ECOLE POLYTECHNIQUE

CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

LINEAR PRICES EQUILIBRIA AND NONEXCLUSIVE INSURANCE MARKET

Frédéric LOSS
Gwenaël PIASER

June 2013

Cahier n° 2013-20

DEPARTEMENT D'ECONOMIE
Route de Saclay
91128 PALAISEAU CEDEX
(33) 1 69333033
http://www.economie.polytechnique.edu/
mailto:chantal.poujouly@polytechnique.edu

Linear Prices Equilibria and Nonexclusive Insurance Market*

Frédéric Loss† Gwenaël Piaser‡

June 20, 2013

Abstract

We consider a competitive insurance market in which agents can privately enter into multi-contractual insurance relationships and undertake hidden actions. We study the existence of linear equilibria when insurance companies do not have any restriction on their pricing rules. We provide conditions under which a linear equilibrium exists. We show that two different types of linear equilibria could exist: A first one in which insurance companies make zero expected profits, and a second one in which they make strictly positive expected profits. We also analyze the welfare properties of the linear equilibria. We show that they are not always second best Pareto optimal.

Keywords: Common Agency, Insurance, Moral Hazard, Perfect Competition, Linear Prices Equilibria.
JEL Classification: D43, D82, D86, G22, L13.

*We would like to thank Andrea Attar, Johanna Etner, Pierre Picard, Casey Rothschild, Anne Marie Tauzin and Bertrand Villeneuve. All remaining errors are our own.

†CNAM - Laboratoire d'économétrie et Ecole Polytechnique.

‡IPAG, Business School, Paris.

1 Introduction

An exclusive contractual relationship implies that a party in a contract can restrict the other party's participations in contractual relationships with other institutions. This requires the institution which designs the contract to be able to perfectly monitor the other party's trades with other institutions and these trades to be observable and verifiable by a court of justice in order to enforce the exclusivity clause of the contract. These are very strong assumptions. Thus, except when it is forbidden by law, insurance contractual relationships are rather nonexclusive. Moreover, most of the time only the insured knows the full set of contracts he has subscribed. In this article, we thus consider a competitive insurance market in which agents can privately contract with several insurance companies and are subject to moral hazard. In this context, we study the existence of linear equilibria when insurance companies do not have any restriction on their pricing rules. Moreover, we also look at the welfare properties of the linear equilibria.

In order to study nonexclusive competition in insurance contracts we consider a model of common agency. We modelize insurance companies as principals offering contracts to agents who can privately choose several contracts among those which are offered and who each exerts an unobservable and costly effort which affects the probability of the accident they are each facing. Our model is close to Hellwig's (1983) and Bisin and Guaitoli's (2004) models except that agents can exert a continuum of effort. This is a crucial assumption, since it allows us to provide a sufficient condition on the existence of linear equilibria (which would not be possible with a discrete effort).

From a methodological point of view, we follow Peters (2001) and Martimort and Stole (2002) who study common agency games. In single-principal models the celebrated revelation principle tells us that one can restrict attention to revealing mechanisms without loss of generality. In other words, in a context of moral hazard a single principal has no reason to offer a menu of contracts. However, in common agency models, such as ours, this is no longer the case. The former authors have then shown that one can assume that principals, here insurance companies, offer menus (which are sets of contracts) and then characterize all the possible equilibria of the game. Hence in our model, insurance companies may offer more than one contract (but they are not constrained to do so).

We first provide conditions under which there exists a pure strategy equilibrium in which insurance companies issue linear contracts. That is, in equilibrium each insurance company offers linear prices, agents choose a repayment and pay a price proportional to it. This equilibrium emerges in a setting in which insurance companies are free to offer any kind of menus of contracts. In particular, they can offer non linear contracts. This differs substantially both from Pauly's (1974) and Arnott and Stiglitz's (1988, and 1991) results which are derived in a setting in which insurance companies are restricted to offer linear contracts. Our result on the existence of linear equilibria implies that both Pauly's and Arnott and Stiglitz's approaches have micro economic foundations. In other words, in this class of models the existence of a pure strategy linear equilibrium can (almost always) be taken as granted. Hellwig (1983) also looks at the existence of linear equilibria. However, he only considers isoelastic utility functions while we study IARA, CARA and DARA utility functions. He shows that linear equilibria always exist if the exponent of the utility function is nonnegative (i.e., if relative risk aversion is not greater than one). With CARA utility functions, we show that linear equilibria always exist since the willingness to pay for more insurance is increasing in wealth. The existence property of linear equilibria is very robust since in particular it does not rely on any assumption on the function which links an agent's probability of accident to the effort he exerts. We show that this result is still valid with IARA utility functions when moral hazard is weak.[1] For DARA utility functions the linear equilibrium existence property still relies on the fact that the willingness to pay for more insurance is increasing in wealth, but it also relies on assumptions on the function which links the probability of accident to the agent's effort.

Second, we show that when moral hazard is quite strong, the linear equilibria are such that insurance companies have strictly positive expected profits. In traditional Bertrand competition model there is no strictly positive (expected) profit in equilibrium since firms could then deviate, make aggressive offers and take all the markets. However, in our model insurance companies cannot do so as they fear agents to take the deviating offer, complement it with other offers (since insurance contracts are not exclusive) and exert a low level of effort which would make the deviating offer unprofitable. We thus show that strictly positive (expected) profits in a competitive setting as in Parlour and Rajan (2001) can also exist when the effort exerted by an agent is continuous and not simply binary.[2]

Finally, we analyze the welfare properties of the linear equilibria. When moral hazard is weak we show that the linear equilibrium could either be second best optimal or not. The (second best) optimality depends on the properties of the function which links the probability of accident to the agent's effort. When moral hazard is quite strong, we then show that the linear equilibria are not *generically* second best Pareto efficient. Moreover, since the linear equilibria are not necessarily second best Pareto optimal, we also discuss the opportunity of a government intervention on the insurance market and for instance the interest to impose exclusive insurance contracts by law.

This paper is related to the literature which studies nonexclusive contracts under a common agency games framework. Common agency games have been a very useful framework to analyze economic issues in particular in the field of insurance. Following an approach close to ours, Ales and Maziero (2009) study the seminal Rothschild and Stiglitz's (1976) adverse selection model when insurance contracts are not exclusive.[3] Kahn and Mookherjee (1998) consider a setting in which agents design their own contracts (insurance companies can only either accept or reject the agents' offers), make contractual decisions sequentially, and have contractual portfolios, which are observable but not contractible upon. Their setting is quite different from ours as well as their results. In particular, in Kahn and Mookherjee's environment insurance companies make zero profits and agents face fair insurance prices in equilibrium while this is not always the case in our setting. Bisin and Guaitoli (2004) allow for negative insurance

[1] We precisely define what the meaning of weak moral hazard is in the paper.

[2] Parlour and Rajan (2001) use a common agency games framework to analyze credit relationships. They obtain strictly positive expected profits, in equilibrium, in the case of a competitive credit market.

[3] They show that there exists only a linear equilibrium in which all insurance companies offer the same menu of contracts.

contracts (i.e., insurance contracts that pay when there is no accident).[4] They thus obtain equilibria which are based on latent contracts, that is, on contracts which are offered by insurance companies but never subscribed in equilibrium. On the contrary, the linear equilibria we obtain are not based on latent contracts. Attar and Chassagnon (2009) in the same setting as that of Bisin and Guaitoli (2004) look at the welfare properties of the equilibria. They show that market equilibria may fail to be- even -third best efficient. We also look at the welfare properties of the linear equilibria we get, but we obtain different results. Finally, Attar, Campioni, Chassagnon and Rajan (2006) restrict the set of contracts that insurance companies can offer to linear or partially linear contracts while we do not restrict insurance contracts at all.

The paper proceeds as follows. Section 2 presents the model. Section 3 analyses the agent's indifference curves. Section 4 analyses the linear equilibria with non exclusivity. Section 5 looks at the welfare properties of the linear equilibria. Section 6 presents concluding remarks. Formal proofs are relegated to the Appendix.

2 The Model

We consider a competitive insurance market which is derived from Mossin's (1968) classical model of insurance. There is a continuum of identical agents of measure 1 and a finite number of identical insurance companies, indexed by i with $i \in I = \{1; \ldots; N\}$.

2.1 Agents

Each agent faces a fixed-damage accident denoted L. The accident is distributed by a Bernoulli distribution. There are two states of the world: either no accident or accident. Without insurance the agent's consumption corresponds to w if there is no accident, while it is equal to $w - L$ if there is an accident. However, an agent can also subscribe insurance contracts. Insurance contracts are nonexclusive (i.e., each agent can buy several contracts from different insurance companies) and private information to the agent. The insured agent's consumption if there is no accident is $w - P$, where $P = \sum_{i=1}^{N} p_i$ corresponds to the sum of all premia p_i paid by the agent to the insurance companies. However, if there is an accident the insured agent's consumption is $w - L + R - P$, where $R = \sum_{i=1}^{N} r_i$ corresponds to the sum of all repayments r_i from the insurance companies. The probability of accident $\pi(e)$ depends on the effort e devoted by the agent to avoid the accident. The probability $\pi(e)$ is decreasing at an increasing rate, that is, $\pi'(e) < 0$ and $\pi''(e) > 0$. We assume that $e \in [\underline{e}; \overline{e}]$, where $\underline{e}$, $\overline{e}$ respectively denote the lowest and the highest level of effort that the agent can exert. Effort is unobservable by the insurance companies and costly for the agent. Finally, we consider that the agent's utility function is separable in consumption and effort, and is event-independant.[5] The expected utility of an agent can be written as:

[4] Note that Kahn and Mookherjee (1998) as well as Bisin and Guaitoli (2004) also study credit relationships.

[5] This means that the occuring event does not affect the utility derived from consumption (i.e., the accident does not alter tastes).

$$U(w, L, R, P, e) = \pi(e)\, u(w - L + R - P) + (1 - \pi(e))\, u(w - P) - e, \tag{1}$$

where $u(.)$ is a von Neumann-Morgenstern utility function, with $u'(.) > 0$, $u''(.) < 0$. Without loss of generality we assume that $u(0) = 0$ and $w > L$.

2.2 Insurance Companies

The insurance market is competitive. We thus assume that the number of insurance companies N is large enough and each insurance company has no market power. There is no restriction on the kind of contracts (r_i, p_i) that insurance companies can offer. However, it is impossible to write a contract contingent either on an agent's effort or on his insurance contracts with other insurance companies since effort and insurance contracts are both private information to the agent.[6] The expected profit of an insurance company associated with an insurance contract (r_i, p_i) is equal to:

$$\Pi_i(r_i, p_i, e) = p_i - \pi(e)\, r_i. \tag{2}$$

2.3 The Contracting Game

Following the literature on common agency we allow insurance companies to offer "menus" of contracts. Each insurance company offers a subset of all possible contracts, i.e., offers a subset of $\mathbb{R}^2$. To make the problem tractable, we do assume that menus are closed subsets of $\mathbb{R}^2$. Given the offered menus of contracts, the agent must choose a portfolio of contracts and an effort. We assume that each agent must choose one and only one contract from each insurance company. Moreover, we assume that an agent can always choose $(r_i, p_i) = (0; 0)$, which corresponds to no insurance at all from company i. This means that we constrain insurance company to propose menus containing contract $(0, 0)$.[7]

The set of strategies of insurance company i is denoted by $\mathcal{M}_i$ and corresponds to the set of all closed subsets of $\mathbb{R}^2$ containing $(0; 0)$. By M_i we denote a generic element of $\mathcal{M}_i$. By M we denote a collection of menus offered by all the insurance companies, $M = \times_{i \in I} M_i$, and by M_{-i} we denote the collection of menus offered by all insurance companies but company i: $M_{-i} = \{M_1, \ldots, M_{i-1}, M_{i+1}, \ldots, M_N\}$. Hence, we have $M = (M_i, M_{-i})$. In the same way we will use the notation $\mathcal{M} = \times_{i \in I} \mathcal{M}_i$.

For an agent, a strategy is a mapping $\sigma(.)$ that maps the set of menus M to an element of the set $M \times [\underline{e}; \overline{e}]$. By $[p_i(M), r_i(M), e(M)]$ we denote the contract chosen by the agent in the menu offered by insurance company i when he faces the collection of menus M. It follows that $\sigma(M) = [p_i(M), r_i(M), e(M)]_{i \in I}$.

The timing of the game is the following:

[6] However, each insurance company does know all the contracts that it has signed with an agent.

[7] Considering that an agent must choose one and only one contract from each insurance company's menus of contracts is not restrictive, since several contracts from the same insurance company, can always be pooled into a single contract. Moreover, by constraining each insurance company to offer contract $(0, 0)$ we implicitly assume that the agent has also the possibility to choose "no contract at all" (not to be insured) from any insurance company.

1. Insurance companies simultaneously offer menus of insurance contracts.

2. Each agent privately chooses insurance contracts and the level of effort he exerts.

3. Either the accident occurs or not and payoffs are paid accordingly.

In the following, we are going to use the concept of subgame perfect Nash equilibrium: Given the menus of contracts issued by the insurance companies M, each agent (privately) chooses which contracts to buy and his level of effort. This determines his consumption in the two states of the world: No accident or accident. Each agent chooses both the set of insurance contracts and the level of effort he exerts so as to maximize his expected utility, which is given in equation (1). Anticipating the choices of each agent, as a function of the set of contracts they offer, insurance companies strategically choose which contracts they issue so as to maximize their profit, which is given in equation (2).

In formal terms, the agent's equilibrium strategy is a collection of mappings $\sigma^*(.)$ such that

$$\sigma^*(M) \in \underset{(r_i,p_i)_{i\in I}\in M,\ e\in[\underline{e},\overline{e}]}{\arg\max}\ \pi(e)u\left(w - L - \sum_{i\in I} p_i + \sum_{i\in I} r_i\right) + [1-\pi(e)]\,u\left(w - \sum_{i\in I} p_i\right) - e. \tag{3}$$

Following the preceding notation, for all M in $\mathcal{M}$, we use the notation

$$\sigma^*(M) = \left[p_i^*\left(M\right), r_i^*\left(M\right), e^*\left(M\right)\right]_{i\in I}.$$

Then equilibrium strategies of the principals are defined by:

$$\forall i \in I,\ M_i^* \in \underset{M_i\in\mathcal{M}_i}{\arg\max}\ p_i\left(M_i, M_{-i}^*\right) - \pi\left[e^*\left(M_i, M_{-i}^*\right)\right] r_i\left(M_i, M_{-i}^*\right). \tag{4}$$

The two preceding conditions (3) and (4) define an equilibrium in our game.

In this article, we are interested in the existence of linear equilibria. A linear equilibrium is an equilibrium in which the ratio insurance premium over repayment in case of accident is constant whatever the repayment chosen. Thus, we are going to look at the existence of equilibria in which each insurance company issues a continuum of linear contracts. In other words, in equilibrium, each company i offers menus M_i^* in which every contract (r_i, p_i) of the menu satisfies $p_i = \alpha r_i$, where α is a positive constant. However, we do not restrict the insurance companies' deviations to be linear. Deviations can be of any type and in particular nonlinear.

In order to be able to study equilibria in the insurance market we must first determine the indifference curves' properties. Indeed, the agent's optimal choice in terms of insurance and his optimal level of effort are both going to depend on the indifference curves' properties.

3 Indifference Curves Properties

Consider an agent's indirect utility function for a given (global) insurance contract (R, P). The indirect utility function, which we denote $V(R, P)$, is computed for $e = e^*$, where e^* corresponds to the effort which maximizes the agent's expected utility. Thus,

$$V(R,P) = \pi(e^*)u(w-L+R-P) + (1-\pi(e^*))\,u(w-P) - e^*, \tag{5}$$

and any indifference curve is characterized by:

$$V(R,P) = A, \text{ with } A \in \mathbb{R}. \tag{6}$$

3.1 Agent's Optimal level of Effort

For any given insurance contract (R,P) an agent exerts the level of effort which maximizes his expected utility. Using Equation (1) and computing the first order condition with respect to e, we obtain the optimal level of effort exerted by an agent when the maximization program admits an interior solution. In this case, e^* is such that

$$\pi'(e^*)\,[u(w-L+R-P) - u(w-P)] = 1. \tag{7}$$

There are also two possible corner solutions: $e^* = \overline{e}$ or $e^* = \underline{e}$. For instance, if there is full insurance $(R = L)$ an agent's consumption is the same whether there is accident or not. The agent has thus no incentive to exert an effort higher than $\underline{e}$ since, for a given probability of accident, his expected utility is decreasing with the effort he exerts. Moreover, for any given P an agent also exerts $e^* = \underline{e}$ when $R > L$. Consider now that $R < L$. Let us define $\underline{R}(P)$ as the implicit solution of Equality (7) for $e = \underline{e}$. For any given P, $\underline{R}(P)$ is thus such that

$$\pi'(\underline{e})\,[u(w-L+\underline{R}-P) - u(w-P)] = 1. \tag{8}$$

An agent optimally chooses to exert an effort $\underline{e} < e^* \leq \overline{e}$ when $R < \underline{R}(P)$, while he optimally exerts the lowest effort for $R \geq \underline{R}(P)$. Applying the implicit function theorem to Equality (8), we obtain:

$$\frac{d\underline{R}(P)}{dP} = \frac{u'(w-L+R-P) - u'(w-P)}{u'(w-L+R-P)} > 0,$$

since $u'(.) > 0$, but $u''(.) < 0$. This implies that the minimal repayment $\underline{R}(P)$ above which an agent chooses to exert the lowest effort $\underline{e}$ increases with P.

Let us now consider the particular case in which an agent is not insured, $R = 0$ (and thus $P = 0$). Equality (7) indicates that the agent optimally chooses to exert an effort $\underline{e} < e^* \leq \overline{e}$ if and only if:

$$u(w) - u(w-L) > -\frac{1}{\pi'(\underline{e})}. \tag{C_I}$$

Thus, condition C_I implies that for any positive premium $P \geq 0$, we have $\underline{R}(P) \in \,]0; L]$. We are going to assume that condition C_I is satisfied henceforth. This condition allows to simplify the analysis of the model without qualitatively altering the results.

In order to fully study the function $\underline{R}(P)$ two cases should be distinguished. Consider first that $\lim_{e \to \underline{e}} \pi'(e) = -\infty$. Then, $\underline{R}(P = 0)$ is such that

$$\pi'(\underline{e})\,[u(w-L+\underline{R}) - u(w)] = 1. \tag{9}$$

However, for any $R < L$ (even for R very close to L) we necessarily have

$$\pi'(\underline{e})\left[u(w-L+R)-u(w)\right] > 1$$

In other words, the marginal gain of effort is strictly higher than the marginal cost of effort. This implies that for any $R < L$ the agent exerts an effort $\underline{e} < e^* \leq \overline{e}$. We thus have $\underline{R}(P=0)=L$. Therefore, for any $P \geq 0$ we necessarily also have $\underline{R}(P) = L$ since $\underline{R}(P=0) = L$ and $\underline{R}(P)$ is increasing in P. Consider now that $\lim_{e\to\underline{e}} \pi'(e) \neq -\infty$. Then, for $R < L$, but $R \simeq L$ we necessarily have $\pi'(\underline{e})\left[u(w-L+R)-u(w)\right] < 1$ by continuity (since $\pi'(\underline{e})\left[u(w-L+R)-u(w)\right] < 1$ for $R = L$). This implies that $\underline{R}(P=0) < L$ in this case.

The following lemma sums up the previous results.

Lemma 1 *For any given $P \geq 0$ an agent exerts an effort $\underline{e} < e^* \leq \overline{e}$ for $R < \underline{R}(P)$, while he exerts the lowest effort $e^* = \underline{e}$ for $R \geq \underline{R}(P)$. Moreover, under condition C_I an agent exerts an effort $\underline{e} < e^* \leq \overline{e}$ in the case of no insurance ($R = 0 = P$). When $\lim_{e\to\underline{e}} \pi'(e) = -\infty$ then: $\forall P$, $\underline{R}(P) = L$. However, when $\lim_{e\to\underline{e}} \pi'(e) \neq -\infty$ then $\underline{R}(P=0) < L$ and $\underline{R}(P)$ is strictly increasing in P with $\underline{R}(P) < L$.*

Now that we know whether the agent exerts an effort higher or just equal to $\underline{e}$ we can look at the differentiability of the indifference curves and at the properties of the agent's marginal rate of substitution between R and P.

3.2 Differentiability of the Indifference Curves and Marginal Rate of Substitution between R and P

Consider first that $R \geq \underline{R}(P)$. Any indifference curve is then "well shaped" and exhibits the standard properties, since the effort is then fixed ($e^* = \underline{e}$). This implies that for $R \geq \underline{R}(P)$ any indifference curve is always differentiable in R. Consider now that $R \leq \underline{R}(P)$. The inequality $\pi''(e) > 0$ implies that $\pi'(e)$ is strictly increasing. Thus, for a given $\{w; L; R; P\}$ the first order condition (i.e., Equality 7) admits a unique solution. Suppose the contrary and consider that the first order condition admits two solutions at point A: $(R_A; P_A)$. Denote e^*_{Left} the agent's optimal effort at the left, but close to point A. Moreover, denote e^*_{Right} (with $e^*_{Left} \neq e^*_{Right}$) the agent's optimal effort at the right, but close to point A. Then, by continuity both e^*_{Left} and e^*_{Right} should correspond to the agent's optimal effort, and should both satisfy Equality 7. We thus obtain a contradiction. Note that when $R_A = \underline{R}(P_A)$ the same reasoning applies with, in this case, $e^*_{Right} = \underline{e}$. Suppose now that an indifference curve is not differentiable at point A (i.e., the indifference curve has a kink at $(R_A; P_A)$). Consider the constrained indifference curve for which the agent is constrained to exert the level of effort which is optimal at point A (i.e., $e^*(R_A; P_A)$). We denote this indifference curve $CIC(e^*(R_A; P_A))$. By definition the unconstrained indifference curve corresponds to the constrained indifference curve only at point A. Consider now another constrained indifference curve associated to $\widehat{e} \neq e^*(R_A; P_A)$. We denoted this indifference curve $\widehat{CIC}$. By definition of $\widehat{e}$ (which is different from $e^*(R_A; P_A)$, since $e^*(R_A; P_A)$ is unique), for $\widehat{CIC}$ to be associated to the same utility as $CIC(e^*(R_A; P_A))$, $\widehat{CIC}$ should be located strictly below $CIC(e^* (R_A; P_A))$ in the mark (R, P), since an agent's utility function is, *ceteris paribus*, increasing in R but decreasing in P. Thus, in the neighborhood of point A, $\widehat{e}$ cannot correspond to the optimal level of effort. This reasoning applies for any level of effort even close to $e^* (R_A; P_A)$. We thus have a contradiction, since the optimal level of effort exerted by an agent is continuous in R and P.

This leads us to the following lemma.

Lemma 2 *For $R \geq 0$, any indifference curve is always differentiable in R.*

Let us now study the agent's marginal rate of substitution between R and P (denoted MRS_{agent} hereafter). The differentiability of the indifference curves implies that MRS_{agent} is always defined. Moreover, using Equality (6) we obtain that MRS_{agent} is equal to

$$\frac{dP}{dR} = \frac{\pi(e^*)u'(w - L + R - P)}{\pi(e^*)u'(w - L + R - P) + [1 - \pi(e^*)]\, u'(w - P)}. \tag{10}$$

MRS_{agent} is positive since the utility of an agent is increasing in R, but decreasing in P. Note that $MRS_{agent}(R = L) = \pi(e^*) = \pi(\underline{e})$ since the agent exerts the lowest effort when he is fully insured. Moreover, any indifference curve is "well shaped", that is, increasing at a decreasing rate when $R \geq \underline{R}(P)$. Thus, MRS_{agent} along the indifference curve only depends on the properties of the utility function $u(.)$ when $R \geq \underline{R}(P)$. However, for any $R \in [0; \underline{R}[$ an indifference curve is not necessarily well shaped since MRS_{agent} also depends on e^* which is affected by P and R. As it has already been demonstrated by Helpman and Laffont (1975) moral hazard can give rise to non-convexity of the indifference curves.

However, even if the indifference curves are not necessarily convex for $R \in [0; \underline{R}[$ we are wondering if, for a given R , the agent's marginal rate of substitution is strictly decreasing with P. In other words, we are wondering if for a given reimbursement (with $R < \underline{R}(P)$) the marginal willingness to pay decreases when the price to pay P, increases. This is an important property that we are going to use when studying the existence of linear equilibria.

For a given repayment R with $R < \underline{R}(P)$, $MRS_{agent}(R, P)$ is going to be strictly decreasing in P if and only if $\frac{\partial MRS_{agent}(R,P)}{\partial P} < 0$ or equivalently if $\frac{\partial MRS_{agent}(R,P)}{\partial w} > 0$, since P and $-w$ play the same role in the agent's utility function. Using relations (7) and (10), $\frac{\partial MRS_{agent}(R,P)}{\partial w} > 0$ if and only if

$$-\frac{u''(w - P)}{u'(w - P)} + \frac{\pi'(e^*)}{\pi(e^*)\,[1 - \pi(e^*)]}\frac{de^*}{dw} > -\frac{u''(w - L + R - P)}{u'(w - L + R - P)}. \tag{11}$$

For $R < \underline{R}(P)$ we know that $\underline{e} < e^* \leq \overline{e}$. Thus, $\frac{de^*}{dw} = -\frac{\pi'(e^*)[u'(w-L+R-P)-u'(w-P)]}{\pi''(e^*)[u(w-L+R-P)-u(w-P)]} < 0$, which implies that $\frac{\pi'(e^*)}{\pi(e^*)[1-\pi(e^*)]}\frac{de^*}{dw} > 0$. For CARA utility functions, the Arrow-Pratt measure of absolute risk-aversion, $-\frac{u''}{u'}$, is constant. Thus, $-\frac{u''(w-P)}{u'(w-P)} = -\frac{u''(w-L+R-P)}{u'(w-L+R-P)}$ and inequality (11) is then necessarily satisfied. For IARA utility functions the Arrow-Pratt measure of absolute risk-aversion is strictly increasing. If $R < \underline{R}(P) \leq L$ then $w - P > w - L + R - P$ and thus $-\frac{u''(w-P)}{u'(w-P)} > -\frac{u''(w-L+R-P)}{u'(w-L+R-P)}$. Therefore, inequality (11) is also necessarily satisfied in the case of IARA utility functions. For DARA utility functions the Arrow-Pratt measure of absolute risk-aversion is strictly decreasing. This implies that $-\frac{u''(w-P)}{u'(w-P)} < -\frac{u''(w-L+R-P)}{u'(w-L+R-P)}$ since $R < \underline{R}(P) \leq L$. Thus, for DARA utility functions inequality (11) is satisfied if and only if

$$\frac{\pi'(e^*)}{\pi(e^*)\left[1-\pi(e^*)\right]}\frac{de^*}{dw} > -\frac{u''(w-L+R-P)}{u'(w-L+R-P)} - \left(-\frac{u''(w-P)}{u'(w-P)}\right)$$

or equivalently

$$\frac{[\pi'(e^*)]^2}{\pi(e^*)[1-\pi(e^*)]\pi''(e^*)} > \left[\frac{u''(w-P)}{u'(w-P)} - \frac{u''(w-L+R-P)}{u'(w-L+R-P)}\right] \times \frac{u(w-P)-u(w-L+R-P)}{u'(w-L+R-P)-u'(w-P)}. \qquad (C_{II})$$

We thus obtain a condition on the function $\pi(e)$, which should be satisfied at $e = e^*$. Under this condition MRS_{agent} is strictly decreasing with respect to P for a given repayment R with $R < \underline{R}(P)$ even in the case of DARA utility functions.[8]

The following lemma summarizes the previous results.

Lemma 3 *For any given repayment R with $R < \underline{R}(P)$, the agent's marginal rate of substitution is strictly decreasing with respect to P in the case of:*

- *CARA utility functions,*
- *IARA utility functions,*
- *DARA utility functions if and only if the function $\pi(e)$ satisfies condition C_{II}.*

The intuition of the previous lemma is the following. For a given repayment R an increase in the premium to pay P is equivalent to a decrease in the wealth of the agent. But, *ceteris paribus* the agent's effort strictly increases when the agent's wealth decreases when $R < \underline{R}(P)$, that is, when $\underline{e} < e^* \leq \overline{e}$. This result comes from the properties of the utility function. Consider Equality (7) which indicates the optimal effort exerted by the agent. For a given (R, P) the difference in utility when the accident occurs or when it does not $(u(w-L+R-P) - u(w-P))$ increases when w decreases, since the utility function $u(.)$ is increasing and concave. Therefore, *ceteris paribus* the agent's optimal effort increases when w decreases, since the marginal gain of effort increases while the marginal cost of effort does not change. This implies that for a given repayment R with $R \leq \underline{R}(P)$ the accident probability strictly decreases when the premium P increases. Consider now first the case of CARA utility functions. For a given R with $R \leq \underline{R}(P)$, the decrease in the probability of accident implies

[8] $u(w)$ is a DARA utility function if and only if the degree of absolute risk aversion: $-\frac{u''(w)}{u'(w)}$, is decreasing in w. Making computations, one can show that this is the case if and only if: $-\frac{u'''(w)}{u''(w)} > -\frac{u''(w)}{u'(w)}$ (with $u'''(w) > 0$), that is if the degree of absolute prudence is strictly greater than the degree of absolute risk aversion. Therefore, C_{II} is necessarily satisfied if the degree of absolute prudence is close to the degree of absolute risk aversion, which amounts to say that the utility function is not "too" DARA. Indeed, in this case the RHS of C_{II} is going to be close to 0 (since we would have $\frac{u''(w-P)}{u'(w-P)} \simeq \frac{u''(w-L+R-P)}{u'(w-L+R-P)}$) while the LHS is strictly positive.

that the agent's marginal willingness to pay in order to buy insurance, which corresponds to MRS_{agent}, decreases when the premium P increases. Second, consider the case of IARA utility functions. For a given R with $R \leq \underline{R}(P)$, the agent's willingness to pay necessarily decreases when P increases since the agent exerts a higher effort which decreases the probability of accident and becomes less risk averse (an increase in P is equivalent to a decrease in w). Finally, consider the case of DARA utility functions. For a given R with $R \leq \underline{R}(P)$, the effect of an increase in P on the willingness to pay of the agent is now ambiguous. On the one hand, the decrease in the probability of accident goes for a decrease in the willingness to pay. However, an agent characterized by a DARA utility function becomes more risk averse when P increases. Overall, the latter effect is dominated by the former effect when condition C_{II} is satisfied.

Note that Lemma 3 is equivalent to say that, *ceteris paribus*, the agent 's willingness to pay for more insurance is increasing in wealth when $R < \underline{R}(P)$. Consider first, the case of CARA utility functions. When the wealth of an agent increases, *ceteris paribus*, the effort he exerts decreases. He is thus willing to buy more insurance in order to compensate the increase in the probability of accident he faces. Consider now the case of IARA utility functions. The agent is then more willing to buy more insurance when his wealth increases, since the probability of accident increases and he becomes more risk averse. Finally, consider the case of DARA utility functions. There are thus two countervailing effects. However, under condition C_{II} the increase in the probability of accident dominates the decrease in the agent's risk aversion associated with an increase in wealth.

Before looking at the linear equilibria with non exclusivity, consider the following remark about condition C_{II}.

Remark 1 *Condition C_{II} is written with respect to e^* which is endogenous. Nevertheless, we can also write a condition with respect to exogenous variables. More precisely, the agent's marginal rate of substitution is strictly decreasing with respect to P if and only if, $\forall e$:*

$$\frac{[\pi'(e)]^2}{\pi(e)\,[1-\pi(e)]\,\pi''(e)} > \left[\underbrace{\frac{u''(w-P)}{u'(w-P)} - \frac{u''(w-L+R-P)}{u'(w-L+R-P)}}_{Term1}\right] \times \underbrace{\frac{u(w-P)-u(w-L+R-P)}{u'(w-L+R-P)-u'(w-P)}}_{Term2}. \tag{C_{III}}$$

Note that the RHD of condition C_{III} is always finite. Indeed, for R strictly different from P, $Term1$ and $Term2$ are both defined and finite. Moreover, for R close to P, we have:

$$Term2 = \frac{\frac{u(w-P)-u(w-L+R-P)}{(w-P)-(w-L+R-P)}}{-\frac{u'(w-P)-u'(w-L+R-P)}{(w-P)-(w-L+R-P)}} = -\frac{u'(w-P)}{u''(w-P)}.$$

$Term2$ is thus well defined and finite. Since $Term1$ tends to zero as R tends to P, condition C_{III} then becomes $\frac{[\pi'(e)]^2}{\pi(e)[1-\pi(e)]\pi''(e)} > 0$. Therefore, it is always possible to define a function $\pi(e)$ which satisfies condition C_{III}. In the following, we are going to consider condition C_{II} only, since condition C_{II} is less restrictive than condition C_{III}.

We can now look at the linear equilibria with non exclusivity.

4 Linear Equilibria with Non Exclusivity

In order to study an equilibrium we should take into account any deviation of any insurance company. However, it is almost impossible to do so since the set of all insurance companies' strategies, $\mathcal{M}_i$, is a set of closed subsets of $\mathbb{R}^2$. In the following lemma we show that considering single offers only at the deviation stage is not restrictive. This simplifies drastically the analysis.

Lemma 4 *If an insurance company has no profitable deviation toward a single offer, then no deviation toward any menu of contracts is profitable.*

Proof. Suppose the contrary. Company i has no profitable single offer deviation, but there is a more sophisticated menu of contracts, denoted M_i, which gives it a higher payoff. Then, if the agent buys contract (p_i, r_i) in M_i, offering M_i or simply the "menu" $\{(p_i, r_i)\}$ is equivalent for company i. Indeed, in the two cases the agent chooses contract (p_i, r_i) eventually. Finally, if the agent does not buy any contract in M_i, the single contract $(0, 0)$ is equivalent to M_i for company i. Therefore, considering single offers only at the deviation stage is not restrictive. ■

We are now going to look at the linear equilibria. Consider that each insurance company issues a continuum of linear contracts (r_i, p_i) with $p_i = \pi(\underline{e}) r_i$. Then, given these insurance contracts maximizing the expected utility of an agent amounts to finding the highest indifference curve tangent to $P(\underline{e})$, where $P(\underline{e})$ is defined as the straight half-line $P(\underline{e}) = \pi(\underline{e}) R$ in the mark (R, P), starting from $(0; 0)$. Denote by $\underline{IC}$ the indifference curve which is tangent to $P(\underline{e})$ when there is full insurance, that is, for $R = L$. The indifference curve $\underline{IC}$ corresponds to all the couples (R, P) which are such that the indirect utility function $V(R, P) = u(w - \pi(\underline{e}) L) - \underline{e} \equiv \underline{u}$. There are two possibilities, which should be distinguished. Either $\underline{IC}$ has no other intersection (or tangent) point with $P(\underline{e})$ for $R < \underline{R}(P(\underline{e}))$, or $\underline{IC}$ has at least one other intersection (or tangent) point with $P(\underline{e})$ for $R < \underline{R}(P(\underline{e}))$.

Consider the (particular) constrained indifference curve, for which the agent is constrained to exert a particular level of effort e, with $e \in]\underline{e}; \overline{e}]$ and which is associated to the level of utility $\underline{u}$. We denote this indifference curve $CIC(e, \pi(e), \underline{u})$. $CIC(e, \pi(e), \underline{u})$ is then well shaped and depending on the value $\pi(e)$ either intersects or not the straight half-line $P(\underline{e})$. There is thus necessarily a particular value $\pi(e) \equiv \pi^T$ for which there exists a value of reimbursement R^T such that $CIC\left(e, \pi(e) = \pi^T, \underline{u}\right)$ is tangent to $P(\underline{e})$. Characterizing the tangent point to $P(\underline{e})$ amounts to finding the couple $\left(R^T; \pi^T\right)$ which satisfies:

$$\begin{cases} \pi^T u\left(w - L + R^T - \pi(\underline{e}) R^T\right) + \left(1 - \pi^T\right) u\left(w - \pi(\underline{e}) R^T\right) - e = \underline{u}, \\ MRS_{agent}\left(CIC(e, \underline{u}) \text{ computed at } R = R^T\right) = \pi(\underline{e}). \end{cases} \quad \text{(System 1)}$$

Doing the same for all $e \in]\underline{e}; \overline{e}]$ and defining $\pi^T = \pi(\underline{e}) = \beta$ with $\beta \in]0, 1[$ for $e = \underline{e}$ we have thus built a particular function $\pi^T(e)$ for any $e \in [\underline{e}; \overline{e}]$.[9] Let us now consider the unconstrained indifference

[9] Note that $\pi^T(e)$ is decreasing in e. Indeed, when the effort increases the first equality of System 1 implies that π should decrease so as to have the LHD equals to the RHS of the equality since at the tangent point we necessarily have $R < L$ for any $e \in]\underline{e}; \overline{e}]$.

curve associated to $\pi^T(e)$ and $\underline{u}$. This particular indifference curve, which we denote $\underline{IC}\left(\pi^T(e)\right)$ is, by construction, confounded with a segment of $P(\underline{e})$ for $e \in]\underline{e};\overline{e}[$.

Comparing $\pi(e)$ to $\pi^T(e)$ we are now able to indicate whether $\underline{IC}$ has either no other intersection point or has at least one other intersection point with $P(\underline{e})$, for $R < \underline{R}(P(\underline{e}))$.

Lemma 5 *If $\forall e \in]\underline{e};\overline{e}]$ we have $\pi(e) > \pi^T(e)$, then $\underline{IC}$ has no other intersection (or tangent) point with $P(\underline{e})$ for $R < \underline{R}(P(\underline{e}))$.*

Contrarily, if there is at least one level of effort $e \in]\underline{e};\overline{e}]$ for which $\pi(e) \leq \pi^T(e)$, then $\underline{IC}$ either intersects or is tangent to $P(\underline{e})$ for at least one level of effort $e > \underline{e}$.

Proof. See Appendix. ■

The intuition of the previous lemma is the following. Consider the extreme case in which the probability of accident does not depend on the level of effort: $\forall e\ \pi(e) = \pi(\underline{e})$. The agent thus exerts the lowest level of effort $\underline{e}$ since exerting an effort is costly. In this case $\underline{IC}$ is well shaped and never intersects $P(\underline{e})$ for any $R < L$. Consider now that the hazard moral is weak in the sense that the effort only slightly affects the probability of accident, that is, $\pi(e)$ is decreasing in e but $\pi(\underline{e}) \simeq \pi(\overline{e})$. Then, by continuity with the previous case $\underline{IC}$ does not intersects $P(\underline{e})$ for any $R < L$. Consider now that the hazard moral problem is strong in the sense that $\pi(e)$ is decreasing in e and $\pi(\overline{e})$ is significantly (strictly) lower than $\pi(\underline{e})$. Then, the risk averse agent can choose to exert an effort strictly higher than $\underline{e}$ in order to decrease significantly the probability of effort. Moreover, $\underline{IC}$ is going to be so badly shaped that it intersects $P(\underline{e})$.

Let us now start with the case in which $\underline{IC}$ has no other intersection (or tangent) point with $P(\underline{e})$ for $R < \underline{R}(P(\underline{e}))$.

4.1 Full Insurance Equilibrium

When the indifference curve $\underline{IC}$ has no other intersection point with $P(\underline{e})$ for $R < \underline{R}(P(\underline{e}))$, then the highest indifference curve tangent to $P(\underline{e})$ is precisely $\underline{IC}$.[10] Therefore, the agent optimally chooses to be fully insured and exerts an effort $e^* = \underline{e}$. Given the agent's optimal effort and insurance choice $(e^* = \underline{e}, R = L)$ the insurance pricing is actuarially fair, and the expected profit of an insurance company is equal to 0.

This situation is an equilibrium if and only if, taking into account any kind of deviation (that is, not only linear deviations), there is no profitable deviation for any insurance company. Consider that an insurance company deviates and issues a single contract $C_{d_1} = (r_{d_1}, p_{d_1})$ with $\frac{p_{d_1}}{r_{d1}} < \pi(\underline{e})$, and $r_{d_1} < L$. Then, the agent can buy this contract, and, since insurance contracts are not exclusive, can complete his insurance coverage with other contracts from the non-deviating insurance companies. Each agent thus faces a new continuum of linear contracts which belong to the straight half-line $P'(\underline{e})$ starting from C_{d_1} and parallels but strictly below $P(\underline{e})$. Assuming that $\underline{IC}$ has no intersection point with $P(\underline{e})$ but

[10] The indifference curve $\underline{IC}$ has no intersection point with $P(\underline{e})$ neither for $R < L$ nor for $R > L$ and is tangent to $P(\underline{e})$ for $R = L$.

$R = L$ amounts to consider that for any contract along the straight half-line $P(\underline{e})$ with a repayment in case of accident $R < L$, the agent's marginal willingness to pay is strictly higher than $\pi(\underline{e})$.

For a given repayment R a decrease in the premium to pay P is equivalent to an increase in the wealth of the agent. But, for a given repayment R the agent's effort strictly decreases when his wealth increases for $R < \underline{R}(P)$.[11] This implies that for a given repayment $R < \underline{R}(P)$ the accident probability strictly increases when the premium P decreases.

Consider first the case of CARA utility functions. For a given R with $R < \underline{R}(P)$ the increase in the probability of accident implies that the agent's marginal willingness to pay in order to buy insurance increases when the premium P decreases (cf., Lemma 3). Moreover, for $R \geq \underline{R}(P)$ the agent exerts the lowest effort and the indifference curves exhibit the standard properties for CARA utility functions. In particular, for a given R, the marginal rate of substitution does not change when P decreases. This implies that the willingness to pay in order to buy insurance does not change when P decreases while R is fixed. Therefore, for $R < L$ the agent's willingness to pay along the straight half-line $P'(\underline{e})$ is strictly higher than the marginal price of insurance $\pi(\underline{e})$. Therefore, the agent also chooses to be fully insured when he faces the new continuum of linear contracts which belongs to the straight half-line $P'(\underline{e})$. Indeed, for a repayment $R = L$ there exists an indifference curve which is tangent to $P'(\underline{e})$ since the marginal rate of substitution of any indifference curve is equal to $\pi(\underline{e})$ when $R = L$. Thus, if an insurance company deviates and issues contract C_{d_1} the agent buys a global insurance contract C_1 for which he is fully insured. The agent thus exerts the lowest effort $e^* = \underline{e}$ which implies that the expected profit of the deviating insurance company is negative.

Consider now the case of IARA utility functions. For $R \geq \underline{R}(P)$ the indifference curves exhibits the standard properties in the case of IARA utility functions. Thus, for a given R with $\underline{R}(P) \leq R < L$ the willingness to pay increases when P decreases, since a decrease in P is equivalent to an increase in w and the agent becomes more risk averse when his wealth increases. For a given R with $R < \underline{R}(P)$, the agent's willingness to pay necessarily increases when P decreases since the agent exerts a lower level of effort which increases the probability of accident and he becomes more risk averse (cf., Lemma 3). Therefore, if an insurance company deviates and proposes contract C_{d_1} the agent buys a global insurance contract for which he is fully insured. He thus exerts the lowest effort and the deviation is not profitable for the deviating insurance company.

Finally, consider the case of DARA utility functions. For $R \geq \underline{R}(P)$ the indifference curves exhibits the standard properties in the case of DARA utility functions. This implies that for a given R with $\underline{R}(P) \leq R < L$ the willingness to pay now decreases when P decreases, since a decrease in P is equivalent to an increase in w and the agent becomes less risk averse when his wealth increases. For $R < \underline{R}(P)$ there are two countervailing effects when P decreases while R does not change. The wealth effect goes for a decrease in the willingness to pay. However, the agent exerts a lower effort, which

[11] For a given (R, P) the difference in utility when the accident occurs or when it does not $(u(w - L + R - P) - u(w - P))$ decreases when w increases, since the utility function $u(.)$ is increasing and concave. Therefore, *ceteris paribus* the agent's optimal effort decreases when w increases, since the marginal gain of effort decreases while the marginal cost of effort does not change.

increases the probability of accident. This effect goes for an increase in the willingness to pay. Finally, Lemma 3 indicates that the latter effect dominates the former effect when condition C_{II} is satisfied. Thus, under condition C_{II} the willingness to pay increases when P decreases while R does not change and $R < \underline{R}(P)$. This implies that for a given R when P decreases the willingness to pay increases when $R < \underline{R}(P)$ but it decreases when $\underline{R}(P) \leq R < L$. However, when the probability of accident is sufficiently decreasing with the level of effort for $e = \underline{e}$ (e.g., when $|\pi'(\underline{e})|$ is high enough) the marginal gain of effort is very important and the agent exerts an effort strictly higher than $\underline{e}$ even when R is close to L. At the limit when $\lim_{e \to \underline{e}} \pi'(e) = -\infty$ then $\underline{R}(P) = L$ (cf. Lemma 1). Thus, when $|\pi'(\underline{e})|$ increases the interval $\underline{R}(P) \leq R < L$ shrinks and for $|\pi'(\underline{e})|$ high enough it almost disappears. Therefore, for $|\pi'(\underline{e})|$ high enough the agent's willingness to pay almost always increases when P decreases while R does not change. This implies that if an insurance company deviates and proposes contract C_{d_1} the agent buys a global insurance contract for which he is fully insured. He then exerts the lowest effort and the deviation is not profitable for the deviating insurance company. Any deviation is thus not profitable for an insurance company.

The following proposition sums up the previous results.

Proposition 1 *For any CARA, IARA utility functions and for any DARA utility function satisfying* C_{II} *when* $|\pi'(\underline{e})|$ *is high enough, there exists a linear equilibrium when* $\underline{IC}$ *has no other intersection point with* $P(\underline{e})$ *for* $R < \underline{R}(P(\underline{e}))$. *This linear equilibrium has the following properties:*

- *Each insurance company proposes a continuum of linear contract* (r_i, p_i) *with* $p_i = \pi(\underline{e}) r_i$.
- *Each agent chooses to be fully insured:* $R^* = L$, *and exerts the lowest effort* $e^* = \underline{e}$.
- *Each insurance company makes an expected profit equal to* 0.

Proof. See Appendix. ■

Proposition 1 is illustrated in Figure 2 below, in which the global insurance contract C_1 the agent would buy if an insurance company deviated and issued contract C_{d_1} is also represented.

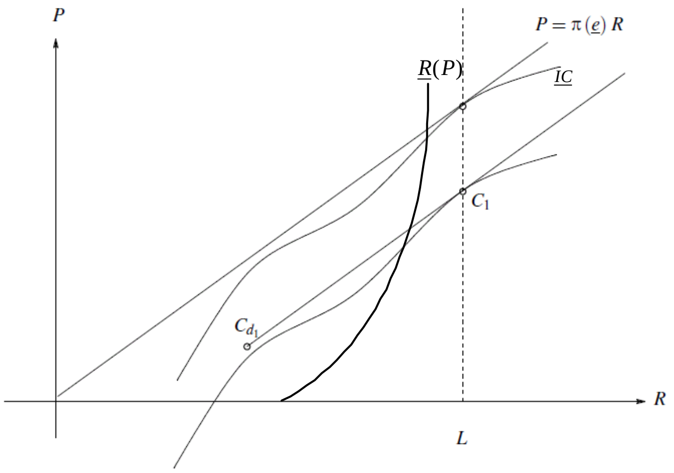

Figure 1: Linear equilibrium and global insurance contract C_1

Let us now look at the case in which $\underline{IC}$ has other intersection (or a tangent) points with $P(\underline{e})$ for $R < \underline{R}(P(\underline{e}))$.

4.2 Partial Insurance Equilibria

When $\underline{IC}$ has at least one other intersection point with $P(\underline{e})$ for $R < \underline{R}(P(\underline{e}))$ the linear equilibrium previously described is no longer an equilibrium. Indeed, the agent's optimal effort and insurance choice associated with a continuum of linear contracts (r_i, p_i) with $p_i = \pi(\underline{e})\ r_i$ is no longer $(e^* = \underline{e}, r_i = L)$. For instance, as illustrated in Figure 2 below the agent strictly prefers to buy insurance contract C' which belongs to an indifference curve superior to $\underline{IC}$.

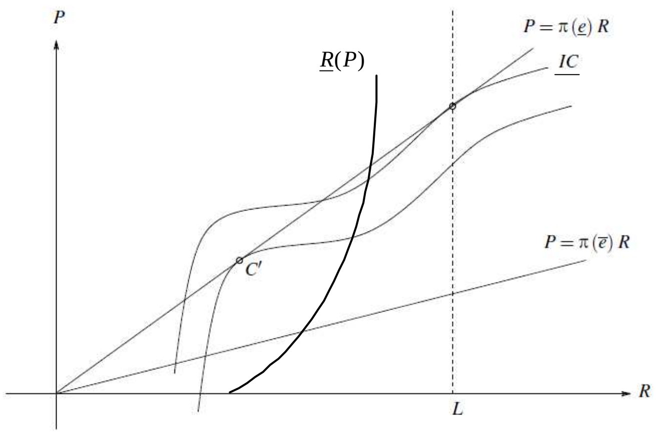

Figure 2: Contract C' is preferred to $(R = L; P = \pi(\underline{e})\, L)$

This case corresponds to the case in which the moral hazard problem is strong, in the sense that the probability of accident significantly decreases when the agent exerts a level of effort strictly higher than the lowest level of effort: $e > \underline{e}$ (cf. Lemma 5).

Taking into account that $\underline{IC}$ which is tangent to $P(\underline{e})$ for $R = L$ has at least one other intersection point with $P(\underline{e})$ for $R < \underline{R}(P(\underline{e}))$, the differentiability of the indifference curves (cf. Lemma 2) implies that there must exist at least one level of effort $e \in [\underline{e}; \overline{e}]$ for which the straight half-line $P(e)$ admits one indifference curve which is tangent to it both for $R < \underline{R}(P)$ and for $R > L$. We however show in the Appendix (See Proof of Lemma 6) that the straight half-line $P(\overline{e})$ cannot admit an indifference curve which is tangent to it for $R < \underline{R}(P(\overline{e}))$.

We thus have the following Lemma.

Lemma 6 *When $\underline{IC}$ has at least one other intersection point with $P(\underline{e})$ for $R < \underline{R}(P(\underline{e}))$ there exists at least one level of effort $e \in [\underline{e}; \overline{e}[$ such that the straight half-line $P(e)$ admits one indifference curve, denoted IC_e, which is tangent to $P(e)$ both for $R < \underline{R}(P(e))$ and for $R > L$.*

Proof. See Appendix. ■

Consider now that each insurance company issues a continuum of linear contracts (r_i, p_i) with $p_i = \pi(e_P) r_i$ where e_P is a level of effort such that $P(e_P)$ admits an indifference curve IC_{e_P} which is tangent to $P(e_P)$ both for $R < \underline{R}(P(e_P))$ and for $R > L$. Lemma 6 implies that $e_P < \overline{e}$. Maximizing the expected utility of the agent amounts to choosing one of the two levels of insurance for which IC_{e_P} is tangent to $P(e_P)$. Since the agent is indifferent between these two possibilities, we can consider that he chooses to be only partially insured. Consider now the effort exerted by the agent and let us constraint the agent to choose an effort $e \leq e_P$. Given this constraint, the probability of accident is $\pi(e) \geq \pi(e_P)$. Then, the agent's optimal choice of insurance is characterized by $R \geq L$. Hence, we can state that for an insurance contract for which $P(e_P)$ admits an indifference curve which is tangent to it both for $R < \underline{R}(P(e))$ and for $R \geq L$ the agent exerts an effort $\overline{e} \geq e^* > e_P$ when he is only partially insured. Thus, the expected profit of each insurance company which issues this type of contract is strictly positive since the agent exerts an effort strictly higher than the effort on which the insurance premium p_i is based.

This situation is an equilibrium if and only if any deviation by an insurance company is not profitable. Consider that an insurance company deviates and issues a single contract $C_{d_2} = (r_{d_2}, p_{d_2})$ with $\frac{p_{d2}}{r_{d2}} < \pi(e_P)$ and $r_{d_2} < L$. Then, each agent faces a new continuum of linear contracts which belongs to the straight half-line $P'(e_P)$ starting from C_{d_2} and parallels but strictly below $P(e_P)$. However, *ceteris paribus* the agent's effort strictly decreases with the agent's wealth for $R < \underline{R}(P)$. This implies that, for a given repayment $R < \underline{R}(P)$, the accident probability strictly increases when the premium P decreases.

Consider first the case of CARA utility functions. For a given repayment $R < \underline{R}(P)$, the agent's marginal willingness to pay in order to buy insurance coverage strictly increases when the premium P decreases (cf., Lemma 3). However, for a given repayment $R \geq \underline{R}(P)$ the agent's marginal willingness to pay so as to buy insurance is not modified when the premium P decreases, since the agent then exerts the lowest effort $\underline{e}$. In other words, when the agent is subject to moral hazard the standard property of parallel indifference curves associated with CARA utility function is only valid when $R \geq \underline{R}(P(e))$. This property is no longer true when $R < \underline{R}(P(e))$. In this later case, the indifference curves become steeper when the premium P decreases. Therefore, there is no longer any indifference curve which is tangent to $P'(e_P)$ for $R < \underline{R}(P'(e_P))$. Indeed, the indifference curve which is tangent to $P'(e_P)$ for $R \geq L$ corresponds first to the translated indifference curve of IC_{e_P}- denoted $IC_{e_P}^T$ henceforth -for $R \geq \underline{R}(P'(e_P))$ and then becomes steeper and is thus located strictly below $IC_{e_P}^T$ for $R < \underline{R}(P'(e_P))$. This latter indifference curve has thus no other tangent point with $P'(e_P)$ for $R < \underline{R}(P'(e_P))$. Therefore, if an insurance company deviates and issues a contract C_{d_2} the agent buys this contract and completes his insurance coverage until he reaches a global insurance contract C_2 for which $R > L$. He thus exerts the lowest effort $e^* = \underline{e}$ and the expected profit of the insurance company which deviates is negative.

Consider now the case of DARA utility functions. For $R > L$, the agent looses money when the accident does not occur. Therefore, when P increases his willingness to buy (over)insurance decreases. Indeed, for DARA utility functions an agent becomes more risk averse when P increases, since an increase in P is equivalent to a decrease in wealth. Thus, for $R > L$, $MRS_{agent}(R, P)$ is now strictly decreasing in P (see equations (10) and (11) for $e^* = \underline{e}$). In other words, when P decreases the indifference curves becomes steeper for a given repayment with $R > L$. The indifference curve which goes through the point where $IC_{e_P}^T$ is tangent to $P'(e_P)$ is necessarily steeper than $IC_{e_P}^T$ and intersects $P'(e_P)$. This implies that the indifference curve which is tangent to $P'(e_P)$ is tangent at the right compare to the point where $IC_{e_P}^T$ is tangent to $P'(e_P)$. Therefore, the indifference curve which is tangent to $P'(e_P)$ is located strictly below $IC_{e_P}^T$ for a repayment $R = L$. For $\underline{R}(P) \leq R \leq L$ the indifference curves becomes flatter when P decreases while R does not change. However, when $R < \underline{R}(P)$ Lemma 3 indicates that when condition C_{II} is satisfied the indifference curves becomes steeper when P decreases while R does not change. Moreover, we know from Lemma 1 that the interval $\underline{R}(P) \leq R < L$ shrinks when $|\pi'(\underline{e})|$ increases and that it disappears at the limit when $\lim_{e \to \underline{e}} \pi'(e) = -\infty$ for which $\underline{R}(P) = L$. This implies that for $|\pi'(\underline{e})|$ high enough there is no longer any indifference curve which is tangent to $P'(e_P)$ for $R \leq L$. Indeed, the indifference curve which is tangent to $P'(e_P)$ for $R \geq L$ is then strictly steeper than $IC_{e_P}^T$ for any $R < L - \varepsilon$ where $\lim_{|\pi'(\underline{e})| \to +\infty} \varepsilon = 0$. The indifference curve which is tangent to $P'(e_P)$ for $R \geq L$ is thus also located strictly below $IC_{e_P}^T$ for $R \leq L$. Therefore, if an insurance company deviates and issues a contract C_{d_2} the agent buys this contract and completes his insurance coverage until he reaches a global insurance contract C_2 for which $R > L$. Thus, each agent exerts the lowest effort $e^* = \underline{e}$ which implies that the expected profit of the insurance company which deviates is negative.

The previous results are summarized in the following proposition.

Proposition 2 *For any CARA utility function and for any DARA utility function satisfying C_{II} when $|\pi'(\underline{e})|$ is high enough, there exists a linear equilibrium when $\underline{IC}$ has at least one other intersection point with $P(\underline{e})$ for $R < \underline{R}(P(\underline{e}))$. This linear equilibrium has the following properties:*

- *Each insurance company proposes a continuum of linear contract* (r_i, p_i) *with* $p_i = \pi(e_P)\, r_i$.

- *Each agent chooses to be only partially insured:* $R^* < L$, *and exerts an effort* $e^* > e_P$.

- *Each insurance company makes a strictly positive expected profit.*

Proof. See Appendix. ■

Proposition 2 is illustrated in Figure 3 below. Moreover, the global insurance contract C_2 that the agent would buy if an insurance company deviated and issued contract C_{d_2} is also represented in Figure 3.

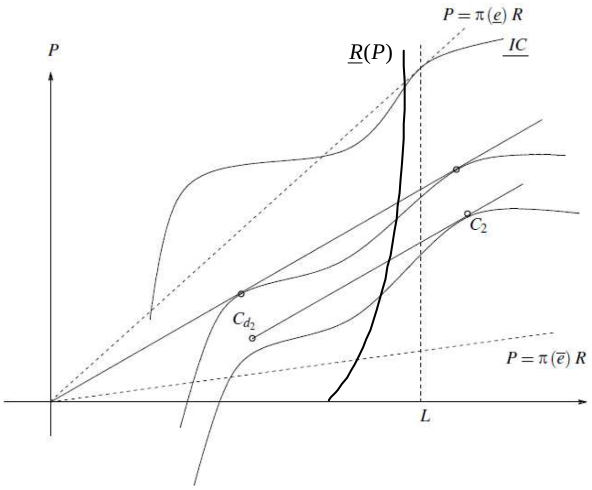

Figure 3: Linear equilibrium and global insurance contract C_2

Finally, consider the case of IARA utility functions. For $R > L$, that is, in the case in which the agent looses money when the accident does not occur, the agent's willingness to buy (over)insurance increases when P increases. Indeed, for IARA utility functions an agent becomes less risk averse when P increases, since an increase in P is equivalent to a decrease in wealth. Thus, for $R > L$, $MRS_{agent}(R, P)$ is now strictly increasing in P (see equations (10) and (11) for $e^* = \underline{e}$). In other words, when P decreases the indifference curves becomes flatter for a given repayment with $R > L$. The indifference curve which goes through the point where $IC^T_{e_P}$ is tangent to $P'(e_P)$ is necessarily flatter than $IC^T_{e_P}$ and intersects $P'(e_P)$. This implies that the indifference curve which is tangent to $P'(e_P)$ is tangent at the left compare to the point where $IC^T_{e_P}$ is tangent to $P'(e_P)$. Therefore, it is no longer the case that the indifference curve which is tangent to $P'(e_P)$ is located below $IC^T_{e_P}$ for a repayment $R = L$, contrarily to the cases of CARA and DARA utility functions. This implies that even if, for a repayment $R \leq L$, $MRS_{agent}(R, P)$ is strictly decreasing in P we cannot state that the indifference curve which is tangent to $P'(e_P)$ is located strictly below $IC^T_{e_P}$ and thus does not intersect or is tangent to $P'(e_P)$ for $R \leq L$. The deviation associated to C_{d_2} might thus be profitable and a partial insurance equilibrium might thus not exist in the case of IARA utility function. Of course, by continuity a partial insurance equilibrium should exist for IARA utility functions close to CARA utility functions. However, we cannot state general results in the case of IARA utility functions.

We are now going to study the second best efficiency of the linear equilibria.

5 Second Best Efficiency of the Linear Equilibria

Insurance company i's isoprofit curve is such that

$$p_i - \pi(e)r_i = B, \tag{12}$$

where $B \in \mathbb{R}^+$. Differentiating Equality 12 with respect to r_i and to p_i indicates that

$$\frac{dp_i}{dr_i} = \frac{\pi(e) + \pi'(e)\frac{de^*}{dr_i}r_i}{1 - \pi'(e)\frac{de^*}{dp_i}r_i}, \tag{13}$$

where $\frac{dp_i}{dr_i}$ indicates how the insurance premium must increase when the repayment increases marginally so that insurance company i's profit stays constant. $\frac{dp_i}{dr_i}$ corresponds to the marginal rate of substitution between r_i and p_i for insurance company i (denoted MRS_{InComp} hereafter). Note that $MRS_{InComp} \geq 0$, since $\pi'(e) < 0$, $\frac{de^*}{dr_i} \leq 0$ and $\frac{de^*}{dp_i} > 0$.

An equilibrium is going to be second best Pareto efficient if it is impossible to increase both the expected utility of an agent and the expected profit of an insurance company, or if it is impossible to increase the expected utility of an agent (the expected profit of an insurance company) without decreasing the expected profit of an insurance company (the expected utility of an agent). This is the case if and only if, in equilibrium:

$$MRS_{InComp} = MRS_{agent}.$$

Without loss of generality, we are going to consider that in equilibrium an agent chooses to contract with only one insurance company, since he is indifferent between contracting with one or with several insurance companies. Using equations (10) and (13) we thus obtain that $MRS_{InComp} = MRS_{agent}$ is equivalent to

$$\frac{\pi(e^*) + \pi'(e^*)\frac{de^*}{dR}R}{1 - \pi'(e^*)\frac{de^*}{dP}R} = \frac{\pi(e^*)u'(w - L + R - P)}{\pi(e^*)u'(w - L + R - P) + [1 - \pi(e^*)]\, u'(w - P)}. \tag{14}$$

We are now going to study the case of the full insurance equilibrium.

5.1 Full Insurance Equilibrium

Consider that moral hazard is weak. The equilibrium then implies full insurance $R = L$ and the agent exerts the lowest level of effort $e^* = \underline{e}$. Consider now any global insurance contract for which $R \geq \underline{R}(P(\underline{e}))$. We then have $\frac{de^*}{dR} = 0 = \frac{de^*}{dP}$. The LHS of Equality 14 reduces to $\pi(\underline{e})$. Moreover, for $R = L$ and $P = \pi(\underline{e})L$ the RHS of Equality 14 also reduces to $\pi(\underline{e})$. Equality 14 is thus satisfied in this case. The full insurance equilibrium is thus *locally* second best Pareto efficient. Determining whether the full insurance equilibrium is globally second best Pareto efficient amounts to find if $\underline{IC}$- the indifference curve which goes through $(R = L; P = \pi(\underline{e})L)$ -intersects or not the isoprofit curve which goes through $(R = L; P = \pi(\underline{e})L)$. Consider that the moral hazard problem is particularly weak in the sense that $\pi(e) \simeq \pi(\underline{e})$, $\forall e \in]\underline{e}, \overline{e}]$. Then by continuity with the case in which there is no moral hazard, the full insurance equilibrium must be (second best) Pareto optimal. Consider now that $\underline{IC}$ is tangent to $P(\underline{e}) = \pi(\underline{e})R$ both for $R = L$ and for $R < \underline{R}(P(\underline{e}))$. The isoprofit curve is confounded with the straight half-line $P(\underline{e}) = \pi(\underline{e})R$ for $R \geq \underline{R}(P(\underline{e}))$, but is located strictly below $P(\underline{e}) = \pi(\underline{e})R$ for $R < \underline{R}(P(\underline{e}))$, that is, when $e^* > \underline{e}$. Therefore, close to the second tangent point to $P(\underline{e})$, for which the agent is not fully insured, $\underline{IC}$ is necessarily located strictly above the insurance company's isoprofit. This implies that the full insurance equilibrium is not second best Pareto optimal, as it is illustrated on Figure 4 below. Indeed, any point in area A is preferred both by the agent and by the insurance company to the full insurance equilibrium. Therefore, by continuity, when $\underline{IC}$ is only tangent to $P(\underline{e})$ when $R = L$, but admits at least one other repayment point, with $R < \underline{R}(P(\underline{e}))$, for which $\underline{IC}$ is close to $P(\underline{e})$, then the full insurance equilibrium is not second best Pareto optimal. This implies that depending on the function $\pi(e)$ the full insurance equilibrium could be either second best Pareto optimal or not.

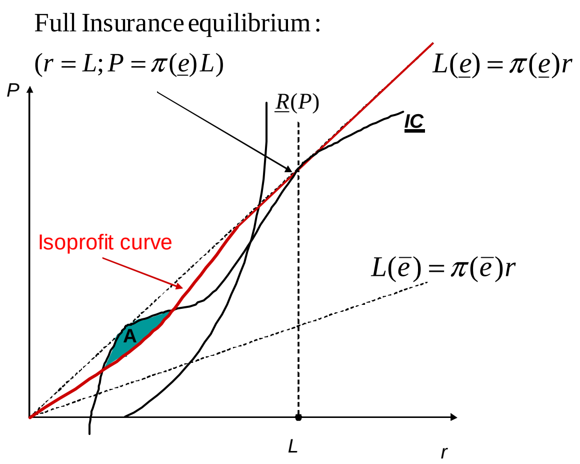

Figure 4: The Full Insurance Equilibrium is not Second Best Pareto Optimal

The following proposition sums up the previous results.

Proposition 3 *The full insurance equilibrium could be or not second best Pareto optimal. For instance, when the moral hazard problem is particularly weak in the sense that* $\pi(e) \simeq \pi(\underline{e})$, $\forall e \in]\underline{e}, \overline{e}]$, *then the full insurance equilibrium is second best Pareto optimal. Contrarily, when there is at least one repayment point, with* $R < \underline{R}(P(\underline{e}))$, *for which* $\underline{IC}$ *is close to* $P(\underline{e}) = \pi(\underline{e})R$, *then the full insurance equilibrium is not second best Pareto optimal.*

We are now going to study the case of partial insurance equilibria.

5.2 Partial Insurance Equilibria

Consider now that hazard moral problem is strong. Then in equilibrium there is partial insurance $R^* < L$. Suppose that a partial insurance equilibrium is second best Pareto efficient. This implies that equality 14 is satisfied in equilibrium. Consider now that the expected profit function of an insurance company is modified and becomes $\Pi_i(\alpha, r_i, p_i, e) = p_i - \alpha\pi(e)r_i$, with, for instance, $\alpha > 1$ (e.g. α corresponds to a functioning cost of the insurance company). Modifying the insurance profit function does not modify the (partial insurance) equilibrium since it is determined by the agent's preferences which are still the same. However, now

$$MRS_{InComp} = \frac{\alpha\left[\pi(e^*) + \pi'(e^*)\frac{de^*}{dr_i}r_i\right]}{1 - \alpha\pi'(e^*)\frac{de^*}{dp_i}r_i}.$$

Therefore, if the equality $MRS_{InComp} = MRS_{agent}$ was verified with $\Pi_i(r_i, p_i, e) = p_i - \pi(e)r_i$, this equality is necessarily not verified with $\Pi_i(\alpha, r_i, p_i, e) = p_i - \alpha\pi(e)r_i$ for any $\alpha \neq 1$ and the partial insurance equilibrium is no longer second best Pareto optimal. Therefore, any partial insurance equilibrium is not generically second best Pareto efficient.

We thus have the following proposition.

Proposition 4 *Any partial insurance equilibrium is not generically second best Pareto efficient.*

We thus have shown that when insurance contracts are not exclusive linear equilibria are not necessarily second best Pareto optimal. This result has two important consequences. First of all, a "laissez-faire" policy is not necessarily the best choice for a government. A government intervention in the insurance market could increase the social welfare of the economy. Moreover, our results also suggest that it could be better for the economy in terms of social welfare to oblige people to sign exclusive contracts only with the insurance companies, since any equilibrium would then be second best Pareto optimal. Note that even if exclusive contracts might be preferable for the economy, insurance companies might prefer non-exclusive contracts since, when partial insurance equilibria exist, they then obtain a strictly positive expected equilibrium profit which would not be the case in a perfectly competitive insurance market with exclusive contracts.

6 Conclusion

In this article, we use a common agency games framework to study linear prices equilibria in a competitive insurance market in which contracts are nonexclusive, insurance companies do not face any restriction on their pricing rules and agents are subject to moral hazard. We show that a linear equilibrium always exists in the case of CARA utility functions since an agent's willingness to pay for more insurance is increasing in wealth. For IARA utility functions a linear equilibrium always exists under the previous property when moral hazard is weak, while it might not exist when moral hazard is strong. Finally, in the case of DARA utility functions, the existence of a linear equilibrium relies on the previous property but also on a property on the function which links the probability of accident to the effort exerted by the agent. We also show that two different types of linear equilibria can exist. When moral hazard is weak then we have a full insurance equilibrium in which each agent is fully insured, exerts the lowest effort and each insurance company's expected profit is equal to zero. Contrarily, when moral hazard is strong there exist partial insurance equilibria in which each agent is only partially insured, exerts an effort strictly higher than the lowest effort and each insurance company's expected profit is strictly positive.

These results give microeconomics foundations to any work which restricts insurance companies to offer linear contracts only. Moreover, for CARA utility functions the linear equilibrium existence property is very robust since it does not rely on any assumption relative to the function which links the probability of accident to the effort exerted by the agent. The same result holds for IARA utility functions when moral hazard is weak. However, for DARA utility functions a linear equilibrium could not be taken as always granted since the linear equilibrium existence property does rely on an assumption on the function linking the probability of accident to the agent's effort.

We also provide a welfare analysis of the linear equilibria. We show that the full insurance equilibrium can either be, or not, second best Pareto optimal depending on the properties of the function which links the probability of accident to the agent's effort. Moreover, any partial insurance equilibria is not generically second best Pareto optimal.

These results could thus justify a government intervention on the insurance market and for instance the imposition of exclusive contractual relationship with insurance companies so as to restore the (second best) Pareto optimality.

A Proof of Lemma 5

Consider the constrained indifference curve $CIC(e, \pi(e), \underline{u})$ in which the agent is constrained to exert the level of effort e with $e \in]\underline{e}; \overline{e}]$ and which is associated to the level of utility $\underline{u}$, where $\underline{u} = u(w - \pi(\underline{e}) L) - \underline{e}$. $CIC(e, \pi(e), \underline{u})$ is well shaped and either intersects or not the straight half-line $P(\underline{e}) = \pi(\underline{e}) R$. There is thus necessarily a particular value $\pi(e) \equiv \pi^T$ for which there exists a value of reimbursement R^T, with $R^T < L$, such that $CIC\left(e, \pi(e) = \pi^T, \underline{u}\right)$ is tangent to $P(\underline{e})$. The couple $\left(R^T; \pi^T\right)$ corresponds to the solution of System 1, and is such that:

$$\begin{cases} \pi^T u\left(w - L + R^T - \pi(\underline{e}) R^T\right) + \left(1 - \pi^T\right) u\left(w - \pi(\underline{e}) R^T\right) - e = \underline{u}, \\ \frac{\pi^T u'\left(w - L + R^T - \pi(\underline{e}) R^T\right)}{\pi^T u'(w - L + R^T - \pi(\underline{e}) R^T) + [1 - \pi^T] u'(w - \pi(\underline{e}) R^T)} = \pi(\underline{e}). \end{cases} \qquad \text{(System 2)}$$

Doing the same for all $e \in]\underline{e}; \overline{e}]$ and defining $\pi^T = \pi(\underline{e}) = \beta$ with $\beta \in]0, 1[$ for $e = \underline{e}$ we have built a particular function $\pi^T(e)$ (as well as $R^T(e)$) for any $e \in [\underline{e}; \overline{e}]$.[12] Consider now the unconstrained indifference curve associated to $\pi^T(e)$ and $\underline{u}$. This particular indifference curve, which we denote $\underline{IC}\left(\pi^T(e)\right)$ is, by construction, confounded with a segment of $P(\underline{e})$ for $e \in]\underline{e}; \overline{e}[$.

The proof of Lemma 5 is in three steps.

Step 1: Constrained indifference curves

For any given $e \in]\underline{e}; \overline{e}]$, $CIC(e, \pi(e), \underline{u})$ is such that

$$\pi(e) u(w - L + R - P) + (1 - \pi(e)) u(w - P) - e = \underline{u}. \tag{15}$$

Consider now that $\pi(e) > \pi^T(e)$. Then for $R = R^T(e)$, System 2's first equality can only be satisfied if and only if P decreases. This implies that $CIC(e, \pi(e), \underline{u})$ is strictly below $CIC(e, \pi(e) = \pi^T(e), \underline{u})$. Thus, when $\pi(e) > \pi^T(e)$, $CIC(e, \pi(e), \underline{u})$ does not intersect $P(\underline{e})$. This implies that when $\pi(e) > \pi^T(e)$ $\forall e \in]\underline{e}; \overline{e}]$, none of the constrained indifference curves intersects $P(\underline{e})$.

Consider now that $\pi(e) < \pi^T(e)$. Then for $R = R^T(e)$, System 2's first equality can only be satisfied if and only if P increases. This implies that $CIC(e, \pi(e), \underline{u})$ is strictly above $CIC(e, \pi(e) = \pi^T(e), \underline{u})$. Thus, when $\pi(e) < \pi^T(e)$, $CIC(e, \pi(e), \underline{u})$ does intersect $P(\underline{e})$. Therefore, in order to have at least one constrained indifference curve $CIC(e, \pi(e), \underline{u})$ which is tangent or which intersects $P(\underline{e})$ it suffices to have for one level of effort (at least) $e \in]\underline{e}; \overline{e}]$, $\pi(e) \leq \pi^T(e)$.

Step 2: Comparing the unconstrained indifference curve $\underline{IC}$ to $CIC(e, \pi(e), \underline{u})$

Suppose that $\underline{IC}$ intersects one constrained indifference curve $CIC(e, \pi(e), \underline{u})$ at point $(R_{inter}; P_{inter})$. This implies that $\underline{IC}$ goes through a point- denoted $(R_{below}; P_{below})$ -which necessarily belongs to a constrained indifference curve associated to the same level of effort but located strictly below $CIC(e, \pi(e), \underline{u})$. We thus have a contradiction. Indeed any constrained indifference curve located strictly below $CIC(e, \pi(e), \underline{u})$ and associated to the same level of effort is necessarily associated to a level of utility strictly higher

[12] For $e = \underline{e}$ we know that $CIC(\underline{e}, \pi(\underline{e}), \underline{u})$ is necessarily tangent to $P(\underline{e})$ for $R^T = L$.

than $\underline{u}$. Thus, $(R_{inter}; P_{inter})$ and $(R_{below}; P_{below})$ cannot both belong to $\underline{IC}$. Doing the same reasoning for any level of effort $e \in [\underline{e}; \overline{e}]$ we obtain that $\underline{IC}$ cannot intersect any constrained indifference curve associated with a level of utility $\underline{u}$.

Step 3: Using Step 2 and Step 3

Consider that $\forall e \in [\underline{e}; \overline{e}]$ we have $\pi(e) > \pi^T(e)$. Thus, from Step 1, we know that none of the constrained indifference curves $CIC(e, \pi(e), \underline{u})$ intersects $P(\underline{e})$. Thus, $\underline{IC}$ which is by construction built from points which belongs to the constrained indifference curves $CIC(e, \pi(e), \underline{u})$ does not intersect $P(\underline{e})$. Consider now that there is at least one level of effort $e \in]\underline{e}; \overline{e}]$ for which $\pi(e) \leq \pi^T(e)$. Then, from Step 1 we know that there is (at least) one constrained indifference curve which either intersects or is tangent to $P(\underline{e})$ for a level of effort $e \in]\underline{e}; \overline{e}]$. Moreover, we know from Step 2 that $\underline{IC}$ never intersects any constrained indifference curve $CIC(e, \pi(e), \underline{u})$. Thus, since $\underline{IC}$ is tangent to $P(\underline{e})$ for $R = L$ and $\frac{dP}{dR} \geq 0$, $\underline{IC}$ necessarily either intersects or is tangent to $P(\underline{e})$ for a level of effort $e \in]\underline{e}; \overline{e}]$.

B Proof of Proposition 1

Let insurance company i offers a menu of linear contracts $M_i = (r_i, p_i)$ with $p_i = \pi(\underline{e}) r_i$. Since $\underline{IC}$ has no intersection point with $P(\underline{e})$ for any $R \neq L$, each agent optimally chooses $R = L$ and exerts the lowest level of effort $e^* = \underline{e}$. The expected profit of each insurance company is thus equal to 0: $\forall i \in I$, $\mathbb{E}\Pi_i(r_i = L, p_i = \pi(\underline{e}) L, e^* = \underline{e}) = \pi(\underline{e}) L - \pi(\underline{e}) L = 0$.

Let now an insurance company deviate and offer a single contract characterized by the couple (r_{d_1}, p_{d_1}) with $\frac{p_{d_1}}{r_{d_1}} < \pi(\underline{e})$ and $r_{d_1} < L$. Let $P'(\underline{e})$ be the straight half-line starting from (r_{d_1}, p_{d_1}) and parallels but strictly below $P(\underline{e})$. The line $P'(\underline{e})$ represents the set of insurance contracts which are now available to the agent. Let us define $\underline{IC}^T$ as the translated of $\underline{IC}$ tangent to $P'(\underline{e})$ at point $(R = L, \pi(\underline{e}) L - \gamma)$. For $R \geq R(P'(\underline{e}))$ we know that all indifference curves are "well shaped", that is, increasing at a decreasing rate and that $MRS_{agent}(R = L) = \pi(\underline{e})$. Thus, there exists one and only one (unconstrained) indifference curve which is tangent to $P'(\underline{e})$ at point $(R = L, \pi(\underline{e}) L - \gamma)$. We denote $IC(P'(\underline{e}))$ this indifference curve. The indifference curve $IC(P'(\underline{e}))$ has by definition no intersection or tangent point to $P'(\underline{e})$ for $R > L$. Consider now that $R < L$. For CARA utility functions $IC(P'(\underline{e}))$ and $\underline{IC}^T$ are confounded one with the other for $R(P(e)) \leq R < L$, while Lemma 3 implies that $IC(P'(\underline{e}))$ is strictly below $\underline{IC}^T$ for $0 \leq R < R(P(e))$. For IARA utility functions $IC(P'(\underline{e}))$ is always strictly below $\underline{IC}^T$ when $R < L$. For $R(P(e)) \leq R < L$ this is due to the properties of IARA utility functions, since MRS_{agent} then increases when P decreases. For $0 \leq R < R(P(e))$ Lemma 3 implies that $IC(P'(\underline{e}))$ is strictly below $\underline{IC}^T$. Finally, consider the case of DARA utility functions. Lemma 3 implies that under condition C_{II}, for a given R with $R < \underline{R}(P)$, the unconstrained indifference curves become steeper when P decreases. Moreover, Lemma 1 indicates that $\underline{R}(P) = L \;\forall\; P$ when $\lim_{e \to \underline{e}} \pi'(e) = -\infty$. Thus, when $\lim_{e \to \underline{e}} \pi'(e) = -\infty$ the unconstrained indifference curve which is tangent to $P'(\underline{e})$ at point $(R = L, \pi(\underline{e}) L - \gamma)$ is necessarily below $\underline{IC}^T$. This result still holds for $|\pi'(\underline{e})|$ high enough by continuity. Therefore, for CARA, IARA and DARA utility functions satisfying condition

C_{II} when $|\pi'(\underline{e})|$ is high enough, if an insurance company deviates and offers contract (r_{d_1}, p_{d_1}) the agent optimally buys this contract and completes it in order to get $(R = L, \pi(\underline{e}) L - \gamma)$. The agent thus exerts the lowest level of effort $e^* = \underline{e}$ and the expected profit of the deviating insurance company offering contract (r_{d_1}, p_{d_1}) is strictly negative, since $\frac{p_{d_1}}{r_{d_1}} < \pi(\underline{e})$. The deviation is not profitable.

C Proof of Lemma 6

For $R \geq \underline{R}(P)$ the agent exerts an effort $e^* = \underline{e}$ and each indifference curve is then "well shaped" (i.e. increasing and at a decreasing rate). This implies that for any straight half-line $P(e)$ there exists a unique indifference curve tangent to it. Moreover, $MRS_{agent} = \pi(\underline{e})$ for $R = L$ and $MRS_{agent} > \pi(\underline{e})$ for $R \in [\underline{R}(P); L[$. Thus, the indifference curve tangent to $P(\underline{e})$ is necessarily tangent for $R = L$, while for any $e > \underline{e}$ the indifference curve tangent to $P(e)$ is necessarily tangent for $R > L$.

Consider now that $e = \overline{e}$. Then, there is no indifference curve with several intersection points to $P(\overline{e})$ for $R < \underline{R}(P(\overline{e}))$. Suppose the contrary and consider Point A on Figure 5 below.

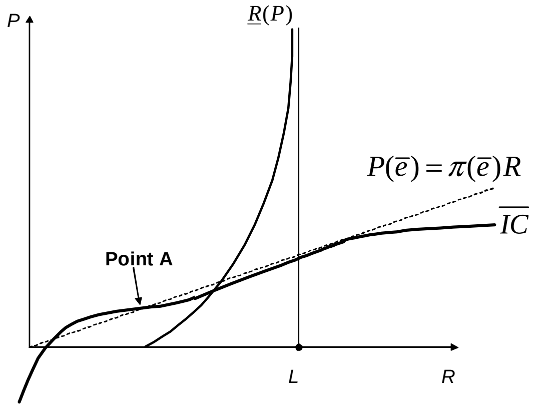

Figure 5

Consider the constrained indifference curve going through Point A and for which the agent is constrained to exert an effort equal to e_A, where $e_A \equiv e^*(\textit{Point A})$. The slope of this constrained indifference curve is equal to $\pi(e_A)$ for $R = L$ and strictly higher than $\pi(e_A)$ for $R < L$. Moreover, the slope of the non-constrained indifference curve going through Point A is equal to the slope of the constrained indifference curve going through Point A only at Point A. Thus, MRS_{agent} computed at Point A is strictly higher than $\pi(e_A)$. Besides, Point A in Figure 5 above is by construction such that $\pi(\overline{e}) > MRS_{agent}$ at Point A. Therefore, we should have:

$$\pi(\overline{e}) > MRS_{agent} \text{ computed at } (\textit{Point A}) > \pi(e_A), \tag{16}$$

which is impossible, since by definition $\overline{e} \geq e_A$ and thus $\pi(\overline{e}) \leq \pi(e_A)$. A contradiction.

Consider now that there exists one indifference curve which is tangent to $P(\overline{e})$ for $R < \underline{R}(P(\overline{e}))$. Denote $\left(\widetilde{P}, \widetilde{R}\right)$ the tangent point with $\widetilde{R} < \underline{R}(P(\overline{e}))$ and $\widetilde{e} \equiv e^* \left(\widetilde{P}, \widetilde{R}\right)$. Consider the constrained indifference curve going through $\left(\widetilde{P}, \widetilde{R}\right)$ and for which the agent is constrained to exert an effort equal to $\widetilde{e}$. The slope of this constrained indifference curve is equal to $\pi(\widetilde{e})$ for $R = L$ and is strictly higher than $\pi(\widetilde{e})$ for $R < L$. Moreover, the slope of the non-constrained indifference curve going through $\left(\widetilde{P}, \widetilde{R}\right)$ is only equal to the slope of the constrained indifference curve going through $\left(\widetilde{P}, \widetilde{R}\right)$ precisely at the point $\left(\widetilde{P}, \widetilde{R}\right)$, since then the agent optimally exerts the effort $\widetilde{e}$. Thus, MRS_{agent} computed at $\left(\widetilde{P}, \widetilde{R}\right)$ is strictly higher than $\pi(\widetilde{e})$. Furthermore, since $\left(\widetilde{P}, \widetilde{R}\right)$ is the tangent point to $P(\overline{e})$, MRS_{agent} computed at $\left(\widetilde{P}, \widetilde{R}\right)$ should also be equal to $\pi(\overline{e})$. Therefore, we should have:

$$\pi(\overline{e}) = MRS_{agent} \text{ computed at } \left(\widetilde{P}, \widetilde{R}\right) > \pi(\widetilde{e}), \tag{17}$$

which is impossible, since by definition $\overline{e} \geq \widetilde{e}$ and thus $\pi(\overline{e}) \leq \pi(\widetilde{e})$. A contradiction.

Therefore, since for $e = \underline{e}$, there is one indifference curve which is tangent to $P(\underline{e})$ for $R = L$ and which has at least one other intersection point with $P(\underline{e})$ for $R < \underline{R}(P(\underline{e}))$, the differentiability of the indifference curves (cf. Lemma 2) implies by continuity that there must exist at least one level of effort e, with $\underline{e} \leq e < \overline{e}$, such that the straight half-line $P(e)$ admits an indifference curve which is tangent to $P(e)$ both for $R < \underline{R}(P(e))$ and for $R > L$.

D Proof of Proposition 2

Let insurance company i offers a menu of linear contracts $M_i = (r_i, p_i)$ with $p_i = \pi(e_P) r_i$, where e_P is such that there exists an indifference curve, denoted IC_{e_P} which has two tangent points with $P(e_P)$. There is one tangent point for $R < \underline{R}(P(e_P))$ and the agent exerts an effort $\overline{e} \geq e^* > e_P$. We denote this tangent point $\left(R^T_{R<\underline{R}}, P^T_{R<\underline{R}}\right)$ henceforth. Besides, there is another tangent point for $R > L$ and the agent exerts the lowest effort $e^* = \underline{e}$. We denote this other tangent point $\left(R^T_{R>L}, P^T_{R>L}\right)$ henceforth. Since the agent is indifferent between the two tangent points we can consider that he chooses $\left(R^T_{R<\underline{R}}, P^T_{R<\underline{R}}\right)$. Moreover, we have $\pi(e_P) R^T_{R<\underline{R}} > \pi\left(e^*\left(R^T_{R<\underline{R}}, P^T_{R<\underline{R}}\right)\right) R^T_{R<\underline{R}}$ since the agent exerts an effort $\overline{e} \geq e^*\left(R^T_{R<\underline{R}}, P^T_{R<\underline{R}}\right) > e_P$. The expected profit of an insurance company is thus strictly positive.

Consider now that an insurance company deviates and offers the single contract (r_{d_2}, p_{d_2}) with $\frac{p_{d_2}}{r_{d_2}} < \pi(e_P)$. Let $P'(e_P)$ be the straight half-line starting from (r_{d_2}, p_{d_2}) and parallels but strictly below $P(e_P)$. The line $P'(e_P)$ represents all the insurance contracts which are now available to the agent. For $R \geq R(P'(e_P))$ all indifference curves are "well shaped", that is, increasing at a decreasing rate. Moreover, $MRS_{agent}(R = L) = \pi(\underline{e}) > \pi(e_P)$ implies that there exists one and only one (unconstrained) indifference curve which is tangent to $P'(e_P)$ for a repayment $R > L$. We denote $IC(P'(e_P))$ this indifference curve. Let $IC^T_{e_p}$ be the translated of IC_{e_p} tangent to $P'(\underline{e})$ both at points $\left(R^T_{R<\underline{R}}, P^T_{R<\underline{R}} - \kappa\right)$ and $\left(R^T_{R>L}, P^T_{R>L} - \kappa\right)$. For CARA utility functions $IC^T_{e_p}$ and $IC(P'(e_P))$ are confounded one with the other for $R \geq R(P(e))$. However, for $R \leq R(P(e))$ Lemma 3 implies that

$IC\left(P'\left(e_P\right)\right)$ is strictly below $IC^T_{e_p}$. Therefore, $IC\left(P'\left(e_P\right)\right)$ which is the only indifference curve tangent to $P'\left(e_P\right)$ for $R > L$ (at $\left(R^T_{R>L}, P^T_{R>L} - \kappa\right)$) never intersects or is tangent to $P'\left(e_P\right)$ for $R \leq L$.

Consider now DARA utility functions. For $R > L$, $e^* = \underline{e}$ and MRS_{agent} is then equal to

$$\frac{dP}{dR} = \frac{\pi\left(\underline{e}\right) u'\left(w - L + R - P\right)}{\pi\left(\underline{e}\right) u'\left(w - L + R - P\right) + \left[1 - \pi\left(\underline{e}\right)\right] u'\left(w - P\right)}. \tag{18}$$

For a given repayment R with $R > L$, $MRS_{agent}\left(R, P\right)$ is going to be strictly decreasing in P if and only if $\frac{\partial MRS_{agent}(R,P)}{\partial P} < 0$ or equivalently if $\frac{\partial MRS_{agent}(R,P)}{\partial w} > 0$, since P and $-w$ play the same role in the agent's utility function. Using equation (18), $\frac{\partial MRS_{agent}(R,P)}{\partial w} > 0$ if and only if

$$-\frac{u''\left(w - P\right)}{u'\left(w - P\right)} > -\frac{u''\left(w - L + R - P\right)}{u'\left(w - L + R - P\right)},$$

which is the case since $R > L$ and $-\frac{u''(\ )}{u'(\ )}$ is decreasing in the case of DARA utility functions.[13] Thus, the indifference curve which goes through $\left(R^T_{R>L}, P^T_{R>L} - \kappa\right)$ is necessarily steeper than $IC^T_{e_p}$ and intersects $P'\left(e_P\right)$. This implies that the indifference which is tangent to $P'\left(e_P\right)$, i.e., $IC\left(P'\left(e_P\right)\right)$, is necessarily tangent to $P'\left(e_P\right)$ for a repayment $R > R^T_{R>L}$ (i.e., the tangent point is located at the right of $\left(R^T_{R>L}, P^T_{R>L} - \kappa\right)$). Moreover, for $L \leq R \leq R^T_{R>L}$ $IC\left(P'\left(e_P\right)\right)$ is necessarily strictly below $IC^T_{e_p}$ since $MRS_{agent}\left(R, P\right)$ is strictly decreasing in P for $R > L$. For $R < L$ we know that, for a given R with $R < \underline{R}\left(P\right)$, the unconstrained indifference curves become steeper when P decreases when condition C_{II} is satisfied (cf. Lemma 3). Moreover, Lemma 1 indicates that $\underline{R}\left(P\right) = L \ \forall \ P$ when $\lim_{e \to \underline{e}} \pi'\left(e\right) = -\infty$. Thus, when $\lim_{e \to \underline{e}} \pi'\left(e\right) = -\infty$ the unconstrained indifference curve which is tangent to $P'\left(e_P\right)$ is also strictly below $IC^T_{e_p}$. Therefore, $IC\left(P'\left(e_P\right)\right)$ does not intersect or is not tangent to $P'\left(e_P\right)$ for $R < L$. For $\left|\pi'\left(\underline{e}\right)\right|$ is high enough this result still holds by continuity.

Thus, for CARA and DARA utility functions satisfying condition C_{II} when $\left|\pi'\left(\underline{e}\right)\right|$ is high enough, if an insurance company deviates and offers contract $\left(r_{d_1}, p_{d_1}\right)$ the agent optimally buys this contract and completes it in order to get a repayment $R > L$. The agent thus exerts the lowest level of effort $e^* = \underline{e}$ and the expected profit of the deviating insurance company offering contract $\left(r_{d_2}, p_{d_2}\right)$ is strictly negative, since $\frac{p_{d_2}}{r_{d_2}} < \pi\left(e_P\right)$. The deviation is not profitable.

[13] For $R > L$, $e^* = \underline{e}$ and $\frac{de^*}{dw} = 0$.

References

[1] Ales, L., Maziero, 2009. Adverse selection and nonexclusive contracts, *Carnegie Mellon University, Tepper School of Business, Working Paper 574.*

[2] Arnott, R., Stiglitz, J., E., 1988. The Basic Analytics of Moral Hazard, *Scandinavian Journal of Economics*, 90(3), 383-413.

[3] Arnott, R., Stiglitz, J., E., 1991. Price equilibrium, efficiency, and decentralizability in insurance markets, *Working Paper NBER*, N°.3642.

[4] Attar, A., Campioni, E., Chassagnon, A., Rajan, 2007. Incentives and competition under moral hazard, *Mimeo IDEI, Toulouse.*

[5] Attar, A., Chassagnon, A., 2009. On moral hazard and nonexclusive contracts, *Journal of Mathematical Economics*, 45(9-10), 511–525.

[6] Bisin, A., Guaitoli, D., 2004. Moral hazard and nonexclusive contracts, *RAND Journal of Economics*, 35, 306-328.

[7] Hellwig, M., 1983. On the moral hazard and non-price equilibria in competitive insurance markets, Discussion Paper N°109, Institut fur Gesellschafts.

[8] Helpman, E., and Laffont, J.-J., 1975. On moral hazard in general equilibrium, *Journal of Economic Theory*, 10, 8-83.

[9] Kahn, C., Mookherjee, D., 1998. "Competition and incentives with nonexclusive contracts, *RAND Journal of Economics*, 29(3), 443–465.

[10] Martimort, D., Stole, L., 2002. The revelation and delegation principles in common agency games, *Econometrica*, 1659–1673.

[11] Mossin, J., 1968. Aspects of rational insurance purchasing, *Journal of Political Economy*, 76(4), 553–568.

[12] Parlour, C., Rajan, R., 2001. Competition in loan contracts, *American Economic Review*, 91(5), 1311–1328.

[13] Pauly, M., 1974. Overproduction and public provision of insurance, *Quarterly Journal of Econcomics*, 88, 44-62.

[14] Peters, M., 2001. Common agency and the revelation principle, *Econometrica*, 69(5), 1349–1372.

[15] Rotschild, M., Stiglitz, J., 1976. Equilibrium in competitive insurance markets: An essay on the economics of imperfect information, *Quarterly Journal of Economics*, 90(4), 629–649.