ECOLE POLYTECHNIQUE
CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

Tax avoidance and the political appeal of progressivity

Matías Núñez

September 2007

Cahier n° 2007-14

LABORATOIRE D'ECONOMETRIE
1rue Descartes F-75005 Paris
(33) 1 55558215
http://ceco.polytechnique.fr/
mailto:lyza.racon@shs.poly.polytechnique.fr

Tax avoidance and the political appeal of progressivity[1]

Matías Núñez[2]

September 2007

Cahier n° 2007-14

Résumé: Empiriquement, on observe que la majorité des Etats a fait le choix d'un système d'imposition progressif. Pourtant, le fondement théorique de ce choix n'est pas évident. Si l'on interprète le système d'imposition appliquée comme le résultat d'un jeu entre deux partis politiques Downsiens, le fait que la majorité de la population soit relativement pauvre permet de conforter l'observation empirique. Cependant, des résultats théoriques récents montrent que, à l'équilibre, des électeurs purement égoïstes ne font pas toujours ce choix-là.
Cet article tente de raffiner ces derniers modèles théoriques pour proposer une autre explication au choix d'un système progressif. Notre thèse est la suivante : la présence d'évasion fiscale - caractéristique importante des systèmes d'imposition sur le revenu - a des effets sur l'équilibre du jeu politique en modifiant les préférences des individus les plus riches de la société. Dans un premier temps, l'ensemble des équilibres en stratégies mixtes du jeu est caractérisé (pour des systèmes d'imposition de type quadratique), et on montre alors que l'évasion fiscale renforce l'élection de systèmes progressifs d'imposition. Dans un deuxième temps, on analyse un cas de système d'imposition de type "wiggling" en montrant que l'évasion fiscale mène, quand elle est suffisamment importante, à l'élection de systèmes progressifs d'imposition avec certitude.

Abstract: Different theories have attempted to explain why contemporary societies have adopted marginal-rate progressive taxation schemes. One possible way of justifying this fact is to interpret the choice of a taxation scheme as the outcome of a political game between two office-seeking Downsian political parties. One can think that in such a game the existence of a majority of relatively poor voters will be enough to justify this choice. However, recent results show that at equilibrium self-interested voters do not always choose marginal-rate progressive taxation schemes in this game. We provide a refinement of the basic theory by introducing tax avoidance and showing how it affects the equilibrium outcome of the political game. We first characterize the set of mixed-strategy equilibria of the game in the case of quadratic taxation schemes, showing that tax avoidance enhances the election of marginal-rate progressive tax schemes. Second, we analyse a ``wiggling" taxes' case proving that tax avoidance leads, when ``efficient" enough, to the election of progressive taxation schemes with probability one.

Mots clés : Taxation du revenu progressive, Evasion fiscale, Compétition electorale, Equilibre en stratégies mixtes.

Key Words : Progressive Income taxation, Tax avoidance, Electoral competition, Mixed strategy equilibrium.

Classification JEL: H23; H31; D72; D78.

---

[1] I wish to thank Jean-François Laslier, Eve Ramaekers, Clemence Christin and Pierre Boucard for their help and valuable comments.

[2] Département d' Economie, Ecole Polytechnique, 1 Rue Descartes 75005, Paris, France.
E-mail address: matias.nunez@shs.polytechnique.fr

1 Introduction

As pointed out by Roemer (1998), "the framers of the US constitution" believed that, if universal suffrage was introduced, the poor would soon expropriate the rich. If this were true, democratic societies with such a suffrage would have set up an income tax schedule with a nil tax rate for the poor voters (citizens with an income lower than the mean) and a tax rate equal to one for the rich ones, as the former constitute a large majority of the population.

As a matter of fact, income tax schedules are in practice marginal-rate progressive and this expropriation has not taken place, even if universal suffrage is widely applied in contemporary societies. Different theories have been proposed to justify this feature of taxation schemes. One possible way to study this phenomenon is to interpret the taxation scheme as the outcome of a political-economic model. In such a model, the choice of the taxation scheme is determined by the electoral incentives of office-seeking political parties. This theory - called sometimes "positive theory of income taxation" - is based on the assumption that each political party proposes different tax schedules and self-interested voters choose their preferred one. Therefore, at equilibrium, political parties have an incentive to propose taxation schemes which maximize popular support.

However, the theory is still quite inconclusive regarding the progressivity of taxation schemes. Marhuenda and Ortuño Ortín (1995) ([10]) show that if the median income is below the mean income of the population, then any concave tax scheme obtains less popular support than any convex tax scheme provided that the latter treats the poorest agent no worse than the former. However, this theorem does not give much information concerning the set of equilibria of the political game. This fact implies that this theorem is not helpful to escape from voting cycles (see Hindriks (2001)).

In the literature, there has been several approaches to determine the structure of the equilibria of this political game. One of the main difficulties is the lack of existence of a Pure Strategy Nash equilibrium as stated by Myerson (1993). Indeed, in political environments, only Pure strategy equilibria were considered as conceptually acceptable. However, the novel interpretation of electoral mixed strategies, stated by Laslier (2000), paved the way for the study of the set of mixed strategy Nash equilibria. Under this approach, a mixed strategy profile in a two-party game can be interpreted as reflecting the parties' platform, where the probability that a policy alternative is offered equals the fraction of voters identifying a party with this alternative. In this sense, we could say that "ambiguity is a rational behavior for the parties". Among this strand of the literature, we can refer to De Donder (2000), Lizzeri and Persico (2001), Laslier and Picard (2002), Roberson (2006), Kvasov (2007) and Bernhardt, Duggan and Squintani (2007).

Carbonell-Nicolau and Ok (2007) ([3]) study the taxation schemes that arise at equilibrium from electoral competition between two Downsian parties. [3] make two fundamental contributions to this field. They first show that if political parties restrict the proposed tax schemes to progressive and regressive ones then progressive taxation is proposed with probability one. More formally, they show that the set of mixed strategy equilibria lies within the set of marginal-progressive tax schemes under this restriction. Secondly they claim that, when this restriction is softened, the equilibrium outcome does not consist of progressive taxation schemes anymore. They also provide an example where, at equilibrium, regressive taxes are more likely to be elected than progressive ones. This example is based on the introduction of what they call "wiggling" taxes. "Wiggling" taxes can be understood as tax schedules that are neither progressive nor regressive. Their main conclusion is that "majority support of progressive taxation cannot be only due to the majority of the population being relatively poor". Finally, [3] suggest looking for some other reasons to justify this claim.

In the present work, we take part in this debate by suggesting that tax avoidance could be one of the missing arguments to justify this empirically verified fact. Tax avoidance can be defined as the wide variety of activities people engage in where the only purpose is to lower the individual tax burden. Not much attention has been paid to it in the literature on voting over income taxation. To our knowledge, only one paper deals with this phenomenon. Roine (2006) ([13]) studies the effects of tax avoidance when voting over a linear tax scheme. When tax avoidance is taken into account, he shows that a plethora of political equilibria can emerge and that if tax avoidance is effective enough, a coalition of the poor and the very richest individuals can favor a higher tax rate.

According to [13], tax avoidance represents 2 to 7% of GDP; and 5 to 20 % of the population avoid 10 to 20% of their official tax payments. [13] also estimates that the share of total income which is not taxed due to tax avoidance is close to 4.6 %. These estimations are in line with classic estimates but not in line with Feldstein (1999) who estimates a deadweight loss of as much as 30 % from using a tax on labor income instead of a lump-sum transfer. Although judging which of these estimates is correct is out of the scope of this paper, we have decided to use [13]'s estimates which are significantly lower. This choice is not random and, indeed, using [4]'s ones will strengthen our results as we will show throughout.

Tax avoidance can be understood in two ways: either as a change on the preferences of the richest individuals or as some kind of distortion in the payoffs of the political game. We show that it enhances popular support for progressive taxation. We assume that there is a continuum of individuals which differ in endowed income. These individuals face two decisions: how to vote and whether to avoid taxes or not. We assume voters are self-interested and maximize their after-tax income. To keep things simple, the available tax schemes are the quadratic ones. We first show (Section 2) that the popular support for progressive taxation increases with the presence of tax avoidance. Following on from this result, Section 3 provides a characterization result of mixed strategy Nash equilibria that arise in the political game when taxes are quadratic. Finally, we study the counterexample with the "wiggling" tax under tax avoidance. We show that (Section 4), when tax avoidance is efficient enough, progressive taxation scheme is elected with probability one at equilibrium.

2 The model

We consider an exchange economy with a continuum of individuals. Individuals differ in endowed income, $x \in [0, 1]$, which is distributed according to a distribution function F.

The function F is such that the median income $m = F^{-1}(\frac{1}{2})$ is lower than the mean income $\mu = \int_0^1 x dF$. Furthermore, F is assumed to be an increasing and differentiable continuous function with $F(0) = 0$ and $F(1) = 1$. By definition, p_F denotes the Lebesgue-Stieltjes probability measure induced by F on [0,1], i.e. $p_F(S) = \int_S dF$ for any $S \subseteq [0, 1]$.

We shall assume here that, given $F \in \mathcal{F}$, tax functions are designed to collect an exogenously given amount of tax revenue $0 < r < \mu$.

A map $t \in \mathbf{C}\,[0, 1]$ is a tax scheme if it satisfies the following two properties:

- $0 \leq t(x) \leq x$ for all x
- $x \longmapsto t(x)$ and $x \longmapsto x - t(x)$ are increasing maps on [0,1]

The set of tax schemes is denoted by $\mathcal{T}$. The set of quadratic tax functions $\mathcal{Q}$ is a subset of $\mathcal{T}$, defined by $\mathcal{T} \supset \mathcal{Q} = \{t \in \mathcal{T} |\, t(x) = ax^2 + bx \text{ and } a \neq 0\}$. The set $\mathcal{Q}$ can be divided as follows

$$\mathcal{Q} = \mathcal{P} \cup \mathcal{R}$$

where $\mathcal{P} = \{t \in \mathcal{Q} \,|\, t \text{ is convex}\}$ and $\mathcal{R} = \{t \in \mathcal{Q} \,|\, t \text{ is concave}\}$.

As we shall see, due to the existence of some tax avoidance technology, taxpayers do not "really" face the same tax schemes, or at least, do not have the same preferences over them.

2.1 A simple individual decision concerning tax avoidance

Individuals in this economy face two decisions. A political decision on how to vote in the majority election that determines the tax rate, and a binary choice as far as tax avoidance is concerned.

Indeed, individuals' objective is to maximize their after-tax income $x - t(x)$. However, individuals can avoid declaring a share (1-δ), $\delta \in [0, 1]$, of their taxable income through investing a lump-sum $A \in [0, 1]$ in tax avoidance. The option to invest in tax avoidance is available to everyone, and the parameters describing the avoidance technology (A, δ) are assumed to be exogenously given. The aim of this work is to determine the existence of equilibria of this simple voting game and to evaluate the effects of tax avoidance in the set of equilibria[1]. This assumes that the government does not know the true parameters of (A, δ). This assumption even if not perfectly realistic, captures the intuition that tax administration cannot restructure legal schemes in order to eliminate tax avoidance.

As will be shown, the main effect of tax avoidance is to enhance the popular support for progressive taxation schemes. Indeed, the presence of a tax avoidance system implies that individuals in the highest quantile of income have a tendency to vote for progressive rather than for regressive taxation schemes.

Following [13], assuming that the avoidance decision can be taken at any point in time, while the election date is fixed, implies that it is made in response to the electoral outcome. Given this sequence, the optimal avoidance decision is simple. For any given tax rate, an

[1] Thus, it seems quite natural to adopt a framework similar to the one used in [3] to ensure the existence of equilibrium in the case without tax avoidance

individual chooses to invest in tax avoidance if that choice results in higher utility than paying full taxes, that is if

$$x - t(\delta x) - A > x - t(x)$$

Thus, we can give a formal definition of the critical income x_t as follows.

Definition 2.1 (Critical Income) *Given $F \in \mathcal{F}$, let (A, δ) represent the tax avoidance possibilities in the economy. Let t(x) represent the taxation scheme faced by individuals. We define the critical income x_t as the minimum income which is the solution to the indifference condition. Formally,*

$$x_t \equiv \min_{x \in [0,1]} t(x) - t(\delta x) = A$$

This formula can be interpreted as follows. The critical income (or income threshold) x_t is the income level where the entry cost to tax avoidance equals the utility gain of avoiding taxes.

Thus, the tax scheme the individual faces is

$$t^*(x) = \begin{cases} t(x) & \text{if } x \leq x_t \\ A + t(\delta x) & \text{if } x > x_t \end{cases}$$

When tax avoidance is taken into consideration, there are two types of taxation systems: the ones which are proposed to voters by political parties, denoted for instance by t, and the ones that voters *really* face and therefore express their opinion about, denoted with the same symbol with the superscript (*), i.e. t^*.

We denote by E an element of $\mathcal{E}$, the taxation environment which consists on the following set

$$\mathcal{E} = \{(F, r, A, \delta) : F \in \mathcal{F}\,, 0 < r < \mu,\ (A, \delta) \in [0,1] \times [0,1]\}$$

where $\mathcal{F}$ denotes the set of admissible distribution functions.

Modelling tax avoidance One of the main drawbacks of the way we introduce tax avoidance in this model is its simplicity. One could argue that this simple binary choice does not really mimic the tax avoidance behaviour. This modelling of tax avoidance is extensively discussed by [13]. He shows that this simple binary formulation simulates the behavioral response of the taxpayers, and that its qualitative results are robust to the introduction of more sophisticated and “realistic” expressions. Besides, it could also be argued, due to the actual complexity of taxation systems, that high-income individuals can more easily take advantage of tax avoidance than low-income individuals. For instance, as [6] says, “hiring a tax consultant to file one’s tax return may only be profitable if one has sufficiently high income”. Thus, even if this assumption could seem quite “crude”, the introduction of more advanced behavioral hypotheses will not qualitatively change the outcome, which is that the richer an individual is, the more he can benefit from tax avoidance. For instance, we can show that the results presented in this paper hold under an alternative modelling assumption. We assume (following [6]) that there exists two different technologies of tax avoidance: a cheap and inefficient one and other which is more expensive and efficient. We assume every taxpayer can access both technologies. We denote them by (δ_c, A_c) for the cheap technology and (δ_e, A_e) for the expensive one with $\delta_c > \delta_e$ and $A_c < A_e$. This formulation seems to be realistic as according to estimates 10 to 20 % of the population avoid from 10 to 20% of their official tax payments and a very small fraction (less than 0.5 %) completely avoids taxes. This more subtle characterization of the tax avoidance phenomenon will enhance the effects described on this work.

2.2 Quadratic tax functions

The next lemma is useful for the purpose of simplicity. The proof is given in the appendix.

Lemma 2.1 *Let $t \in \mathcal{P}$ and $\tau \in \mathcal{R}$. The intersection of both curves is denoted by $\theta \in [0,1]$. Furthermore, θ depends only on the income distribution $F \in \mathcal{F}$ and $\theta = \frac{\mu_2}{\mu}$.*

where μ_2 stands for $\int x^2 dF$. Obviously, the intersection θ is lower bounded by m as shown by [10]. The main aim of our formulation is to get an explicit formula.

Throughout this work, we will mostly use quadratic tax functions. In the case of quadratic taxation schemes, i.e. $t(x) = ax^2 + bx$, we can write that

$$x_t = \frac{1}{2a(\delta^2 - 1)} \left(b(1-\delta) \pm \sqrt{\frac{b^2}{(1-\delta)^2} + \frac{4aA}{1-\delta^2}} \right)$$

This means that at any positive tax rate, there is a unique income, x_t which splits the population into two separate parts. Those with an income below x_t pay full taxes, while those with a higher income choose to avoid a share δ of their tax payment.

The following proposition characterizes the partial orderings of the critical incomes in the case of quadratic taxation.

Proposition 2.1 *Let $E \in \mathcal{E}$ and $(t, \tau) \in \mathcal{P} \times \mathcal{R}$. Then, we can write that*

$$1.\ x_\tau \geq \frac{\theta}{(1+\delta)} \Longrightarrow 0 \leq x_t \leq x_\tau.$$
$$2.\ x_\tau < \frac{\theta}{(1+\delta)} \Longrightarrow 0 < x_\tau \leq x_t < \frac{\theta}{1+\delta}.$$

Proof: It is easy to see that x_t and x_τ verify the following equalities

$$a(1-\delta^2)x_t^2 + b(1-\delta)x_t = A$$
$$c(1-\delta^2)x_\tau^2 + d(1-\delta)x_\tau = A$$

Thus, comparing both income thresholds is equivalent to determining the intersections of both quadratic curves with the line of equation $y = A$. However, we can interpret this problem as follows. By assumption, we know that

$$a\mu_2 + b\mu = c\mu_2 + d\mu = r$$

Thus, we can express the initial problem differently and we can write that x_t and x_τ verify

$$a(1-\delta^2)x_t^2 + (\frac{r - a\mu_2}{\mu})(1-\delta)x_t = A$$
$$c(1-\delta^2)x_\tau^2 + (\frac{r - c\mu_2}{\mu})(1-\delta)x_\tau = A$$

That is equivalent to

$$a[(1-\delta^2)x_t^2 - \frac{\mu_2}{\mu}(1-\delta)x_t] + \frac{r}{\mu}(1-\delta)x_t = A$$
$$c[(1-\delta^2)x_\tau^2 - \frac{\mu_2}{\mu}(1-\delta)x_\tau] + \frac{r}{\mu}(1-\delta)x_\tau = A$$

It should be noted that the following equivalence is true for $i = t, \tau$:

$$(1-\delta^2)x_i^2 - \frac{\mu_2}{\mu}(1-\delta)x_i \geq 0 \Longleftrightarrow x_i \geq \frac{\theta}{(1+\delta)}.$$

Let us assume first that $x_\tau \geq \frac{\theta}{(1+\delta)}$ and that $x_t > x_\tau$. By assumption, we know that

$$a[(1-\delta^2)x_t^2 - \frac{\mu_2}{\mu}(1-\delta)x_t] + \frac{r}{\mu}(1-\delta)x_t = c[(1-\delta^2)x_\tau^2 - \frac{\mu_2}{\mu}(1-\delta)x_\tau] + \frac{r}{\mu}(1-\delta)x_\tau$$

which is equivalent to

$$a[(1-\delta^2)x_t^2 - \frac{\mu_2}{\mu}(1-\delta)x_t] - c[(1-\delta^2)x_\tau^2 - \frac{\mu_2}{\mu}(1-\delta)x_\tau] = \frac{r}{\mu}(1-\delta)(x_\tau - x_t)$$

However, as we have assumed that $x_t > x_\tau$, we know that the right side of this equality is negative. Besides, as both x_t and x_τ are greater than $\frac{\theta}{(1+\delta)}$, we can write that

$$a[(1-\delta^2)x_t^2 - \frac{\mu_2}{\mu}(1-\delta)x_t] > 0$$
$$-c[(1-\delta^2)x_\tau^2 - \frac{\mu_2}{\mu}(1-\delta)x_\tau] > 0$$

which implies that the left side of the equality is positive. This leads us to a contradiction. Similarly, it can be proven that if $x_\tau < \frac{\theta}{(1+\delta)}$, then the following inequality holds,

$$0 \leq x_\tau \leq x_t < \frac{\theta}{1+\delta}.$$

In order to show that there does not exist a tax avoidance technology (A, δ) such that $x_\tau < \frac{\theta}{1+\delta}$ and $x_t > \frac{\theta}{1+\delta}$, we solve the following equation

$$a(1-\delta^2)x_t^2 + b(1-\delta)x_t = c(1-\delta^2)x_\tau^2 + d(1-\delta)x_\tau$$

which is equivalent to

$$x_t = \frac{-b + \sqrt{b^2 - 4a(1+\delta)(-dx_\tau - cx_\tau^2(1+\delta))}}{2a(1+\delta)}$$

as we know that $x_t \in [0,1]$. And, therefore, we can write that

$$x_t - x_\tau > 0 \Longrightarrow 0 < x_\tau < \frac{d-b}{(a-c)(1+\delta)} = \frac{\theta}{1+\delta} \qquad \square$$

3 The voting game

Take any taxation environment $E \in \mathcal{E}$, and consider two political parties who are engaged in a competition to hold office. Each party proposes a mixed strategy over a finite number of different taxation schemes which they will apply if elected. If party 1 proposes the tax policy t and party 2 proposes τ, the share of individuals that strictly prefer t over τ in the population is determined as

$$w(t,\tau) = \mathbf{p}_F\{x \in [0,1] : t(x) < \tau(x)\}$$

Of course, in this case the share of individuals who strictly prefer the victory of party 2 is $w(t,\tau)$, following [3]. When tax avoidance is allowed, this is denoted as

$$w^*(t,\tau) = w(t^*,\tau^*) = \mathbf{p}_F\{x \in [0,1] : t^*(x) < \tau^*(x)\}$$

We assume parties' purpose is to maximize their utility u_i which is understood as the popular support of the proposed taxation scheme. That is, we suppose that

$$u_i(t,\tau) = \begin{cases} w(t,\tau) - w(\tau,t) & \text{if } i = 1 \\ w(\tau,t) - w(t,\tau) & \text{if } i = 2 \end{cases}$$

By assumption, we consider that parties propose a finite number of taxation schemes. This assumption is made in order to ensure the existence of equilibrium in the game. We let $S = \{s_1, s_2, \ldots, s_m\}$ represent the set of allowed taxation schemes. We assume that

$$s_j \in \mathcal{P} \text{ if } j \in \{1, \ldots, n.\}.$$
$$s_j \in \mathcal{R} \text{ if } j \in \{n+1, \ldots, m.\}.$$

We let σ denote a typical mixed strategy, i.e., a probability measure over any finite subset $S \subset \mathcal{Q}$. Formally,

$$\sigma : S \rightarrow [0,1]$$
$$s_j \longmapsto \sigma^j$$

with $\sigma(S) = \sum_{j=1}^{m} \sigma^j = 1$ and $\sigma_j \geq 0 \; \forall \, j$. Therefore, the expected payoff to party 1 (similarly to party 2) from the mixed strategy pair (σ_1, σ_2) is

$$U_1(\sigma_1, \sigma_2) = \sum_{i,j} \sigma_1^i \sigma_2^j u_1(s_i, s_j)$$

Therefore, we focus on the following two-person symmetric zero-sum game,

$$H = (\sigma_1(\mathcal{Q}) \times \sigma_2(\mathcal{Q}), (U_1, U_2))$$

for some couple of mixed strategies (σ_1, σ_2). It is not a difficult task to show that this zero-sum symmetric game does not always have a pure strategy Nash equilibrium. The main objective is to characterize the set of mixed strategy Nash equilibria of the game.

As players face a zero-sum symmetric game, it is easy to see (by min max) that if the pair (σ_1, σ_2) constitutes a mixed strategy Nash equilibrium of H, then for all mixed strategies $\hat{\sigma}$ and for both players $i = 1, 2$,

$$U_i(\hat{\sigma}, \sigma_2) \leq U_i(\sigma_1, \sigma_2)$$
$$U_i(\sigma_1, \hat{\sigma}) \geq U_i(\sigma_1, \sigma_2)$$

3.1 The effect of tax avoidance

The main objective in this section is to study the function $w^*(t,\tau)$, that accounts for the popular support when tax avoidance is taken into consideration, and its relationship with $w(t,\tau)$. Indeed, the following theorem summarizes the main intuition.

Theorem 3.1 *For any taxation environment $E \in \mathcal{E}$,*

$$w^*(t,\tau) \geq w(t,\tau) > 1/2 \text{ whenever } (t,\tau) \in \mathcal{P} \times \mathcal{R}$$

Theorem 3.1 enforces the result of Marhuenda and Ortuño Ortín [10] when voters are allowed to avoid taxes. The proof is given in the appendix. Indeed, when individuals can avoid taxes, the part of the population who vote for progressive taxes is higher than in the status quo (without tax avoidance). As a first consequence of this theorem, we can state the following corollary which will be helpful in the proof of the main theorem. No proof is provided as it is a straightforward consequence of the previous result.

Corollary 3.1 *Let $E \in \mathcal{E}$ and take any (σ_1, σ_2). If $\sigma_1(\mathcal{P}) > 0$ and $\sigma_2(\mathcal{R}) > 0$, then*

$$\sum_{i=0}^{n} \sum_{j=n+1}^{m} \sigma_1^i \sigma_2^j u_1(s_i, s_j) > 0$$

We now introduce one of the main results of this work. At equilibrium, both parties propose progressive taxation schemes with probability one.

Theorem 3.2 *For any taxation environment $E \in \mathcal{E}$, there exists a mixed strategy equilibrium of h_E and for any equilibrium (σ_1, σ_2) of this game, we have*

$$\sigma_1(\mathcal{P}) = 1 = \sigma_2(\mathcal{P})$$

Proof: In this section, we show that the mixed strategy equilibria in h_E are such that progressive taxes are chosen with probability one.

Take a pair of probability distributions (σ_1, σ_2) over the set of strategies $S = \{s_1, s_2, \ldots, s_m\}$. Let us suppose that $\sigma_2(\mathcal{R}) > 0$, i.e. there exists a $j \in \{n+1, \ldots, m\}$ such that $\sigma_2^j > 0$ with (σ_1, σ_2) being a mixed strategy Nash equilibrium. Then we have 2 cases: either $U_1(\sigma_1, \sigma_2) > 0$ or $U_1(\sigma_1, \sigma_2) \leq 0$.
Suppose $U_1(\sigma_1, \sigma_2) > 0$. Then $U_2(\sigma_1, \sigma_1) = 0$ and $U_2(\sigma_1, \sigma_2) < 0$, so (σ_1, σ_2) is not a Nash equilibrium, as player 2 can increase her expected payoff by playing σ_1 instead of σ_2.
Suppose now that $U_1(\sigma_1, \sigma_2) \leq 0$ and $\sigma_2(\mathcal{P}) = 0$. Then,

$$\begin{aligned} U_1(\sigma_1, \sigma_2) &= \sum_{i,j} \sigma_1^i \sigma_2^j u_1(s_i, s_j) \\ &= \sum_i \sum_{j=0}^{n} \sigma_1^i \sigma_2^j u_1(s_i, s_j) + \sum_i \sum_{j=n+1}^{m} \sigma_1^i \sigma_2^j u_1(s_i, s_j) \end{aligned}$$

However, as we have assumed that $\sigma_2(\mathcal{P}) = 0$, we have $\sigma_2^j = 0 \, \forall \, j \in \{0, \ldots, n\}$. Thus, we can write that

$$U_1(\sigma_1, \sigma_2) = \sum_i \sum_{j=n+1}^{m} \sigma_1^i \sigma_2^j u_1(s_i, s_j) \leq 0$$

But, let us suppose that $\sigma_1^j = 1$ when $j = j_0$ for some $j_0 \in \{0, \dots, n\}$ and $\sigma_1^j = 0$ elsewhere. Then,

$$U_1(\sigma_1, \sigma_2) = \sum_{j=n+1}^{m} \sigma_2^j u_1(s_{i_0}, s_j)$$

But, we know from Theorem 3.1 that $u_1(s_i, s_j) > 0$ whenever $s_i \in \mathcal{P}$ and $s_j \in \mathcal{R}$. Thus, we have that $U_1(\sigma_1, \sigma_2) > 0$. So, σ cannot be a Nash equilibrium.
Finally, we turn to the case $U_1(\sigma_1, \sigma_2) \leq 0$ and $\sigma_2(\mathcal{P}) > 0$. Define the probability measure ρ_1 on $\mathcal{Q}$ by

$$\rho_1^j = \frac{\sigma_2^i}{\sum_{j=0}^{n} \sigma_2^j} \text{ if } j \in \{0, \dots, n\}$$
$$\rho_1^j = 0 \text{ if not}$$

Observe that

$$\begin{aligned} U_1(\rho_1, \sigma_2) &= \sum_{i,j} \rho_1^i \sigma_2^j u_1(s_i, s_j) \\ &= \sum_{i=0}^{n} \sum_{j=0}^{m} \rho_1^i \sigma_2^j u_1(s_i, s_j) \\ &= \sum_{i=0}^{n} \sum_{j=0}^{n} \rho_1^i \sigma_2^j u_1(s_i, s_j) + \sum_{i=0}^{n} \sum_{j=n+1}^{m} \rho_1^i \sigma_2^j u_i(s_i, s_j) \\ &> \sum_{i=0}^{n} \sum_{j=0}^{n} \rho_1^i \sigma_2^j u_1(s_i, s_j) \\ &= \frac{1}{\sum_{i=0}^{n} \sigma_2^i} \sum_{i=0}^{n} \sum_{j=0}^{n} \sigma_2^i \sigma_2^j u_1(s_i, s_j) \end{aligned}$$

where the last inequality is a consequence of the corollary 3.1. Let us remark that, as H is a symmetric zero-sum game, we can write that $U(\rho_1, \rho_1) = 0$. We can also write that

$$U_1(\rho_1, \rho_1) = \frac{1}{(\sum_{i=0}^{n} \sigma_2^i)^2} \sum_{i=0}^{n} \sum_{j=0}^{n} \sigma_2^i \sigma_2^j u_1(s_i, s_j)$$

Finally, we can write that

$$U_1(\rho_1, \sigma_2) > 0 \geq U_1(\sigma_1, \sigma_2)$$

where the last inequality holds by assumption. This is a contradiction, as by definition, for any mixed strategy $\hat{\sigma}$, if (σ_1, σ_2) is a mixed strategy Nash equilibrium of the game, we have that

$$U_1(\hat{\sigma}, \sigma_2) \leq U_1(\sigma_1, \sigma_2)$$

Therefore, the proof is complete. $\square$.

4 Breaking the cycle from the inside

Carbonell-Nicolau and Ok (2007) propose an interesting example where, at equilibrium, regressive taxes are chosen with a higher probability than progressive ones. The main intuition of their counterexample is that, when introducing tax functions that "wiggle" in the sense of being regressive on certain zones and progressive over others, the equilibrium outcome of the game significantly changes, and in fact opens the door to regressive taxation. The following example is analysed in their work in order to show that the admissibility of "wiggling taxes" might even result in the emergence of regressive taxes in equilibrium. We study the same game with the introduction of some tax avoidance technology (A, δ). As will be shown, tax avoidance perturbs the equilibrium, erasing the fact that the highest probability of the mixed strategy Nash equilibrium is put on the regressive tax. Indeed, it leads to the election of marginally-progressive tax rates with probability 1 when this phenomenon is significant enough.
Let $r = 0.15$ and consider a taxation environment (F, r) where F has the following density:

$$f(x) = 2 - 2x, \quad 0 \leq x \leq 1.$$

Consider the tax functions t^1, t^2 and t^3 defined as follows,

$$t^1(x) = \begin{cases} \frac{x}{4} & \text{if } 0 \leq x \leq \frac{1}{4} \\ \alpha(x - \frac{1}{4}) + \frac{1}{16} & \text{if } \frac{1}{4} < x \leq 1 \end{cases} \quad t^2(x) = \begin{cases} \frac{x}{2} & \text{if } 0 \leq x \leq \frac{1}{4} \\ \beta(x - \frac{1}{4}) + \frac{1}{8} & \text{if } \frac{1}{4} < x \leq 1 \end{cases}$$

and

$$t^3(x) = \begin{cases} \frac{(4x)^\gamma}{8} & \text{if } 0 \leq x \leq \frac{1}{4} \\ \frac{x}{2} & \text{if } \frac{1}{4} < x \leq 1 \end{cases}$$

where $\alpha = \frac{391}{540}$, $\beta = \frac{103}{270}$ and $\gamma = \frac{4\sqrt{6}}{3} + 1$. We know from [3] that $w(t^1, t^2) = 0.67$ and hence if the game was played with the action spaces of both parties reduced to $\{t^1, t^2\}$, then the unique mixed strategy equilibrium would be the one in which both parties choose tax t^1 with probability 1.

Now, consider the situation in which we also allow for a third, "wiggling", tax function t^3. That is, the game is played with the action spaces of both parties being restricted to $\{t^1, t^2, t^3\}$. We know from [3] that $w(t^1, t^3) = 0.41$ and $w(t^2, t^3) = 0.57$. In this situation, The unique mixed strategy equilibrium is $(\frac{7}{33}, \frac{9}{33}, \frac{17}{33})$. That is, as [3] emphasizes, "the probability that the regressive tax would be proposed by a party is larger than that of the progressive tax" at equilibrium.

Proposition 4.1 *Let $E \in \mathcal{E}$ with $\mathcal{T} = \{t^1, t^2, t^3\}$. Then tax avoidance introduces a disturbance on the equilibrium which leads to the election of progressive taxes with probability 1, i.e. $\mu_1(t^1) = \mu_2(t^1)$.*

Proof: The main challenge of this example is that none of the tax schemes is a Condorcet winner when tax avoidance is not allowed: t_1 is socially preferred to t_2, t_2 is socially preferred to t_3, and t_3 is socially preferred to t_1, leading to a cycle. However, we claim that if tax avoidance technology is present enough in society, this cycle can be broken.

In order to show the effect of tax avoidance in the equilibrium, we keep computation simple by focusing on the case where the income thresholds (x_{t_1}, x_{t_2} and x_{t_3}) are located[2] above $\frac{1}{4}$. Then, it is straightforward to show that

$$\frac{1}{4} < x_{t^1} < x_{t^3} < x_{t^2}$$

First, it is easily verified that, due to the construction of t^2 and t^3, tax avoidance does not affect popular support for this strategy couple, i.e. $w^*(t^2, t^3) = w(t^2, t^3)$ as long as $\frac{1}{4} < x_{t^1}$.

Second, we show that $w^*(t^1, t^2) \geq w(t^1, t^2)$ and that $w^*(t^1, t^3) \geq w(t^1, t^3)$ as long as $\frac{1}{4} < x_{t^1}$.

The first matrix represents the game studied by [3]. For simplicity, the coefficients within the matrix represent the payoffs of player 1. We evaluate the consequences of the introduction of tax avoidance in the taxpayers choice.

	t^1	t^2	t^3
t^1	0	0.34	−0.18
t^2	−0.34	0	0.14
t^3	0.18	−0.14	0

We start by assuming that the tax avoidance parameters are equal to the ones estimated by [13]. According to their estimations, the fixed cost A represents about 22% of the average mean and the percentage of taxes paid is 80%. Given our income distribution, it implies that $A = 0.07333$ and $\delta = 0.8$. Then we have that $w^*(t^1, t^2) = 0.67$, $w^*(t^1, t^3) = 0.45$ and $w^*(t^2, t^3) = 0.57$.

	t^1	t^2	t^3
t^1	0	0.34	−0.10
t^2	−0.34	0	0.14
t^3	0.10	−0.14	0

The unique mixed strategy equilibrium is $(0.241739, 0.172414, 0.586207)$. In equilibrium, we see that the probability assigned to the regressive tax is lower than the one assigned to the progressive one.

We now allow both parameters A and δ to vary slightly, in order to show that this disturbance continues to exist and is even enforced.

4.1 Rising the efficiency

The tax avoidance parameters are such that $A = 0.07333$ and $\delta = 0.7895$.

Then we have that $w^*(t^1, t^2) = 0.67$, $w^*(t^1, t^3) = 0.5$ and $w^*(t^2, t^3) = 0.57$.

[2] This assumption is realistic and not too restrictive. Indeed, F(1/4)=0.4375 and so there is at most 56% of the population who can avoid paying full taxes, in accordance with the estimations of [13].

	t^1	t^2	t^3
t^1	0	0.34	0
t^2	-0.34	0	0.14
t^3	0	-0.14	0

The unique mixed strategy equilibrium is $(0.291667, 0, 0.708333)$. In equilibrium, we see that the lowest probability is placed, this time, on the regressive tax.

When the tax avoidance parameters are such that $A = 0.07333, \delta = 0.75$, then we have that $w^*(t^1, t^2) = 0.71$, $w^*(t^1, t^3) = 0.55$ and $w^*(t^2, t^3) = 0.57$. The unique strategy equilibrium is to choose t_1, the progressive tax with probability 1.

To sum up, the presence of tax avoidance changes the payoffs and the equilibrium of the game. Indeed, as tax avoidance technology becomes more efficient, the game evolves towards the following one:

	t^1	t^2	t^3
t^1	0	0.42	0.1
t^2	-0.42	0	0.14
t^3	-0.1	-0.14	0

Therefore, we know from [3] that at equilibrium, the only mixed strategy Nash equilibrium is $\mu_1(t^1) = \mu_2(t^1) = 1$.

4.2 Decreasing the cost

The tax avoidance parameters are such that $A = 0.06833$ and $\delta = 0.8$.

Then we have that $w^*(t^1, t^2) = 0.67$, $w^*(t^1, t^3) = 0.5$ and $w^*(t^2, t^3) = 0.57$.

	t^1	t^2	t^3
t^1	0	0.34	0
t^2	-0.34	0	0.14
t^3	0	-0.14	0

The unique mixed strategy equilibrium is $(0.291667, 0, 0.708333)$. In equilibrium, we see that the lowest probability is placed, this time, on the regressive tax.

When the tax avoidance parameters are such that $A = 0.06333, \delta = 0.8$, then we have that $w^*(t^1, t^2) = 0.68$, $w^*(t^1, t^3) = 0.52$ and $w^*(t^2, t^3) = 0.57$. The unique strategy equilibrium is to choose t_1, the progressive tax with probability 1.

Again, the presence of tax avoidance changes the game. Indeed, as tax avoidance technology becomes less costly, the game evolves towards the following one:

	t^1	t^2	t^3
t^1	0	0.36	0.04
t^2	-0.36	0	0.14
t^3	-0.04	-0.14	0

Therefore, we know from [3] that at equilibrium, the only mixed strategy Nash equilibrium is $\mu_1(t^1) = \mu_2(t^1) = 1$.

The following diagram illustrates the intuition of the present analysis. The income y is plotted against the tax schemes t^1 (continuous curve) and t^2 (discontinuous curve). We can see that the introduction of tax avoidance modifies the intersection of t^1 and t^2 and therefore the game converges rapidly to a different one from that without tax avoidance where the only equilibrium is to choose the progressive taxation scheme with probability one.

[Figure 1 about here]

[Figure 2 about here]

5 Conclusion

In this work, we have attempted to understand the role of tax avoidance when voting over income taxation. Our main finding is that tax avoidance modifies the preferences of the richest individuals and therefore introduces a disturbance on the equilibrium of the political game. Indeed, we claim that tax avoidance has a clear effect: it increases the tendency of the society to vote for progressive income taxation. However, the main limit of our approach is to give an equilibrium version only for the case of quadratic taxation. The next natural step will be to try to understand whether our claim is true in more general environments.

As pointed out in the last section, tax avoidance leads us to restore the election of progressive taxes at equilibrium as soon as tax avoidance technology is close to the estimated parameters of [13]. These estimates imply in our model around five percent of loss due to the presence of tax avoidance. Using [4]'s estimates, which imply thirty percent of loss, will increase this effect. This confirms our intuition: tax avoidance enhances the political appeal of progressivity.

References

[1] Banks J.S., Duggan J. and Le Breton M. (2002), Bounds for Mixed strategy equilibria and the spatial model of elections, *Journal of Economic Theory* , 103: 88-105.

[2] Bernhardt D., Duggan J. and Squintani F. (2007), Electoral competition with privately-informed candidates, *Games and Economic Behavior* , 58: 1-29.

[3] Carbonell-Nicolau O. and Ok E.A. (2007), Voting over income taxation, *Journal of Economic Theory*, 134: 249-286.

[4] Feldstein, M. (1999), Tax avoidance and the Deadweight Loss of the Income tax, *Review of Economics and Statistics*, 81: 674-680.

[5] Hindriks, J. (2001), Is there a demand for income tax progressivity ?, *Economics Letters*, 73: 43-50.

[6] Kvasov, D. (2007), Contests with limited resources, *Journal of Economic Theory*, forthcoming.

[7] Laslier, J.F. (2000), Interpretation of electoral mixed strategies, *Social Choice and Welfare*, 17: 283-292.

[8] Laslier, J.F. and Picard N. (2002), Distributive politics and electoral competition, *Journal of Economic Theory*, 103: 106-130.

[9] Lizzeri, A. and Persico, N. (2001), The provision of public goods under alternative electoral incentives, *American Economic Review*, 91: 225-239.

[10] Marhuenda, F. and Ortuño Ortín I. (1995), Popular support for progressive taxation, *Economics Letters*, 48: 319-324.

[11] Myerson, R. (1993), Incentives to cultivate favored minorities under alternative electoral systems, *American Political Science Review*, 87: 856-869.

[12] Roberson, P. (2006), The Colonel Blotto Game, *Economic Theory*, 29: 1-24.

[13] Roine, J. (2006), The political economics of not paying taxes, *Public choice*, 126: 107-134.

[14] Roemer, J. (1998), Why the poor do not expropriate the rich: an old argument in the new garb, *Journal of Public Economics*, 70: 399-424.

A Appendix

A.1 Preliminary concepts

Proof of Lemma 2.1: Let $t \in \mathcal{P}$ and $\tau \in \mathcal{R}$. As they both belong to $\mathcal{Q}_a$, we can write that

$$\int_0^1 t(y)dF(y) = \int_0^1 \tau(y)dF(y) = r$$

which implies that

$$a\mu_2 + b\mu = c\mu_2 + d\mu \Longleftrightarrow (a-c) = \frac{\mu_2}{\mu}(d-b)$$

where $\mu = \int_0^1 xdF(x)$ and $\mu_2 = \int_0^1 x^2dF(x)$. Denoting by θ the intersection between t and τ, we can give an explicit formulation for θ that only depends on the income distribution. Formally,

$$\exists\, \theta \in (0,1] \text{ s.t. } t(\theta) = \tau(\theta) \Longrightarrow \theta = \frac{d-b}{a-c} = \frac{d-b}{\frac{\mu}{\mu_2}(d-b)} = \frac{\mu_2}{\mu} > \mu \quad \square.$$

A.2 Proof of the Theorem 3.1.

For simplicity, we denote by θ^s the intersection between the modified taxation schemes t^* and τ^* in reference to the intersection θ between the proposed taxation schemes t and τ.
Case 1: When there exists neither x_t nor x_τ

In this case, both income thresholds are located either above the maximal income or below the minimal one. Thus, the tax avoidance affects in the same way both taxation schemes. Therefore, we are either in the status quo situation or in a case where all the population avoids taxes. In the status quo situation, we have that,

$$t^*(y) = t(y) \ \forall\, y \in [0,1]$$
$$\tau^*(y) = \tau(y) \ \forall\, y \in [0,1]$$

Thus, as shown by Lemma 2.1, we have that the intersection $\theta^s = \theta$ is located above m and then $w^*(t,\tau) = w(t,\tau) > \frac{1}{2}$. In the situation where all the population avoids taxes, the modified taxation schemes t^* and τ^* are such that:

$$t^*(y) = A + t(\delta y) \ \forall\, y \in [0,1]$$
$$\tau^*(y) = A + \tau(\delta y) \ \forall\, y \in [0,1]$$

Thus, the intersection θ^s is such that

$$t^*(\theta^s) = \tau^*(\theta^s) \Longrightarrow A + t(\delta\theta^s) = A + \tau(\delta\theta^s)$$
$$\Longrightarrow \theta^s = \frac{\theta}{\delta} > \theta > m$$

So, we can write that $w^*(t,\tau) > w(t,\tau) > \frac{1}{2}$.

Case 2: When there exists both x_τ and x_t

Lemma A.1 *Let $E \in \mathcal{E}$ and suppose $x_\tau \geq \frac{\theta}{(1+\delta)}$. We can write that $w^*(t,\tau) > w(t,\tau) > \frac{1}{2}$ whenever $(t,\tau) \in \mathcal{P} \times \mathcal{R}$.*

Proof:
Case 1.1. Suppose first that $x_t > \theta$ and so, $x_\tau > \theta$. Then it is easy to show to write that $w^*(t,\tau) \geq w(t,\tau)$.
Case 1.2. Suppose now that $x_\tau > \theta$ and $x_t < \theta$. If we show that $t^*(\theta) < \tau^*(\theta)$, we will be done as the intersection between t^* and τ^* will take place after θ. But, given that $x_t < \theta$, we can write

$$t^*(\theta) = A + t(\delta\theta) < t(\theta) = \tau(\theta) = \tau^*(\theta)$$

where the last equality comes from the fact that $x_\tau > \theta$.
Case 2.1. Suppose now that $x_\tau \in [\frac{\theta}{1+\delta}, \theta]$. As $x_\tau < \theta$, we know that $t(x_\tau) < \tau(x_\tau)$, by the definition of θ. However, by Proposition 2.1, we know that in this case, $x_t \leq x_\tau$. Thus, we can write that

$$t^*(x_\tau) = A + t(\delta x_\tau) < t(x_\tau) < \tau(x_\tau) = \tau^*(x_\tau)$$

Therefore, $\tau^*(x_\tau) > t^*(x_\tau)$. This ensures the fact that the intersection takes place between $A + t(\delta x)$ and $A + \tau(\delta x)$. Thus, $w^*(t,\tau) > w(t,\tau) > \frac{1}{2}$. $\Box$.

Lemma A.2 *Let $E \in \mathcal{E}$ and suppose $x_\tau < \frac{\theta}{(1+\delta)}$. We can write that $w^*(t,\tau) > w(t,\tau) > \frac{1}{2}$ whenever $(t,\tau) \in \mathcal{P} \times \mathcal{R}$.*

Proof:

As Proposition 2.1 shows, assuming that $x_\tau < \frac{\theta}{(1+\delta)}$ implies that $x_\tau < x_t < \frac{\theta}{1+\delta} < \theta$. However, by the definition of θ, we can write that

$$\tau(x_t) > t(x_t)$$

Note that if we show that $\tau^*(x_t) > t(x_t)$, then the intersection will take place between $A + t(\delta y)$ and $A + \tau(\delta y)$. By definition, we know that $t(x_t) = A + t(\delta x_t)$. Therefore, we can write that

$$\begin{aligned} \tau^*(x_t) > t(x_t) &\Longleftrightarrow \\ A + \tau(\delta x_t) > A + t(\delta x_t) &\Longleftrightarrow \\ \tau(\delta x_t) > t(\delta x_t) &\Longleftrightarrow \end{aligned}$$

which is straightforward, as $\delta x_t \in [0,\theta]$.

Therefore, we can write that $\tau^*(x_t) > t(x_t)$ whenever $x_\tau < \frac{\theta}{(1+\delta)}$.

Thus, the only possible intersection between t^* and τ^* which is denoted θ^s is the solution to the equation

$$t(\delta\theta^s) = \tau(\delta\theta^s)$$

Thus, $w^*(t,\tau) > w(t,\tau) > \frac{1}{2}$. $\Box$.

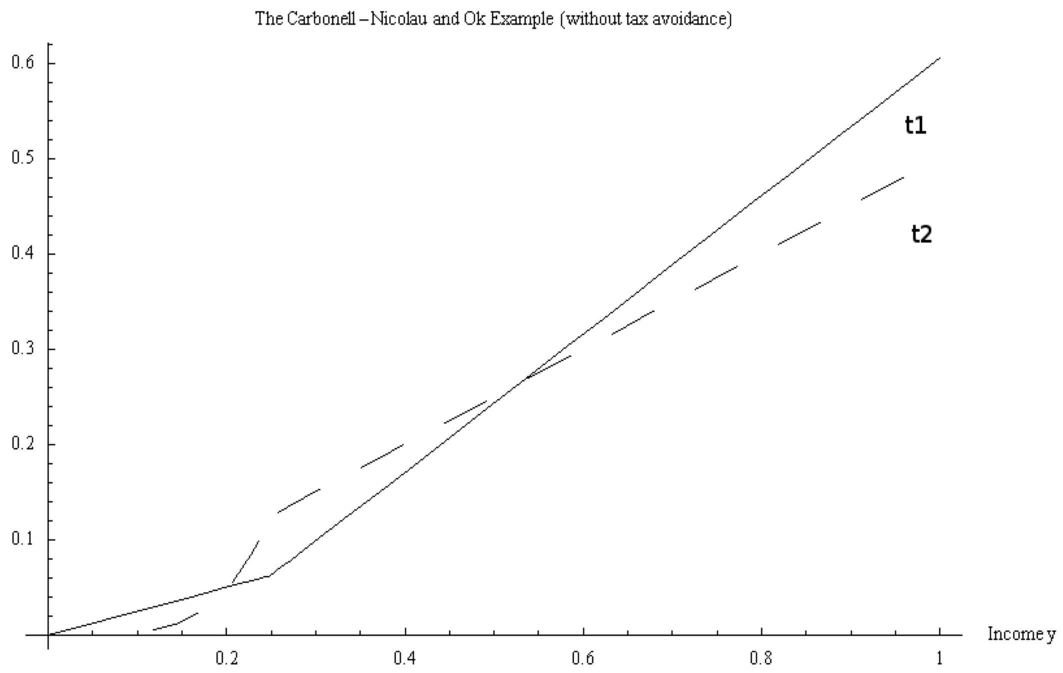

Figure 1: The tax functions t^1 and t^2

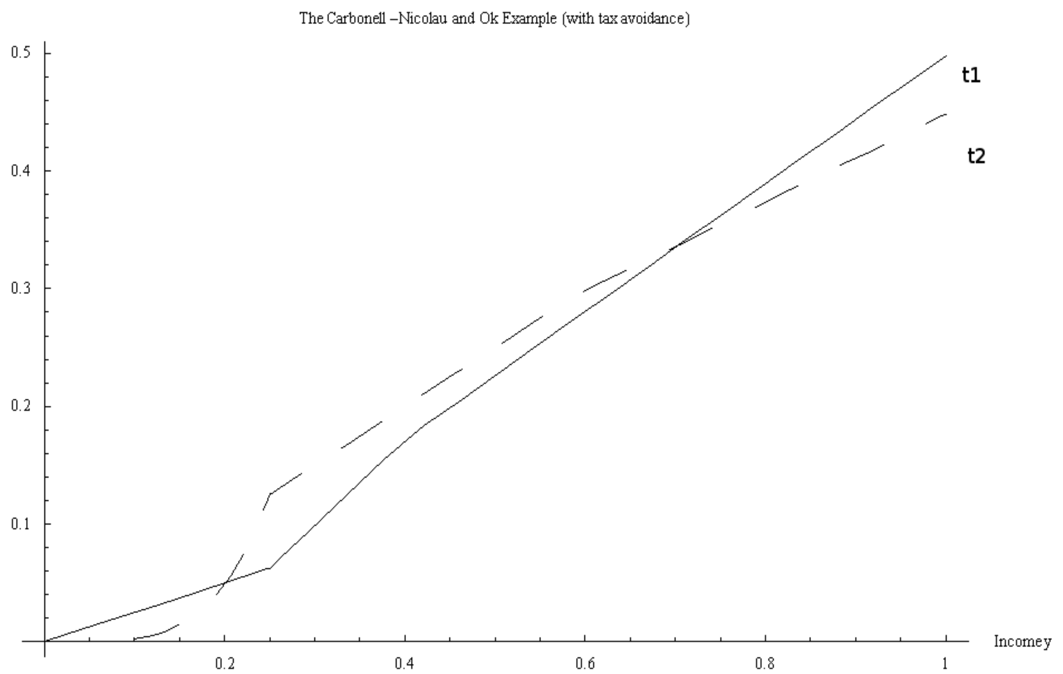

Figure 2: The effect of tax avoidance in t^1 and t^2