Numerical Modeling of Radiation-Dominated and QED-Strong Regimes of Laser-Plasma Interaction.
3. Muon Production as a Fundamental Process in Laser-Based High Energy Physics

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Outline

• We consider effects in strong laser fields at $10^{22} - 10^{24} \text{ W/cm}^2$ and the projection towards $10^{29} \text{ W/cm}^2$
• We discuss the way along which the radiation and its back-reaction on electrons can be described theoretically and simulated numerically.
• QED-strong field ($\chi \sim 100$):
  – No unlimited pair production (limited by the radiation back-reaction)
  – No exponential growth
  – Interesting for pair production, but there is no avalanche.
• Muon-producing field ($\chi \approx 10^7 - 10^8$):
  – Noticeable muon production
  – This is a dominant channel of the rest mass energy production
  – Remarkably high energetic efficiency for electron beam conversion to the muon beam.
Achievements and plans with relativistically strong laser pulses

• An electron in the field of an electromagnetic wave suffers oscillations or rotation depending on the wave polarization.

• The electron energy exceeds $mc^2$ as the wave intensity exceeds $\sim 10^{18} \text{ W/cm}^2$. So strong waves are referred to as ‘relativistically strong’.

• Actually achieved intensities: $\sim 2 \cdot 10^{22} \text{ W/cm}^2$. In such fields even ions are almost relativistic. Planned to be achieved soon $\sim 5 \cdot 10^{23}$-$\sim 3 \cdot 10^{24} \text{ W/cm}^2$ within European Extreme Light Infrastructure (ELI) project.

• QED-strong fields - one of the definitions: the electron in that strong field emits the photons, their energy being comparable with the electron energy. Generally, the electron motion in such fields should be treated within the framework of QED (probability calculations).

• Even if the field is not QED strong, still the emission processes can be significant (‘the radiation-dominated regime’). The radiation force may exceed the Lorentz force (in some sense) and its account is inevitable.
I-1 Introduction

• There is a pronounced similarity between processes in strong electromagnetic fields, in different environments.

• Particularly, for a relativistic electron interacting with the magnetic field of a neutron star, and for interactions of counter-propagating electrons with relativistically strong laser pulses the same dimensionless parameter,

\[ \chi \sim \frac{E_0}{E_s} \]

is essential, which is the ratio of the electric field, experienced by an electron, in the frame of reference, co-moving with the electron,

\[ E_0 = \begin{cases} \frac{|p \times B|}{mc} \\ \frac{|dA/d\xi| \omega (\varepsilon - p ||) / c} \end{cases} \]

to the Schwinger field,

\[ E_s = \frac{m_e c^2}{|e| \Delta c} \]
In the astrophysical environments, $\chi > 1$ occurs in relation to:
(1) the high-field limit of gyro-synchrotron emission,
(2) possible pair creation,
(3) significant electron recoil while emitting.

$\chi > 1$ can be achieved when 600MeV electron interacts with a counter-propagating laser pulse of intensity $2 \times 10^{22} \text{W/cm}^2$. QED effects including radiation back-reaction on the particle motion come to power.
At $\chi > 1$ the following inequalities hold:

1. Electric field acting on the particle in the co-moving frame exceeds $\left(\frac{2}{3}\right)E_s$.

   That is, this field is QED-strong.

2. Formally calculated within the classical theory the typical energy of emitted photons $\eta \omega_c$ exceeds the electron energy $\epsilon mc^2$ so that:

   $\eta \omega_c > \epsilon mc^2$

   That is, the electron recoil should be accounted for.

3. The numerical model to account for the QED effects within the framework of Particle-In-Cell method is reasonably mature now (see I.V. Sokolov et al, Physics of Plasmas 18, 093109 (2011))

$$E_s = mc^2 / e \Delta_c$$
QED is not compatible with the traditional approach to the radiation force in classical electrodynamics (LAD equation and its known approximations).

An alternative equation of motion for a radiating electron with respect to proper time has been suggested:

\[
\frac{dp^i}{d\tau} = \frac{e}{c} F_{ik} \frac{dx_k}{d\tau} - \frac{I_{QED}}{mc^2} p^i
\]

\[
\frac{dx^i}{d\tau} = \frac{p^i}{m} + \tau_0 \frac{I_{QED}}{I_{cl}} \frac{e F^{ik} p_k}{m^2 c}
\]

The derivation of these equations is given:

- by re-normalizing the mass operator [1];
- from the conservation law for a single-photon emission: for the electron to acquire the extra energy from the classical field, an extra classical current must be generated [2];
- from QED in the classical limit, for 1D wave [3].

\[\tau_0 = 2e^2 / (3mc^3)\]

[1] I.V. Sokolov, JETP 109, 207 (2009);
### I-4 Emission spectra

\[
Q(r_0, \chi) = \frac{r_0 \left( \int_{r_\chi}^{\infty} K_{\frac{5}{3}}(y) \, dy + r_0 r_\chi \chi^2 K_{\frac{2}{3}}(r_\chi) \right)}{\int_{0}^{\infty} dr_0 r_0 \left( \int_{r_\chi}^{\infty} K_{\frac{5}{3}}(y) \, dy + r_0 r_\chi \chi^2 K_{\frac{2}{3}}(r_\chi) \right)}
\]

For various values of \( \chi \):

- \( \log_{10}(I/I_C) \)
- \( \log_{10}(\omega/\omega_c) \)

- \( r_\chi = \frac{r_0}{1 - \chi r_0} \)
- \( r_0 = \frac{\omega}{\omega_c} \)
- \( \omega_c = \varepsilon c \chi / \Delta_C \)

\( \chi = \sqrt{I_{cl} / I_C} \)

\( I_C = \frac{8 e^2 c}{27 \Delta_C^2} \)
I-4 Emitted radiated power

\[
\frac{dI}{d\Omega dr_0} = I_{cl} \delta \left( \Omega - \frac{p}{p} \right) \left( \frac{I_{QED}}{I_{cl}} \right) Q(r_0, \chi)
\]

\[
\frac{I_{QED}}{I_{cl}} = \frac{9\sqrt{3}}{8\pi} \int_0^\infty dr_0 r_0 \left( \int_{r_\chi}^{\infty} K_{\frac{\chi}{3}}(y)dy + r_0 r_\chi \chi^2 K_{\frac{\chi}{3}}(r_\chi) \right)
\]

Interpolation formula (dashed line):

\[
I_{QED} = \frac{I_{cl}}{\left(1 + 1.04 \sqrt{I_{cl}/I_C}\right)^{4/3}}
\]
a_0 = 100, circular polarization, t = 100T, plasma: n_0 = 10n_c, m_i = 2m_p

Electrons escape the ponderomotive potential if the ratio \( a_0/(n_0/n_c) \) becomes too large. The radiation losses stabilize the piston.

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I-6 Validating QED effects in the model

- Validation of QED effects in the model can be done for: \( \chi = [0.1, 10] \).
- In the example is shown emission spectrum for: 600 MeV electrons interacting with 30-fs laser pulse of intensity \( 2 \times 10^{22} \) W/cm\(^2\). Here \( \chi \approx 1 \).
- We see that physically absurd prediction (dashed curve) that the maximum photon energy approaches 1 GeV is eliminated by the QED effects.
Backscattered light in simulation of the interaction of QED-strong laser fields with plasmas

\[ I_0 = 2 \times 10^{23} \text{ W/cm}^2 \]

\[ a_0 = 300 \]

Linear pol.

\[ n_0 = 30 \ n_c \]

\[ m_i = 2 \ m_p \]
Conclusions - I

• For laser intensities of $10^{23} - 10^{24}$ W/cm$^2$ gamma-photons dominate (as the loss mechanism, as the most abundant sort of particles, as the main effect in the electron motion).
• Electrons are driven by QED-strong field.
• All reactions with incoming or outgoing electrons have modified probabilities.
• Intensities $10^{23}$ W/cm$^2$ combined with the oppositely propagating 1GeV electron beam could allow us to foresee what will occur at $10^{24}$ W/cm$^2$.
• Challenges in modeling:
  – Gamma-emission spectrum, radiation back-reaction, QED corrections
  – Radiation transport, gamma-to-pair absorption.
II.1 QED-strong fields vs laser intensity & electron energy

In QED an electric field, $E$, should be treated as strong if it exceeds the Schwinger limit:

$$\chi \sim \frac{E_0}{E_s} \geq 1,$$

$$E_s = \frac{mc^2}{|e| \Delta c}$$

Counterpropagating particle in a 1D wave $a(\xi)$, $\xi = \omega t - (k \cdot x)$ may experience a QED-strong field, because the laser frequency is Doppler-upshifted in the frame of reference comoving with the electron:

$$E_0 = |dA/d\xi| \omega (E_e - p_\parallel) / c$$

$$\chi \approx 2.7 \times \frac{E_e}{\text{GeV}} \times \sqrt{\frac{I}{10^{23} \text{W/cm}^2}}$$
II.2 Radiation back-reaction

• In high-energy electron beam interaction with the ultrastrong laser pulse, treated as 1D plane wave field, the initially high value of $\chi \gg 1$ ensures multiple pair creation.
  - The radiation back-reaction, however, splits the initially high value of the invariant, $(k \cdot p)$, between all newborn particles.
  - The reduced values of $(k \cdot p)$ result in smaller values of $\chi$.
  - The cascade terminates, when all particles have $\chi < 1$ and become incapable of creating new pairs.
  - The radiation losses, thereby limit the cascading pair creation.
• The discussed processes are described by the kinetic equations for the involved particles (electrons, positrons, $\gamma$-photons).
• For circularly polarized wave the convenient choice of the independent variables in these kinetic equations is: the wave phase (instead of spatial-temporal variables) and $\chi$ –parameter (instead of energies)

II.3 Numerical example for $\chi=90$

Possible parameters: counterpropagating 46.6 GeV electron beam (SLAC $e^-$-beam) & laser pulse of intensity $\sim5\times10^{22}$ W/cm$^2$

- The initial electron beam is rapidly converted into $\gamma$-photons with high $\chi$, which then rapidly produce pairs.
- The larger fraction of the particles is born at $\chi<1$, with strongly reduced pair production rate.
II.4 Numerical example for $\chi=90$

- At the initial stage:
\[
\frac{dN_{e^-e^+}}{d\xi} = \frac{dW_{\text{absorption}}}{d\xi} N_{\text{rigid photons}}
\]
\[
\frac{dN_{\text{rigid photons}}}{d\xi} = \frac{dW_{\text{emission}}}{d\xi} N_{e^-,\text{initial}}
\]
\[
N_{e^-,\text{initial}} = \text{Const}
\]
\[
N_{\text{rigid photons}} \propto \xi
\]
\[
N_{e^-e^+} \propto \xi^2
\]

- 10-40 pairs are born per single beam electron.

- Slow absorption of photons with $\chi \sim 1-2$ maintains pair production even at tens of wave periods.
Conclusions - II

• Laser-beam interaction may be accompanied by multiple pair production.
• Electron beam initial energy is efficiently spent for creating pairs with significantly lower energies as well as softer photons.
• This effect may be used:
  - for producing a pair plasma,
  - for deactivation after-use electron beams.
• The way to solve the kinetic equations:
  - is accurate,
  - does not employ the Monte Carlo method.
• The solution can be used to benchmark numerical methods designed to simulate processes in QED-strong laser fields.

References:
III. Muon-Producing fields

• (Electron-Positron) Pair-Producing Fields

\[ \chi \approx 2.7 \times \frac{E_e}{\text{GeV}} \times \sqrt{\frac{I}{10^{23} \text{ W/cm}^2}} \geq 10 \]

• Muon-Producing Fields

\[ \chi_\mu = \chi \left( \frac{m_e}{m_\mu} \right)^3 \approx 1.2 \cdot 10^{-7} \chi \geq 10, \quad \frac{m_e}{m_\mu} \approx \frac{1}{206.7}, \]

\[ \frac{\chi}{10^6} \approx 2.7 \times \frac{E_e}{\text{TeV}} \times \sqrt{\frac{I}{10^{29} \text{ W/cm}^2}} \sim 30 - 100 \]

• So, we should think (dream) about the 1D wave fields of Schwinger intensity and multi-TeV particles!
III.1. Muon Production Rate

• Scaling with mass

\[ \frac{dw}{dt d\chi} = U_{\gamma \rightarrow \mu, \mu^+} \left[ \chi_\gamma, \chi_\mu \right] = U_{\gamma \rightarrow e, p} \left[ \left( \frac{m_e}{m_\mu} \right)^3 \chi_\gamma, \left( \frac{m_e}{m_\mu} \right)^3 \chi_\mu \right] \]

• Matrix element

\[ U_{\gamma \rightarrow \mu, \mu^+} \left[ \chi_\gamma, \chi_\mu \right] = \left( \frac{m_e}{m_\mu} \right)^4 \Delta \chi U_{\gamma \rightarrow e, p} \left[ \left( \frac{m_e}{m_\mu} \right)^3 \chi_\gamma, \left( \frac{m_e}{m_\mu} \right)^3 \chi_\mu \right] \]
III.2. Mass production rate

- Production rates – electrons vs muons
III.3. “Optimistic” run with muon production

- Electron beam collision with 1D wave

\[ a \approx 50000, \quad I = 10^{29} W/cm^2, \quad E_e \approx 40 TeV, \quad \chi_0 = 10^8 \]
Conclusion-III

- In the limit of high-field-high-energy the “radiation reaction” converts a strongly emitting electron to a weakly radiating muon.
- The energetic efficiency of this conversion may be very high – up to 30%.
- Open issues:
  - 1D wave field model is not realistic
  - Electro-weak interaction theory should be used?
  - Neutrino production?