BH} = 2$, $\theta_{AH} = 3$, $q = 0.3$, $\psi_A = 1.1$, and $\psi_B = 2$. Let δ be any number in $(0, 1)$. Then an equilibrium exists, as follows. Market prices are $p_Y = p_O = 1$. Market proportions are $q_Y = q_O = \alpha = 0$. Threshold prices are $p_{AH} = 3$, $p_{BH} = 2$, $p_{AL} = p_{BL} = 1$, $p_{AY} = 1.75$, and $p_{AO} = 1$. Market values are $V_A = V_B = 0$. Beliefs are $\mu_Y = 0.375$ and $\mu_O = 0$. The price of a practice is given by $(1 - \delta)k = 0.75$. It is straightforward to verify that V_H^{PP} is the best payoff for type-H pros and that V_L^{NN} and V_L^{PN} are the best ones for type-L pros. All type-A clients are in practices. Type-B clients hire both young and old pros; some hire no one. All type-H pros buy practices when young and hold them for two periods (PP). A measure 0.5 of young type-L pros buy practices when young and sell them after one period (PN); the rest never buy practices. Figure 3 illustrates. Since $p_O = p_{BL}$, B-L matches may or may not be renewed; in the figure they are.

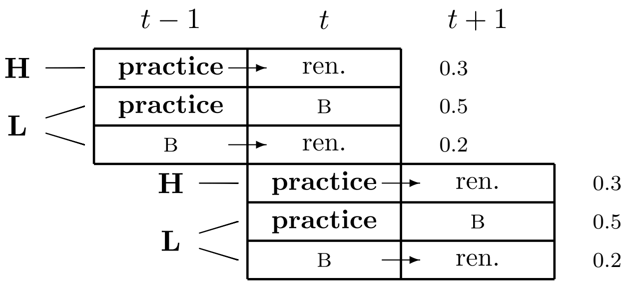

Figure 3. Professionals' histories in Example 1. The word "practice" means the purchase of a practice. The letter B means being hired by a type-B client during the market phase. Arrows indicate matches renewed during the negotiation phase.

What we may distill from this result, and from condition (25) in particular, is that it is hard, in the absence of government intervention, to exclude type-L professionals from the market for practices. Efficiency requires that all type-H pros buy practices. It is impossible, however, to incite all of them to do so without also inciting the type-L ones. That is why there need to be *more* type-A clients than type-H professionals in the economy. How much more depends on S, which summarizes the utility parameters generating the incentives.

The following theorem, combined with the previous, guarantees that an equilibrium always exists.

Proposition 3. *An (inefficient) equilibrium exists if (25) is not met.*

Example 2. Let $\theta_L = 1$, $\theta_{BH} = 2$, $\theta_{AH} = 5$, $q = 0.3$, $\psi_A = 0.4$, and $\psi_B = 2$. Let δ be any number in $(0, 1)$. Then an equilibrium exists, as follows. Market prices are $p_Y = 1.2$ and $p_O = 1$. Market proportions are $q_Y = 0.2$ and $q_O = \alpha = 0$. Threshold prices are $p_{AH} = 2.8$, $p_{BH} = 2$, $p_{AL} = 0.8$, $p_{BL} = 1$, $p_{AY} = 2$, and $p_{AO} = 0.8$. Market values are $(1 - \delta)V_A = 0.2$ and $V_B = 0$. Beliefs are $\mu_Y = 0.6$ and $\mu_O = 0$. The price of a practice is given by $(1 - \delta)k = 0.8$. It is straightforward to verify that V_H^{NN} and V_H^{PP} are the best payoffs for type-H pros and that V_L^{NN} and V_L^{PN} are the best ones for type-L pros. All type-A clients are in practices. Type-B clients hire both young and old pros; some hire no one. A measure $n_H = 0.15$ of type-H pros buy practices when young and hold them for two periods (PP). A measure $n_L = 0.1$ of young type-L pros buy practices when young and sell them after one period (PN); the rest never buy practices. Figure 3 illustrates. Since $p_O = p_{BL}$, B-L matches may or may not be renewed; in the figure they are.

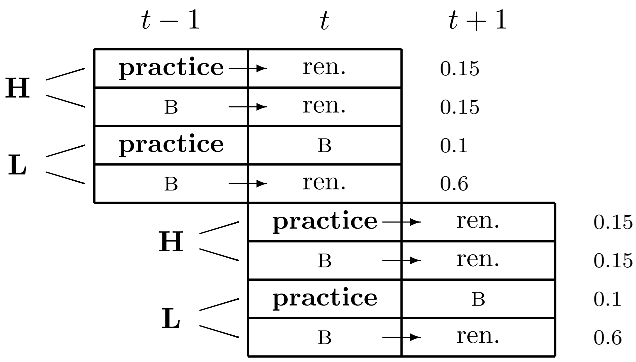

Figure 4. Professionals' histories in Example 2. The word "practice" means the purchase of a practice. The letter B means being hired by a type-B client during the market phase. Arrows indicate matches renewed during the negotiation phase.

In general, equilibrium is not unique. For instance, a pooling equilibrium, in which no one buys practices, can exist under some conditions on parameters. In such an equilibrium, market prices and quantities are the same as those of the benchmark model. Practices are free ($k = 0$) but beliefs are so low ($\mu_Y = \mu_O = 0$) that practices are not worth having.

3.5 Why B-Practices Are Not Traded

We have supposed throughout that practices traded during the practice phase necessarily consist of type-A clients. We now ask whether practices consisting of type-B clients (B-practices, for short) could also be traded.

One might ask: why would someone pay to be matched with a type-B client, given that one can always find one (or a type-A client, which is better) on the open market? A professional might prefer to buy a B-practice in order to send the client a signal that he is a type-H professional. That way he can secure the price p_{BH} rather than the (presumably lower) p_Y. So there *is* a potential incentive for professionals to purchase B-practices.

The problem is that if such practices had any value, there would be an excess supply of them. To see this, suppose an equilibrium exists in which only type-H professionals buy B-practices. When a B-practice changes hands, the client believes with probability one that the new owner is type-H, since only type-H professionals buy B-practices. Therefore he is willing to pay up to p_{BH} for the new professional's services. The new owner, then, knows he can charge the client up to p_{BH}.

The foregoing is true regardless of whether it is a type-H or type-L professional selling the practice. The client's beliefs about the new owner's types cannot, in equilibrium, be predicated on the *previous* owner's type. And the buyer of the practice sees no point in finding out what the last price paid by the client was, as in the case of A-practices. The previous owner's type simply does not matter. Consequently, if B-practices have any value, type-L professionals will supply them too. Since only type-H professionals buy them, this is incompatible with steady-state equilibrium.

The same logic applies even if we consider an equilibrium where some type-L pros also buy B-practices.

3.6 Client Welfare

Do clients benefit from the presence of a market for practices? To answer this we need to compare the values obtained by clients in this scenario (practices but no government intervention) with the corresponding values from the benchmark model.

In all equilibria we study, type-B clients obtain $V_B = 0$. The average per-period utility enjoyed by type-A clients is correctly measured by $(1-\delta)V_A$, but this quantity varies from one equilibirum to the next. For the benchmark model, we denote it π_A^b. It is

$$\pi_A^b \quad = \quad q(\theta_{AH} - \theta_{BH}) \quad . \tag{26}$$

As for the current scenario, the equilibria we have described in Theorems 2 and 3 are not necessarily unique. However, we can place an upper bound on type-A clients' average utility.

PROPOSITION 4. *In any equilibrium with practices but without government intervention,* $(1-\delta)V_A \leq q(\theta_{AH}-\theta_{BH}) = \pi_A^b$.

Neither type-A nor type-B clients, therefore, can do better when there is a market for practices than when there is not.

4 Equilibrium With Government Intervention

Here we introduce a social planner with power of taxation, and present a form of taxation which will guarantee the existence of an efficient separating equilibrium, regardless of parameter values.

When we analyzed equilibrium without taxation, the main source of inefficiency was the dilution of the signal by type-L professionals, who purchased practices when young and sold them after only one period. The planner's efforts, then, should focus on eliminating this behavior. If one could simply penalize professionals who hold practices for *one* period, but not those who hold them for *two* periods, the market might achieve separation, leading to efficiency. Type-L professionals would not hold practices for only one period, because of the penalty; and they would not hold them for two periods, because clients would not renew their relationships with them. Hence they would be successfully excluded from buying and selling practices. Type-H professionals would have no problem holding practices for two periods, and would not be excluded.

To this end let us suppose that a proportional tax τ is levied whenever a practice is sold by an old professional, but that practices sold by retiring professionals are exempt from taxation. Payoffs from buying young and selling after one period are now

$$V_L^{PN}(\tau) = V_H^{PN}(\tau) = -k + p_{AY} + \delta k + \delta p_O - \delta\tau k \quad . \tag{27}$$

For type-L pros, strategy PP is also affected:

$$V_L^{PP}(\tau) = -k + p_{AY} + \delta p_{AO} + \delta^2 k - \delta\tau k \quad . \tag{28}$$

All other professional payoffs are the same as before.

We assume the tax is purely dissuasive: the planner is not interested in raising revenue or redistributing surplus. Let us say that the planner sets τ high enough to dissuade both types from buying a practice when young and selling it after a single period. As a result, only type-H professionals buy practices, and when they do so they hold it for two periods. This has the following implications for the open market:

LEMMA 9. *In any equilibrium with optimal taxation,* $p_Y = q_Y\theta_{BH} + (1-q_Y)\theta_L$, $q_O = 0$*, and* $p_O = \theta_L$.

In equilibrium type-L pros will not buy practices, so each practice will be an A-H match. Therefore the number of practices, denoted n, cannot exceed $\min\{\psi_A, 2q\}$, as there are ψ_A type-A clients and $2q$ type-H professionals in the economy. But n cannot be less than $\min\{\psi_A, 2q\}$ either. If it were, there would be type-H professionals without practices and type-A clients not in practices. Some of these would inevitably meet on the market, and each time they did a new practice would be created. In steady-state equilibrium no existing practices are destroyed, so no new ones can be created. The following theorem formalizes this result.

PROPOSITION 5. *In equilibrium with optimal taxation* $n = \min\{\psi_A, 2q\}$. *Equilibrium is socially optimal.*

It remains to find the incentive compatibility conditions (conditions on k and τ) which make this equilibrium possible. These are obtained by setting $V_L^{PN}(\tau) \leq V_L^{NN}$ and $V_H^{PP} \geq V_H^{NN}$. Client beliefs, to be consistent with equilibrium behavior, will be $\mu_Y = 1$ and $\mu_O = 0$.

Let us first consider the case $\psi_A \geq 2q$. In this case $n = 2q$. All type-H professionals hold practices, so there is no chance of meeting any on the market: therefore $q_Y = q_O = 0$ and $p_Y = p_O = \theta_L$. Clients get no rents, since $V_A = V_B = 0$. Threshold prices are $p_{iH} = \theta_{iH}$ and $p_{iL} = \theta_L$.

With these results the condition $V_L^{PN}(\tau) \leq V_L^{NN}$, whereby it is optimal for type-L professionals not to buy practices, boils down to

$$k \;\geq\; \frac{\theta_{AH} - \theta_L}{1 - \delta + \delta\tau} \tag{29}$$

The other condition, $V_H^{PP} \geq V_H^{NN}$, becomes

$$k \;\leq\; \frac{(\theta_{AH} - \theta_L) + \delta(1-\alpha)(\theta_{AH} - \theta_{BH})}{1 - \delta^2 + \alpha\delta^2} \tag{30}$$

once the appropriate substitutions are made. Recall that α is the probability of being hired by a type-A client when going on the market as a young professional. This is of course the proportion of type-A clients among all those hiring young professionals. In equilibrium, however, the $\psi_A - 2q$ type-A clients on the market get $V_A = 0$ whatever they do. They are indifferent among the three options (hire no one, hire young, hire old) and there is no telling which they will choose. So α could be as low as zero, and it could be as high as

$$\bar{\alpha} = \frac{\psi_A - 2q}{1 - q} \quad , \tag{31}$$

which happens when all type-A clients not in practices hire young professionals. The denominator $1-q$ is the measure of young professionals on the market, which in this case means all the type-L ones.

Conditions (29) and (30) can be satisfied simultaneously as long as

$$\tau \quad \geq \quad \frac{\alpha\theta_{AH} - (1-\delta+\alpha\delta)\theta_L + (1-\alpha)(1-\delta)\theta_{BH}}{(1+\delta-\alpha\delta)\theta_{AH} - \theta_L - \delta(1-\alpha)\theta_{BH}} \quad . \tag{32}$$

The right-hand side of (32) reaches its maximum when $\alpha = \bar{\alpha}$. Even then it is less than one, so there is necessarily a level of taxation which ensures that a separating equilibrium exists.

Example 3. Let $\theta_L = 1$, $\theta_{BH} = 2$ and $\theta_{AH} = 5$, $q = 0.3$, $\psi_A = 0.8$, $\psi_B = 2$, and $\delta = 2/3$. As we mentioned, in equilibrium type-A clients on the market (i.e. not in practices) are indifferent among their three options; assume they all hire young professionals, so that $\alpha = \bar{\alpha} = 2/7$. Then conditions (29) and (30) become

$$\frac{6}{1+2\tau} \quad \leq \quad k \quad \leq \quad 156/43 \quad . \tag{33}$$

Setting $\tau \geq 17/52$ ensures that these conditions can be satisfied. In equilibrium k satisfies (33), and the other results are as stated previously, i.e. $q_Y = q_O = 0$, $p_Y = p_O = 1$, and $n = 2q = 0.6$. This is sustained by beliefs $(\mu_Y, \mu_O) = (1, 0)$. Equilibrium is socially efficient. Professionals' histories for this example are depicted in Figure 5. Agents in B-L matches are indifferent between renewing and going on the market; in the diagram these matches are renewed.

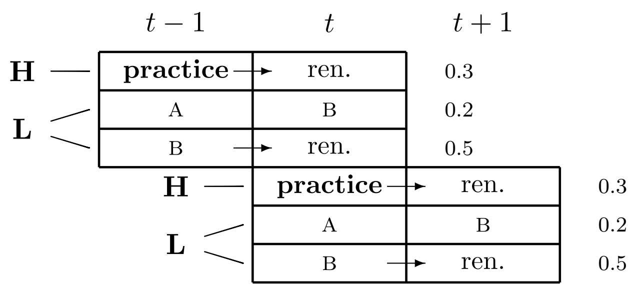

Figure 5. Professionals' histories in Example 3. The word "practice" means the purchase of a practice. The letters A and B mean being hired by that type of client during the market phase. Arrows indicate matches renewed during the negotiation phase.

Now let us consider the case $\psi_A < 2q$. There are $n = \psi_A$ practices, not enough for all high-quality professionals. Each period half the practices are bought by young type-H professionals from retiring ones (the other half are kept by old professionals for a second period). The remaining young type-H agents go on the market, and so

$$q_Y = \frac{q - n/2}{1 - n/2} = \frac{2q - \psi_A}{2 - \psi_A} . \tag{34}$$

This allows us to calculate p_Y, V_A and p_{AH}. Since some young type-H professionals buy practices while others go on the market, they must be indifferent between the two. That is to say, $V_H^{PP} = V_H^{NN}$. All type-A clients are in practices, so it is impossible to meet one on the market. This means $\alpha = 0$, which makes the calculation of V_H^{NN} fairly easy. The condition $V_H^{NN} = V_H^{PP}$, once the relevant substitutions are made, can be manipulated to find the equilibrium price of practices:

$$k = \left[\frac{1 - q_Y}{1 - \delta^2}\right] \Big[(\theta_{AH} - \theta_L) + \delta(\theta_{AH} - \theta_{BH})\Big] . \tag{35}$$

We must again impose $V_L^{PN}(\tau) \leq V_L^{NN}$. This is needed to ensure that buying practices be sub-optimal for type-L professionals. With the new prices found, the condition becomes

$$k \geq \frac{(1 - q_Y)(\theta_{AH} - \theta_L)}{1 - \delta + \delta\tau} \tag{36}$$

Combining (35) and (36) allows us to find the minimum tax level required for equilibrium:

$$\tau \geq \frac{(1 - \delta)(\theta_{BH} - \theta_L)}{(\theta_{AH} - \theta_L) + \delta(\theta_{AH} - \theta_{BH})} . \tag{37}$$

The right-hand side is less than one, therefore taxation *can* once again ensure the existence of a separating equilibrium.

Example 4. Let $\theta_L = 1$, $\theta_{BH} = 2$, $\theta_{AH} = 5$, $q = 0.3$, $\psi_A = 0.4$, $\psi_B = 2$, and $\delta = 2/3$. Then in equilibrium we have $q_Y = 0.125$, $q_O = 0$, $p_Y = 1.125$, $p_O = 1$, and $n = 0.4$. This is sustained by beliefs $(\mu_Y, \mu_O) = (1, 0)$. The price of a practice is $k = 4.25$. From (37), the tax rate must be at least 1/8 for this equilibrium to hold. This outcome is socially efficient. Professionals' histories are shown in Figure 6. Agents in B-L matches are indifferent between renewing and going on the market; in the diagram these matches are renewed.

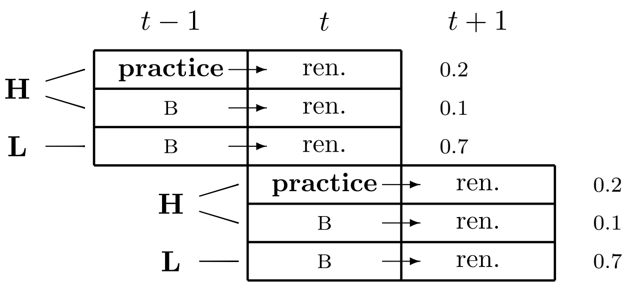

FIGURE 6. Professionals' histories in Example 4. The word "practice" means the purchase of a practice. The letter B means being hired by a type-B client during the market phase. Arrows indicate matches renewed during the negotiation phase.

5 Price Ceilings

A social planner may be interested in more than just efficiency, which is achieved when total surplus in the economy is maximized. The planner may also care about how this surplus is distributed among the agents. In this section we shall look at the average per-period utility enjoyed by type-A clients in the various equilibria we have studied. [Type-B clients obtain zero utility in all our equilibria.] We will consider how the planner can set a ceiling on the price of professional services in an effort to transfer some surplus from type-H to type-A agents, without compromising the efficiency achieved with optimal taxation.

As mentioned in Section 3.6, the average per-period utility enjoyed by a type-A client is measured by $(1-\delta)V_A$. This quantity may actually be lower when the proposed tax is in place than when it is not. To give an example, suppose $\psi_A < 2q$. Then average payoff with the tax is

$$\pi_A^{tax} = \left[\frac{2q-\psi_A}{2-\psi_A}\right](\theta_{AH}-\theta_{BH}) \quad . \tag{38}$$

where we have used (34). This is what type-A clients can get by hiring young professionals on the open market, even though in equilibrium they do not. [In equilibrium they are all in practices, where they get this same value.] The average payoff without tax, denoted π_A^{ni} (*ni* for no intervention), is

$$\pi_A^{ni} = \left[\frac{q-n_H}{1+n_H-\psi_A}\right](\theta_{AH}-\theta_{BH}) \quad ; \tag{39}$$

where the expression in brackets is gotten from (50) and (51). In the first part of the appendix it is shown that $n_H < \psi_A/2$. It follows that $\pi_A^{tax} < \pi_A^{ni}$. Even though the equilibrium with taxation is efficient and the one without taxation is not, the gain in efficiency was made at clients' expense.

Some of the rents captured by professionals can be redistributed to type-A clients if in addition to imposing a tax on early sales of practices the government uses another policy instrument, namely a ceiling on the price of professional services. In what follows we assume that the social planner can distinguish neither clients' nor professionals' types. In fact we will assume that a single ceiling can be set, regardless of the agents' types, the professional's age, or the origin of the match (practice or open market).

In the optimal-taxation equilibrium, the highest price paid for professional services is $p_{AY} = p_{AH} = \theta_{AH}$, the price charged by the owner of a practice. The next highest price is $p_{BH} = \theta_{BH}$, the price a type-B client pays a type-H professional when their types are known to each other. In order for sellers of practices to convince buyers that their clientele is type-A, they must show that these clients paid more than p_{BH}. Given these considerations, the ceiling, which we will call $\bar{p}$, should be set lower than θ_{AH} but higher than θ_{BH}. This way practice owners can charge $\bar{p}$ and still prove that their clients are type-A.[3]

If this is to be done without affecting efficiency, care must be taken to gauge the effects of the price ceiling on the various payoffs to professionals. As long as $\bar{p} > \theta_{BH}$, an efficient equilibrium still exists as described in Section 4. The incentive compatibility conditions change somewhat, as a result of $\bar{p}$ being charged instead of p_{AY} and p_{AH}.[4]

With such a price ceiling (and the appropriate tax) in place, type-A clients in practices get utility $\theta_{AH} - \bar{p}$ per period. Those type-A clients not in practices, if there are any, get nothing. The average per-period payoff for type-A clients in this price-ceiling scenario, denoted π_A^c, is therefore

$$\pi_A^c = \frac{n\ (\theta_{AH} - \bar{p})}{\psi_A} \quad . \tag{40}$$

How does this compare with average payoffs in the other cases? From Theorem 4 we know that π_A^b is the highest average payoff type-A clients can get in equilibium without government intervention. The following theorem states that this can be improved upon.

PROPOSITION 6. *In equilibrium with optimal taxation, there exists a price ceiling $\bar{p}$ such that $\pi_A^c > \pi_A^b$.*

[3]Note that when $\psi_A \geq 2q$, the price $p_{BH} = \theta_{BH}$ is never actually paid in equilibrium, the reason being that all type-H professionals own practices, so B-H matches are never formed. However, the fact that type-H professionals *could* go on the market and be matched with type-B clients (eventually charging tham $p_{BH} = \theta_{BH}$) means that the requirement $\bar{p} > p_{BH}$ is necessary in this case too.

[4]To obtain the new conditions, replace θ_{AH} by $\bar{p}$ in equations (29), (30) and (32); and replace θ_{AH} by $(\bar{p} - q_Y\theta_{BH})/(1 - q_Y)$ in equations (35), (36) and (37), where q_Y is given in (34).

This result ensures that by using the two policy instruments jointly, the government can bring about social efficiency without detriment to clients.

6 Summary and Conclusion

In this paper we present a theoretical model of two related markets: a market for professional services, and a market for the practices in which these services are provided. We show how the workings of these two markets jointly determine which types of clients and professionals get matched with one another. We call an equilibrium efficient if it achieves the maximum possible amount of matches between the professionals who provide the highest-quality services and the clients who stand to benefit the most from them.

We find that without a market for practices, the economy necessarily settles at an inefficient equilibrium. The market for services cannot, by itself, assure the socially optimal allocation of professionals to clients. The main obstacle to efficiency is that clients and professionals do not know each other's types until after they have transacted. Professional services are an experience good, and matches early in a professional's career are made in a context of two-sided uncertainty.

When a market for practices exists, however, social efficiency is possible. The reason is that the practices market can act as a catalyst for the transmission of information. Professionals who buy practices learn their clients' types from the practices' previous owners; a practice is in fact valueless unless this information is provided. Clients in practices also obtain information — albeit imperfect information — about their professionals' types, simply from the fact that the latter bought practices.

The existence of a practices market is not sufficient to guarantee efficiency. If there are too few high-need (i.e. type-A) clients in the economy, then competition between high- and low-quality professionals for the opportunity to serve these clients ends up driving some high-quality pros out of the practices market altogether. These end up being matched with low-need clients, at the same time that some low-quality pros end up serving high-need clients: an obviously inefficient allocation.

In all equilibria without government intervention, buying a practice is always worthwhile for a low-quality professional whenever it is worthwhile for a high-quality one. This means that buying a practice does *not* constitute a clear signal of high quality. More precisely, there does not exist a separating equilibrium in which the incentive compatibility conditions are satisfied strictly.[5]

By taxing the sale of practices in the right way, i.e. taxing those that are sold prior to retirement age, the government can create conditions where it is always profitable for a high-quality pro to buy a practice, but never for a low-quality one. Then the purchase of a practice becomes a perfect signal of quality, and social efficiency follows as a result. There is no distortion, since no taxes are actually collected: the tax is like a fine or penalty.

[5]This can be shown formally, but has been omitted for brevity.

Although taxation can bring about an efficient matching of professional-client matchings, it can at the same time reduce clients' welfare. This can be corrected, however, by placing an appropriate ceiling on the price of professional services. The rents accruing to type-A clients can actually be made greater than what they are in any equilibrium without public intervention. [Type-B clients' rents are zero in all equilibria.] Thus the twofold objective of providing social efficiency and increasing client welfare can be achieved with two policy instruments.

In the United States, taxes are levied when a practice changes hands. There are sales, income, and capital-gains taxes to be paid, according to the components of the practice. To our knowledge, however, there is no major tax advantage offered to retiring professionals.

If anything, the contrary is true. Under Internal Revenue Code Section 1031, a person selling a practice can be exempt from capital-gains taxes if he purchases like-kind property (i.e. another practice) within a short period of time. In our model, this statute would make it more profitable for type-L professionals to choose strategy PP (buying two practices in succession). It would certainly be an added obstacle to efficiency.

Naturally we should not automatically assume the worst of a professional availing himself of IRC 1031. It is a great help to those professionals who simply want to move to a different neighborhood or city. Yet it is certain that others besides these have benefited. Several professionals, advertising on the internet and in trade journals, boast of having bought and resold several practices over their lifetime, to great personal advantage; and they offer to help others do the same. We are of course in no position to judge the quality of professional services offered by these individuals. However, such actions can clearly be undertaken by both good and bad practitioners, and the fact that the tax system facilitates them can only erode the overall efficiency of the system, as we have defined it.

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Appendix

PROOF OF LEMMA 1. Suppose that no type-B clients hire young professionals. Then they must hire old professionals, since there are not enough type-A clients to keep all professionals in the economy employed. So V_B must be the value to a type-B client of hiring an old professional. Since $V_B = 0$, we have $p_O = q_O\theta_{BH} + (1 - q_O)\theta_L$.

And if young professionals are not hired by type-B clients, they must be hired by type-A clients. So V_A must be the value to a type-A client of hiring a young professional; with (5) and (2) this allows us to compute $p_{AH} = (1-q)(\theta_{AH} - \theta_L) + p_Y$. From (5), we must have $p_Y \geq q\theta_{BH} + (1 - q)\theta_L$, otherwise V_B would be positive. This leads to $p_{AH} \geq q\theta_{BH} + (1 - q)\theta_{AH}$. Since $q \geq q_O$, this means $p_{AH} > p_O$. In the negotiation phase, therefore, type-H professionals offer type-A clients p_{AH}, and all A-H matches are renewed.

So if no type-B clients hire young professionals, all young professionals (a measure 1) are hired by type-A clients, and all A-H matches (a measure q) are renewed. In steady state, where both generations of professionals coexist, the total measure of type-A clients must therefore be at least $1 + q$. Since $\psi_A < 1 + q$, this is impossible. Therefore some type-B clients must hire young professionals. Since $V_B = 0$, this means $p_Y = q\theta_{BH} + (1 - q)\theta_L$.

PROOF OF LEMMA 2. Suppose there are old type-H professionals on the open market. Then they have rationally made offers that were rejected. This can only happen if $p_O \geq \min\{p_{AH}, p_{BH}\} = \theta_{BH}$. But if $p_O \geq \theta_{BH}$, the value of hiring an old professional is negative, and no one hires any. This is a situation of excess supply.

PROOF OF LEMMA 3. Since $q_O = 0$, p_O cannot be less than θ_L, otherwise V_B would be positive. Suppose $p_O > \theta_L$. Then the value of hiring an old professional is negative, and no one hires any. But if $p_O > \theta_L$, it is higher than both p_{AL} and p_{BL}, and as a result all A-L and B-L matches are dissolved. Therefore all type-L professionals end up on the market when old. This is a situation of excess supply.

PROOF OF LEMMA 4. Suppose $q_Y > q$. Young professionals either go on the market or buy practices; there are no other possibilities. So if proportionately more type-H than type-L pros go on the market, it means that proportionately fewer type-H than type-L pros buy practices. Hence $q_Y > q$ implies $\mu_Y < q$. As a result,

$$p_{AY} < q\theta_{AH} + (1 - q)\theta_L - (1 - \delta)V_A \quad . \tag{41}$$

Since $(1 - \delta)V_A \geq q_Y\theta_{AH} + (1 - q_Y)\theta_L - p_Y$ by definition, and since we are supposing $q_Y > q$, we find that (41) implies $p_{AY} < p_Y$. At these prices, all young type-L pros will prefer going on the market to buying practices, even if the latter are free. Indeed

$V_L^{NN} > V_L^{PN}$ and $V_L^{NP} > V_L^{PP}$. Consequently $q_Y \leq q$, contrary to what has been supposed. We conclude that $q_Y \leq q$. And as a corollary we can state that $\mu_Y \geq q$.

PROOF OF LEMMA 5. Suppose $q_O > q$. Compared to the population of old pros in the economy, there is a relative abundance of type-H ones on the market. This could be due to two things: either (i) proportionately fewer type-H than type-L pros renewed their matches during the negotiation phase; or (ii) proportionately fewer type-H than type-L pros bought practices when old.

The first is impossible. Since $p_{AH} > p_{AL}$ and $p_{BH} > p_{BL}$, a price p_O high enough to cause the breakup of even *some* A-H matches would necessarily cause the breakup of *all* A-L matches, and similarly for B-H and B-L matches.

The second possibility would entail $\mu_O < q$ as a consistent belief, and so

$$p_{AO} < q\theta_{AH} + (1-q)\theta_L - (1-\delta)V_A \quad . \tag{42}$$

Since $(1-\delta)V_A \geq q_O\theta_{AH} + (1-q_O)\theta_L - p_O$ by definition, and since we are supposing $q_O > q$, we find that (42) implies $p_{AO} < p_O$. At these prices, old type-L pros will not buy practices, even if they are free. The second explanation is therefore impossible also.

PROOF OF LEMMA 6. Suppose the last price paid by the client is p_{AY}. By the same reasoning as that used to derive (12), the most a type-B client would pay in the same circumstances would be $\mu_Y\theta_{BH} + (1-\mu_Y)\theta_L - (1-\delta)V_B$. We need to show that p_{AY} is greater than this last amount. From (12) and the fact that $V_B = 0$, this is equivalent to showing that $\mu_Y(\theta_{AH} - \theta_{BH}) > (1-\delta)V_A$. From Lemma 4 we know that $\mu_Y \geq q$. With equations (14), (15) and (16), and Lemmas 4 and 5, we can show that $(1-\delta)V_A \leq q(\theta_{AH} - \theta_{BH})$. Hence $\mu_Y(\theta_{AH} - \theta_{BH}) > (1-\delta)V_A$, so price p_{AY} fully reveals client type.

Suppose the last price paid by the client is p_{AH}. The most a type-B client would pay under the same circumstances is $p_{BH} = \theta_{BH}$. We need to establish $p_{AH} > p_{BH}$. We know that $p_{AH} = \theta_{AH} - (1-\delta)V_A$. From the previous paragraph we know that $(1-\delta)V_A \leq q(\theta_{AH} - \theta_{BH})$. It follows that $p_{AH} > p_{BH}$; therefore p_{AH} is fully-revealing.

PROOF OF LEMMA 7. In any period there is a measure 2 of professionals employed in the economy (half of them young, half of them old); and there are fewer than $1+q$ type-A clients. So type-B clients must employ more than $1-q$ professionals. If they hire only young ones, then some of those they hire will have to be type-H, since there are only $1-q$ type-L ones; moreover, all old professionals will have to be employed by type-A clients. Thus there will be both B-H and A-L matches in the economy, an inefficient situation. Similarly, if type-B clients hire only old professionals, some of those they hire will have to be type-H; and all young professionals will have to be employed by type-A clients. Again this means there will be both B-H and A-L matches. We conclude that type-B clients must hire both young and old professionals if equilibrium is to be efficient. Since $V_B = 0$, market prices must be as stated.

Proof of Lemma 8. Suppose $\psi_A \leq 2q$ and suppose an efficient equilibrium exists. From Lemmas 4 and 5 we know that $q_Y \leq q$ and $q_O \leq q$, which means there are both young and old type-L professionals on the market. Since no A-L matches are allowed, type-A clients cannot hire anyone on the market: they must all be in practices all the time. So $\alpha = 0$.

The fact that A-L matches cannot take place also means that type-L pros cannot buy practices. In other words, strategy NN must be optimal for them. This means $V_L^{NN} \geq V_L^{PN}$ and $V_L^{NN} \geq V_L^{NP}$, which yield the following two inequalities:

$$-k + p_{AY} + \delta k \leq p_Y \quad ; \tag{43}$$

$$-k + p_{AO} + \delta k \leq p_O \quad . \tag{44}$$

Since there are practices and type-L agents cannot buy them, it has to be optimal for type-H pros to buy them. Lemma 7 tells us that both p_Y and p_O must be less than θ_{BH} in an efficient equilibrium. And $p_{BH} = \theta_{BH}$ is easily obtained from (2) and the fact that $V_B = 0$. These observations, combined with (21), allow us to compute

$$V_H^{NN} = p_Y + \delta\theta_{BH} \quad . \tag{45}$$

Let us compare this to V_H^{PP}. If type-H pros buy practices when young, they are the only ones doing so; consistency therefore requires $\mu_Y = 1$, which implies $p_{AH} = p_{AY}$. Now (43) and (23) tell us that V_H^{PP} cannot exceed $(1+\delta)p_Y$, which is strictly less than V_H^{NN}. V_H^{NP} and V_H^{PN} can also be shown to be less than V_H^{NN} using (43) and (44). For type-H professionals, therefore, strategy NN (never buying a practice) strictly dominates all the others.

So type-L professionals cannot buy practices, type-H ones do not want to, and yet all type-A clients must be in practices. This is of course impossible, and we must conclude that an efficient equilibrium does not exist when $\psi_A \leq 2q$.

Proof of Proposition 2.
Part 1 (necessity). We know from Lemma 8 that an equilibrium cannot be efficient if $\psi_A \leq 2q$. So let us assume $\psi_A > 2q$. For equilibrium to be efficient, all type-H professionals must be employed by type-A clients. Since type-B clients hire both young and old pros (see Lemma 7), and since B-H matches are not allowed, type-H professionals must never go on the market. They must all buy practices and keep them for two periods. This means $q_Y = q_O = 0$, which in turn means $p_Y = p_O = \theta_L$ by Lemma 7.

Since type-H pros never buy practices when old, clients do not expect them to, that is to say $\mu_O = 0$. With $q_Y = q_O = 0$ and $p_Y = p_O = \theta_L$ it becomes impossible for any client to obtain positive utility on the market, and so $V_A = 0$. Threshold prices adjust accordingly: $p_{AH} = \theta_{AH}$, $p_{BH} = \theta_{BH}$, $p_{AY} = \mu_Y \theta_{AH} + (1 - \mu_Y)\theta_L$, and $p_{AO} = \theta_L$.

For strategy PP to be optimal for type-H professionals, we need $V_H^{PP} \geq V_H^{NN}$. With the results obtained so far, this is

$$\mu_Y(\theta_{AH} - \theta_L) - (1 - \delta^2)k + \delta\theta_{AH} \geq \alpha\ \delta(p_{AH} + \delta k) + (1 - \alpha)\ \delta\theta_{BH} \quad . \tag{46}$$

As for type-L professionals, we can see, given the market and threshold prices found, that $V_L^{NP} < V_L^{NN}$ and $V_L^{PP} < V_L^{PN}$. This leaves only NN and PN as viable strategies. Although these young type-L professionals *can* buy practices in equilibrium, they cannot *all* do so. If they did, the number of practices in the economy would have to be at least $1 + q$ (i.e. $1 - q$ for the young type-L pros and another $2q$ for young and old type-H ones); but there are not enough type-A clients for that. At least some of the young type-L professionals must choose NN, therefore $V_L^{NN} \geq V_L^{PN}$ is necessary. This yields

$$(1 - \delta)k \geq \mu_Y(\theta_{AH} - \theta_L) \quad . \tag{47}$$

Combining (46) and (47) we obtain the necessary condition

$$\mu_Y\theta_L + (1 - \mu_Y)\theta_{AH} \geq \alpha(p_{AH} + \delta k) + (1 - \alpha)\theta_{BH} \quad . \tag{48}$$

The left-hand side of (48) reaches its highest possible value, and the right-hand side its lowest possible value, when all type-A clients are in practices, so that $\alpha = 0$. This happens when all type-H pros play strategy PP and a measure $\psi_A - 2q$ of type-L pros play PN and the rest NN. In that case $\mu_Y = q/(\psi_A - q)$. Substituting these values into (48) and rearranging, we obtain (25).

Part 2 (sufficiency). Suppose (25) is met. Then an efficient equilibrium exists, described as follows. The number of practices is $n = \psi_A$. Note that this is greater than $2q$, otherwise (25) would not be met. Each period all young type-H pros and a measure $\psi_A - 2q$ of young type-L pros buy practices; the former will hold on to them for two periods, but the latter will sell them after only one period. No one buys practices when old. Beliefs are $\mu_Y = q/(\psi_A - q)$ and $\mu_O = 0$, which is consistent with the trade in practices just described. The price of a practice is $k = \mu_Y(\theta_{AH} - \theta_L)/(1 - \delta)$.

Conditions on the open market are given by $q_Y = q_O = 0$ and $p_Y = p_O = \theta_L$. Client values are $V_A = V_B = 0$. Since all type-A clients are in practices, only type-B clients are matched on the market, hence $\alpha = 0$. Threshold prices in the negotiation phase are $p_{AY} = \mu_Y\theta_{AH} + (1 - \mu_Y)\theta_L$, $p_{AO} = \theta_L$, $p_{AH} = \theta_{AH}$, $p_{BH} = \theta_{BH}$, $p_{AL} = p_{BL} = \theta_L$.

With these values we can easily show that $V_H^{PP} = \max\{V_H^{NN}, V_H^{NP}, V_H^{PN}, V_H^{PP}\}$ and $V_L^{NN} = V_L^{PN} = \max\{V_L^{NN}, V_L^{NP}, V_L^{PN}, V_L^{PP}\}$, confirming that the behavior described is optimal. Sellers of practices can show potential buyers that their clients are type-A, since $p_{AH} > \theta_{BH}$ and p_{AY} exceeds what a type-B client would be willing to pay a young pro (given q_Y and μ_Y). Since all type-H pros are matched with type-A clients, this equilibrium is efficient.

Proof of Proposition 3. In this case an equilibrium exists, described as follows. Some professionals of each type buy practices when young; the low-quality ones hold them for a single period, the high-quality ones for two. Let n_j denote the measure young type-j pros who buy practices when young. All type-A clients are in practices, therefore $\alpha = 0$ and

$$2n_H + n_L = \psi_A \quad . \tag{49}$$

Average quality of pros on the market is given by

$$q_Y = \frac{q - n_H}{1 - n_H - n_L} \quad , \tag{50}$$

and $q_O = 0$. Beliefs are consistent with professionals' behavior, therefore

$$\mu_Y = \frac{n_H}{n_H + n_L} \quad , \tag{51}$$

and $\mu_O = 0$. Market prices are $p_Y = q_Y\theta_{BH} + (1 - q_Y)\theta_L$ and $p_O = \theta_L$. Client values are $(1 - \delta)V_A = q_Y(\theta_{AH} - \theta_{BH})$ and $V_B = 0$. Threshold prices are calculated in the usual manner, and the price of a practice is given by $(1 - \delta)k = (\mu_Y - q_Y)(\theta_{AH} - \theta_L)$. The values of n_H, n_L, q_Y and μ_Y are found by solving equations (49), (50), (51), and the following:

$$(1 - q_Y)(\theta_{BH} - \theta_L) = (1 - \mu_Y)(\theta_{AH} - \theta_L) \quad . \tag{52}$$

Equation (52) was obtained by setting $V_H^{NN} = V_H^{PP}$ and $V_L^{NN} = V_L^{PN}$. It can be verified easily that these represent optimal strategies for professionals. What is needed to make this equilibrium stand is for the measures n_j to be in the relevant ranges, i.e. $n_H \in (0, q)$ and $n_L \in (0, 1 - q)$. The fact that (25) does not hold ensures this (see below).

Proof that $0 < n_H < q$ and $0 < n_L < 1 - q$ (Proposition 3).

The measure of young type-H professionals buying practices, n_H, is found by reducing (49), (50), (51) and (52) to a single equation. This is the quadratic

$$h(x) \equiv 2x^2 + Fx + G = 0 \quad , \tag{53}$$

where

$$F = 2 + S + qS - 3\psi_A \quad ; \tag{54}$$
$$G = \psi_A(\psi_A - 1 - qS) \quad ; \tag{55}$$

and S is the positive constant $(\theta_{BH} - \theta_L)/(\theta_{AH} - \theta_{BH})$. A solution to (53) exists if and only if $F^2 - 8G \geq 0$, and this inequality is the first thing we have to prove. We find

$$F^2 - 8G \quad = \quad (2 + S + qS)^2 + \psi_A(\psi_A - 4 - 6S + 2qS) \quad . \tag{56}$$

The theorem supposes that $0 < \psi_A < q(2+S)$. The right-hand of (56) is decreasing in ψ_A over this range, and thus approaches its infimum as ψ_A tends towards $q(2+S)$. This infimum is found to be $(2 + S - 2q - 2qS)^2$, a non-negative number. Therefore $F^2 - 8G \geq 0$ holds.

Because $2n_H + n_L = \psi_A$, the conditions $0 < n_H < q$ and $0 < n_L < 1 - q$ boil down to

$$\max\left\{0, \frac{\psi_A - 1 + q}{2}\right\} \quad < \quad n_H \quad < \quad \min\left\{q, \frac{\psi_A}{2}\right\} \quad . \tag{57}$$

The function $h(x)$ is a convex parabola centered at $x_c \equiv -F/4$. Given that $\psi_A < q(2 + S)$, it is fairly straightforward to check that $(1/2)(\psi_A - 1 + q)$ is greater than x_c, i.e. right of center. This means that only the *higher* of the two roots of (53) can satisfy (57) — see Figure 7. It is also easy to show that q and $(1/2)\psi_A$ are greater than x_c. We calculate

$$h(q) \quad = \quad [q(2 + S) - \psi_A]\,[(1 + q) - \psi_A] \quad > \quad 0 \quad ; \tag{58}$$
$$h(\psi_A/2) \quad = \quad (1/2)(1 - q)\psi_A S \quad > \quad 0 \quad ; \tag{59}$$
$$h((1/2)(\psi_A - 1 + q)) \quad = \quad (1/2)(1 - q)(1 + S)[\psi_A - (1 + q)] \quad < \quad 0 \quad ; \tag{60}$$

where we have used the restrictions on ψ_A to determine signs. These inequalities establish that $(1/2)(\psi_A - 1 + q) < n_H < \min\{q, (1/2)\psi_A\}$.

It remains to show that $n_H > 0$. To have $n_H \leq 0$ it would be necessary to have $h(0) \geq 0$ and $x_c \leq 0$. This means $F \leq 0$ and $G \leq 0$ would be required. However, $G \geq 0$ implies $\psi_A \geq 1 + qS$, and also $S < 1$, since $\psi_A < 1 + q$ by assumption; and these results imply $F < 0$. Therefore either F or G must be negative, so $n_H > 0$.

Proof of Proposition 4. This follows from equations (14), (15) and (16), and Lemmas 4 and 5.

Proof of Lemma 9. Professionals who hold practices do not end up on the market when old. A type-j professional matched with a type-i client will end up on the

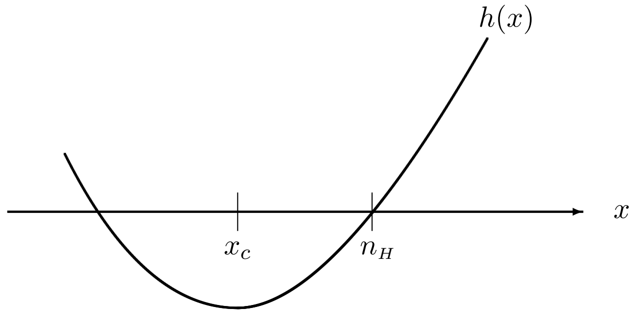

FIGURE 7. The function $h(x)$.

market when old only if $p_O \geq p_{ij}$. Since $p_{AH} > p_{AL}$ and $p_{BH} > p_{BL}$, the proportion of type-H agents who end up on the market when old is no greater than that of type-L agents. Hence $q_O \leq q_Y$. The rest of the proof follows the same lines as those of Lemmas 1, 2 and 3.

PROOF OF PROPOSITION 5. Suppose $n < \min\{\psi_A, 2q\}$. Steady state requires that half the practices be held by the younger generation, hence a measure $n/2 < q$ of young type-H professionals buy practices. This means some young type-H professionals offer their services on the market, so that $q_Y > 0$. Since $n < \psi_A$, some type-A clients are not in practices.

What do the type-A clients not in practices do on the market? Hiring no one gives them zero. Since $q_O = 0$ and $p_O = \theta_L$ (by Lemma 9), hiring old professionals also gives them zero. But since $p_Y = q_Y\theta_{BH} + (1 - q_Y)\theta_L$ (by Lemma 9) and $q_Y > 0$, hiring young professionals gives them positive value. Therefore they hire young professionals.

This means that some young type-H professionals are hired by type-A clients on the market. When they become old, they can charge them p_{AH}. So when they retire, they have a practice to sell. Since all those of their generation who bought practices also have practices to sell, the total number of practices has increased. This cannot happen in steady-state equilibrium.

We conclude that the number of practices must be $n = \min\{\psi_A, 2q\}$. This is also the number of A-H matches. Since it is at its maximum possible level, equilibrium is socially efficient.

PROOF OF PROPOSITION 6. Since $\bar{p}$ can be made arbitrarily close to (but greater than) θ_{BH}, the level of π_A^c can be made greater than π_A^b as long as $(n/\psi_A) > q$. If $\psi_A \leq 2q$ then $n = \psi_A$. The result is obtained, since $(n/\psi_A) = 1$.

If $\psi_A > 2q$, on the other hand, then $n = 2q$. The fact that $\psi_A < 1 + q$ allows us to say that $(n/\psi_A) > 2q/(1+q)$, which is greater than q. Therefore $\pi_A^c > \pi_A^b$ for $\bar{p}$ close enough to θ_{BH}.