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MARKET MICROSTRUCTURE, INFORMATION AGGREGATION AND EQUILIBRIUM UNIQUENESS IN A GLOBAL GAME*

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Abstract. This paper studies the outcome of a two-stage global game wherein a market-based asset price determined at the trading stage of the game provides an endogenous public signal about the fundamental that affects traders' decisions in the coordination stage of the game. The microstructure of the trading stage is one in which informed traders may place market orders –rather than full demand schedules– and where a competitive market-making sector sets the price. Because market-order traders face price execution risk, they trade less aggressively on their private information than demand-schedule traders, which slows down information aggregation and limits the informativeness of the asset price. When all traders place market orders, the precision of the price signal is bounded above and the outcome of the coordination stage is *unique* as the noise in the private signals vanishes. More generally, in an asset market with both market-order and demand-schedule traders, the presence of the former may drastically limit the range of parameters leading to multiple equilibria. This is especially true when traders optimise over their type of order, in which case market-order traders tend to overwhelm the market when the precision of the private signal is large.

Keywords: Market microstructure; Information aggregation; Global game.
JEL Codes: C72, D82, G14.

1. Introduction

In this paper, we study the outcome of a two-stage global game wherein a market-based asset price determined at the trading stage of the game provides an endogenous public signal about the fundamental that affects traders' decision in the coordination stage of the game. Our motivation for doing so is to examine the concern, first raised by Atkeson (2001) and then made formal by Angeletos and Werning (2006), that a publicly observed market price may aggregate dispersed information so effectively as to crowd out private signals in traders' assessment of the fundamental, and in so doing facilitate their coordination on a self-fulfilling outcome. As illustrated by Angeletos and Werning (2006), this may precisely occur as the noise in the private signal vanishes, a result that directly challenges Carlsson and van Damme (1993) and Morris and Shin (1998)'s argument that a small perturbation of the full-information game restores equilibrium uniqueness.

The possibility that a small amount of private noise lead to multiplicity rather than uniqueness of equilibrium outcomes arises when the precision of the endogenous public signal grows faster than that of the underlying exogenous private signals at high levels of precision.[1] In what follows we

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[1]See also Hellwig et al. (2006). Hellwig (2002) emphasised the role of the relative precison of public versus private information in determining the outcome of the game. Angeletos et al. (2006) study global games wherein endogenous public information comes from policy choices rather than an asset price.

show that this property crucially depends on the type of market microstructure that one assumes at the trading stage of the game, and what this microstructure implies for the amount of private information that is aggregated into the asset price. We substantiate this point by considering a market microstructure for the trading stage wherein informed traders may place either full demand schedules or more basic *market orders*, i.e., order to sell or buy a fixed quantity of assets unconditional on the execution price.[2] All orders (from informed and noise traders) are then aggregated into an asset price by a competitive market-making sector. This price provides the endogenous public signal that informed traders may use to coordinate a speculative attack in the second stage of the game.

To summarise, our results are as follows. In a pure market-order market (see Vives, 1995), the precision of the endogneous public signal provided by the asset price is bounded above, even when the precision of the underlying private signals is very (arbitrarily) large. This is due to the competition of two forces. On the one hand, greater precision leads informed traders to trade more aggressively on their private information by opening the possiblity of reaping large payoffs from trading. On the other hand, this very aggressiveness renders the asset price very volatile ex post (after all market orders have irreversibly been aggregated), which raises the conditional volatility of the net payoff, i.e., the terminal dividend minus the trading price of the asset. The first effect makes the informativeness of the price an increasing function of the precision of private signals. The second effect, however, runs counter the first effect: it deters market-order traders, which are exposed to price execution risk, from placing large orders. As the precision of private information increases the strength of the second effect gradually catches up with that of the first effect and the precision of the price signal increases more and more slowly. This boundedness of the information conveyed by the price overturns the result in Angeletos and Werning (2006), because (endogenous) public information can no longer crowd out (exogenous) private information in traders' Bayesian learning of the fundamental. As a consequence, a high level of precision of private information can again uniquely pin down the outcome of the coordination game – and we are back to Morris and Shin (1998). When the share of market-order traders is still exogenous but not necessarily equal to one, our result must be qualified in the following sense. While it is again true that as private information becomes infinitely precise then so does public information, just as in the pure demand schedule/Walrasian auctioneer case of Angeletos and Werning (2006), it is nevertheless the case that for large range degrees of precision the uniqueness region can be greatly expanded relative to pure demand schedule case.

We finally examine the case where informed traders can choose their order type ex ante, where the tradeoff is between placing expensive demand schedules or cheap market orders.[3] We notably study the impact of this choice on the equilibrium share of market-order traders and, by way of consequence, on the outcome of the coordination stage. We show that as private noise vanishes

[2] See Brown and Zhang (1997), Wald and Horrigan (2005) and Vives (2008) for further disscussion of the importance of market orders in actual asset markets. In Challe and Chrétien (2015) we study a market microstructure with a more general information structure than that in the present paper (by considering correlation of private noise across traders, which significantly complicates trader's Bayesian learning), but we do not look at the implications of market orders for the outcome of a coordination game.

[3] In as much as demand schedules allow full conditionality of trades on the realised trading price, they are much more (in fact, infinitely more) complex that market orders (which are not conditional on the price). Therefore, demand schedules should be more expensive, as we assume them to be.

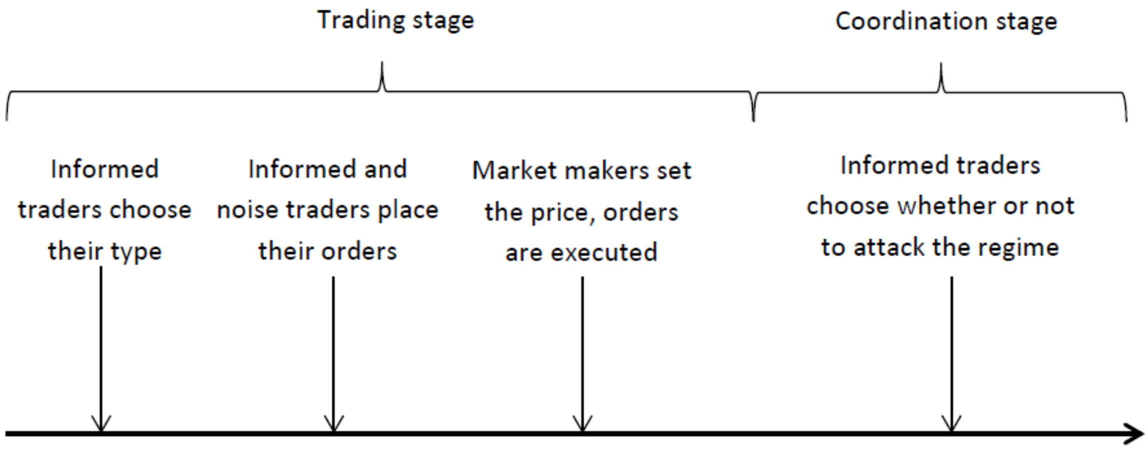

Figure 1: Sequence of events.

the equilibrium is always interior (i.e., market-order and demand-schedule traders are both in positive measure), but market-order traders ultimately overwhelm the market (i.e., their measure tends to one). As a result, the rate of convergence of the precision of the public signal under endogenous order type is half that under exogenous order types. This implies that the endogenous adjustment of the share of market-order traders further reduces the multiplicity region as private noise decreases, relative to the case where this share is exogenous.

The rest of the paper is organised as follows. Section 2 presents two stages of the game. Section 3 analyses the outcome of the game when the shares of market-order and demand-schedule traders are exogenous. Section 4 studies the endogenous determination of those shares, and how this affects the size of the multiplicity versus uniqueness regions. Section 5 concludes the paper. All the proofs appear in the Appendix.

2. The model

Following Angeletos and Werning (2006), we consider a two-stage global game wherein a continuum of informed traders $i \in I = [0,1]$ trade an asset in a *trading stage* before deciding whether to attack the regime in the *coordination stage* –see Figure 1. Before the game starts, an unobserved fundamental θ is drawn from the distribution $\mathcal{N}\left(\bar{\theta}, \alpha_\theta^{-1}\right)$ (which is also the common prior of informed traders) and affects both asset payoffs in the trading stage and the ability of the government to withstand a speculative attack in the coordination stage. Every informed trader gets two noisy signals about θ: an exogenous private signal $x_i = \theta + \alpha_x^{-1/2}\xi_i$, with $\alpha_x > 0$, $\xi_i \sim \mathcal{N}(0,1)$ and $\operatorname{cov}(\theta, \xi) = \operatorname{cov}\left(\xi_i, \xi_{j\neq i}\right) = 0$, and a public signal $z = \theta + \alpha_z^{-1/2}\tilde{\varepsilon}$, with $\tilde{\varepsilon} \sim \mathcal{N}(0,1)$ and $\operatorname{cov}(\tilde{\varepsilon}, \theta) = \operatorname{cov}(\tilde{\varepsilon}, \xi) = 0$. This public signal is taken as given by informed traders in the coordination stage but is endogenously determined in the trading stage of the game.

2.1. Coordination stage. In the coordination stage informed trader i chooses action $a_i \in \{0,1\}$, with $a_i = 1$ $(= 0)$ if the trader is attacking (not attacking) the regime.[4] The mass of attacking traders is thus $A = \int_0^1 a_i \mathrm{d}i$, and it is assumed that the regime collapses whenever

[4] This section parallels Angeletos and Werning (2006), except for the fact that we consider a nondiffuse prior, as is required for the asset demands of market-order traders to be well defined. For the sake of comparability we keep the same notations as theirs whenever this is possible.

$A > \theta$. Trader i's payoff at that stage is $U(a_i, A, \theta) = a_i(\mathbf{1}_{A>\theta} - c)$, where $c \in (0,1)$ is the cost of attacking the regime. Hence, the payoff for a trader who successfully (unsuccessfully) attacks the regime is $1 - c > 0$ $(-c < 0)$, while one who does not attack earns 0 for sure. In equilibrium A only depends on the aggregates (θ, z), i.e., $A = A(\theta, z)$. Trader i's policy function is $a(x_i, z) = \arg\max_{a\in\{0,1\}} \mathbb{E}[U(a, A(\theta,z), \theta) | x_i, z]$, with $A(\theta, z) = \int_{\mathbb{R}} a(x_i, z) f(x_i|\theta)\mathrm{d}x_i$, where $f(x|\theta)$ is the density of $x|\theta$ $(\sim \mathcal{N}(\theta, \alpha_x^{-1}))$.

We can restrict our attention to monotone equilibria, in which informed trader i chooses $a_i = 1$ (i.e., to attack) if and only if $x_i < x^*(z)$ (i.e., the trader is sufficiently pessimistic about θ, given (x_i, z)), where $x^*(z)$ is a strategy threshold common to all traders, to be determined as part of the equilibrium.[5] In such equilibria the mass of traders attacking the regime is $A(\theta, z) = \Pr(x_i < x^*(z)|\theta) = \Phi\left(\sqrt{\alpha_x}(x^*(z) - \theta)\right)$, where $\Phi(.)$ is the c.d.f. of the standard normal. The regime is abandoned whenever $A(\theta, z) > \theta$, or equivalently whenever $\theta < \theta^*(z)$, where $\theta^*(z)$ solves

$$\Phi\left(\sqrt{\alpha_x}\left(x^*(z) - \theta^*(z)\right)\right) = \theta^*(z). \tag{1}$$

It directly follows from the properties of $\Phi(.)$ that the latter equation has a unique solution $\theta^*(z) \in (0,1)$ for all $x^*(z) \in \mathbb{R}$, and that $\theta^*(z)$ is continuous and strictly increasing in $x^*(z)$. This has the following interpretation. The threshold $x^*(z)$ summarises traders' *aggressiveness*, in that for any (θ, z) a greater value of $x^*(z)$ increases the attacking mass A. $\theta^*(z)$ represents the regime's *fragility*, in that for any z a greater value of $\theta^*(z)$ widens the range of realisations of θ leading to the regime's collapse. Hence equation (1) summarises the way in which a greater level of aggressiveness on the part of traders raises the fragility of the regime.

Since the regime collapses if and only if $\theta \leq \theta^*(z)$, trader i's expected payoff from attacking the regime is $\Pr(\theta \leq \theta^*(z)| x_i, z) - c$. In monotone equilibrium the threshold $x^*(z)$ corresponds to the signal received by the marginal trader (i.e. that indifferent between attacking or not) and hence must satisfy $\Pr(\theta \leq \theta^*(z)| x^*(z), z) = c$. Given the assumed information structure, $\theta| z, x$ is normally distributed with variance $\alpha^{-1} \equiv (\alpha_x + \alpha_z + \alpha_\theta)^{-1}$ and mean $\alpha^{-1}\left(\alpha_x x + \alpha_z z + \alpha_\theta \bar{\theta}\right)$. Hence, indifference of the marginal trader requires:

$$\Phi\left(\sqrt{\alpha_x + \alpha_z + \alpha_\theta}\left(\frac{\alpha_x x^*(z) + \alpha_z z + \alpha_\theta \bar{\theta}}{\alpha_x + \alpha_z + \alpha_\theta} - \theta^*(z)\right)\right) = 1 - c \tag{2}$$

The latter equality implicitly defines traders' aggressiveness $x^*(z) \in \mathbb{R}$ as a continuous, strictly increasing function of the regimes' fragility $\theta^*(z) \in (0,1)$ –i.e., a fragile regime makes it safer to bet on its collapse, thereby inducing a rightward shift in $x^*(z)$. Solving both (1) and (2) for $x^*(z)$ and equating the two gives the equation $G(\theta^*) = \Gamma(z)$, where

$$G(\theta^*) \equiv \Phi^{-1}(\theta^*) - \frac{\alpha_z + \alpha_\theta}{\sqrt{\alpha_x}}\theta^*,\ \Gamma(z) = \sqrt{1 + \frac{\alpha_z + \alpha_\theta}{\alpha_x}}\Phi^{-1}(1 - c) - \frac{\alpha_\theta}{\sqrt{\alpha_x}}\bar{\theta} - \frac{\alpha_z}{\sqrt{\alpha_x}}z,$$

so we have $\theta^*(z) \in G^{-1}(\Gamma(z))$. When $G : (0,1) \to \mathbb{R}$ is monotonically increasing, it necessarily crosses the $\Gamma(z)$ line exactly once whatever the value of z. When $G(.)$ is non-monotonic there are values of z such that $G(.)$ crosses the $\Gamma(z)$ more than once. It then follows from the minimal value

[5] See, e.g., Morris and Shin (2004, Lemma 1).

of $\partial G/\partial\theta$ that there exists a unique Bayesian Nash equilibrium for all $z \in \mathbb{R}$ if and only if:

$$\sqrt{2\pi\alpha_x} \geq \alpha_z + \alpha_\theta. \tag{3}$$

2.2. Trading stage. Let us now turn to the trading stage, which will determine both the distribution (ex ante) and the realisation (ex post) of the public signal z. We assume that informed traders have access to two assets: (i) a riskless bond in perfectly elastic supply and paying out a constant interest rate; and (ii) a risky asset with trading price p and payoff θ. Aside from informed traders, noise traders place a net asset demand for the risky asset of $\varepsilon \sim \mathcal{N}\left(0, \alpha_\varepsilon^{-1}\right)$. Following Vives (1995) and Medrano (1996), we consider a market microstructure wherein (a) all or some traders place *market orders* (rather than full demand schedules), and (b) a (competitive, risk-neutral) market-making sector sets the price p. In contrast to a demand schedule, a market order is conditional on the private signal x_i but not on the execution price p; once placed, it is executed irrevocably at whatever value of p is set by market makers. The market-making sector observes the order book $L(.)$ emanating from informed and noise traders and sets the price p; competition among risk-neutral market makers then causes them to undercut each other until $p = \mathbb{E}(\theta | L(.))$. Note that $L(.)$ is itself a function of p whenever a positive mass of informed traders places demand schedules.

Let us call $M \subset I$ the set of market-order traders and $I \backslash M$ the complementary set of demand-schedules traders, and define $\nu = \int_{I\backslash M} \mathrm{d}i \in [0,1]$ and $1-\nu$ as the measures of those sets. In what follows we will consider both the case where M and $I\backslash M$ are exogenous (Section 3) and that where they are endogenous (Section 4). All informed traders have zero initial wealth (this is without loss of generality) and preferences $V(w_i;\gamma_i) = -e^{-\gamma_i w_i}$, where γ_i and $w_i = (\theta - p)\,k_i$ are the risk aversion coefficient and end-of-stage wealth of trader i, respectively. Private signals are assumed to be independent of risk tolerance, i.e.,

$$\forall J \subset I, \ \int_J (\xi_i/\gamma_i)\,\mathrm{d}i = 0.$$

An *equilibrium* of the trading stage is a pair of investment functions for demand-schedule $(k_{I\backslash M}(x_i, p; \gamma_i))$ and market-order $(k_M(x_i;\gamma_i))$ traders and a price function $p(\theta,\varepsilon)$ such that:

- $k_{I\backslash M}(.)$ and $k_M(.)$ maximise traders' expected utility:

$$\forall i \in I\backslash M, \ k_{I\backslash M}(x_i, p; \gamma_i) \in \arg\max_{k\in\mathbb{R}} \mathbb{E}[V((\theta-p)k;\gamma_i)|x_i, p], \tag{4}$$

$$\forall i \in M, \ k_M(x_i;\gamma_i) \in \arg\max_{k\in\mathbb{R}} \mathbb{E}[V((\theta-p)k;\gamma_i)|x_i]; \tag{5}$$

- The market-making sector sets $p = \mathbb{E}[\theta|L(.)]$, where

$$L(p) = \int_{I\backslash M} k_{I\backslash M}(x_i, p;\gamma_i)\mathrm{d}i + \int_M k_M(x_i;\gamma_i)\mathrm{d}i + \varepsilon. \tag{6}$$

We then have the following lemma.

Lemma 1. The trading stage has a unique linear Bayesian equilibrium, which is characterised by:

- the investment functions

$$k_{I\backslash M}(x_i, p; \gamma_i) = \frac{\alpha_x}{\gamma_i}(x_i - p), \; k_M(x_i; \gamma_i) = \frac{\beta}{\gamma_i}(x_i - \overline{\theta}), \text{ with } \beta = \frac{1}{\alpha_x^{-1} + \alpha_\theta^{-1} - (\alpha_\theta + B^2\alpha_\varepsilon)^{-1}};$$

- the price function

$$p(\theta, \varepsilon) = (1 - \lambda B)\,\overline{\theta} + \lambda B\left(\theta + B^{-1}\varepsilon\right), \text{ with } \lambda = \frac{B\alpha_\varepsilon}{B^2\alpha_\varepsilon + \alpha_\theta}. \tag{7}$$

In those functions $B > 0$ is the unique real solution to the cubic equation:

$$B = \alpha_x \frac{\nu}{\gamma_{I\backslash M}} + \frac{1-\nu}{\gamma_M}\left(\frac{1}{\alpha_x} + \frac{1}{\alpha_\theta} - \frac{1}{\alpha_\theta + \alpha_\varepsilon B^2}\right)^{-1}, \tag{8}$$

where $\gamma_{I\backslash M}^{-1}$ and γ_M^{-1} are the average risk tolerance coefficients of demand-schedule and market-order traders:

$$\gamma_{I\backslash M}^{-1} = \frac{1}{\nu}\int_{I\backslash M} \gamma_i^{-1} \mathrm{d}i, \; \gamma_M^{-1} = \frac{1}{1-\nu}\int_M \gamma_i^{-1} \mathrm{d}i.$$

Equation (7) implies that observing p is equivalent to observing $\theta + B^{-1}\varepsilon$. Thus, the endogenous public signal z about θ is $z = \theta + B^{-1}\varepsilon$ (i.e., $\tilde{\varepsilon} = B^{-1}\varepsilon$) and has precision $\alpha_z = B^2\alpha_\varepsilon$. We then infer from (3) that equilibrium uniqueness in the coordination stage requires

$$\sqrt{2\pi\alpha_x} \geq B^2\alpha_\varepsilon + \alpha_\theta. \tag{9}$$

Note that when $\alpha_\theta \to 0$ (i.e., the prior is diffuse), $\nu = 1$ and $\gamma_i = \gamma \; \forall i \in [0, 1]$ (i.e., all informed traders share the same preferences and place demand schedules), then equation (8) gives $B = \gamma^{-1}\alpha_x$, so that $p = \theta + \gamma\sigma_x^2\varepsilon$. Condition (9) then becomes $\sqrt{2\pi\alpha_x} \geq \gamma^{-2}\alpha_x^2\alpha_\varepsilon$, which is identical to that in Angeletos and Werning (2006).

3. EXOGENOUS TRADER TYPES

3.1. Markets with a single type. We first consider the case where all informed traders place market orders in the trading stage (as in Vives, 1995) and that where they all place full demand schedules. We then have the following proposition.

Proposition 1. If all informed traders place market orders in the trading stage, then the outcome of the coordination stage is unique as $\alpha_x \to +\infty$. If all informed traders place demand schedules, then are multiple equilibrium outcomes in the coordination stage as $\alpha_x \to +\infty$.

Proposition 1 implies that when the market microstructure of the trading stage is such that traders place market orders and market makers set the price, then one recovers the original property in Morris and Shin (1998), according to which the outcome of the coordination stage is unique as the noise in the private signal vanishes. In contrast, in a pure demand-schedule market one recovers the basic result in Angeletos and Werning (2006), in which a Walrasian auctioneer (rather than a market-making sector) sets the price. The intuition for this difference is as follows. In a pure *demand-schedule* market ($\nu = 1$), informed traders are able to condition their trades on the trading price, so the only source of risk they face concerns the true value of the fundamental. As the precision of the private signals increases, traders collectively trade more aggressively against any discrepancy between the observed price p and the fundamental θ. Formally, from Lemma 1 the total asset demand by informed traders in a pure demand-schedule market is given by:

$$\int_{I\backslash M} \frac{\alpha_x}{\gamma_i}(\theta + \alpha_x^{-1/2}\xi_i - p)\mathrm{d}i = \alpha_x \left(\int_{I\backslash M} \gamma_i^{-1}\mathrm{d}i \right) (\theta - p) = \frac{\alpha_x}{\gamma_{I\backslash M}}(\theta - p),$$

which implies that $B = \gamma_{I\backslash M}^{-1}\alpha_x \to +\infty$, and thus $p \to \theta$, as $\alpha_x \to +\infty$. In the limit p becomes perfectly informative of θ (i.e. $\alpha_z \to +\infty$); this eventually causes every traders to choose a_i based exclusively on p (rather than x_i) in the second stage and thereby facilitates coordination on a self-fulfilling outcome. In contrast, in a pure *market-order* market ($\nu = 0$) informed traders do *not* condition their trades on p and hence face a residual payoff risk even as the x_is get more and more informative of θ. This payoff risk leads market-order traders to trade less aggressively on the basis of their private signal, which limits the amount of information that is aggregated into the price. Formally, from Lemma 1 again the total asset demand by informed traders in a pure market-order market is:

$$\int_{M} \frac{\beta}{\gamma_i}(\theta + \alpha_x^{-1/2}\xi_i - \overline{\theta})\mathrm{d}i = \beta \left(\int_{M} \gamma_i^{-1}\mathrm{d}i \right) (\theta - \overline{\theta}) = \frac{\beta}{\gamma_M}(\theta - \overline{\theta}),$$

In the limit as $\alpha_x \to +\infty$ we have $\alpha_z = B^2\alpha_\varepsilon < +\infty$, i.e. the precision of the public signal is bounded above. In this case *private* signals ultimately determine actions in the second stage of the game, which hinders coordination on a self-fulfilling outcome.

3.2. Market with both types. We now consider the case where both M and $I\backslash M$ have positive measure. For expositional clarity we assume here that traders share the same preferences, i.e., $\gamma_i = \gamma > 0$ $\forall i \in [0,1]$, but the result can straightforwardly be extended to the case of heterogenous γs. We first note that for all $\nu \in [0,1]$ it is necessarily the case that $0 < B \leq \alpha_x/\gamma$, with $B = \alpha_x/\gamma$ when $\nu = 1$ and $B < \alpha_x/\gamma$ when $\nu < 1$.[6] Moreover, the uniqueness condition (9) implies that, for $(\alpha_x, \alpha_\varepsilon, \alpha_\theta)$ given, the uniqueness region expands as B falls. Thus, if for a given set of parameters we are in the uniqueness region when $\nu = 1$ (i.e., the pure demand-schedule case), then we are also in the uniqueness region when $\nu < 1$ (and both types coexist). Total differencing (8) and using the fact that $0 < B \leq \alpha_x/\gamma$, we find that, for any $(\alpha_x, \alpha_\varepsilon, \alpha_\theta)$ given and for all $\nu \in [0,1]$ we have

$$\frac{\partial B}{\partial \nu} = \frac{\alpha_x - \beta}{\gamma + 2(1-\nu)B\alpha_\varepsilon\left(\alpha_\theta + B^2\alpha_\varepsilon\right)^{-2}\beta^2} > 0.$$

In short, the greater the fraction of market-order traders, the larger the uniqueness region. Again, this is because market order traders face price risk and hence trade less aggressively on their private information than demand-schedule traders do. This reduces the amount of private information that is aggregated into p, thereby reducing its weight in traders' assessment of θ and impeding traders' coordination.[7]

[6]Since $\alpha_\theta + B^2\alpha_\varepsilon \geq \alpha_\theta$, we have $\alpha_x^{-1} + \alpha_\theta^{-1} - \left(\alpha_\theta + B^2\alpha_\varepsilon\right)^{-1} \geq \sigma_x^2$ and hence $B \leq \alpha_x/\gamma$.

[7]Note the total effect of ν on B aggregates two effects. First, as ν increases, traders on average trade more aggressively and hence prices become more informative. Second, the aggressiveness of demand-schedule traders

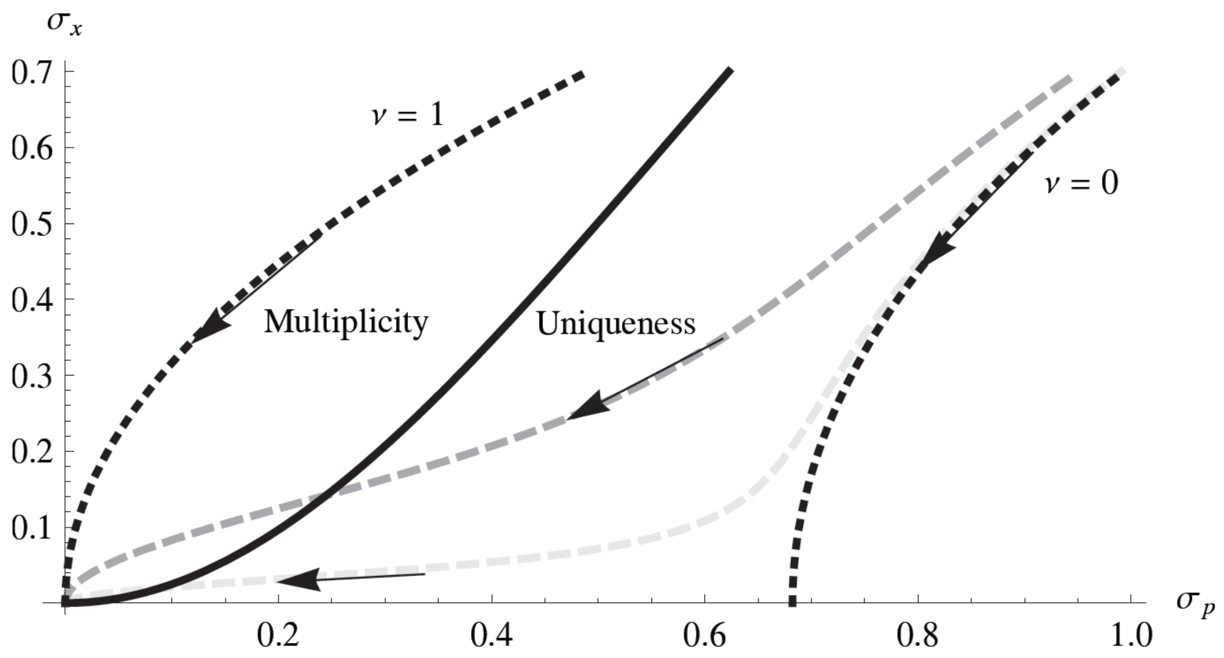

Figure 2: Multiplicity and uniqueness regions under exogenous order types. Note: $\sigma_p \equiv \alpha_z^{-1/2}$ and $\sigma_x \equiv \alpha_x^{-1/2}$ denote the noise in the public and private signals, respectively. The bold line is the uniqueness frontier, while the dotted lines shows how σ_p depends on σ_x for different values of ν.

The role of ν in affecting the multiplicity region is illustrated in Figure 2. From the analysis in Section 2.2 we know that $\alpha_z = B^2\alpha_\varepsilon$. Total differencing equation (8), we find that for $\partial B/\partial\alpha_x > 0$, implying that a greater precision of the private signal tends to raise α_z. The dotted and dashed lines shows the monotone response of $\sigma_p \equiv \alpha_z^{-1/2}$ (i.e., the noise in the public price signal) to changes in $\sigma_x = \alpha_x^{-1/2}$ (i.e., the noise in the price signal) for different values of ν. The bold line represents the multiplicity versus uniqueness boundary (3), i.e. the $\sqrt{2\pi\sigma_x^{-2}} = \sigma_z^{-2} + \alpha_\theta$ line. A smaller value of ν is associated with a smaller uniqueness region as $\sigma_x \to 0$.

4. Endogenous trader types

The analysis above shows that the presence of market-order traders tends to reduce the indeterminacy region by limiting the impact of the price signal on ex post beliefs about the fundamental. We now analyse a trader's choice of order type, and solve for the equilibrium shares of demand-schedule and market-order traders. Essentially, the key tradeoff faced by every informed trader is as follows. On the one hand, placing a demand schedule insulates the expected net payoff of a trader from price risk (since effective trades are conditional on the price). On the other hand, it is more costly than a market order, as it requires to place a large (in fact, infinite) number of limit orders in order to generate a complete conditionality of the quantity traded on the execution price. Following Vives (2008), we capture this tradeoff by normalising the cost of a market order to zero and setting that of a full demand schedule to $c > 0$. We work out the solution to this problem under the maintained assumption that the choice of order type must be made before the traders

tends to increase the price risk faced by market-order traders, thereby pushing them to trade *less* aggressively on their private information as ν increases. The direct effect always dominate, implying that $\partial B/\partial\nu > 0$.

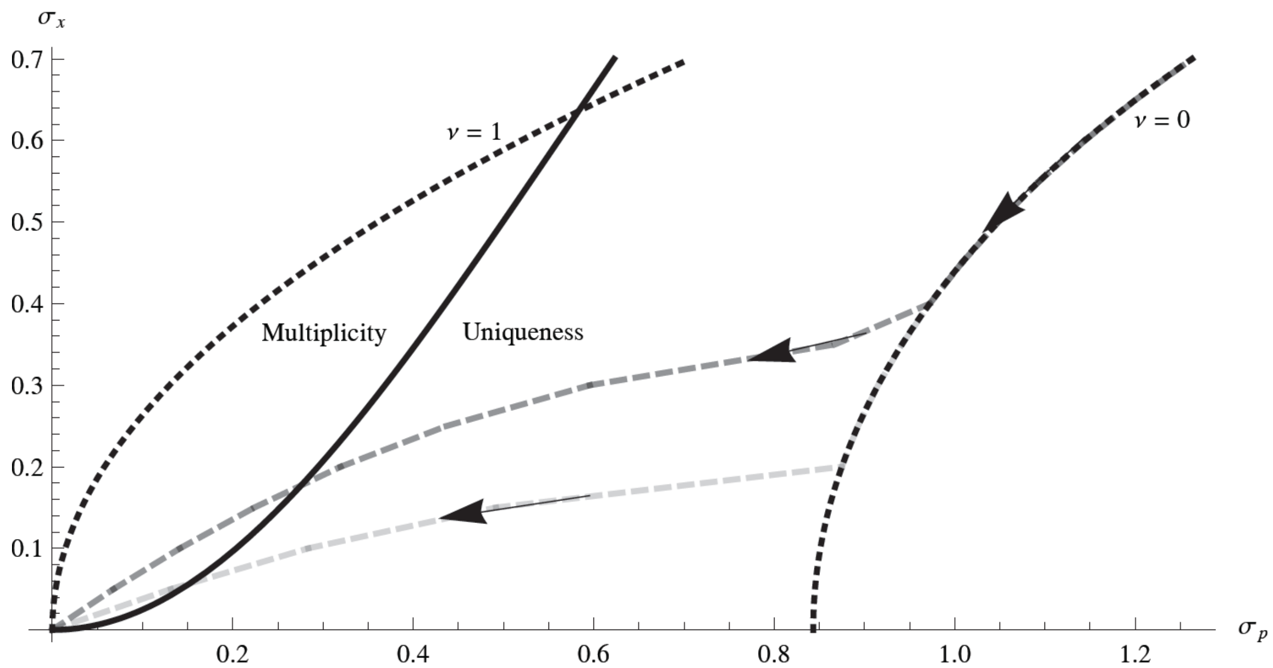

Figure 3: Multiplicity and uniqueness regions under endogenous order types. Note: $\sigma_p \equiv \alpha_z^{-1/2}$ and $\sigma_x \equiv \alpha_x^{-1/2}$ denote the noise in the public and private signals, respectively. The bold line is the uniqueness frontier, while the dotted lines shows how σ_p depends on σ_x for different values of c (the relative cost of demand schedules), taking into acount the endogenous adjustment of ν.

observe their private signal and place their orders–see Figure 1 again.[8]

We rank informed traders in nondecreasing order of risk aversion, define the nondecreasing function $\gamma : [0,1] \to \mathbb{R}_+$, and further assume that $\gamma(.)$ is an increasing homeomorphism and that $\gamma(0) > 0$. We solve for traders' choice of order backwards. First, we compute the expected utility of a trader of each type conditional on its information set (i.e. (x_i, p) $\forall i \in I \backslash M$, and x_i $\forall i \in M$). Second, we compute the unconditional ex ante utility of each type; and third, we compare the two ex ante utilities for a given risk aversion coefficient.

We know from the CARA-Normal framework that the value function associated with the information set G_i is:

$$W(G_i; \gamma_i) \equiv \max_k \mathbb{E}[V(w_i - \kappa c)|G_i; \gamma_i] = -\exp\left[-\frac{\mathbb{E}[\theta - p|G_i]^2}{2\mathbb{V}[\theta - p|G_i]} + \kappa c \gamma_i\right],$$

where $\kappa = 1$ if $G_i = (x_i, p)$ (i.e., the trader places a full demand schedule) or $\kappa = 0$ if $G_i = x_i$ (i.e., the trader places a market order). Using the conditional distributions of θ and $\theta - p$ for demand-schedule and market-order traders (see equations (A1)–(A2) in the Appendix A for details), we

[8] This follows Medrano (1996) and Brown and Zhang (1997). If it were not the case, traders could potentially be willing to adjust their trades (both in terms of order type and amount of trades) depending on the observed shares of demand-schedule and market-order traders, and this would make the signal extraction problem intractable.

find the corresponding value functions to be:

$$W_{I\backslash M}(x_i, p; \gamma_i) = -\exp\left[-\frac{C}{2}(x_i - p)^2\right],\ C \equiv \frac{\alpha_x^2}{\alpha_x + \alpha_\theta + B^2\alpha_\varepsilon}, \tag{10}$$

$$W_M(x_i; \gamma_i) = -\exp\left[-\frac{D}{2}(x_i - \bar{\theta})^2\right],\ D \equiv \beta^2\left(\frac{(1-\lambda B)^2}{\alpha_x + \alpha_\theta} + \frac{\lambda^2}{\alpha_\varepsilon}\right), \tag{11}$$

where β and B are defined in Lemma 1. Let $f(x)$ denote the ex ante (i.e., unconditional) density of the signal x. From the distributions of θ and ξ we have $x \sim \mathcal{N}\left(\theta, \alpha_\theta^{-1} + \alpha_x^{-1}\right)$. Hence, using (11) and rearranging the ex ante utility from being a market-order trader is found to be

$$\mathbb{E}\left[W_M(x_i; \gamma_i)\right] = \int_{\mathbb{R}} W_M(x_i; \gamma_i) f(x_i)\,\mathrm{d}x_i = -\sqrt{\frac{\frac{\alpha_\theta^2}{\alpha_\theta + \alpha_x} + B^2\alpha_\varepsilon}{\alpha_\theta + B^2\alpha_\varepsilon}}. \tag{12}$$

The ex ante utility of demand-schedule traders is computed in a similar way, except that we must first condition their information set (x_i, p) on x_i before computing the unconditional expectation of $W_{I\backslash M}(x_i, p; \gamma_i)$.[9] Applying the law of iterated expectations and rearranging we get:

$$\begin{aligned}\mathbb{E}\left[\mathbb{E}\left[W_{I\backslash M}(x_i; \gamma_i)\middle| x_i\right]\right] &= \int_{\mathbb{R}} \mathbb{E}\left[W_{I\backslash M}(x_i; \gamma_i)\middle| x_i\right] f(x_i)\,\mathrm{d}x_i \\ &= -\mathrm{e}^{c\gamma(i)}\sqrt{\frac{\alpha_\theta + B^2\alpha_\varepsilon}{\alpha_\theta + B^2\alpha_\varepsilon + \alpha_x}} \end{aligned} \tag{13}$$

Trader i chooses chooses to place a full demand schedule if and only if $\mathbb{E}\left[\mathbb{E}\left[W_{I\backslash M}(x_i; \gamma_i)\middle| x_i\right]\right] \geq \mathbb{E}\left[W_M(x_i; \gamma_i)\right]$, i.e., if and only if

$$\gamma(i) \leq \bar{\gamma} = \frac{1}{c}\ln\left(\frac{\sqrt{\left(\frac{\alpha_\theta^2}{\alpha_\theta + \alpha_x} + B^2\alpha_\varepsilon\right)(\alpha_\theta + B^2\alpha_\varepsilon + \alpha_x)}}{\alpha_\theta + B^2\alpha_\varepsilon}\right), \tag{14}$$

where, from Lemma 1,

$$B = \alpha_x \int_0^{\gamma^{-1}(\bar{\gamma})} \gamma(i)^{-1}\,\mathrm{d}i + \left(\frac{1}{\alpha_x} + \frac{1}{\alpha_\theta} - \frac{1}{\alpha_\theta + \alpha_\varepsilon B^2}\right)^{-1}\int_{\gamma^{-1}(\bar{\gamma})}^1 \gamma(i)^{-1}\,\mathrm{d}i, \tag{15}$$

with $\gamma^{-1}(\bar{\gamma}) = 0$ if $\bar{\gamma} < \gamma(0)$ and $\gamma^{-1}(\bar{\gamma}) = 1$ if $\bar{\gamma} > \gamma(1)$. For $(\alpha_x, \alpha_\theta, \alpha_\varepsilon) \in \mathbb{R}_+^3$ given, the properties of the $\gamma(.)$ function imply that the solution $(\bar{\gamma}, B)$ to (14)–(15), if it exists, can be of three types. More specifically, it is either such that $\bar{\gamma} \in [\gamma(0), \gamma(1)]$, in which case the solution is interior (i.e., both M and $I\backslash M$ are nonempty); or $\bar{\gamma} < \gamma(0)$, so that the solution is corner and all traders placing market orders (i.e., $(M, I\backslash M) = (I, \varnothing)$); or $\bar{\gamma} > \gamma(1)$ and all traders place

[9] Here the intermediate step is the computation of $\mathbb{E}\left[W_L(x_i, p; \gamma_i)\middle| x_i\right]$. Using the price function (7) and the fact that $\theta| x_i \sim \mathcal{N}\left(\frac{\alpha_x x_i + \alpha_\theta\bar{\theta}}{\alpha_x + \alpha_\theta}, \frac{1}{\alpha_x + \alpha_\theta}\right)$ we find that

$$W_L(x_i, p; \gamma_i)|x_i \sim \mathcal{N}\left(\frac{\alpha_x[\alpha_x(1-\lambda B) + \alpha_\theta](x_i - \bar{\theta})}{\sqrt{2(\alpha_x + \alpha_\theta + B^2\alpha_\varepsilon)}(\alpha_x + \alpha_\theta)}, \left(\frac{\alpha_x\sqrt{(\lambda B)^2(\alpha_x + \alpha_\theta)^{-1} + \lambda^2\alpha_\varepsilon^{-1}}}{\sqrt{2(\alpha_x + \alpha_\theta + B^2\alpha_\varepsilon)}}\right)\right)^2.$$

full demand schedules (i.e., $(M, I\backslash M) = (\varnothing, I)$). The intuition for this sorting of informed traders according to their degree of risk aversion is that a greater risk aversion lowers trading aggressiveness, and hence the expected benefit from expanding the information set from x_i to (x_i, p).[10]

As before we are interested in the outcome of the coordination stage of the game as α_x becomes large (holding $(\alpha_\theta, \alpha_\varepsilon, c)$ fixed), especially with regard to the way market-order traders alter the size of the uniqueness region. This is summarised in the proposition 2 below.

Proposition 2. For any $(\alpha_\theta, \alpha_\varepsilon, c) \in \mathbb{R}^3_+$, and as $\alpha_x \to +\infty$, (i) both M and $I\backslash M$ have strictly positive measure (i.e., the equilibrium is interior); (ii) $\bar{\gamma} \to \gamma(0)$ (i.e., market-order traders eventually overwhelm the market); (iii) $\alpha_z \underset{\alpha_x \to \infty}{\sim} (\mathrm{e}^{2\gamma(0)c} - 1)^{-1}\alpha_x$, so that α_z goes to infinity as the same rate α_x (while it does at the same rate as α_x^2 when $I\backslash M$ has exogenous, positive measure).

Proposition 2 emphasises several key properties of the equilibrium when α_x is large. Note that the property that the equilibrium is interior as $\alpha_x \to +\infty$ (point (ii)) is valid for any value of the cost c; in contrast, when α_x is small one can easily construct examples of corner solutions with a pure market-order (demand-schedule) market when c is sufficiently high (low). Points (ii) and (iii) are closely related. As discussed in Section 3, market-order traders tend to slow down information aggregation. It is precisely because they crowd out demand-schedule traders as $\alpha_x \to +\infty$ (point (ii)) that the precision of the endogenous public signal grows at the same rate as α_x, instead of α_x^2 when market shares are exogenous (point (iii)). To see how this may expand the uniqueness region, note that under exogenous shares from (8) we have $\alpha_z \underset{\alpha_x \to \infty}{\sim} (\nu/\gamma_{I\backslash M})\alpha_x^2$. Hence for $\nu > 0$ and α_x large enough, since $\left[(e^{2\gamma(0)c} - 1)^{-1}\alpha_x\right] / (\nu/\gamma_{I\backslash M})\alpha_x^2 \underset{\alpha_x \to \infty}{\to} 0$ it is necessarily the case that the precision of the price signal is greater under endogenous orders than under exogenous orders. Hence, whenever the uniqueness condition (3) is satisfied under exogenous shares, it is also so under endogenous shares, but the converse is not true. Figure 3 illustrates the relationship between α_z and α_x when α_x is large (i.e., $\sigma_x = \alpha_x^{-1/2}$ is small) and the shares of market-order and demand-schedules traders are endogenous.

Finally, note from (13) that heterogeneity in the cost c is formally equivalent to heterogeneity in risk aversion. To encompass both cases, rank traders in nondecreasing orders of $c(i)\gamma(i)$, assume that the function $g(i) = c(i)\gamma(i)$ is continuous, strictly increasing, that its reciprocal is continuous, and that $0 < g(0) < g(1) < +\infty$, and solve for the marginal trader exactly in the same way as in the case where $c(i) = c\ \forall i \in I$.

5. CONCLUDING REMARKS

In this paper, we have analysed a two-stage global game wherein a market-based asset price determined at the trading stage of the game provides an endogenous public signal affecting traders' decisions in the coordination stage of the game. By allowing both market-order and demand-schedule traders to coexist, and by letting traders choose their preferred order type, the market microstructure considered here is both richer and more realistic than the usual pure demand schedule/Walrasian auctioneer paradigm. As we have shown, in this context the multiplicity region can be small even when private information is very precise –and especially so when traders optimise over their type of order (in addition to their amount of trade). The reason for this is that the

[10] See Medrano (1996) and Vives (2008) for further discussion.

presence of market-order traders limits information aggregation and hence the precision of the endogenous public signal that may serve as a coordination device when deciding whether or not to attack the regime. In this sense, a lower degree of informational efficiency (in the trading stage) may ultimately be stabilising (in the coordination stage). While this conclusion was derived under a specific barrier to full informational efficiency –market-order traders' willingness to avoid price risk–, we conjecture that it would also apply in a variety of contexts where information aggregation is impeded.[11]

6. Appendix

A. Proof of Lemma 1 We restrict our attention to equilibrium price functions $p(\theta, \varepsilon)$ that are linear in (θ, ε), which implies that p is normally distributed. A trader i with risk aversion coefficient γ_i and information set G_i has a demand for assets $k_i(G_i) = \gamma_i^{-1}\mathbb{E}[\theta - p|G_i]/\mathbb{V}[\theta - p|G_i]$. We may thus write the demands by limit- and market-order traders as follows:

$$\forall i \in I\backslash M,\ k^i_{I\backslash M}(x_i, p) = \gamma_i^{-1} f_{I\backslash M}(x_i, p), \text{ with } f_{I\backslash M}(x_i, p) = \frac{\mathbb{E}[\theta|x_i, p] - p}{\mathbb{V}[\theta|x_i, p]},$$
$$\forall i \in M,\ k^i_M(x_i) = \gamma_i^{-1} f_M(x_i), \text{ with } f_M(x_i) = \frac{\mathbb{E}[\theta - p|x_i]}{\mathbb{V}[\theta - p|x_i]},$$

i.e., within each group asset demands are identical up to a risk tolerance correction γ_i^{-1}. Now conjecture that $f_{I\backslash M}(.)$ and $f_M(.)$ have the form $f_{I\backslash M}(x_i, p) = a(x_i - \overline{\theta}) + \zeta(p)$ and $f_M(x_i) = c(x_i - \overline{\theta})$, where a and b are normalised trading intensities (for a trader with $\gamma_i = 1$) and $\zeta(.)$ is linear. Using the convention that the average signal equals θ a.s., and recalling that γ_i is independent from ξ_i, the limit order book is given by

$$L(p) = \int_{I\backslash M} k^i_{I\backslash M}(x_i, p)\mathrm{d}i + \int_M k^i_M(x_i)\mathrm{d}i + \varepsilon = B\left[\theta + B^{-1}\varepsilon\right] - B\overline{\theta} + \zeta(p)\int_{I\backslash M} \gamma_i^{-1}\mathrm{d}i,$$

where $B = a\nu/\gamma_{I\backslash M} + c(1-\nu)/\gamma_M$. The market making sector observes $L(.)$, a linear function of p, and sets $p = \mathbb{E}[\theta|L(.)] = \mathbb{E}[\theta|z]$, where $z = \theta + B^{-1}\varepsilon$ is the public signal. From standard normal theory we infer that p is indeed linear, normal and given by equation (7).

We now need to identify a and c. From the joint distribution of (p, x_i, θ) we get:

$$\forall i \in I\backslash M, \begin{cases} \mathbb{E}[\theta|p, x_i] = \frac{B^2\alpha_\varepsilon z + \alpha_\theta\overline{\theta} + \alpha_x x_i}{B^2\alpha_\varepsilon + \alpha_\theta + \alpha_x} = \frac{(B^2\alpha_\varepsilon + \alpha_\theta)p + \alpha_x x_i}{B^2\alpha_\varepsilon + \alpha_\theta + \alpha_x}, \\ \mathbb{V}[\theta|p, x_i] = (B^2\alpha_\varepsilon + \alpha_\theta + \alpha_x)^{-1}. \end{cases} \tag{A1}$$

$$\forall i \in M, \begin{cases} \mathbb{E}[\theta - p|x_i] = \frac{(1-\lambda B)\alpha_x}{\alpha_x + \alpha_\theta}(x_i - \overline{\theta}), \\ \mathbb{V}[\theta - p|x_i] = (1-\lambda B)^2\ \mathbb{V}[\theta|x_i] + \frac{\lambda^2}{\alpha_\varepsilon} = \frac{(1-\lambda B)^2}{\alpha_x + \alpha_\theta} + \frac{\lambda^2}{\alpha_\varepsilon}. \end{cases} \tag{A2}$$

Hence, we obtain

$$k^i_{I\backslash M}(x_i, p; \gamma_i) = \frac{\mathbb{E}[\theta|p, x_i] - p}{\gamma_i \mathbb{V}[\theta|p, x_i]} = \frac{\alpha_x}{\gamma_i}(x_i - p),\ \ k^i_M(x_i; \gamma_i) = \frac{\mathbb{E}[\theta - p|x_i]}{\gamma_i \mathbb{V}[\theta - p|x_i]} = \frac{\beta}{\gamma_i}\left(x_i - \overline{\theta}\right),$$

where $\beta = (\alpha_x^{-1} + \alpha_\theta^{-1} - \left(\alpha_\theta + B^2\alpha_\varepsilon\right)^{-1})^{-1}$. In the special case where $\gamma_i = \gamma\ \forall i \in [0, 1]$, we have

[11] For example, inasmuch as impefect competition in the asset market slows down information revelation (Kyle, 1989), we expect it to also expand the uniqueness region in the coordination stage.

$k^i_{I\backslash M}(x_i,p) = \gamma^{-1}\alpha_x(x_i - p)$, $k_M(x_i) = \gamma^{-1}\beta(x_i - \overline{\theta})$ and $p = (1-\lambda B)\overline{\theta} + \lambda B z$, where B solves $B = \nu\gamma^{-1}\alpha_x + (1-\nu)\gamma^{-1}\beta$ and $\lambda = \frac{B\alpha_\varepsilon}{B^2\alpha_\varepsilon + \alpha_\theta}$.

Let us now turn to the parameter B. To establish that B is unique, positive and finite, define the function $f : \mathbb{R} \to \mathbb{R}$ as follows:

$$f : B \to B - \frac{\nu\alpha_x}{\gamma_{I\backslash M}} - \frac{1-\nu}{\gamma_M(\alpha_x^{-1} + \alpha_\theta^{-1} - (\alpha_\theta + B^2\alpha_\varepsilon)^{-1})},$$

so that a root of $f(B)$ is a solution to (8). f is continuous and strictly increasing over $[0,+\infty)$ and such that $f(0) = -\alpha_x\left(\frac{\nu}{\gamma_{I\backslash M}} + \frac{(1-\nu)}{\gamma_M}\right) < 0$ and $\lim_{B\to+\infty} f(B) = +\infty$. Hence f is a bijection that admits a unique root $B_0 > 0$ over $[0,+\infty)$. Moreover, as $B \to \alpha_x^{-1} + \alpha_\theta^{-1} - \left(\alpha_\theta + B^2\alpha_\varepsilon\right)^{-1} > 0$ on $\mathbb{R}$, $f(.)$ is strictly negative on $\mathbb{R}_-$. Hence B_0 is the unique root of f in $\mathbb{R}$. In the numerical implementation of the model we use the exact solution for B, which is found using Cardano's method and gives:

$$\begin{aligned} B \;=\; & \sqrt[3]{\frac{1}{2}(-(2\frac{a_2^3}{27} - \frac{a_1a_2}{3} + a_0) + \sqrt{\frac{4a_1^3 + 4a_0a_2^3 - (a_1a_2)^2}{27} - \frac{2}{3}a_0a_1a_2 + a_0^2})} \\ & + \sqrt[3]{\frac{1}{2}(-(2\frac{a_2^3}{27} - \frac{a_1a_2}{3} + a_0) - \sqrt{\frac{4a_1^3 + 4a_0a_2^3 - (a_1a_2)^2}{27} - \frac{2}{3}a_0a_1a_2 + a_0^2})} - \frac{a_2}{3}, \end{aligned}$$

where

$$a_0 = -\frac{\alpha_\theta}{(\alpha_x^{-1} + \alpha_\theta^{-1})\alpha_\varepsilon}\left(\frac{\nu}{\gamma_{I\backslash M}} + \frac{(1-\nu)}{\gamma_M}\right), \quad a_1 = \frac{\alpha_x^{-1}\alpha_\theta}{(\alpha_x^{-1} + \alpha_\theta^{-1})\alpha_\varepsilon},$$

$$\text{and} \quad a_2 = -\frac{[\frac{\nu}{\gamma_{I\backslash M}} + \frac{(1-\nu)}{\gamma_M} + \frac{\nu}{\gamma_{I\backslash M}}\alpha_x\alpha_\theta^{-1}]}{(\alpha_x^{-1} + \alpha_\theta^{-1})}.$$

B. Proof of Proposition 1 We know from Lemma 1 that $B \in \mathbb{R}_+^*$ uniquely solves (8). When $\nu = 0$, B solves $1/(\gamma_M x) = \alpha_x^{-1} + \alpha_\theta^{-1} - \left(\alpha_\theta + \alpha_\varepsilon x^2\right)^{-1}$. In this case $\lim_{\alpha_x\to+\infty} B$ is finite, hence the uniqueness condition (9) necessarily holds as $\alpha_x \to +\infty$. In contrast, when $\nu = 1$ we have $\lim_{\alpha_x\to+\infty} \frac{B^2}{\sqrt{\alpha_x}} = +\infty$, hence the uniqueness condition (9) is necessarily violated as $\alpha_x \to +\infty$.

C. Proof of Proposition 2 (i) We show that $k \equiv \gamma^{-1}(\bar{\gamma}) \in]0;1[$ for α_x sufficiently high, and that k is unique. Let us first define the function

$$\tilde{f} : \alpha_x, k \to \mathrm{e}^{2\gamma(k)c} - \left(1 - \frac{\alpha_\theta}{\alpha_x + \alpha_\theta}\frac{\alpha_x}{B(k,\alpha_x)^2\alpha_\varepsilon + \alpha_\theta}\right)\left(1 + \frac{\alpha_x}{B(k,\alpha_x)^2\alpha_\varepsilon + \alpha_\theta}\right), \tag{C.1}$$

where $B(k,\alpha_x)$ is the unique solution to

$$B(k,\alpha_x) = \alpha_x\int_0^k \gamma(i)^{-1}\mathrm{d}i + \left(\frac{1}{\alpha_x} + \frac{1}{\alpha_\theta} - \frac{1}{\alpha_\theta + B(k,\alpha_x)^2\alpha_\varepsilon}\right)^{-1}\int_k^1 \gamma(i)^{-1}\mathrm{d}i. \tag{C.2}$$

We have $\tilde{f}(\alpha_x,1) \underset{\alpha_x\to\infty}{\to} \mathrm{e}^{2\gamma(1)c} - 1 > 0$ while $\tilde{f}(\alpha_x,0) \underset{\alpha_x\to\infty}{\to} -\infty < 0$. Hence, by the intermediate value theorem there exists $\underline{\alpha} \in \mathbb{R}_+^*$,such that for all $\alpha_x \geq \underline{\alpha}$, $0 \in]\tilde{f}(\alpha_x,0), \tilde{f}(\alpha_x,1)[$. By continuity,

$\forall \alpha_x \geq \underline{\alpha}$, $\exists k(\alpha_x) \in]0,1[$ such that $\tilde{f}(\alpha_x, k(\alpha_x)) = 0$. In this range of parameter, there exists an interior equilibrium allocation, and the corner solutions are ruled out (otherwise the polar traders would be better off switching positions).

To establish uniqueness, define $\tilde{\alpha} \equiv \alpha_\theta / (\alpha_x + \alpha_\theta)$ and $X \equiv \alpha_x / \left(B(k(\alpha_x), \alpha_x)^2 \alpha_\varepsilon + \alpha_\theta\right)$, and rewrite $\tilde{f}(\alpha_x, k(\alpha_x)) = 0$ as $\mathcal{P}(X) = \tilde{\alpha} X^2 - (1-\tilde{\alpha})X + \mathrm{e}^{2\gamma(k(\alpha_x))c} - 1 = 0$. This polynomial has the following two real roots:

$$s^-, s^+ = \frac{1}{2\tilde{\alpha}} \left[(1-\tilde{\alpha}) \mp \sqrt{(1-\tilde{\alpha})^2 - 4\tilde{\alpha}\left(\mathrm{e}^{2\gamma(k(\alpha_x))c} - 1\right)} \right].$$

We prove by contradiction that $X = s^-$ is the only possible root of $\mathcal{P}(X) = 0$ when α_x becomes large enough. Formally,

$$\exists \underline{\alpha}^1 \geq \underline{\alpha}, \forall \alpha_x \in \mathbb{R}_+^*, \forall k \in]0;1[, \quad \left.\begin{array}{c} \alpha_x \geq \underline{\alpha}^1 \\ \tilde{f}(\alpha_x, k) = 0 \end{array}\right\} \Rightarrow \frac{\alpha_x}{B(k, \alpha_x)^2 \alpha_\varepsilon + \alpha_\theta} = s^-. \tag{C.3}$$

To see this, suppose that $\forall \underline{\alpha}^1 \geq \underline{\alpha}, \exists \alpha_x \in \mathbb{R}_+^*, \exists k \in]0;1[$ such that

$$\left(\alpha_x \geq \underline{\alpha}^1\right) \wedge \left(\tilde{f}(\alpha_x, k(\alpha_x)) = 0\right) \wedge \left(\frac{\alpha_x}{B(k(\alpha_x), \alpha_x)^2 \alpha_\varepsilon + \alpha_\theta} = s^+\right).$$

In particular, for $n \in \mathbb{N}$ large enough (say larger than $n_0 = \lceil \underline{\alpha} \rceil$),

$$\exists \alpha_x, \exists k, (\alpha_x \geq n) \wedge (\tilde{f}(\alpha_x, k) = 0) \wedge \left(\alpha_x / \left(B(k, \alpha_x)^2 \alpha_\varepsilon + \alpha_\theta\right) = s^+\right)$$

For every $n \geq n_0$ we pick an α_x, and an associated $k(\alpha_x)$, satisfying $(\alpha_x \geq n) \wedge (\tilde{f}(\alpha_x, k(\alpha_x)) = 0) \wedge \left(\alpha_x / \left(B(k(\alpha_x), \alpha_x)^2 \alpha_\varepsilon + \alpha_\theta\right)\right)$ and denote it α_n (resp. $k(\alpha_n)$), thereby constructing the series $(\alpha_n)_{n \geq n_0}$ (resp. $(k(\alpha_n))_{n \geq n_0}$). As $\alpha_n \underset{n\to\infty}{\to} \infty$, and since $k(\alpha_n)$ must belong to $[0;1]$ we have

$$\frac{4\tilde{\alpha}}{(1-\tilde{\alpha})^2}\left(\mathrm{e}^{2\gamma(k(\alpha_n))c} - 1\right) = \frac{4\alpha_\theta}{\alpha_n + \alpha_\theta}\left(\frac{\alpha_n + \alpha_\theta}{\alpha_n}\right)^2 \left(\mathrm{e}^{2\gamma(k(\alpha_n))c} - 1\right) \underset{n\to\infty}{\to} 0, \tag{C.4}$$

and hence

$$\frac{X\tilde{\alpha}}{1-\tilde{\alpha}} = \frac{1}{2}\left[1 + \sqrt{1 - \frac{4\tilde{\alpha}}{(1-\tilde{\alpha})^2}\left(\mathrm{e}^{2\gamma(k(\alpha_n))c} - 1\right)}\right] \underset{n\to\infty}{\to} 1.$$

This in turn implies that

$$\frac{\alpha_n}{B(k(\alpha_n), \alpha_n)^2 \alpha_\varepsilon + \alpha_\theta} \underset{n\to\infty}{\sim} \frac{\alpha_n}{\alpha_\theta},$$

that is, $B(k(\alpha_n), \alpha_n)^2 \alpha_\varepsilon \underset{n\to\infty}{\to} 0$. Since for each $n \geq n_0$, $B(k(\alpha_n), \alpha_n)^2 \geq B(0, \alpha_n)^2$ while $\left(B(0, \alpha_n)^2\right)_{n \in \mathbb{N}}$ admits a finite, non-zero limit as $n \to \infty$, we have a contradiction that proves (C.3). To summarise, for every given set of parameters, the function

$$k \to \frac{\alpha_x}{B(k, \alpha_x)^2 \alpha_\varepsilon + \alpha_\theta} - \frac{1}{2\alpha}\left[(1-\tilde{\alpha}) - \sqrt{(1-\tilde{\alpha})^2 - 4\alpha\left(\mathrm{e}^{2\gamma(k)c} - 1\right)}\right] \tag{C.4}$$

is strictly decreasing and thus has a unique root. For every $\alpha_x \geq \underline{\alpha}^1$, we also have that $\alpha_x \geq \underline{\alpha}$, hence we know that there exists $k \in]0;1[$ such that $\tilde{f}(\alpha_x, k) = 0$. Moreover, as $\alpha_x \geq \underline{\alpha}^1$, we know that k is the unique root of (C.4). Hence, $k \in]0;1[$ exists and is unique. We will denote it $k(\alpha_x)$.

(ii) From above, $\forall \alpha_x \geq \underline{\alpha}^1$ we have $\alpha_x / \left(B(k(\alpha_x), \alpha_x)^2 \alpha_\varepsilon + \alpha_\theta\right) = s^-$. Since $k(\alpha_x) \in]0;1[$ we have

$$\frac{\alpha_x}{B(k(\alpha_x), \alpha_x)^2 \alpha_\varepsilon + \alpha_\theta} \geq \frac{(1-\tilde{\alpha}) - \sqrt{(1-\tilde{\alpha})^2 - 4\tilde{\alpha}\left(\mathrm{e}^{2\gamma(0)c} - 1\right)}}{2\tilde{\alpha}},$$

or, rearranging,

$$B(k(\alpha_x), \alpha_x)^2 \leq \frac{2\tilde{\alpha}}{(1-\tilde{\alpha}) - \sqrt{(1-\tilde{\alpha})^2 - 4\tilde{\alpha}\left(\mathrm{e}^{2\gamma(0)c} - 1\right)}} \frac{\alpha_x}{\alpha_\varepsilon}.$$

Moreover, from (15) we know that

$$\left(\frac{k\left(\alpha_x\right)}{\gamma\left(1\right)}\alpha_x\right)^2 \leq \alpha_x^2 \left(\int_0^{k(\alpha_x)} \gamma(i)^{-1} \mathrm{d}i\right)^2 \leq B(k(\alpha_x), \alpha_x)^2$$

Hence, $\forall \alpha_x \geq \underline{\alpha}^1$,

$$0 \leq k\left(\alpha_x\right) \leq \frac{\gamma\left(1\right)}{\alpha_x} \sqrt{\frac{\alpha_x}{\alpha_\varepsilon} \frac{2\tilde{\alpha}}{(1-\tilde{\alpha}) - \sqrt{(1-\tilde{\alpha})^2 - 4\tilde{\alpha}\left(\mathrm{e}^{2\gamma(0)c} - 1\right)}}}$$

We know from (C.4) that $\frac{4\tilde{\alpha}}{(1-\tilde{\alpha})^2}\left(\mathrm{e}^{2\gamma(0)c} - 1\right) \underset{\alpha_x \to \infty}{\to} 0$. It follows that

$$\frac{(1-\tilde{\alpha}) - \sqrt{(1-\tilde{\alpha})^2 - 4\tilde{\alpha}\left(\mathrm{e}^{2\gamma(0)c} - 1\right)}}{2\tilde{\alpha}} = \frac{(1-\tilde{\alpha}) - (1-\tilde{\alpha})\sqrt{1 - \frac{4\tilde{\alpha}}{(1-\tilde{\alpha})^2}\left(\mathrm{e}^{2\gamma(0)c} - 1\right)}}{2\alpha}$$
$$= \frac{(1-\tilde{\alpha})}{4\tilde{\alpha}} \frac{4\tilde{\alpha}}{(1-\tilde{\alpha})^2}\left(\mathrm{e}^{2\gamma(0)c} - 1\right) + \underset{\alpha_x \to \infty}{o}(1) = \frac{\mathrm{e}^{2\gamma(0)c} - 1}{1-\tilde{\alpha}} + \underset{\alpha_x \to \infty}{o}(1) \underset{\alpha_x \to \infty}{\to} \mathrm{e}^{2\gamma(0)c} - 1,$$

We infer that

$$\frac{\gamma\left(1\right)}{\alpha_x} \sqrt{\frac{\alpha_x}{\alpha_\varepsilon} \frac{2\tilde{\alpha}}{(1-\tilde{\alpha}) - \sqrt{(1-\tilde{\alpha})^2 - 4\tilde{\alpha}\left(\mathrm{e}^{2\gamma(0)c} - 1\right)}}} \underset{\alpha_x \to \infty}{\sim} \frac{\gamma}{\alpha_x} \sqrt{\frac{\alpha_x / \alpha_\varepsilon}{\mathrm{e}^{2\gamma(0)c} - 1}} \underset{\alpha_x \to \infty}{\to} 0,$$

Hence $k(\alpha_x) \underset{\alpha_x \to \infty}{\to} 0$, and (by the continuity of $\gamma\left(.\right)$) $\bar{\gamma} = \gamma\left(k(\alpha_x)\right) \underset{\alpha_x \to \infty}{\to} \gamma\left(0\right)$.

(iii) Using (C.3) above we get

$$\frac{\alpha_x}{B(k(\alpha_x), \alpha_x)^2 \alpha_\varepsilon + \alpha_\theta} = s^- = \frac{\mathrm{e}^{2\gamma(k(\alpha_x))c} - 1}{1-\tilde{\alpha}} + \underset{\alpha_x \to \infty}{o}\left(\frac{\mathrm{e}^{2\gamma(k(\alpha_x))c} - 1}{1-\tilde{\alpha}}\right) \underset{\alpha_x \to \infty}{\to} \mathrm{e}^{2\gamma(0)c} - 1.$$

We infer that

$$\frac{B(k(\alpha_x), \alpha_x)^2 \alpha_\varepsilon}{\alpha_x} = \frac{B(k(\alpha_x), \alpha_x)^2 \alpha_\varepsilon + \alpha_\theta}{\alpha_x} - \frac{\alpha_\theta}{\alpha_x} \underset{\alpha_x \to \infty}{\to} \frac{1}{\mathrm{e}^{2\gamma(0)c} - 1}$$

We conclude that $\alpha_z = B(k(\alpha_x), \alpha_x)^2 \alpha_\varepsilon \underset{\alpha_x \to \infty}{\sim} (\mathrm{e}^{2\gamma(0)c} - 1)^{-1} \alpha_x$.

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