Uniqueness for a system of SDE, in the context of Scaling limits of G-W with sex

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The following question was raised by Vincent last year in his presentation at *Berlin-Paris Young Researchers Workshop, second edition Stochastic Analysis with applications in Biology and Finance (May 2-4 2018)*:

Does the following system of SDEs

$$X_t = x_0 + \alpha_1 \int_0^t \mathfrak{S}_s ds + \sigma_1 \int_0^t \sqrt{\mathfrak{S}_s}\, dB_s^1 + \int_0^t \int \mathbf{1}_{\theta \le \mathfrak{S}_{s-}} z \mathbf{1}_{z \le 1}\, d\widetilde{N}^1 + \int_0^t \int \mathbf{1}_{\theta \le \mathfrak{S}_{s-}} z \mathbf{1}_{z \ge 1}\, dN^1$$

$$Y_t = y_0 + \alpha_2 \int_0^t \mathfrak{S}_s ds + \sigma_2 \int_0^t \sqrt{\mathfrak{S}_s}\, dB_s^2 + \int_0^t \int \mathbf{1}_{\theta \le \mathfrak{S}_{s-}} z \mathbf{1}_{z \le 1}\, d\widetilde{N}^2 + \int_0^t \int \mathbf{1}_{\theta \le \mathfrak{S}_{s-}} z \mathbf{1}_{z \ge 1}\, dN^2$$

has a **unique solution**? Here $\mathfrak{S}_t = X_t \wedge Y_t$, $B^i, i = 1,2$ are BM and $N^i, i = 1,2$ are Poisson processes (independent??) with intensities $d\nu^i(ds, d\theta, dz) = ds\, d\theta\, \lambda_i(dz)$.

Note that this system appears as scaling limits in the context of G-W with sex (work in progress V. Bansaye, M.E. Caballero and S. Méléard).

We shall prove uniqueness of this system under the hypotheses

$$\int_{\mathbb{R}_+} (z^2 \wedge 1)\, \lambda_i(dz) < \infty.$$

plus a technical condition to be explained.