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AND FOOD PRICE VOLATILITY

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Agricultural Trade, Biodiversity Effects and Food Price Volatility

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Abstract

Production risks in agriculture due to biotic elements such as pests create biodiversity effects that impede productivity. Pesticides reduce these effects but are damaging for the environment and human health. When regulating farming practices, governments weigh these side-effects against the competitiveness of their agriculture. In a Ricardian two-country setup, we show that free trade results in an incomplete production specialization, that restrictions on pesticides are generally more stringent than under autarky and that trade increases the price volatility of crops produced by both countries and some of the specialized crops. If biodiversity effects are large, the price volatility of all crops is larger than under autarky.

Keywords: agricultural trade, food prices, agrobiodiversity, pesticides.

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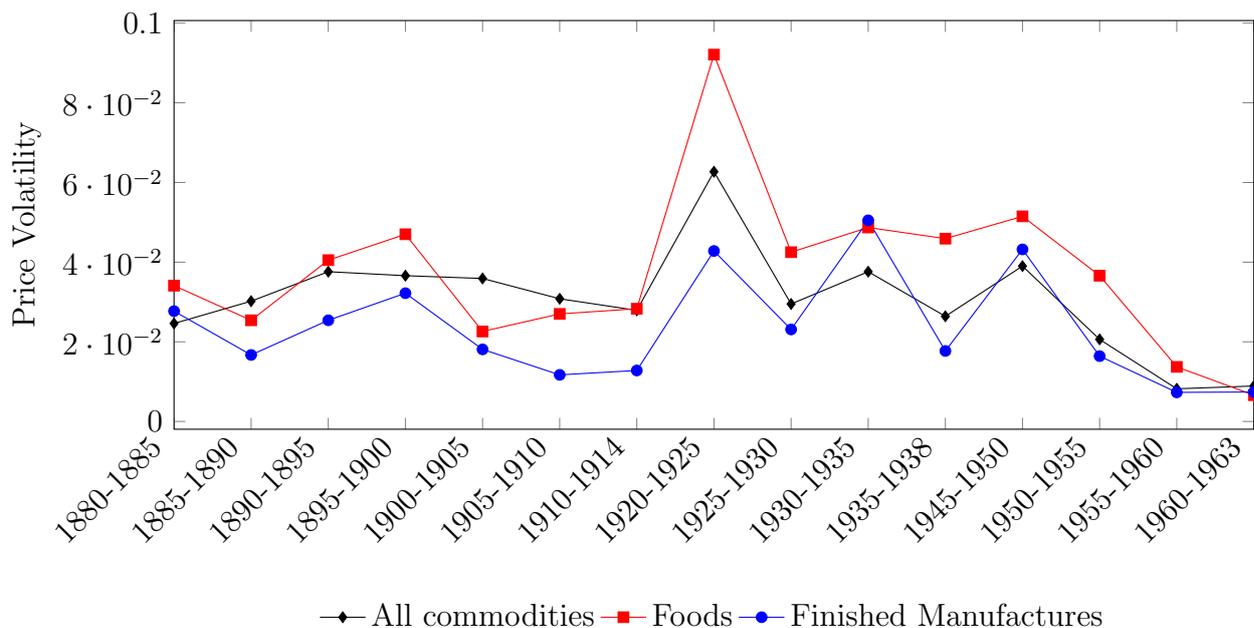


Figure 1: Volatility of U.S. export prices, 1880–1963 (from Jacks et al., 2011, table 6).

1 Introduction

World prices of major agricultural commodities are subject to busts and booms that regularly trigger worries about the functioning of world markets and raise questions about the mechanisms underlying the trend of food prices and their volatility.¹ Price variations are not specific to agricultural markets but the volatility of agricultural commodities (and particularly food commodities) is higher than that of manufactures as illustrated on figure 1 which plots the U.S. export price volatility of foods and manufactures over the period 1880—1963.

Various causes for price fluctuations have been proposed, but economic models usually neglect stochastic elements that may affect production processes. While this is probably innocuous for industrial goods, which are rarely subject to hazardous conditions, it is more questionable regarding agricultural products. Indeed, agricultural goods face a large range of

¹One recent example of public concern was given during the G-20 leaders summit meeting of November 2010: The participants requested several international institutions “to develop options on how to better mitigate and manage the risks associated with the price volatility of food and other agriculture commodities, without distorting market behavior, ultimately to protect the most vulnerable.” See Gilbert & Morgan (2010) and Wright (2011) for overviews on food price volatility and examinations of the causes of the recent spikes.

potential threats that may severely undercut farms' production. Two types of factors may interfere with the agricultural production: Biotic factors, called pests, which are harmful organisms, either animal pests (insects, rodents, birds, worms, etc), pathogens (virus, bacteria, fungi, etc), or weeds, and abiotic factors such as water stress, temperature, irradiance and nutrient supply that are often related to weather conditions (Oerke, 2006). Many studies show that crop losses due to pests are critical.²

Our analysis highlights the importance of the stochastic elements affecting agricultural production in determining the pattern of food trade, and particularly the biotic ones that relate the biodiversity of cultivated crops to their productivity and create what are dubbed *biodiversity effects* in the following. Because farmland has specificities that differ from one country to the other, trade must result in specialized international crop productions according to the implied competitive advantages. In this Ricardian setup, opening to trade should not help much to smooth out production risks since the world is exposed to the idiosyncratic risk of the only country producing each crop. However, we show that biodiversity effects create a production externality that impedes productivity and results in an incomplete specialization at equilibrium under free trade. To reduce the productive loss due to pests, farmers have recourse to agrochemicals (pesticides, fungicides, herbicides and the like),³ but these products also have damaging side-effects on the environment and human health that spur governmental intervention.⁴ However, restrictions on the use of pesticides may impede

²Oerke (2006) estimates that the potential losses due to pests (i.e., without crop protection of any kind) between 2001 and 2003 for wheat, rice, maize, soybeans and cotton were respectively 49.8%, 77%, 68.5%, 60% and 80% of the potential yield. Savary et al. (2000) also finds significant losses: combined average level of injuries led to a decrease of 37.2% in rice yield (if injuries were considered individually, the cumulative yield loss would have been 63.4%). Fernandez-Cornejo et al. (1998) compile similar data on previous periods and find that expected losses relative to current or potential yield from insects and diseases range between 2 to 26% and that losses from weeds range between 0% and 50%.

³Practices controlling biotic factors include physical (mechanical weeding or cultivation...), biological (crop rotations, cultivar choice, predators...) and chemical measures. They allow a reduction of losses, but do not totally avoid them. In the case of wheat, they reduce potential losses of 50% to average actual losses of about 29% (from 14% in Northwest Europe up to 35% and more in Central Africa and Southeast Asia). Actual losses for soybean average around 26%. The order of magnitude of losses for maize and rice is greater, with actual losses of respectively 40% and 37% (Oerke, 2006).

⁴Pimentel (2005) reports more than 26 million cases worldwide of non-fatal pesticide poisoning and approximately 220 000 fatalities. He estimates that the effects of pesticides on human health cost about 1.2 billion US\$ per year in the United States. Mammals and birds are also affected. Farmland bird population decreased by 25% in France between 1989 and 2009 (Jiguet et al., 2012), and a sharp decline was also

the competitiveness of agriculture on international markets. Governments, when designing these regulations, thus face a tradeoff: either they contain the externality on consumers and tolerate a high level of negative production externality, which diminishes the competitiveness of their agricultural sector, or they are more permissive in the utilization of pesticides to enhance the productivity of their agricultural sector to the detriment of their consumers. We show that when governments use these regulations strategically to enhance the comparative advantages of their agriculture, trade results in an environmental “dumping” situation where regulations are more lenient than they otherwise would be. Nevertheless, restrictions on pesticides under free trade are generally more stringent than under autarky. The resulting effect of trade on crop price distributions is ambiguous: because of more restrictive policies and the intensification of production, the expected market price of crops produced exclusively by one country is increased for its domestic consumers. For crops produced by both countries, the change in price depends on the share of farmland devoted to these crops compared to under autarky. Trade also unambiguously increases the production volatility of crops produced by both countries. Some specialized productions (the ones for which comparative advantages are large) could see a reduction in their volatility, but that supposes very small biodiversity effects.

To investigate the importance of biodiversity effects on the pattern of trade and on price volatility, we re-examine the standard Ricardian model of trade as developed by Dornbusch et al. (DFS, 1977), which considers the impact of trade on two economies that produce a continuum of goods. The productive efficiencies of these economies determine their comparative advantages, resulting in a division of the production into two specific ranges of products, one for each country. Here, we consider two-sector economies, an industrial/service sector that produces a homogeneous good with equal productivity in the two countries (which serves as the numeraire), and an agricultural sector that produces a range of goods with different

observed in the whole EU during the same period (EEA, 2010). Pesticides also contaminate water and soils and significantly affect water species both locally and regionally (Beketov et al., 2013). Many countries have adopted regulations that forbid the most harmful molecules, set provisions on the use and storage of pesticides, and intend to promote their sustainable use. For example, only a fourth of the previously marketed molecules passed the safety assessment made within the European review process ended in 2009 (EC, 2009). The use of pesticides seems to follow a decreasing trend: in Europe, quantities sold decreased by 24% between 2001 and 2010 (ECP, 2013).

potential yields, the effective crop yields depending on biotic and abiotic factors. Many agro-ecology studies show that genetic diversity within and among crop species improves the resilience of agricultural ecosystems to adverse climatic conditions and harmful organisms. Crop biodiversity reduction, observed in many modern agricultural systems, increases the vulnerability of crops to these stresses.⁵ To account for these relationships in a tractable way, we follow Weitzman (2000): the larger the share of farmland dedicated to a crop, the more its parasitic species proliferate, and thus the more fields of that crop are at risk of being wiped out.⁶ More specifically, farming a given plot of land is a Bernoulli trial in which the probability of success depends on the way land is farmed in the country: the larger the share of farmland devoted to a particular crop, the higher the probability that the plots of that crop are destroyed by biotic or abiotic factors. At the country level, these biodiversity effects translate into crop production that follows normal distributions parameterized by farming intensities. Pesticides have a positive impact at both the individual level (direct effect) and regional level (indirect, cross-externality effect): by destroying pests on her field, a farmer improves the resilience of her plot and reduces the risk borne by the adjacent fields. This cross-externality between fields of the same crop thus attenuates the negative biodiversity effects. We abstract from risk-aversion by assuming that farmers and consumers are risk-neutral.⁷ Although by assumption potential crop yields are different, consumer preferences and the other characteristics of the two economies are such that the autarky case displays symmetric situations: the agricultural revenue, land rent and environmental tax are at the same levels; only crop prices differ. We then consider the free trade situation and show that

⁵This is demonstrated in Tilman et al. (2005) using simple ecological models that describe the positive influence of diversity on the biomass produced by both natural and managed (i.e. agricultural) ecosystems, and corroborated by empirical results detailed in Tilman & Downing (1994) and Tilman et al. (1996). See also Smale et al. (1998), Di Falco & Perrings (2005) or Di Falco & Chavas (2006) for econometric investigations.

⁶Weitzman (2000) makes an analogy between parasite-host relationships and the species-area curve that originally applies to islands: the bigger the size of an island, the more species will be located there. He compares the total biomass of a uniform crop to an island in a sea of other biomass. A large literature in ecology uses the species-area curve which is empirically robust not only for islands but for uniform regions more generally (May, 2000; Garcia Martin & Goldenfeld, 2006; Drakare et al., 2006; Plotkin et al., 2000; Storch et al., 2012).

⁷Risk-aversion would induce government to reduce specialization under trade to diversify production risks among countries as demonstrated by Gaisford & Ivus (2014). Abstracting from this diversification effect allows us to isolate the impact of biodiversity on specialization.

biodiversity effects result in an incomplete specialization of the countries. The intuition is as follows: without biodiversity effects, free trade results in the same specialization as described by DFS, a threshold crop delimiting the production range specific to each country. With biodiversity effects, such a clear-cut situation with one threshold crop can no longer exist because the intensification of farming reduces expected crop yields: an increase in acreage devoted to a crop reduces its expected yield and, symmetrically, a diminution of its acreage increases it. As a result, the two countries produce a whole range of crops delimited by two threshold crops, their shares of production evolving according to the relative yields. This range of crops depends on the environmental taxes, which also determine the country's share of the worldwide agricultural revenue. Although we consider competitive economies and free trade, the free trade outcome is the result of a strategic game played by the two countries similar to the “strategic trade policy” problem first investigated by Brander & Spencer (1985) (see Helpman & Krugman, 1989, for a detailed analysis), where environmental taxes are used as a substitute for trade policies.⁸ This is the case with or without biodiversity effects, since pesticides have a direct effect on production, but distinguishing between these two cases allows us to detail the biodiversity effects more thoroughly. Two countervailing effects are at work when accounting for the biodiversity effects: as specialization increases the negative production externality, governments are induced to lower the tax on pesticides, but since the externality limits trade specialization, the effect of the tax on prices concerns a reduced set of crops, which induces governments to increase the tax.

To solve this game when biodiversity effects are at work, and to compare its equilibrium to the autarky case, we focus on a family of relative potential yield functions that allows for symmetric equilibria. We also characterize a fictitious non-strategic situation where the governments are unaware that regulations have an impact on their revenues. Compared to this non-strategic situation, we show that environmental taxes are lower at the equilibrium

⁸It is thus a “strategic environmental policy game” as analyzed by Barrett (1994) in an oligopoly setup à la Brander & Spencer (1985) where firms pollute and governments are prevented from using trade policy instruments. He shows that market share rivalry results in weak environmental standards. Trade policies in the DFS setup have been analyzed in Itoh & Kiyono (1987), Opp (2010) and Costinot et al. (2013). The latter study shows that the optimal trade policy is a mix of uniform import tariffs and of export subsidies that are weakly decreasing with respect to comparative advantage.

of the strategic policy game. Hence, to give a competitive advantage to their farmers, the governments are permissive regarding the use of pesticides. That leniency depends on the way the relative potential yield evolves along the range of crops: the more rapidly the productivity differential increases, the tighter the regulation.

However, we also show that environmental taxes are generally above the autarky level. Indeed, under autarky, governments are willing to accept a more intensive use of pesticides because they allow a decrease in the prices of all crops sold in the country. Domestic consumers bear all the costs and reap all the benefits of the agrochemical use. This is no longer true under free trade because of the (partial) specialization of each country. Domestic agricultural production concerns only a reduced range of products, and some of these crops are also produced abroad: prices of these goods are less reactive to the domestic regulation.⁹

Finally, the distribution of crop production allows us to characterize the market price distribution in each case. Approximations of the expected value, of the standard deviation and of confidence intervals for the market prices are derived using the variation coefficient (the ratio of the standard deviation over the expected value) of the crop production. At a country level, because shocks affecting plots are independent, an increase in a crop's share of farmland creates a "scale effect" that counteracts the biodiversity effect of intensification, resulting in a decrease in production volatility. However, at the worldwide level, the discrepancy in the countries' farmland productivity creates a "yield effect" that increases volatility. We show that compared to the volatility of the aggregate production under autarky, trade increases the volatility of crops produced by both countries as well as some of the specialized productions: the production volatility only of those crops for which countries have high comparative advantages could be reduced by trade. However, if biodiversity effects are large, the volatility of all crops is increased.

Models of food trade incorporating production shocks generally assume that the distri-

⁹Markusen et al. (1995) and Kennedy (1994) obtain comparable results in an imperfect competition framework. Compared to the market share rivalry analyzed by Barrett (1994), which tends to lower environmental tax policies below Pigovian levels, they show that there is a "pollution shifting" effect that works in the opposite direction when the competition takes place on domestic markets: although under autarky an environmental tax increase is detrimental for consumption, this is not the case when domestic consumers have access to the goods produced in the foreign country. Governments are thus induced to (unilaterally) increase their environmental tax.

butions of idiosyncratic shocks are not affected by trade policies and that free trade allows the diversification of risk. This is the case e.g. in Newbery & Stiglitz (1984) which shows that when financial markets are missing, production choices of risk-averse farmers under free trade are different from those made under autarky and may result in welfare losses. These models assume that the potential productivity of farmland is the same whatever the country. We take the opposite viewpoint and adopt the Ricardian setup of DFS to account for farmland discrepancies between countries. This setup allows us to describe how idiosyncratic shocks evolve with trade. The literature of the effect of volatility in Ricardian setups dates back to Turnovsky (1974) and is mostly focused on asserting the impact of risk aversion on the pattern of trade.¹⁰ Recent Ricardian models of trade involving more than two countries (Eaton & Kortum, 2002; Costinot & Donaldson, 2012) incorporate a stochastic component together with trade costs to specify the pattern of trade and avoid complete specialization. These components are not related to the production process (they are “exogenous” and of no use in explaining price volatility). Our setup offers such features endogenously: biotic and abiotic factors affect stochastically production which generates price volatility and causes productivity losses that prevent complete specialization at equilibrium.

The rest of this article is organized as follows: the next section details how we integrate biodiversity effects in the agricultural production process. We derive the autarky situation and show that biodiversity effects result in an incomplete specialization under free trade. Section 3 is devoted to the tax policy under free trade. Strategic and non strategic tax policies are derived to analyze the impact of the biodiversity effects on public policy. Their implications on price volatility are exposed in section 4. We discuss the impacts of trade on fertilizer use in section 5. The last section concludes.

¹⁰This is the case e.g. in Gaisford & Ivus (2014) which introduces stochastic shocks to foreign technologies in a DFS setup. In their model, given high risk, risk aversion and no domestic and international risk sharing, tariff protection encourages the sectoral diversification of domestic production and increases expected welfare.

2 The model

Consider two countries (Home and Foreign) whose economies are composed of two sectors; industry and agriculture. Our focus being on agriculture, the industrial/service sector is summarized by a constant return to scale production technology that allows production of one item with one unit of labor. The industrial good serves as the numeraire which implies that the wage in these economies is equal to 1. The agricultural sector produces a continuum of goods indexed by $z \in [0, 1]$ using three factors: land, labor and agrochemicals (pesticides, herbicides, fungicides and the like) directed to control pests and dubbed “pesticides” in the following.¹¹ Both countries are endowed with total labor force and farmland denoted by L and N for Home, and L^* and N^* for Foreign (asterisks are used throughout the paper to refer to the foreign country). As farming one plot requires one unit of labor, industry employs $L - N$ workers.

Technical coefficients in agriculture differ from one crop to the other, and from one country to the other. More precisely, absent production externality and adverse meteorological or biological events, this mere combination of one unit of labor with one unit of land produces $\bar{a}(z)$ good z in Home and $\bar{a}^*(z)$ in Foreign. The crops are ranked in decreasing order for Home: the relative crop yield $A(z) \equiv \bar{a}^*(z)/\bar{a}(z)$ satisfies $A'(z) > 0$, $A(0) < 1$ and $A(1) > 1$. Hence, on the basis of these differences in potential yield, Home is more efficient producing goods belonging to $[0, z_s)$ and Foreign over $(z_s, 1]$ where $z_s = A^{-1}(1)$. However, the production of crops is affected by various factors such as weather conditions (droughts or hail) and potential lethal strains (pests, pathogens), resulting in land productivity lower than its potential. We suppose in the following that a unit plot of crop z is destroyed with probability $1 - \psi(z)$ from one (or several) adverse conditions and that, otherwise, with probability $\psi(z)$ the crop survives and the plot produces the yield $\bar{a}(z)$ independently of the fate of the other plots in the region.¹² Here, $\psi(z)$ is a survival probability that depends on many factors, among which a biodiversity factor that evolves with the way land is farmed in the country,

¹¹In order to streamline the analysis, we don't consider fertilizers. They are discussed section 5.

¹²Pests and/or meteorological events do not necessarily totally destroy a plot, but rather affect the quantity of biomass produced. This assumption allows for tractability, our random variable being the number of harvested plots rather than the share of harvested biomass.

and more specifically, the intensity of each crop cultivation as measured by the proportion of land devoted to it, i.e. $B(z) \equiv N(z)/N$ where $N(z)$ is the number of crop z plots.¹³ The biodiversity effect we consider relates positively the intensity of a crop with the probability that pests affect each individual plot of this crop if farmers do not take appropriate actions to diminish their exposure. This biodiversity effect thus corresponds to a negative production externality between farmers of the same crop, which engenders a decreasing returns to scale production technology in farming: the more land (and labor) that is devoted to a crop at the national level, the lower its expected yield per acre.

However, to counteract the impact of external events on her plot, a farmer can avail herself of a large range of chemicals. All these individual actions put together may in turn have a positive impact on the survival probability of plots by impeding the pest infection in the region, which limits the negative externality due to the intensity of the cultivation. To synthesize these elements in a manageable way, we assume that at the farmer i 's level, the survival probability of her plot is given by

$$\psi(z, \pi_i, \bar{\pi}(z), B(z)) = \mu_0 \eta(\pi_i, z) / [1 + \hat{\kappa}(\bar{\pi}, z) B(z)] \quad (1)$$

where π_i is the quantity of pesticides used by farmer i , and $\bar{\pi}(z)$ the average quantity of pesticides used by the other farmers of crop z in the country. Here, $\eta(\pi_i, z)$ corresponds to the impact of farmer i 's pesticides on the resilience of her plot, with $\eta(0, z) = 1$, $\eta'_z(0, z) > 0$, $\eta''_{zz} < 0$, and $\mu_0 [1 + \hat{\kappa}(\bar{\pi}, z) B(z)]^{-1}$ the survival probability of her plot if she does not use pesticides. This probability is nevertheless affected by a cross-external positive impact of the other farmers' treatments, with $\hat{\kappa}(\bar{\pi}, z) > 0$ and $\hat{\kappa}'_{\bar{\pi}}(\bar{\pi}, z) < 0$. Hence, the expected resilience of individual plots decreases with B , the intensity of the crop, but increases with the level of pesticides used $\bar{\pi}$. In the following, we consider the convenient specifications $\eta(\pi_i, z) = e^{\pi_i(\theta(z) - \pi_i/2)}$ where $\theta(z)$ corresponds to the level of pesticides that maximizes the expected yield of the crop, $\hat{\kappa}(\bar{\pi}, z) = e^{(\theta(z) - \bar{\pi})^2/2} \kappa(z)$ where $\kappa(z) \equiv \kappa_0 e^{-\theta(z)^2/2}$ is the

¹³By assuming independence between plots of the same country, we abstract from macroeconomic effects other than the biodiversity ones. Also, to simplify our setup, we abstract from potential positive or negative spillovers across different varieties of crops.

lowest cross externality factor for crop z , when all crop- z farmers use pesticide level $\theta(z)$, and $e^{-(\theta(z)-\bar{\pi})^2/2}$ is the negative effect on the crop's resilience induced by an average use of pesticides $\bar{\pi} \leq \theta(z)$. We assume that μ_0, κ_0 and $\theta(z)$ are the same in both countries and such that $\psi(\cdot) \leq 1$.¹⁴

Because of the externality due to pesticides (on human health and the environment), governments restrict their use. To ease the exposition and simplify the following derivations, we model the governmental policy as a tax which results in a pesticide's price τ , and we suppose that the governments complement this tax policy with a subsidy that allows farmers to cope with the environmental regulation: at equilibrium, it is financially neutral for farmers.¹⁵

We assume that farmers are risk-neutral and may only farm one crop (the one they want) on one unit plot. Given her crop choice, farmer i chooses the intensity of the chemical treatment of her field in order to reach expected income

$$r_i(z) \equiv \max_{\pi} E[\tilde{p}(z)\tilde{y}_i(z)] - \tau\pi + T(z) - c$$

where $\tilde{y}_i(z)$ is her stochastic production level, either equal to 0 or $\bar{a}(z)$, $T(z)$ is the subsidy for crop z , which is considered as a lump sum payment to farmer i , and c the other input costs, i.e. the sum of the wage (one unit of labor is necessary to farm a plot of land) and of the rental price of land, which is the same whatever crop is farmed. Denote by $y(z) \equiv E[\tilde{y}(z)]$ and by $p(z) \equiv E[\tilde{p}(z)]$ the expected production of crop z and the corresponding expected

¹⁴Parameter μ_0 may be considered as resulting from factors beyond the control of the collective action of farmers, such as meteorological events, which may differ from one crop to the other and from one country to the other, i.e. we may rather have two functions $\mu(z)$ and $\mu^*(z)$ taking different values. However, as competitive advantages are assessed by comparing $a(z) = \mu_0\bar{a}(z)e^{\theta(z)^2/2}$ to its foreign counterpart, assuming that it is a parameter is innocuous.

¹⁵This tax-neutral policy is equivalent to implementing a restriction on pesticide use, i.e. a quota policy where farmers of crop z are allowed to spread only $\pi \leq \pi(z)$ on their fields. τ then corresponds to the "shadow price" of the pesticides quota and is the same for all farmers.

market price, and define the crop z reference price as¹⁶

$$\bar{p}(z) \equiv E[\tilde{p}(z)\tilde{y}(z)]/y(z) = p(z) + cov(\tilde{p}(z), \tilde{y}(z))/y(z). \quad (2)$$

This reference price corresponds to the per unit average price of crop z for farmers. As $cov(\tilde{p}(z), \tilde{y}(z)) < 0$, it is lower than the expected market price due to the correlation between total production $\tilde{y}(z)$ and market price $\tilde{p}(z)$. With atomistic individuals (N is so large that the yield of a single plot has a negligible effect on the market price), the crop z farmer's program can be rewritten as

$$r(z) = \max_{\pi} \bar{a}(z)\psi(z, \pi, \bar{\pi}(z), B(z))\bar{p}(z) - \tau\pi + T(z) - c.$$

The first-order condition leads to an optimal level of pesticides $\pi(z)$ that satisfies

$$\eta'_{\pi}(\pi(z), z) = \tau[1 + \hat{\kappa}(\pi(z), z)B(z)]/[\mu_0\bar{a}(z)\bar{p}(z)]$$

at the symmetric Nash equilibrium between crop z farmers. As the subsidy allows farmers to break even, we have $T(z) = \tau\pi(z)$.¹⁷ Competition in the economy (notably on the land market) leads to $r(z) = 0$ for all z at general equilibrium, which gives using $\eta'_{\pi}(\pi, z) = (\theta(z) - \pi)\eta(\pi, z)$,

$$\pi(z) = \theta(z) - \tau/c. \quad (3)$$

Competition also implies that the reference price $\bar{p}(z)$ corresponds to the farmers' break-even price: we have

$$\bar{a}(z)\psi(z)\bar{p}(z) = c \quad (4)$$

¹⁶This definition holds whatever the case at hand, autarky or free trade, with $\tilde{y}(z)$ and $y(z)$ replaced by $\tilde{y}(z) + \tilde{y}^*(z)$ and $y(z) + y^*(z)$ in the latter case.

¹⁷For the sake of simplicity, we consider neither the production nor the market of agrochemicals in the following. Implicitly, farmers thus are "endowed" with a large stock of agrochemicals that farming does not exhaust, leading to prices equal to 0.

where $\psi(z) \equiv \psi(z, \pi(z), \pi(z), B(z))$, which leads to

$$E[\tilde{p}(z)\tilde{y}(z)] = \bar{p}(z)\bar{a}(z)\psi(z)NB(z) = cNB(z), \quad (5)$$

i.e., the expected value of the crop z domestic production is equal to the sum of the wages and the land value involved in its farming.

Defining the tax index $t \equiv e^{(\tau/c)^2/2} = e^{-(\theta(z)-\pi(z))^2/2}$ where $\pi(z)$ is given by (3), which measures the negative effect of the restricted use of pesticides on the crop's resilience (it diminishes with $\pi(z)$ and is equal to 1 when $\pi(z) = \theta(z)$), we get

$$\psi(z) = \frac{\mu_0 e^{\theta(z)^2/2}}{t[1 + t\kappa(z)B(z)]}. \quad (6)$$

Denoting the crop z maximum expected yield $a(z) \equiv \bar{a}(z)\mu_0 e^{\theta(z)^2/2}$, we get the convenient expressions

$$\bar{p}(z) = ct[1 + t\kappa(z)B(z)]/a(z) \quad (7)$$

and

$$y(z) = \frac{a(z)NB(z)}{t[1 + t\kappa(z)B(z)]} \quad (8)$$

for the crop z reference price and domestic expected production level. Production of crop z decreases with t because of two effects: the corresponding reduction in the use of pesticides has a direct negative impact on the productivity of each plot but also an indirect negative cross-externality effect between plots.

The representative individuals of the two countries share the same preferences over the goods. Their preferences are given by the Cobb-Douglas utility function

$$U = b \ln x_I + (1 - b) \int_0^1 \alpha(z) \ln \tilde{x}(z) dz - hZ$$

where $\int_0^1 \alpha(z) dz = 1$ and $h > 0$. The first two terms correspond to the utility derived from the consumption of industrial and agricultural goods respectively, while the last term

corresponds to the disutility of the environmental damages caused by a domestic use of

$$Z = N \int_0^1 B(z)\pi(z)dz$$

pesticides by farmers. The corresponding demand for the industrial good is $x_I = bR$ where R is the revenue per capita. The rest of the revenue, $(1 - b)R$, is spent on agricultural goods with demand for crop z given by

$$\tilde{x}(z) = \alpha(z)(1 - b)R/\tilde{p}(z) \quad (9)$$

where $\tilde{p}(z)$ depends on the realized production level $\tilde{y}(z)$ and $\alpha(z)$ is the share of the food spending devoted to crop z . We assume that consumers are risk-neutral and thus evaluate their *ex ante* welfare at the average consumption level of crop z , $x(z) \equiv E[\tilde{x}(z)]$. Using (9), the corresponding reference price is $E[1/\tilde{p}(z)]^{-1}$. At market equilibrium under autarky (the same reasoning applies under free trade), as $L\tilde{x} = \tilde{y}$, we obtain from definition (2) and the fact that $L\tilde{x}\tilde{p} = \alpha(z)(1 - b)LR$ that $\bar{p}(z) = E[1/\tilde{p}(z)]^{-1}$, i.e. the consumers' and producers' reference prices are the same. The representative consumer's indirect utility function can be written as¹⁸

$$V(R, Z) = \ln(R) - (1 - b) \int_0^1 \alpha(z) \ln \bar{p}(z) dz - hZ. \quad (10)$$

where $R = (L - N + cN)/L$ since there is no profit at equilibrium.

The government determines the optimal policy by maximizing this utility, taking account of the relationship between pesticides and the value of land.

2.1 Equilibrium under autarky

The agricultural production cost under autarky c_A (the sum of the rental price of the agricultural land and of the wage), is derived from the equilibrium condition on the market for industrial goods.¹⁹ Indeed, due to the constant returns to scale in the industrial sector,

¹⁸Up to a constant given by $b \ln(b) + (1 - b) \int_0^1 \alpha(z) \ln \alpha(z) dz$.

¹⁹Subscript "A" indexes equilibrium values under autarky.

the total spending on industrial products must be equal to the total production cost at equilibrium, i.e.,

$$bLR = b[Nc_A + L - N] = L - N$$

which gives $c_A = (\ell - 1)(1 - b)/b$ where $\ell \equiv L/N > 1$. Consequently, the value of land is positive if $N < (1 - b)L$, a condition assumed to hold in the following. Observe that this value depends neither on the use of pesticides nor on the crops' prices and is the same in both countries in spite of their differences in terms of crop yields. This is due to the Cobb-Douglas preferences (in addition to the constant productivity in the industrial sector). Total domestic revenue is given by

$$LR = Nc_A + L - N = N(\ell - 1)/b$$

for both countries. Market equilibrium on the crop z market implies that total expenses are equal to total production cost, i.e.

$$\alpha(z)(1 - b)LR = NB(z)c_A$$

which gives $B(z) = \alpha(z)$. Using (3) and $\tau/c = \sqrt{2 \ln t}$, we get

$$Z = N \int_0^1 \alpha(z)\theta(z)dz - N\sqrt{2 \ln t}.$$

The optimal level of pesticide (or, equivalently, the optimal pesticides tax index) is determined by maximizing the utility of the representative consumer (10) which reduces to

$$\min_t (1 - b) \int_0^1 \alpha(z) \ln(t[1 + t\kappa(z)\alpha(z)])dz + hZ,$$

a program that applies to both countries. We obtain that the optimal tax index under autarky, t_A , solves

$$\sqrt{2 \ln t_A} \left[1 + \int_0^1 \frac{t_A \kappa(z) \alpha(z)^2}{1 + t_A \kappa(z) \alpha(z)} dz \right] = \frac{Nh}{1 - b}. \quad (11)$$

As $\kappa(z) = \kappa_0 e^{-\theta(z)^2/2}$, the optimal tax decreases when κ_0 increases, with a maximum given by $\tau_A = (L - N)h/b$.

While acreage and pesticide levels are the same in both countries, the average production of each is different because of the differences in crop yields. The revenue being the same in both countries, crop demands are identical but because average production levels are different, break-even prices are also different.

2.2 Free trade equilibrium

First consider the equilibrium on the market for the industrial good: the condition of equalization of total spending with the total production cost is given by

$$b(Nc + L - N + N^*c^* + L^* - N^*) = L - N + L^* - N^*$$

where $L = L^*$ and $N = N^*$. We obtain $c + c^* = 2(\ell - 1)(1 - b)/b$, hence that the worldwide agricultural revenue is the same as under autarky. This is also the case for the total revenue, given by

$$LR + L^*R^* = N[c + c^* + 2(\ell - 1)] = 2N(\ell - 1)/b.$$

As under autarky, the share of the agricultural sector of this revenue is unchanged, given by $1 - b$. For Home, it results in a per-individual revenue given by

$$R = (\ell - 1)[1 + 2q(1 - b)/b], \tag{12}$$

which depends on the fraction $q \equiv c/(c + c^*)$ obtained at equilibrium. This share depends on crop yields and on the environmental tax policies implemented in each country.

In addition to tax policies, competitive advantages also depend on biodiversity effects that evolve with the way land is farmed. Indeed, for a given tax level, because of the production externality effect, the higher the intensity of the farming of a crop (i.e. the more land is devoted to that crop), the lower the average productivity of the land: intensification undermines the competitive advantages apparent under autarky. More precisely, if crop z is

produced by Home only, the market equilibrium condition implies that worldwide expenses on crop z are equal to total production cost, i.e.,

$$2\alpha(z)(\ell - 1)(1 - b)N/b = NB(z)c$$

which can be written as $\alpha(z)/q = B(z)$. Opening to trade could thus correspond to a large increase of the acreage devoted to that crop: e.g., if $q = 1/2$, the total farmland doubles which may seriously impair Home's land productivity for crop z . On the other hand, crop z is produced by both countries if their break-even prices are equal, or using (7), if

$$A(z) = \frac{c^* t^*}{c t} \frac{1 + t^* \kappa(z) B^*(z)}{1 + t \kappa(z) B(z)}. \quad (13)$$

At market equilibrium, worldwide expenditure on crop z is equal to the sum of production costs of the two countries:

$$2\alpha(z)N(\ell - 1)(1 - b)/b = cNB(z) + c^*N^*B^*(z),$$

which can also be written as

$$\alpha(z) = qB(z) + q^*B^*(z). \quad (14)$$

Crop z is thus produced by both countries if there exist $B(z) > 0$ and $B^*(z) > 0$ which solve (13) and (14). As stated formally in the following proposition, this is true for a whole range of crops. More precisely,

Proposition 1 *Specialization is incomplete under free trade: Assuming κ_0 is not too large, both countries produce crops belonging to (\underline{z}, \bar{z}) , $0 \leq \underline{z} < \bar{z} \leq 1$ satisfying*

$$A(\bar{z}) = \frac{t^* q^* + t^* \kappa(\bar{z}) \alpha(\bar{z})}{t q} \quad (15)$$

and

$$A(\underline{z}) = \frac{t^* q^*}{t q + t \kappa(\underline{z}) \alpha(\underline{z})}. \quad (16)$$

The intensity of these crops is given by

$$B(z) = \chi(z)[1 - q\phi(z)]/q \quad (17)$$

where

$$\phi(z) \equiv \frac{1 + A(z)t/t^*}{1 + \alpha(z)t^*\kappa(z)}, \quad (18)$$

$$\chi(z) \equiv \frac{1 + t^*\alpha(z)\kappa(z)}{t\kappa(z)[A(z)t/t^* + t^*/t]} \quad (19)$$

for Home and symmetric expressions hold for Foreign (with $A(z)$ replaced by $1/A(z)$). Crops belonging to $[0, \underline{z}]$ ($[\bar{z}, 1]$) are produced by Home (Foreign) only, with intensity $B(z) = \alpha(z)/q$.

Proof: see the appendix.

Using (15) and (16) with $\kappa_0 = 0$, we end up with $A(\bar{z}) = A(\underline{z}) = (q^*/q)(t^*/t)$ and thus a unique threshold index and complete specialization. With $\kappa_0 > 0$, we have $A(\underline{z}) < A(\bar{z})$ and since A is strictly increasing, $\underline{z} < \bar{z}$: albeit technical differences exist between the two countries, comparative advantages are trimmed by the negative externality that affects national production of each country.

Using (8) and (17), we obtain that Home's expected production of crop $z \in (\underline{z}, \bar{z})$ is

$$y_T(z) = Na(z) \frac{q^* + \alpha(z)\kappa(z)t^* - qA(z)t/t^*}{t\kappa(z)[qt^* + q^*t + tt^*\alpha(z)\kappa(z)]}.$$

Using the symmetric expression for Foreign, we obtain that total expected production under free trade is given by

$$\begin{aligned} y_T^W &= N \frac{\alpha(z)[a^*(z)t_T/t_T^* + a(z)t_T^*/t_T]}{qt_T^* + q^*t_T + \alpha(z)t_T t_T^* \kappa(z)} \\ &= y_A(z) \frac{t_A}{t_T} \frac{1 + \alpha(z)t_A \kappa(z)}{1 + \alpha(z)t_T^* \kappa(z) - q(1 - t_T^*/t_T)} + y_A^*(z) \frac{t_A}{t_T^*} \frac{1 + \alpha(z)t_A \kappa(z)}{1 + \alpha(z)t_T \kappa(z) - q^*(1 - t_T/t_T^*)}. \end{aligned} \quad (20)$$

Observe that with unchanged tax index ($t_T = t_A$) in both countries, we would have $y_T^W = y_A^W$. More generally, compared to the production of these crops under autarky, trade operates as if two distortive effects were at work on domestic productions: a direct tax effect

and a cross-externality effect. The first one, given by the ratio t_A/t_T , measures the impact of the change in the environmental policy on the use of pesticides. The second one also reflects the impact of the policy change on the production externality, but it is not a simple ratio: there is a correcting term given by $q(1 - t_T^*/t_T)$ which also depends on the relative wealth.

For these crops produced by both countries, the break-even price is

$$\bar{p}_m(z) = \frac{2(1-b)(\ell-1)}{ba(z)} \frac{q(t^* - t) + t[1 + \alpha(z)t^*\kappa(z)]}{t^*/t + A(z)t/t^*}. \quad (21)$$

For the other crops, the corresponding expected productions and break-even prices are given by

$$y(z) = \frac{a(z)N\alpha(z)}{t[q + t\kappa(z)\alpha(z)]}$$

and

$$\bar{p}_s(z) = \frac{2(\ell-1)(1-b)t[q + t\kappa(z)\alpha(z)]}{ba(z)} \quad (22)$$

for all $z \leq \underline{z}$ and by symmetric expressions for all crops $z \geq \bar{z}$.

From these preliminary results, it is possible to isolate a first effect due to the production externality by supposing that the environmental taxes and the agricultural revenues of the two countries are unchanged at equilibrium.

Lemma 1 *Suppose that $t_T = t_T^* = t_A$ and $q = 1/2$ at equilibrium under free trade. Then, the break-even prices of the crops produced by only one country are higher for this country than under autarky and are reduced for the other country. For crops produced by both countries, the price for Home (Foreign) is lower (larger) for all crops $z \in (A^{-1}(1), \bar{z}]$ and larger (lower) for all $z \in [\underline{z}, A^{-1}(1))$.*

Proof: see the appendix.

These increases in break-even prices affecting crops produced more intensively under trade than under autarky are the consequences of the cross-externality effects between plots: while the input prices do not change by assumption, the marginal production cost increases, whereas in DFS it is constant.

Of course, environmental taxes and the sharing of the total agricultural revenue evolve with international trade. Using (3), we obtain that the level of pesticides under free trade is given by

$$Z_T = \frac{N}{q} \left\{ \int_0^{\underline{z}} \alpha(z)\theta(z)dz + \int_{\underline{z}}^{\bar{z}} \chi(z)[1 - q\phi(z)]\theta(z)dz \right\} - N\sqrt{2\ln t} \quad (23)$$

which depends on the crops farmed at equilibrium and on how the worldwide agricultural revenue is shared.

To determine this share, we can use the fact that the domestic revenues come from the sale of the goods produced nationally.²⁰ On interval $[0, \underline{z}]$, all revenues spent are collected by Home, while it is only a share $s(z)$ of them on $[\underline{z}, \bar{z}]$. We thus have

$$LR = L - N + (1 - b)(LR + L^*R^*) \left\{ \int_0^{\underline{z}} \alpha(z)dz + \int_{\underline{z}}^{\bar{z}} s(z)\alpha(z)dz \right\}$$

which simplifies to

$$q = \int_0^{\underline{z}} \alpha(z)dz + \int_{\underline{z}}^{\bar{z}} s(z)\alpha(z)dz \quad (24)$$

where $s(z) \equiv y_T(z)/y_T^W(z)$ corresponds to Home's share of expected production relative to the total. Using (5), (14) and (17), we get

$$s(z) = \frac{qB(z)}{qB(z) + q^*B^*(z)} = \frac{qB(z)}{\alpha(z)} = \frac{[1 - q\phi(z)]\chi(z)}{\alpha(z)} \quad (25)$$

and thus

$$q = \frac{\int_0^{\underline{z}} \alpha(z)dz + \int_{\underline{z}}^{\bar{z}} \chi(z)dz}{1 + \int_{\underline{z}}^{\bar{z}} \phi(z)\chi(z)dz}. \quad (26)$$

3 Environmental tax policy and trade

The free trade equilibrium depends on the environmental tax policies (hence t and t^*) that determine the sharing of the worldwide agricultural revenue between the countries. We

²⁰The same expression can be derived using the equilibrium condition on the land market.

model this problem as a Nash equilibrium of a two-stage game where Home and Foreign governments choose simultaneously their tax policies in the first stage, knowing that in the second stage farmers decide which crops to sow and how much pesticide to use. Home's government problem when defining its tax policy corresponds to the following program:

$$\max_t \ln(R) - (1 - b) \left\{ \int_0^{\underline{z}} \alpha(z) \ln \bar{p}_s(z) dz + \int_{\underline{z}}^{\bar{z}} \alpha(z) \ln \bar{p}_m(z) dz + \int_{\bar{z}}^1 \alpha(z) \ln \bar{p}_s^*(z) dz \right\} - hZ \quad (27)$$

where $\bar{p}_s^*(z)$ is given by (22) with t and q replaced by t^* and $q^* = 1 - q$. The optimal environmental policy resulting from this program depends on t^* : maximizing (27) gives Home's best-response to Foreign's policy t^* . We suppose that Foreign's government is in the symmetric situation, i.e. that opening the borders to trade results in a Nash equilibrium of a game where both governments act simultaneously in a strategic way. For the sake of argument, we consider two cases in the following: one in which governments ignore the relationship between their tax policies and their share of the worldwide agricultural revenue, called "non-strategic" free trade equilibrium, and the more realistic case where they reckon that q and thus R depend on t and t^* , which leads to trade equilibrium dubbed "strategic". One may easily show that q is negatively related to t and positively related to t^* . Indeed, a total differentiation of (24) yields, using $s(\bar{z}) = 0$ and $s(\underline{z}) = 1$,

$$\frac{dq}{dt} = \frac{\int_{\underline{z}}^{\bar{z}} [B(z)(d\chi(z)/dt)/\chi(z) - \chi(z)(d\phi(z)/dt)] dz}{1 + \int_{\underline{z}}^{\bar{z}} \phi(z)\chi(z) dz} \quad (28)$$

where it is straightforward from (19) and (18) that $d\chi(z)/dt < 0$ and $d\phi(z)/dt > 0$. Hence, we have $dq/dt < 0$ and since $q + q^* = 1 - b$, $dq^*/dt = -dq/dt > 0$. Because of the strategic substitutability of the environmental taxes, pesticides are used more intensively at the strategic trade equilibrium than when governments act non strategically. To detail this competition effect and assess its interaction with the biodiversity externality that affects production, we consider in the following the case where $\alpha(z) = 1$ and $\theta(z) = \theta$ for all z which implies that $\kappa(z) = \kappa_0 e^{-\theta^2/2} \equiv \kappa$ for all z . Hence, neither the demand nor the externality on consumers' utility distinguishes crops, so we have $Z = N\theta - N\sqrt{2 \ln t}$ whatever the case at hand. We

also suppose that $A(z)$ allows us to obtain symmetric equilibria so that $q = 1/2 = z_s$ at equilibrium. We analyze the two types of free trade equilibria (non-strategic and strategic) assuming first that there is no biodiversity effect, i.e. $\kappa = 0$. While there are no cross-externality effects between fields of the same crop, farmers still have an incentive to spread pesticides on their plots to increase their expected yield. We then introduce the negative production externality ($\kappa > 0$), which induces decreasing returns to scale in the agricultural sector at the national level.

3.1 Trade without biodiversity effect

When $\kappa = 0$, we obtain using (11), $\sqrt{2 \ln t_A} = \tau_A/c_A$ and $c_A = (\ell - 1)(1 - b)/b$, that the environmental tax under autarky is given by $\tau_A = (L - N)h/b$. Under free trade, each country specializes on one segment of the range of crops delimited by threshold z_s which satisfies $A(z_s) = (q^*t^*)/(qt)$ using (13). Equilibrium on the land market, $\int_0^{z_s} B(z)dz = \int_0^{z_s} (1/q)dz = 1$, leads to $q = z_s$: Home's share of the worldwide agricultural revenue is equal to the range of crops produced domestically. Consequently z_s solves $\xi(z_s) = t^*/t$ where $\xi(z) \equiv A(z)z/(1 - z)$ is strictly increasing. In the non-strategic situation, governments do not take into account the effect of their environmental taxes on the sharing of the agricultural revenue. The effect of the tax policies on z_s, q and R are neglected when solving (27). The problem simplifies to

$$\min_t (1 - b)q \ln t - hN\sqrt{2 \ln t}$$

where q is considered as a constant. The first-order condition leads to an optimal tax index that solves

$$\sqrt{2 \ln t} = Nh/[q(1 - b)]. \quad (29)$$

Using $q = z_s$, we get $t = e^{(\frac{Nh}{(1-b)z_s})^2/2}$ and thus a threshold crop that solves

$$\xi(z_s) = e^{\frac{(Nh)^2(1-2z_s)}{2[(1-b)z_s(1-z_s)]^2}}.$$

As $\sqrt{2 \ln t} = \tau/c$ and $c = 2qc_A$, (29) allows us to obtain $\tau = 2\tau_A$ whatever the country's

share of the worldwide agricultural revenue. Stated formally:

Proposition 2 *Suppose that there are no biodiversity effects. Then, at the non-strategic trade equilibrium, the environmental tax is doubled compared to under autarky.*

The intuition is as follows. The environmental policy affects only crops produced domestically. As the range of domestic products is smaller under free trade than under autarky, the impact of the environmental policy on consumer welfare is reduced on the consumption side (prices affected by the tax are only those produced by Home) while it is unchanged on the environmental side. It is thus optimal to raise the tax compared to under autarky. Trade creates a *pollution shifting* effect: while consumers benefit from the low prices allowed by pesticides used abroad, they want pesticides use restricted domestically to reduce pollution. Observe that the resulting situation is not Pareto optimal: indeed, if the two countries could agree on tax levels, each would have to account for the price effect of its tax on the other country's consumers. In our setup, the resulting Pareto optimal tax level is the autarky one.²¹

Now suppose that governments are strategic in the sense that they take into account the effect of the tax on their shares of agricultural revenue. Using (12) and (27) we obtain that Home's best-response to t^* solves

$$\max_{t,q} \left\{ \ln \left(1 + \frac{2q(1-b)}{b} \right) - (1-b) \left[\int_0^q \ln \bar{p}_s(z) dz + \int_q^1 \ln \bar{p}_s^*(z) dz \right] - hZ : q = \xi^{-1} \left(\frac{t^*}{t} \right) \right\}.$$

It is implicitly defined by

$$\sqrt{2 \ln t} \left\{ q + \left[\frac{2}{b + 2q(1-b)} \right] \frac{t^*}{t \xi'(z_s)} \right\} = \frac{Nh}{1-b}. \quad (30)$$

At a symmetric equilibrium, i.e. $q = 1/2 = z_s$, $t = t^*$, which implies that $A(1/2) = 1$ and thus $\xi'(z_s) = A'(1/2) + 4$, we get

$$\sqrt{2 \ln t} = \frac{Nh}{1-b} \left[1 + \frac{A'(1/2)}{A'(1/2) + 8} \right].$$

²¹Indeed, for any sharing $(q, 1-q)$ of the agricultural revenue $2w_A$, the Pareto optimal tax levels solve $\min_t 2(1-b)q \ln t - hN\sqrt{2 \ln t}$ and the equivalent program for Foreign.

The following proposition characterizes the optimal policy at equilibrium:

Proposition 3 *Suppose that there are no biodiversity effects. Then, at the symmetric strategic trade equilibrium, the environmental tax τ verifies $2\tau_A > \tau \geq \tau_A$, with $\tau = \tau_A$ in the limit case where $A'(1/2) = 0$. Moreover, the steeper the comparative advantage function $A(z)$, the larger the environmental tax.*

Proof: see the appendix.

When comparative advantages are not too different, allowing farmers to use more pesticides could have a large impact on the country's market share of agricultural products. At the symmetric equilibrium, countries do not gain market share, but as this rivalry counteracts the *pollution-shifting* effect described above, this ineffective competition in terms of market share results in a situation which is a Pareto improvement compared to the non-strategic one.

3.2 Biodiversity effects

Biodiversity effects create two countervailing distortions in the governments' trade-off we have described above: On the one hand, as specialization increases the production externality, governments should be induced to lower the tax on pesticides. On the other hand, as the externality limits specialization, the effect of the tax on prices concerns a reduced set of crops, which should induce governments to increase the tax. To give a comprehensive appraisal of these countervailing effects, we consider that the relative potential yield function is given by

$$A(z) = \frac{1 + m(2z - 1)}{1 - m(2z - 1)} \quad (31)$$

where $0 < m < 1$. We have $0 < A(0) < 1$, $A(1) > 1$, $A(1/2) = 1$ and

$$A'(z) = \frac{4m}{[1 - m(2z - 1)]^2}.$$

Hence, the larger m , the larger the discrepancy between the countries' relative potential yield away from $z = 1/2$ (graphically, the relative potential yield curve becomes steeper

when m increases).

At a symmetric equilibrium, threshold crops given by (15) and (16) simplify to

$$\bar{z} = \frac{1}{2} + \frac{t\kappa}{2m(1+t\kappa)} \quad (32)$$

and

$$z = \frac{1}{2} - \frac{t\kappa}{2m(1+t\kappa)}. \quad (33)$$

They are equally distant from the center of the range of crops ($1/2$), and the length of the subset of crops produced by both countries,

$$\bar{z} - z = \frac{1}{m} \frac{t\kappa}{1+t\kappa}, \quad (34)$$

increases with κ and t and decreases with the relative potential yield parameter m .

In the non-strategic case, the condition that determines t at the symmetric equilibrium can be written as $(\partial V/\partial t)_{t^*=t} = 0$ where

$$\left. \frac{\partial V}{\partial t} \right|_{t^*=t} = -\frac{1-b}{t} \left[\frac{1+4t\kappa}{1+2t\kappa} z + (\bar{z} - z) \frac{1+2t\kappa}{2(1+t\kappa)} \right] - h \frac{dZ}{dt}. \quad (35)$$

The last term corresponds to the environmental impact on consumers, which is positive since $dZ/dt < 0$. It leads the government to increase the environmental tax. The bracketed term is composed of two elements, the first one corresponding to the price effect on the goods produced locally and the second one to the price effect on the goods produced by both countries. In these terms, biodiversity effects are ambiguous. Indeed, using (33) and (34), the effect on goods produced locally can be rewritten as

$$\frac{1+4t\kappa}{1+2t\kappa} z = \left(1 + \frac{2t\kappa}{1+2t\kappa} \right) \left(\frac{1}{2} - \frac{\bar{z} - z}{2} \right)$$

which shows that compared to the case where $\kappa = 0$, intensification of farming has an ambiguous impact: in the first bracket, the fraction $2t\kappa/(1+2t\kappa)$ tends to reduce the tax on the crops produced locally while the second bracketed term highlights that the range of

crops specific to Home is not half of the total but is reduced by $(\bar{z} - \underline{z})/2$, which tends to increase the tax. This effect due to the decrease in the range of specific crops exceeds the one concerning crops produced by both countries: indeed, we have

$$\frac{\bar{z} - \underline{z}}{2} \left(1 + \frac{2t\kappa}{1 + 2t\kappa} - \frac{1 + 2t\kappa}{1 + t\kappa} \right) = \frac{\bar{z} - \underline{z}}{2} \left(\frac{2t\kappa}{1 + 2t\kappa} - \frac{t\kappa}{1 + t\kappa} \right) > 0.$$

The fact that both countries are producing crops belonging to (\underline{z}, \bar{z}) thus tends to increase the tax level compared to the case where $\kappa = 0$. As a result, the environmental tax could be larger or lower than $2\tau_A$, depending on the relative potential yields of crops. More precisely, we have the following result:

Proposition 4 *Suppose that the relative potential yield function is given by (31). Then, at the symmetric non-strategic trade equilibrium, biodiversity effects result in a reduction of the environmental tax compared to the case where $\kappa = 0$ unless m is very small. Overall, the environmental tax is greater than under autarky.*

Proof: see the appendix.

When the discrepancy in relative potential yields is large between the two countries, specialization is important (the range of crops produced by both is relatively small), and the cross externality effect is optimally contained by an intensive use of pesticides.

In the strategic case, there is a marginal effect of the environmental tax on the share of the agricultural revenue that induces governments to reduce their environmental tax. Indeed, the marginal effect of the tax policy on welfare is given by

$$\frac{dV}{dt} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial q} \frac{dq}{dt}$$

which entails an additional term compared to the non-strategic case, where

$$\frac{\partial V}{\partial q} = \frac{2(1-b)}{b + 2q(1-b)} - (1-b) \left[\frac{\underline{z}}{q + t\kappa} - \frac{1 - \bar{z}}{1 - q + t^*\kappa} + \frac{(\bar{z} - \underline{z})(t^* - t)}{q(t^* - t) + t(1 + t^*\kappa)} \right]. \quad (36)$$

The first term corresponds to the direct effect on welfare due to the increase in revenue

while the remaining terms concern the effects on the price of crops produced domestically, abroad and by both countries respectively. At a symmetric equilibrium, the price effects cancel out, leading to $(\partial V/\partial q)_{t^*=t, q=1/2} = 2(1 - b)$. Using (31), we obtain that (28) is expressed as

$$\left. \frac{dq}{dt} \right|_{t^*=t, q=1/2} = -\frac{3 + 9t\kappa + 4(t\kappa)^2}{12t(1 + t\kappa)[m(1 + t\kappa) + 1]} \quad (37)$$

which decreases with κ : the greater the biodiversity effects, the stronger the negative impact of the environmental tax on the share of the agricultural revenue. However, we show in the appendix that

Proposition 5 *Suppose that the relative potential yield function is given by (31). Then, at the symmetric strategic trade equilibrium the environmental tax is generally greater than under autarky.*

Proof: see the appendix.

4 Trade and volatility

We compare the volatility of crop production through the variation coefficient (VC) which is defined for random variable \tilde{X} as $v(\tilde{X}) \equiv \sigma(\tilde{X})/E[\tilde{X}]$. Assuming that plots are independently affected by pests, the variance of crop z domestic production is given by $V(\tilde{y}(z)) = \bar{a}(z)^2 NB(z)\psi(z)(1 - \psi(z))$. Denoting $\mu(z) \equiv \mu_0 e^{\theta(z)^2/2}$ and using (6) and (8), we get for a crop produced by Home

$$v(\tilde{y}(z)) = \left[\frac{t[1 + t\kappa(z)B(z)] - \mu(z)}{\mu(z)NB(z)} \right]^{1/2}, \quad (38)$$

which increases with t and κ and decreases with N and B . Due to independence, both the variance and the mean increase linearly with N , which results in a negative “scale” effect on volatility. There is also a scale effect associated to intensification (an increase in B) that dominates biodiversity effects. The VCs of the worldwide production under autarky are

given by

$$v(\tilde{y}_A^W(z)) = \frac{[V(\tilde{y}_A(z)) + V(\tilde{y}_A^*(z))]^{1/2}}{\tilde{y}_A(z) + \tilde{y}_A^*(z)} = \frac{[(\bar{a}(z)^2 + \bar{a}^*(z)^2)N\alpha(z)\psi(z)(1 - \psi(z))]^{1/2}}{[\bar{a}(z) + \bar{a}^*(z)]N\alpha(z)\psi(z)}.$$

Using (6), we get

$$v(\tilde{y}_A^W(z)) = \left[1 - \frac{2A(z)}{[1 + A(z)]^2}\right]^{1/2} \left[\frac{t_A[1 + t_A\kappa(z)\alpha(z)] - \mu(z)}{\mu(z)N\alpha(z)}\right]^{1/2}. \quad (39)$$

While the second bracketed term in (39) is similar to (38), the first term reveals a yield effect on production volatility when the same crops are produced by both countries: as $A(z)/[1 + A(z)]^2$ is cap-shaped with a maximum at $A(z) = 1$, this effect is decreasing for $z < 1/2$ and increasing for $z > 1/2$. Hence, the yield effect on volatility is higher the larger the difference between the crop yields of the two countries.²² Assuming symmetry, $\alpha(z) = 1$ and $\theta(z) = \theta$ (which implies $\mu(z) = \mu$ and $\kappa(z) = \kappa$ for all z), the intensification effects are the same for all crops under autarky and for all exclusive crops under trade. This is not the case for crops produced by both countries under trade since farmland intensities vary: while the total share of land devoted to crops at the symmetric equilibrium is the same ($B(z) + B^*(z) = 2$ in any case), the relative importance of Home is decreasing with z (from $B(z) = 2$ to $B(\bar{z}) = 0$), whereas it is constant under autarky. Using $V(\tilde{y}^W(z)) = V(\tilde{y}_T(z)) + V(\tilde{y}_T^*(z))$ we have

$$v(\tilde{y}^W(z))^2 = s(z)^2v(\tilde{y}(z))^2 + s^*(z)^2v(\tilde{y}^*(z))^2,$$

which gives

$$v(\tilde{y}^W(z)) = \left[\frac{(1 + t\kappa)(1 + 2t\kappa) - \mu\kappa}{2\mu N\kappa} - \frac{2(1 + t\kappa)^2}{\mu N\kappa} \frac{A(z)}{[1 + A(z)]^2}\right]^{1/2}. \quad (40)$$

As under autarky, there is a yield effect at work: the volatility index is decreasing over $[\underline{z}, 1/2)$, increasing over $(1/2, \bar{z}]$, and thus reaches a minimum at $z = 1/2$. Comparing the

²²With identical yields, i.e. $A(z) = 1$, this term is equal to $\sqrt{2}/2$, the scale effect of a doubling of farmland.

VCS under autarky and trade, we obtain the following result:

Proposition 6 *Without biodiversity effects, trade could potentially reduce the production volatility of all crops. However, because of a higher environmental tax than under autarky only the volatility of crops for which countries have large comparative advantages are reduced (if any). With biodiversity effects, trade increases the production volatility of crops produced by both countries and of the specialized crops with moderate competitive advantage. The volatility of large comparative advantage crops is reduced only if biodiversity effects are small and the environmental tax not too different from its autarky level.*

Proof: see the appendix.

Because the probability of the Bernoulli distribution $\psi(z)$ doesn't depend on N , the distribution of crop z production converges to the normal distribution $\mathcal{N}(y(z), \sigma(\tilde{y}(z)))$ when N is large. This allows us to derive some characteristics of the price distributions. In particular, the break-even price $\bar{p}(z)$ corresponds to crop z 's median price: we have $\Pr\{\tilde{p}(z) \leq \bar{p}(z)\} = \Pr\{\tilde{y}(z) \geq y(z)\} = 1/2$ since the normal distribution is symmetric. Consequently, as $\bar{p}(z)$ is lower than the average market price $p(z)$ due to the correlation between prices and quantities, the price distribution is asymmetric.

This asymmetry is also revealed by the upper and lower bounds of confidence intervals implied by the distributions of crop production. Denoting by $y_d^\gamma(z)$ and $y_u^\gamma(z)$ the lower and upper quantity bounds of the confidence interval at confidence level $1 - \gamma$, the corresponding price bounds are derived from $\Pr\{y_d^\gamma(z) \leq \tilde{y}(z) \leq y_u^\gamma(z)\} = \Pr\{p_d^\gamma(z) \leq \tilde{p}(z) \leq p_u^\gamma(z)\}$: we have $p_u^\gamma(z) \equiv \alpha(z)(1 - b)LR/y_d^\gamma(z)$ and $p_d^\gamma(z) \equiv \alpha(z)(1 - b)LR/y_u^\gamma(z)$. Because of the symmetry of the quantity distributions, $y_d^\gamma(z)$ and $y_u^\gamma(z)$ are equally distant from $y(z)$, but since prices and quantities are inversely related, this is not the case for $p_d^\gamma(z)$ and $p_u^\gamma(z)$. The following proposition completes these general features of the price distributions with some useful approximations:²³

Proposition 7 *The expected value and the standard deviation of crop prices are approxi-*

²³Where Φ denotes the cumulative distribution function of the standard normal distribution.

mated by

$$p(z) \approx \bar{p}(z)[1 + v(\tilde{y}(z))^2] \quad (41)$$

and

$$\sigma(\tilde{p}(z)) \approx \bar{p}(z)v(\tilde{y}(z))(1 - v(\tilde{y}(z))^2)^{1/2}.$$

Confidence intervals at confidence level $1 - \gamma$ are delimited by $p_u^\gamma(z) = p(z) + s_u^\gamma\sigma(\tilde{p}(z))$ and $p_d^\gamma(z) = p(z) + s_d^\gamma\sigma(\tilde{p}(z))$ with

$$s_u^\gamma \approx \frac{v(\tilde{y}(z)) + s_\gamma}{(1 - v(\tilde{y}(z))^2)^{1/2}(1 - s_\gamma v(\tilde{y}(z)))} \quad (42)$$

and

$$s_d^\gamma \approx \frac{v(\tilde{y}(z)) - s_\gamma}{(1 - v(\tilde{y}(z))^2)^{1/2}(1 + s_\gamma v(\tilde{y}(z)))} \quad (43)$$

where $s_\gamma \equiv \Phi^{-1}(1 - \gamma/2)$. They are approximately equal to

$$p_u^\gamma(z) \approx \bar{p}(z) \left(1 + v(\tilde{y}(z))^2 + v(\tilde{y}(z)) \frac{v(\tilde{y}(z)) + s_\gamma}{1 - s_\gamma v(\tilde{y}(z))} \right) \quad (44)$$

and

$$p_d^\gamma(z) \approx \bar{p}(z) \left(1 + v(\tilde{y}(z))^2 + v(\tilde{y}(z)) \frac{v(\tilde{y}(z)) - s_\gamma}{1 + s_\gamma v(\tilde{y}(z))} \right). \quad (45)$$

Proof: see the appendix.

Because prices and quantities are inversely related, we have $s_d < s_u$, i.e the price distribution is skewed to the right: its right tail is longer and fatter than its left tail. The consequences on volatility are that the chances that a crop price is very low compared to the expected price, i.e., $\tilde{p}(z) \leq \bar{p}(z) < p(z)$, are larger than the chances of a high price, i.e. $\tilde{p}(z) > p(z)$, since $1/2 = \Pr\{\tilde{p}(z) \geq \bar{p}(z)\} > \Pr\{\tilde{p}(z) > p(z)\}$, but the possible range of high prices is wider than the range of low prices: $p_u(z) - p(z) > p(z) - p_d(z) > \bar{p}(z) - p_d(z)$.²⁴

Figures 2 and 3 illustrate these findings. The solid curves with the marks depict the

²⁴This is of course dependent on the convexity of the demand function. Indeed, the condition $p_u(z) - p(z) > p(z) - p_d(z)$, is equivalently written $p(z) < [p_u(z) + p_d(z)]/2 = [D^{-1}(y_d(z)) + D^{-1}(y_u(z))]/2$. As $p(z) > \bar{p}(z) = D^{-1}(y(z))$ where $y(z) = [y_d(z) + y_u(z)]/2$, a necessary condition is $D^{-1}([y_d(z) + y_u(z)]/2) \leq [D^{-1}(y_d(z)) + D^{-1}(y_u(z))]/2$, hence $D^{-1}(y)$ must be convex from the Jensen's inequality.

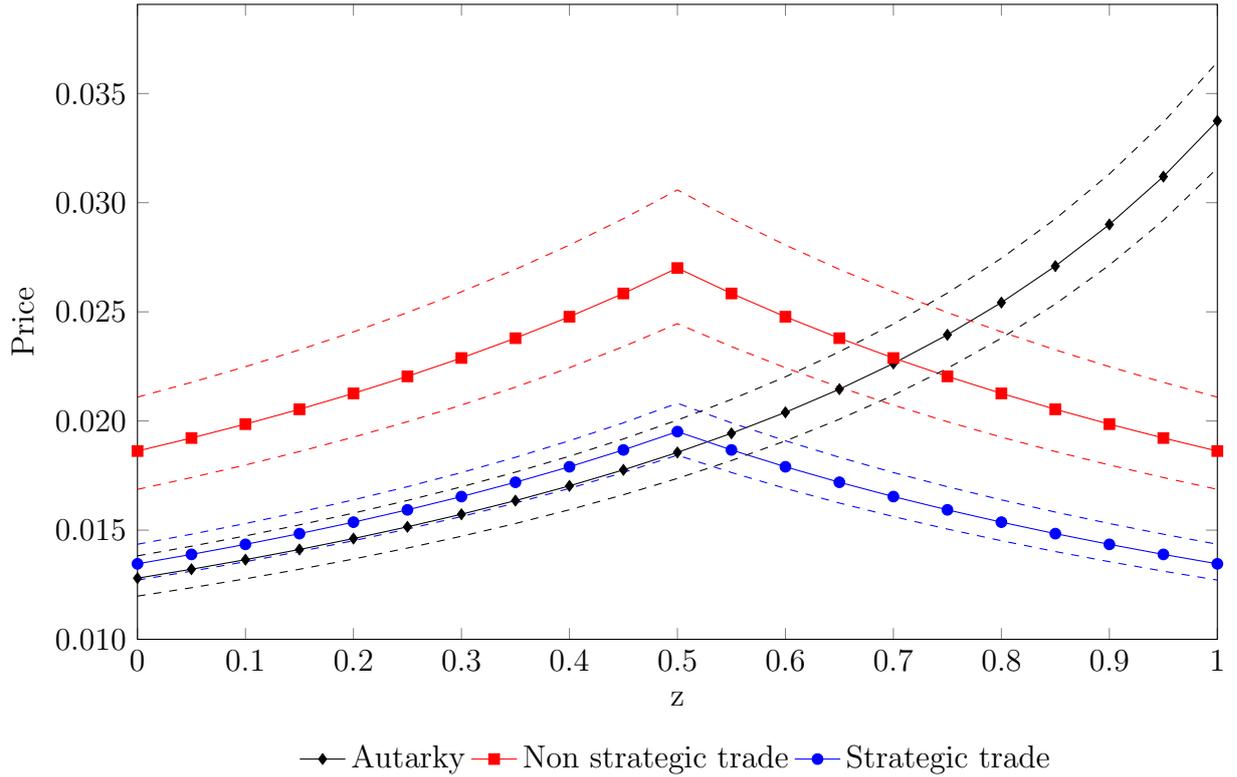


Figure 2: Average prices and price volatility (confidence interval at 95% confidence level) without biodiversity effects. $\kappa = 0$, $N=100$, $h = 10^{-3}$, $b = 0.8$, $\ell = 20$, $m = 0.45$, $\mu = 1$, $a(1/2) = 29$.

crop average price in each case as indicated (depicted are Home's autarky prices). The corresponding dashed curves depict the approximate values of $p_d(z)$ and $p_u(z)$. The vertical distance between these curves defines a confidence interval for a crop price at a confidence level equal to 95%. Compared to autarky, the non-strategic average prices are larger for almost all crops, and the confidence intervals are very large. This is due to the tightening of the pesticide regulations mentioned above. The strategic effects that loosen these regulations induce lower average prices and confidence intervals. Biodiversity effects are reflected in Fig. 3 by strategic and non-strategic average price curves that encompass a flat portion around $z = 1/2$ which corresponds to the mix-production range. Confidence intervals over these ranges are smaller the closer the crop is to $z = 1/2$.

Tables 1 and 2 assess the way the biodiversity effects and potential yield differentials affect the trade pattern, the environmental tax policy, the food price average level and the

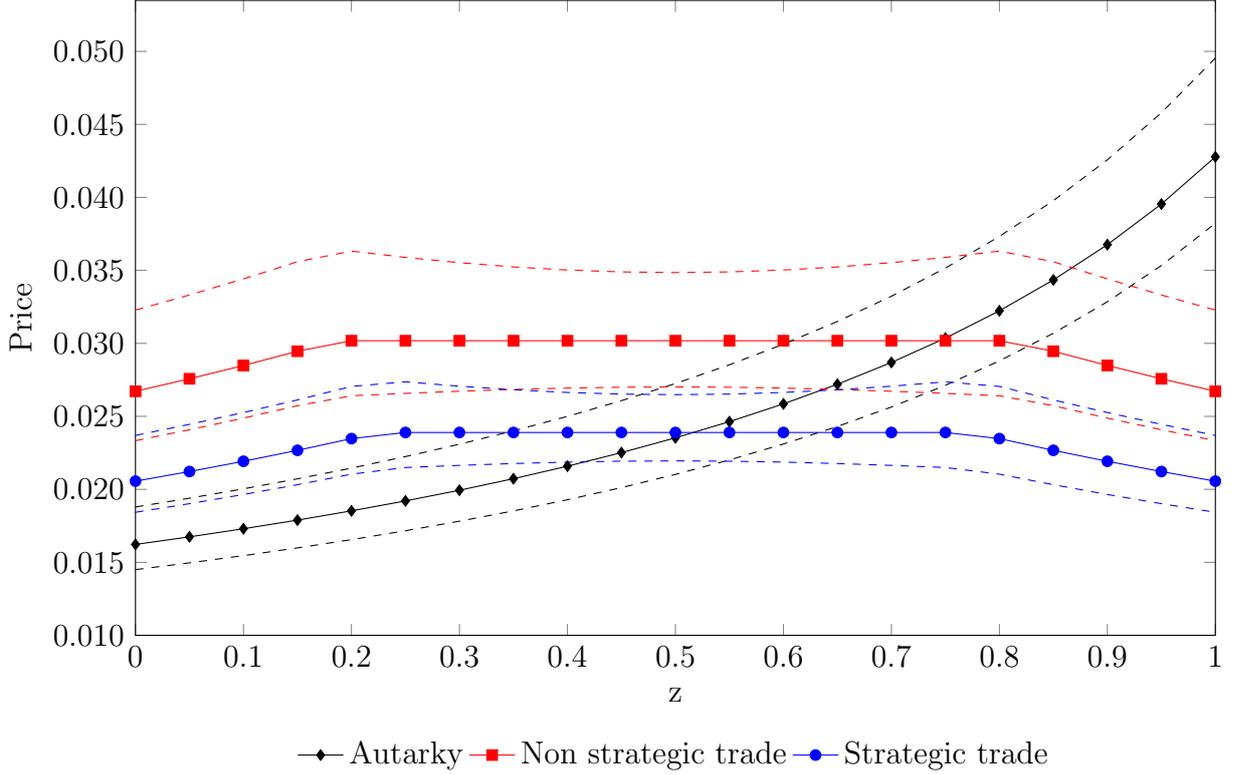


Figure 3: Average prices and price volatility (confidence interval at 95% confidence level) with biodiversity effects. $\kappa = 0.3$, $N=100$, $h = 10^{-3}$, $b = 0.8$, $\ell = 20$, $m = 0.45$, $\mu = 1$, $a(1/2) = 29$.

crop price volatilities.²⁵

Table 1 summarizes the impacts due to the biodiversity effects: the larger κ is, the larger the negative impact of the cultivation intensity on the survival probability of plots. Biodiversity effects play against the specialization induced by trade: the range of crops produced by both countries increases when κ rises. Since biodiversity effects impede production, prices increase with κ as shown by the trend in the food price index reported in table 1. The larger κ is, the more effective pesticides are in reducing the negative externality due to the cultivation

²⁵The tables report the tax as a percentage of production costs τ/c (c is the same whatever the case at hand). The food price index is defined as $\int_0^1 p(z)y(z)dz / \int_0^1 y(z)dz$ and approximated as detailed in the appendix. A food price index of 2 means that the average price of agricultural goods equals 2% of the industrial goods price. The VC of domestic crop prices is the same for all crops under autarky. Worldwide VCs indicated under autarky are derived from a fictitious price distribution corresponding to the sum of Home and Foreign productions. Worldwide VCs evolve between minimum value $v(\tilde{p}^W(1/2))$ and maximum values $v(\tilde{p}^W(0))$ (autarky) and $v(\tilde{p}^W(\underline{z}))$ (trade).

intensity and, therefore, the lower the environmental tax. Nevertheless, pesticides do not allow farmers to eradicate the biodiversity effect. Thus, even if more pesticides are applied when κ increases, the food price volatility increases. The tax on pesticides is more stringent under trade than under autarky, even when trade is strategic, for the reasons detailed above. In our simulations, non strategic taxes are more than 58% higher than strategic ones. Food price index levels reported in table 1 show that the decrease in food prices due to trade is lower the larger the biodiversity effects. The ratio between the VC levels at $z = 1/2$ of the worldwide production and the domestic production under autarky gives the size of the scale effect on prices (e.g. for $\kappa = 0.1$, it corresponds to $5.46/8.62=63.3\%$ which is lower than on the production side $\sqrt{2}/2 \approx 70.7\%$). The yield effect is maximum for crop $z = 0$: for $\kappa = 0.1$, it corresponds to an additional $(6.24-5.46)/8.62=9\%$. Table 1 also illustrates that the price volatility increases for crops produced by both countries even for small values of κ because of the significant increases in the environmental tax under strategic trade compared to autarky: even for $\kappa = 0.1$, the volatility of the specialized crops (7.07%) is larger than under autarky (6.24%).

Table 2 describes the impact of the potential crop yield differentials: the larger m is, the larger the difference in the potential yield of each crop (away from $z = 1/2$), and the greater the difference in comparative advantages of the two countries. The range of crops that are produced by both countries under free trade ($\bar{z} - \underline{z}$) decreases with m . Effects of m on the environmental tax depend on whether trade is strategic or not. Under non strategic trade, the impact of the environmental policy on consumer welfare goes through the crop prices only. Therefore, the larger m is, the larger the specialization and the lower the tax on pesticides. Under strategic trade, the use of pesticides may have a significant impact on the market share of the country, particularly when comparative advantages are sufficiently close. This is reflected in table 2: the smaller m is, the lower the environmental tax. The way the food price index varies with m also differs between the non strategic and the strategic case. Two effects play on the quantities produced: on the one hand, when m raises, the productivity increases and, on the other hand, when the environmental tax increases, the quantities produced decrease. In the non strategic case, when m increases, the tax decreases.

Table 1: Sensitivity analysis on parameter κ

Variables (%)	Values of κ				
	0.1	0.3	0.5	0.7	0.9
Autarky					
τ/c	45.46	40.15	37.07	35.05	33.61
Food price index	2.03	2.38	2.73	3.09	3.45
$v(\tilde{p}(\cdot))$	8.62	10.10	11.39	12.53	13.57
$v(\tilde{p}^W(1/2))$	5.46	6.29	6.99	7.60	8.14
$v(\tilde{p}^W(0))$	6.24	7.16	7.94	8.61	9.20
Non strategic trade					
τ/c	83.29	73.23	69.03	66.49	64.68
Food price index	2.28	2.83	3.35	3.85	4.32
$v(\tilde{p}^W(1/2))$	8.00	8.84	9.76	10.62	11.42
$v(\tilde{p}^W(\underline{z}))$	8.82	10.61	12.19	13.55	14.75
$\bar{z} - \underline{z}$	20.66	46.96	64.70	77.69	87.65
Strategic trade					
τ/c	52.51	44.83	40.99	38.53	36.76
Food price index	1.77	2.25	2.68	3.08	3.46
$v(\tilde{p}^W(1/2))$	6.39	7.37	8.28	9.10	9.85
$v(\tilde{p}^W(\underline{z}))$	7.07	8.91	10.42	11.70	12.84
$\bar{z} - \underline{z}$	17.16	41.51	58.71	71.64	81.76

$$m = 0.6, N=100, h = 10^{-3}, b = 0.8, \ell = 20, \mu = 0.7, a(1/2) = 290.$$

Then, the two effects go in the same direction, the quantities produced increase and prices decline. Under strategic trade, m and the environmental tax both increase, their effects are countervailing on the quantities produced. The impact of the increase in the productivity prevails and the food price index decreases, even if the environmental tax is raised. Finally, in both strategic and non strategic cases, price volatility increases when the use of pesticides declines, i.e. when the environmental tax raises.

5 Fertilizers

Our focus being on biodiversity effects in agricultural production, we have so far not discussed the impact of trade on the use of fertilizers. However, because they have considerable effects on both crop yields and on the environment (and human health), changes in the openness of

Table 2: Sensitivity analysis on parameter m

Variables (%)	Values of m			
	0.2	0.4	0.6	0.8
Autarky				
τ/c	45.46	45.46	45.46	45.46
Food price index	2.03	2.03	2.03	2.03
$v(\tilde{p}(\cdot))$	8.62	8.62	8.62	8.62
$v(\tilde{p}^W(1/2))$	5.46	5.46	5.46	5.46
$v(\tilde{p}^W(0))$	5.55	5.83	6.24	6.76
Non strategic trade				
τ/c	86.02	83.94	83.29	82.97
Food price index	2.70	2.47	2.28	2.12
$v(\tilde{p}^W(1/2))$	8.19	8.05	8.00	7.98
$v(\tilde{p}^W(\underline{z}))$	9.02	8.87	8.82	8.80
$\bar{z} - \underline{z}$	63.23	31.13	20.66	15.46
Strategic trade				
τ/c	46.82	49.83	52.51	54.81
Food price index	2.00	1.88	1.77	1.68
$v(\tilde{p}^W(1/2))$	6.18	6.29	6.39	6.49
$v(\tilde{p}^W(\underline{z}))$	6.84	6.96	7.07	7.17
$\bar{z} - \underline{z}$	50.19	25.43	17.16	13.01

$\kappa = 0.1, N=100, h = 10^{-3}, b = 0.8, \ell = 20, \mu = 0.7, a(1/2) = 290.$

countries to trade are likely to impact the way their use is regulated.²⁶ We may thus expect that crop price volatility is also affected through this channel. It is possible to analyze these changes by considering that crop z 's potential yield $\bar{a}(z)$ is the result of the intrinsic quality of land and the quantity of fertilizers spread on the field, $g(z)$. Denoting by $a_0(z)$ the potential crop z yield absent any treatment, we have $\bar{a}(z) = a_0(z)f(g(z))$ with $f(0) = 1, f'(g) > 0$ and $f''(g) < 0$. Total use of fertilizers, given by $G = N \int_0^1 B(z)g(z)dz$, has a negative impact on consumer welfare due to environmental damages. As pesticides, fertilizers have a direct positive impact on crop yields, but unlike pesticides, their productive impact is limited to

²⁶Commercial fertilizers are responsible for 30% to 50% of crop yields (Stewart et al., 2005). Sutton et al. (2011a,b) find that half of the nitrogen added to farm fields ends up polluting water or air. Excess of nitrogen and phosphorus in freshwater increases cancer risk and creates aquatic and marine dead zones through eutrophication. In the air, nitrates contribute to ozone generation which causes respiratory and cardiovascular diseases. Sutton et al. (2011a) estimates that in the European Union the benefits of nitrogen for agriculture through the increase in yields amount to 25 to 130 billion euros per year and that they cause between 70 and 320 billion euros per year in damage.

the field they are spread on. Hence, the trade-off that defines the fertilizer policy is similar to the one of the pesticide regulation without biodiversity effects. While under autarky domestic consumers bear all the costs and reap all the benefits of the fertilizers used by their fellow farmers, this is no longer the case in free trade: they benefit from the crops produced abroad and share the advantages of a productive national sector with foreign consumers. As a result, restrictions on fertilizers are tighter under free trade than under autarky, with the same caveat as for pesticides: governments may use the fertilizer policy strategically. How lenient they are depends on the impact of fertilizers on relative yields: the more responsive is the relative yields function, i.e. the larger $f'(g)$, the lower the restrictions.

6 Conclusion

We have shown that biotic risk factors such as pests that affect the productivity of farming create biodiversity effects that modify standard results of trade models. An explicit account of their effects on production allows us to clarify the distribution of idiosyncratic risks affecting farming which depends on the countries' openness to trade. These production shocks translate into the price distribution and are impacted by environmental policies. Of course, these productive shocks are not the only ones affecting food prices, but they are an additional factor in their distributions that may explain their greater volatility compared to manufacture prices.

While these effects are analytically apparent within the standard two-country Ricardian model, an extension to a more encompassing setup involving a larger number of countries as permitted by Eaton & Kortum (2002) and applied to agricultural trade by Costinot & Donaldson (2012) and Costinot et al. (2012) is necessary to investigate their scope statistically. These studies incorporate a stochastic component to determine the pattern of trade but it is not related to the production process and somehow arbitrary. Our analysis offers an interesting route to ground these approaches at least in the case of agricultural products. Introducing these biodiversity effects should allow for a better assessment of the importance of trade costs in determining the pattern of trade.

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Appendix

A Proof of Proposition 1

For given t, t^*, c, c^* , (13) and (14) define a system of two linear equations with two unknowns. Solving this system gives (17). By definition of threshold crops \underline{z} and \bar{z} , we must have $B(z) = 0$ for all $z \geq \bar{z}$ and $B^*(z) = 0$ for all $z \leq \underline{z}$. This implies that we must have $\phi(z) \geq 1/q$ or all $z \geq \bar{z}$ and $\phi^*(z) \geq 1/q^*$ for all $z \leq \underline{z}$. Differentiating (18) and its counterpart for Foreign, we get

$$\dot{\phi}(z) \equiv \frac{\phi'(z)}{\phi(z)} = \frac{A'(z)}{t^*/t + A(z)} - t^* \kappa_0 e^{-\theta(z)^2/2} \frac{\alpha'(z) - \theta'(z)\alpha(z)}{1 + t^* \alpha(z)\kappa(z)}$$

and

$$\dot{\phi}^*(z) = -\dot{A}(z) + \frac{A'(z)}{t^*/t + A(z)} - t \kappa_0 e^{-\theta(z)^2/2} \frac{\alpha'(z) - \theta'(z)\alpha(z)}{1 + t^* \alpha(z)\kappa(z)}.$$

Suppose $\kappa_0 = 0$: As $\dot{A}(z) > 0$, we have $\dot{\phi}(z) = A'(z)/[t^*/t + A(z)] > 0$ and $\dot{\phi}^*(z) = -\dot{A}(z)(t^*/t)/[t^*/t + A(z)] < 0$. Both conditions are thus satisfied if κ_0 is sufficiently small. They are also satisfied whatever the value of κ_0 if $\alpha(z)$ and $\theta(z)$ are constant as supposed in the symmetric case considered in section 3. Eq. (15) and (16) are derived from $\phi(\bar{z}) = 1/q$ and $\phi^*(\underline{z}) = 1/q^*$ respectively. Using these equations, we obtain $A(\bar{z})/A(\underline{z}) = [q^* + t^* \kappa(\bar{z})\alpha(\bar{z})][q + t \kappa(\underline{z})\alpha(\underline{z})]/(qq^*) > 1$. As $A(z)$ is increasing, we thus have $\bar{z} > \underline{z}$.

B Proof of Lemma 1

Consider the crops produced more intensively by Home (symmetric results hold for Foreign). For all $z < \underline{z}$, it is obvious by setting $t = t_A$ and $q = 1/2$ in (22) that $\bar{p}(z) > \bar{p}_A(z)$ given by (7) where $c = (\ell - 1)(1 - b)/b$ and $B(z) = \alpha(z)$. For Foreign, we have $\bar{p}(z) < \bar{p}_A^*(z)$ iff

$$\frac{1 + t_A \kappa(z)\alpha(z)}{1 + 2t_A \kappa(z)\alpha(z)} > A(z).$$

As $\phi^*(z) > 1/q^*$ for all $z < \underline{z}$, we obtain by setting $t = t_A$ and $q^* = 1/2$ that

$$\frac{1 + 1/A(z)}{1 + t_A \kappa(z) \alpha(z)} > 2$$

for all $z < \underline{z}$, which gives, re-arranging terms

$$A(z) < \frac{1}{1 + 2t_A \kappa(z) \alpha(z)} < \frac{1 + t_A \kappa(z) \alpha(z)}{1 + 2t_A \kappa(z) \alpha(z)}$$

hence $\bar{p}(z) < \bar{p}_A^*(z)$ for all $z < \underline{z}$. For crops z in $[\underline{z}, \bar{z}]$, we obtain using (21) that $\bar{p}(z) < \bar{p}_A(z)$ iff $2/[1 + A(z)] < 1$, hence the result.

C Proof of Proposition 3

Using (22) which simplifies to

$$\bar{p}_s(z) = \frac{2(\ell - 1)(1 - b)tq}{ba(z)}$$

and denoting by ϖ the Lagrange multiplier associated with the constraint, the first-order condition with respect to t gives

$$-(1 - b)q + \frac{Nh}{\sqrt{2 \ln t}} - \varpi \frac{1}{\xi'(z_s)} \frac{t^*}{t} = 0$$

and the one with respect to q

$$\varpi = \frac{2(1 - b)}{b + 2q(1 - b)}.$$

Plugging the latter expression into the former and rearranging terms gives (30). At a symmetric equilibrium, using $\xi'(z_s) = A'(1/2) + 4$ and $\tau_A/c_A = Nh/(1 - b)$, we get

$$\frac{\tau}{c} = \frac{\tau_A}{c_A} \left[1 + \frac{A'(1/2)}{A'(1/2) + 8} \right]$$

where $c = c_A$. We thus have

$$\tau = \tau_A \left[1 + \frac{A'(1/2)}{A'(1/2) + 8} \right].$$

Denoting $M \equiv A'(1/2)$, we get $\lim_{M \rightarrow 0} \tau = \tau_A$, $\lim_{M \rightarrow +\infty} \tau = 2\tau_A$ and

$$\frac{d\tau}{dM} = \frac{8\tau_A}{(M + 8)^2} > 0.$$

D Proof of Proposition 4

Differentiating (27) with respect to t , we obtain the following FOC

$$\frac{\partial V}{\partial t} = -(1-b) \left[\int_{\underline{z}}^{\bar{z}} \frac{q + 2t\kappa}{t(q + t\kappa)} + \frac{(\bar{z} - \underline{z})(q^* + t^*\kappa)}{q(t^* - t) + t(1 + t^*\kappa)} - \int_{\underline{z}}^{\bar{z}} \frac{A(z)/t^* - t^*/t^2}{A(z)t/t^* + t^*/t} dz \right] - h \frac{dZ}{dt}.$$

At a symmetric equilibrium, using $(A(z) - 1)/(A(z) + 1) = m(2z - 1)$ and integrating gives (35). Using (34) in (35) and collecting terms, we arrive at

$$\frac{\partial V}{\partial t} = \frac{Nh}{t\sqrt{2\ln t}} - (1-b) \left[\frac{1}{2t} + \kappa \frac{2m(1 + t\kappa)^2 - t\kappa}{2m(1 + 2t\kappa)(1 + t\kappa)^2} \right].$$

Denote by t_0 the optimal tax when there is no cross-externality effects, i.e. $\kappa = 0$. It verifies (29) where $q = 1/2$. We have

$$\frac{\partial V}{\partial t} \Big|_{t=t_0} = -(1-b)\kappa \frac{2m(1 + t_0\kappa)^2 - t_0\kappa}{2m(1 + 2t_0\kappa)(1 + t_0\kappa)^2}$$

which is positive if

$$m \leq \frac{t_0\kappa}{2(1 + t_0\kappa)^2} \leq 1/8.$$

We also have

$$\begin{aligned}
\frac{1}{1-b} \frac{\partial V}{\partial t} \Big|_{t=t_A} &= \frac{1}{2t_A} + \frac{\kappa}{1+t_A\kappa} - \kappa \frac{2m(1+t_A\kappa)^2 - t_A\kappa}{2m(1+2t_A\kappa)(1+t_A\kappa)^2} \\
&= \frac{1}{2t_A} + \frac{t_A\kappa^2}{1+t_A\kappa} \frac{2m(1+t_A\kappa) + 1}{2m(1+2t_A\kappa)(1+t_A\kappa)} > 0
\end{aligned} \tag{46}$$

hence $t > t_A$ at the optimum.

E Proof of Proposition 5

At a symmetric equilibrium (36) simplifies to

$$\frac{\partial V}{\partial q} = 2(1-b) + 2(1-b) \frac{1 - (\bar{z} + \underline{z})}{1 + 2t\kappa}$$

where $\bar{z} + \underline{z} = 1$, hence $\partial V/\partial q = 2(1-b)$. We also obtain using (28) that at a symmetric equilibrium

$$\frac{dq}{dt} = - \frac{\frac{2(1+t\kappa)}{t} \int_{\underline{z}}^{\bar{z}} \frac{A(z)}{[A(z)+1]^2} dz - \frac{1}{2t} \int_{\underline{z}}^{\bar{z}} \frac{A(z)}{A(z)+1} dz}{t\kappa + \bar{z} - \underline{z}}.$$

Denoting $m_0 = (m+1)/2$, using

$$\begin{aligned}
\int_{\underline{z}}^{\bar{z}} \frac{A(z)}{[A(z)+1]^2} dz &= \int_{\underline{z}}^{\bar{z}} [m_0 - mz - (m_0 - mz)^2] dz \\
&= (\bar{z} - \underline{z})(m_0 - m/2) + \frac{(m_0 - m\bar{z})^3 - (m_0 - m\underline{z})^3}{3m} \\
&= \frac{t\kappa}{2m(1+t\kappa)} \frac{2(1+t\kappa)^2 + 1 + 2t\kappa}{6(1+t\kappa)^2}
\end{aligned}$$

and

$$\int_{\underline{z}}^{\bar{z}} \frac{A(z)}{A(z)+1} dz = \int_{\underline{z}}^{\bar{z}} (1 - m_0 + mz) dz = (\bar{z} - \underline{z})(1 - m_0 + m/2) = \frac{t\kappa}{2m(1+t\kappa)}$$

yields (37). Differentiating, we get

$$\frac{dq^2}{dt d\kappa} = -\frac{(4-m)(1+t\kappa)^2 + 4m(1+t\kappa) + 2}{12(1+t\kappa)^2[m(1+t\kappa) + 1]^2} < 0.$$

Using (46) we obtain

$$\frac{1}{1-b} \frac{dV}{dt} \Big|_{t=t_A} = \frac{1}{2t_A} + \frac{t_A \kappa^2 [2m(1+t_A \kappa) + 1]}{2m(1+2t_A \kappa)(1+t_A \kappa)^2} - \frac{3 + 9t_A \kappa + 4(t_A \kappa)^2}{6t_A(1+t_A \kappa)[m(1+t_A \kappa) + 1]} \quad (47)$$

where the second term diverges when m tends to 0 and decreases with m while the last term increases with m (decreases in absolute value). Assuming that m is large enough and $\kappa \approx 0$, we have

$$\begin{aligned} \frac{1}{1-b} \frac{dV}{dt} \Big|_{t=t_A} &= \frac{1}{2t_A} - \frac{1 + 3t_A \kappa}{2t_A [m(1 + 2t_A \kappa) + 1 + t_A \kappa]} + o(\kappa) \\ &= \frac{1}{2t_A} \left[\frac{m(1 + 2t_A \kappa) - 2t_A \kappa}{m(1 + 2t_A \kappa) + (1 + t_A \kappa)} \right] + o(\kappa) \end{aligned}$$

where the bracketed term is negative if

$$m < \bar{m} \equiv \frac{2t_A \kappa}{1 + 2t_A \kappa}.$$

However, as the second term of (47) diverges when m approaches 0, we cannot neglect this term unless m is above some threshold $\underline{m} > t_A \kappa / (1 + t_A \kappa)$. Indeed, in the case where $m = t_A \kappa / (1 + t_A \kappa)$, we have

$$\begin{aligned} \frac{1}{1-b} \frac{dV}{dt} \Big|_{t=t_A} &= \frac{1}{2t_A} + \frac{\kappa}{2(1+t_A \kappa)} - \frac{3 + 9t_A \kappa + 4(t_A \kappa)^2}{6t_A(1+t_A \kappa)^2} \\ &= \frac{(t_A \kappa)^2}{3t_A(1+t_A \kappa)^2} > 0. \end{aligned}$$

Hence, assuming κ is small enough, cases where the environmental tax at equilibrium is

lower than under autarky correspond to $m \in [\underline{m}, \bar{m}]$ where

$$\bar{m} - \underline{m} < \frac{2t_A\kappa}{1+2t_A\kappa} - \frac{t_A\kappa}{1+t_A\kappa} = \frac{t_A\kappa}{(1+t_A\kappa)(1+2t_A\kappa)} \leq \frac{\sqrt{2}}{4+3\sqrt{2}} \approx 0.17.$$

F Proof of Proposition 6

Without biodiversity effects, using (38) and (39) with $\kappa = 0$, we obtain that $v(\tilde{y}_A^W(z)) \geq v(\tilde{y}^W(z))$ iff

$$1 - \frac{4A(z)}{[1+A(z)]^2} \geq \frac{t-t_A}{t_A-\mu}.$$

As $A(z)/[1+A(z)]^2$ is cap-shaped with a maximum equal to $1/4$ at $z = 1/2$, this condition is satisfied for all z only if $t = t_A$. With $t > t_A$, it could be satisfied for z belonging only to one of the extremes of the crops' range, i.e. for z either close to 0 or close to 1, if $t - t_A$ is small enough. With biodiversity effects, for $z \in [\underline{z}, \bar{z}]$, using (40) and assuming that $t \geq t_A$, we obtain that $v(\tilde{y}_A^W(z)) \geq v(\tilde{y}^W(z))$ iff

$$\frac{4A(z)}{[1+A(z)]^2} - 1 \geq \frac{t\kappa(1+t\kappa) - \kappa t_A(1+t_A\kappa)}{(1+t\kappa)^2 - \kappa t_A(1+t_A\kappa) + \mu\kappa}$$

which is impossible unless $t = t_A$ and $z = 1/2$ since the last term is positive. For all $z \in [0, \underline{z}] \cup [\bar{z}, 1]$, using (38), we have $v(\tilde{y}_A^W(z)) \geq v(\tilde{y}^W(z))$ iff

$$2 - \frac{4A(z)}{[1+A(z)]^2} \geq \frac{t(1+2t\kappa) - \mu}{t_A(1+t_A\kappa) - \mu}.$$

A necessary condition is given by $2 > [t(1+2t\kappa) - \mu]/[t_A(1+t_A\kappa) - \mu]$, or re-arranging terms $t_A - \mu > (t - t_A)[1 + 2\kappa(t + t_A)]$ which is satisfied only if $t - t_A$ is not too large and κ sufficiently small.

G Proof of Proposition 7

A second-order approximation gives

$$\begin{aligned}\tilde{p}(z) &= \frac{\alpha(z)(1-b)LR}{\tilde{y}(z)} \approx \frac{\alpha(z)(1-b)LR}{y(z)} \left[1 - \frac{\tilde{y}(z) - y(z)}{y(z)} + \left(\frac{\tilde{y}(z) - y(z)}{y(z)} \right)^2 \right] \\ &= \bar{p}(z) \left[2 - \frac{\tilde{y}(z)}{y(z)} + \left(\frac{\tilde{y}(z) - y(z)}{y(z)} \right)^2 \right]\end{aligned}$$

and thus

$$p(z) \approx \bar{p}(z) \left[1 + E \left(\frac{\tilde{y}(z) - y(z)}{y(z)} \right)^2 \right] = \bar{p}(z)[1 + v(\tilde{y}(z))^2].$$

A first order approximation yields

$$\frac{E[(\tilde{p}(z) - \bar{p}(z))^2]^{1/2}}{\bar{p}(z)} \approx E \left[\left(1 - \frac{\tilde{y}(z)}{y(z)} \right)^2 \right]^{1/2} = v(\tilde{y}(z)),$$

which gives

$$\begin{aligned}\sigma(\tilde{p}(z)) &\approx E \left[\tilde{p}(z) - \bar{p}(z) - \bar{p}(z)v(\tilde{y}(z))^2 \right]^{1/2} = \left(E \left[(\tilde{p}(z) - \bar{p}(z))^2 \right] - \bar{p}(z)^2 v(\tilde{y}(z))^4 \right)^{1/2} \\ &= \bar{p}(z)v(\tilde{y}(z))(1 - v(\tilde{y}(z))^2)^{1/2}.\end{aligned}$$

From $p_u^\gamma(z) = p(z) + s_u^\gamma \sigma(\tilde{p}(z))$ we get

$$\begin{aligned}s_u^\gamma &\approx \frac{1}{\sigma(\tilde{p}(z))} \left(\frac{\alpha(z)(1-b)LR}{y(z) - s_\gamma \sigma(\tilde{y}(z))} - \bar{p}(z)(1 + v(\tilde{y}(z))^2) \right) \\ &= \frac{\bar{p}(z)}{\sigma(\tilde{p}(z))} \frac{y(z)(1 + v(\tilde{y}(z))^2) - y(z) + s_\gamma \sigma(\tilde{y}(z))}{y(z) - s_\gamma \sigma(\tilde{y}(z))} \\ &\approx \frac{y(z)v(\tilde{y}(z))^2 + s_\gamma \sigma(\tilde{y}(z))}{v(\tilde{y}(z))(1 - v(\tilde{y}(z))^2)^{1/2}(y(z) - s_\gamma \sigma(\tilde{y}(z)))} \\ &= \frac{v(\tilde{y}(z)) + s_\gamma}{(1 - v(\tilde{y}(z))^2)^{1/2}(1 - s_\gamma v(\tilde{y}(z)))}\end{aligned}$$

which gives (44). Similar derivations for $p_d^\gamma(z) = E[\tilde{p}(z)] - s_d^\gamma \sigma(\tilde{p}(z))$ yield (43) and (45).

H Food Price Index

The food price index is defined as

$$F = \int_0^1 y(z)p(z)dz / \int_0^1 y(z)dz$$

where $p(z)$ is approximated by (41). As we also have $\bar{p}(z)y(z) = cNB(z)$, we get $p(z)y(z) \approx cNB(z)[1 + v(\tilde{y}(z))^2]$. In autarky, $c = (\ell - 1)(1 - b)/b$. Using $B(z) = 1$, $\theta(z) = \theta$, which implies $\mu(z) = \mu$, and $y(z) = a(z)N/[t(1 + t\kappa)]$ gives

$$\int_0^1 p(z)y(z)dz = \frac{N(\ell - 1)(1 - b)}{b} \left[1 + \frac{t(1 + t\kappa) - \mu}{N\mu} \right].$$

Using $a(z) = \mu_0 e^{\theta^2/2} [1 - m(2z - 1)]$ yields

$$\int_0^1 y(z)dz = \frac{N}{t(1 + t\kappa)} \int_0^1 \mu_0 e^{\theta^2/2} [1 - m(2z - 1)] dz = \frac{N\mu_0 e^{\theta^2/2}}{t(1 + t\kappa)}$$

and thus

$$F_A = \frac{(\ell - 1)(1 - b)t(1 + t\kappa)}{b\mu_0 e^{\theta^2/2}} \left[1 + \frac{t(1 + t\kappa) - \mu}{N\mu} \right].$$

Under free trade, we have $\bar{p}(z)y(z) = 2N(\ell - 1)(1 - b)/b$. $B(z) = 2$ for crops produced by only one country and we get, using $\underline{z} + \bar{z} = 1$,

$$\int_0^{\underline{z}} p_s(z)y(z)dz + \int_{\bar{z}}^1 p_s(z)y^*(z)dz = \frac{4\underline{z}N(\ell - 1)(1 - b)}{b} \left[1 + \frac{t(1 + 2t\kappa) - \mu}{2\mu N} \right].$$

For crops produced by both countries we obtain

$$\begin{aligned} \int_{\underline{z}}^{\bar{z}} p_m(z)y_W(z)dz &= \int_{\underline{z}}^{\bar{z}} \frac{2N(\ell - 1)(1 - b)}{b} \left[1 + \frac{(1 + t\kappa)(1 + 2t\kappa) - \mu\kappa}{2\mu N\kappa} - \frac{2(1 + t\kappa)^2}{\mu N\kappa} \frac{A(z)}{[1 + A(z)]^2} \right] dz \\ &= \frac{2N(\ell - 1)(1 - b)}{b} \left\{ \left[1 + \frac{(1 + t\kappa)(1 + 2t\kappa) - \mu\kappa}{2\mu N\kappa} \right] (\bar{z} - \underline{z}) - \frac{t[2(1 + t\kappa)^2 + 1 + 2t\kappa]}{\mu N m \delta (1 + t\kappa)} \right\}. \end{aligned}$$

The corresponding quantities are given by

$$\begin{aligned} \int_0^{\underline{z}} y(z)dz + \int_{\bar{z}}^1 y(z)dz &= \frac{2N}{t(1+2t\kappa)} \left\{ \int_0^{\underline{z}} a(z)dz + \int_{\bar{z}}^1 a^*(z)dz \right\} \\ &= \frac{2N\mu_0 e^{\theta^2/2}}{t(1+2t\kappa)} [\underline{z}(m+1-m\underline{z}) + 1 - \bar{z}(1+m\bar{z}-m)] \end{aligned}$$

and

$$\int_{\underline{z}}^{\bar{z}} y_W(z)dz = \frac{N}{t(1+t\kappa)} \int_{\underline{z}}^{\bar{z}} [a^*(z) + a(z)]dz = \frac{2N\mu_0 e^{\theta^2/2}(\bar{z} - \underline{z})}{t(1+t\kappa)} = \frac{2N\mu_0 e^{\theta^2/2}\kappa}{m(1+t\kappa)^2}.$$