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AGENDA SELECTION WITH UNCERTAIN ISSUES

Raphael GODEFROY
Eduardo PEREZ-RICHET

DEPARTEMENT D'ECONOMIE

Choosing Choices:
Agenda Selection with Uncertain Issues[*]

Raphael Godefroy[†]   Eduardo Perez-Richet[‡]

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Abstract

We study *selection rules*: voting procedures used by committees to choose whether to place an issue on their agenda. At the *selection stage* of the model, committee members are uncertain about their final preferences. They only have some private information about these preferences. We show that, conditional on the event that a voter is pivotal, increasing the voting rule at the selection stage increases the probability that the proposal is adopted if she eventually prefers the proposal at a faster rate than the same probability if she eventually prefers the status quo. To compensate for that, voters become more conservative when the *selection rule* itself becomes more conservative. The *decision rule* has the opposite effect. We compare these voting procedures to the designation of an agenda setter among the committee, and to a utilitarian social planner with all the ex interim private information.

Keywords: Selection Rules, Strategic Voting, Asymmetric Information, Agenda Setting, Citizens' Initiative.
JEL classification: D72, D83.

1   Introduction

Before they can be decided according to a majority rule, cases brought to the Supreme Court of the United States need to be approved for selection by at least four of the nine justices. This Rule of Four, which is rather a custom than a constitutional requirement, was used as a defense by the justices when in the mid-1930s the Court came under fire from the President and the

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Congress. It was accused, among other charges, of "using its discretionary jurisdiction to duck important cases,"[1] to which the justices responded that they use a submajority rule precisely because they prefer "to be at fault in taking jurisdiction rather than to be at fault in rejecting it."[2] The argument of the justices seems obvious at first, it is easier to gather four votes than five. Yet it is not so clear once we take strategic behavior into account: wouldn't the justices offset the effects of the selection rule by adjusting their individual behavior? We show that it is not the case by presenting a model in which rational individual behavior strengthens the effects of the selection rule: voters become more conservative as the rule becomes more stringent. More generally we look at the effect of democratizing the agenda by comparing the use of a voting rule at the selection stage to letting a member of the committee select the agenda, and to socially optimal selection.

Selection rules are not limited to the Supreme Court[3]. For instance, any member of the French Assemblée Nationale can place a proposal on the agenda of the parliament as long as the proposed law doesn't increase expenditure for the government. In the United States Congress, bills must be approved by vote in a specialized standing committee before they can be brought to the floor. Citizens' initiatives, which allow a group of citizens to obtain the organization of a referendum by way of petitions, are another form of selection rules. A general concern about citizens' initiatives is that they tend to bring too many issues to the agenda. Our study suggests that outcomes may be particularly sensitive to the selection rule that is chosen because of the positive feedback between the direct effect of a change in the rule and the indirect effect on behavior. Finally, recruiting committees often use selection rules as well.

Our model allows us to analyze and compare these rules. To our knowledge, it is the first formal analysis of selection rules in a rational voting framework. Our three working assumption

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[1] 81 Cong. Rec. 2809-2812 (1939).

[2] Hearings on S. 2176 before the Senate Judiciary Committee, 74th Cong. 1st sess., 9-10 (1935) (statement of Justice Van Devanter). We found a discussion of these events and the citations in Epstein and Knight (1998) p.86 who refer to a memorandum titled "The Rule of Four" that justice Marshall circulated to conference Sept. 21, 1983. For a detailed account of the selection procedure at the Supreme Court, see Perry (1994).

[3] State Supreme Courts also use selection rules. In California, for example, the justices use a supermajority rule of four out of seven justices.

are (i) that voters are uncertain about their preferences at the selection stage; (ii) that they have private information; and (iii) that their preferences are uncorrelated (conditional on the public information embedded in the prior), so that we are in a pure private value framework[4].

At least two arguments support the assumption that voters are uncertain about their final preferences. First, voters are likely to have less information about the issue at the selection stage than at the decision stage. Once an issue is selected, hearings of experts and stakeholders may be organized, public attention and the media may help produce and aggregate information about the issue itself and the preferences of the people which may affect those of their representatives. Second, the process leading to the final proposal is often complex and tends to generate uncertainty at the outset about the nature of the final proposal. In parliaments, when a bill is introduced to the floor, it usually goes through long series of amendments that often modify the text of the proposal substantially and unpredictably. Similarly, at the Supreme Court, there is uncertainty about which of the justices will be assigned to write the opinion and about which exact policy relevant points will be raised. Whereas the literature on agenda setting has generally focused on the process leading from the initial proposal to its final version, we are more interested in how initial proposals (issues) are selected and placed on the agenda in the first place. Our approach is to black-box this transformation process and merely assume that it creates uncertainty about what will be voted on in the final stage.

There are two rounds of voting. In the first round, the *selection stage*, committee members vote to select an issue. In the second round, the *decision stage*, they decide whether to adopt a proposal or maintain the status quo. Even though voters' preferences are private, one's expected utility at the selection stage depends indirectly on the preferences, hence on the private information, of other voters since they determine the probability that the proposal will pass the final round if it is selected. Therefore, the selection stage aggregates strategically

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[4]It is arguably more natural to assume private information in a framework with heterogeneous preferences like ours than in the homogeneous preferences framework of the literature on pivotal voting where individuals have private information about a common event. Indeed, while deliberation can be expected to make all the information public in the case of homogeneous preferences, there is no particular reason to assume that it would do so in general in the case of heterogeneous preferences. For an analysis of deliberation that supports this claim see Austen-Smith and Feddersen (2006).

relevant information about the probabilities of different outcomes. Rational voters condition their decision on the event that their vote is pivotal. The exact information conveyed by the pivotal event, however, depends on the selection rule. When a rule requires a higher tally of votes to select an issue, the event that a single vote is pivotal conveys the information that more voters are likely to favor the proposal at the decision stage. Therefore, conditional on being pivotal at the selection stage, a voter who votes to select an issue faces a higher chance that the status quo will be reversed when the selection rule is more stringent. When selecting an issue, however, a voter also keeps the option to vote against change in the second round so this increased probability is not sufficient to explain her behavior. Rather, the voter compares the probability that the proposal passes when she eventually prefers it to when she doesn't. We show that the probability that the proposal passes given that the voter does not support it increases at a higher rate with the selection rule than the same probability given that the voter supports the proposal. In order to compensate for that, voters become individually more conservative when the rule itself is more conservative[5]. Remarkably, this result does not depend on the particular distribution of preferences or on the size of the committee. It derives from a general property of sums of independent Bernoulli random variables.

These properties also allow us to compare voting rules to other selection procedures. We show that an individual agenda setter chosen from the committee is always individually more conservative than a committee member under the one-vote selection rule, but less conservative than a committee member under the unanimous selection rule. We also compare the equilibrium choice under a voting rule to the choice of a utilitarian social planner with all the private information at the selection stage. The least conservative equilibrium under the one-vote selection rule is always too accepting, while the most conservative equilibrium under the unanimous selection rule is always too conservative.

Related Literature. The seminal literature on voting under asymmetric information[6] (Austen-

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[5]This account leaves some details aside. Indeed the equilibria that we consider (symmetric) are not unique in general and, in standard practice, the exact comparative statics result is that both the minimum and the maximum equilibrium thresholds move as explained with the selection rule.

[6]More recent contributions include Duggan and Martinelli (2001), Laslier and Weibull (2009), and Li, Rosen

Smith and Banks, 1996; Feddersen and Pesendorfer, 1996, 1997, 1998; Myerson, 1998) focused on the jury model in which agents have common preferences (with possibly heterogeneous intensities) conditional on an unknown state of the world, and private information about this state of the world. An important insight of this literature is that a strategic voter should reason as if her vote were pivotal since it is the only event in which her vote has any effect on the collective decision. Under any voting rule, the pivotal event conveys some information about the votes of others, and therefore about their private information and what it means about the state of the world. In our model, each voter's payoff is independent from the information of others. Because of the two-round procedure, however, a voter who is uncertain about her final preferences cares about the preferences of others as they carry information about the chances of the proposal in the final round. To model voters' uncertainty about their own preferences, we draw on the setup of Barbera and Jackson (2004) to which we add asymmetric information.

Our comparative statics result is reminiscent of Feddersen and Pesendorfer (1998), but its spirit is quite different. In their model, the harder the voting rule makes it to convict, the more people vote to convict, possibly against their signal. This story is one of very strong negative feedback that can offset the purpose of a change in the rule. Our story is one of positive feedback that makes the outcome very sensitive to a change in the rule.

This paper is related to the literatures on group experimentation and conservatism. The most closely related papers are Fernandez and Rodrik (1991) and Strulovici (2010). Fernandez and Rodrik (1991) show that majority voting in groups leads a status quo bias in the presence of individual uncertainty on the benefits of a reform, even if there is no uncertainty that reform will be beneficial on aggregate. Strulovici (2010) consider a more general dynamic framework in which individuals may learn that they gain from the reform at any point in time. In both models, the asymmetry of information is very particular: at any point in time, some individuals know that they gain from the reforms while those that are uncertain have homogeneous beliefs about whether they win or lose from the reform. In these frameworks, no information is

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and Suen (2001). Iaryczower and Shum (2012) estimate a related model on Supreme Court data.

aggregated when voting. With a more general structure, the event that a voter is pivotal conveys information about the preferences of other voters that matters because it affects the chances that the reform passes in later rounds. While our result about the effect of the first-stage voting rule relies entirely on these effects, our result about the effect of the second-stage rule (the decision rule) is related to Li (2001): agents are more willing to experiment and bear the cost of gathering information when the final acceptance rule is more conservative.

Several authors have built on the pivotal voting literature to model multiple-round elections (Piketty, 2000; Razin, 2003; Iaryczower, 2008; Shotts, 2006; Meirowitz and Shotts, 2008). In these papers, voting in earlier stages provides information to the voters later stages. This creates a signaling motive for voters in the first stages. By contrast, the signaling channel is completely absent from our two-round model. An early paper looking at sequential voting procedures with private information is Ordeshook and Palfrey (1988). Through examples, they identify both the signaling channel and the possibility to learn about the preferences of others, but agents are certain about their own preferences. A more recent paper, Hummel (2009), considers a model of repeated elections with three candidates in which, as in our model, the outcome of earlier rounds is informative about the distribution from which the preferences of other voters are drawn. In his model, however, voters learn their own preferences at the outset.

Our work is also connected to the literature on agenda setting[7], foremost because the selection stage of our model is a process of endogenous agenda selection, but also because of the use of sequential elections in this literature. The topic has been treated from the point of view of legislative bargaining, and by the literature on sequential agenda. While this literature aims at modeling the whole process of amendments and modifications of a proposal, we only model the initial decision of placing an issue on the agenda, and account for the process between the selection and the decision stage with the assumption that it generates uncertainty at the outset about the final proposal. Other papers that endogenize the selection of proposals are Barbera and Coelho (2010) in which a committee votes on a list of names among which a third party

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[7]See the introduction of Bernheim, Rangel and Rayo (2006) for a nice review of this literature.

can then make a choice, and Copic and Katz (2007) in which some members of the committee propose a certain move from the status quo along an idiosyncratic dimension, and a speaker then decides which of these moves will be considered in the decision voting procedure.

Finally, in Godefroy and Perez-Richet (2012), we analyze the asymptotic behavior of our model, and explain how to choose voting rules optimally to let the voters internalize the cost of organizing the second round.

2 The Model

Voting Procedure. We consider a committee of $n \geq 2$ voters indexed by i. The voting procedure has two stages, the *selection stage* and the *decision stage*. At the selection stage, an issue is placed on the agenda if at least n_s committee members select it, where $n_s \in \{1, \cdots, n\}$ is the *selection rule*. If the issue is not selected, the status quo is maintained. If it is selected, the agents vote again to decide whether to adopt a proposal or maintain the status quo. The proposal is adopted if more than n_d committee members vote in favor, where $n_d \in \{1, \cdots, n\}$ is the *decision rule*.

Preferences and Information. If an issue is selected, the voters face a pair of alternatives: the *status quo* and the *proposal*. Since there are two alternatives, we need only keep track of the difference in payoffs between them. It is therefore without loss of generality that we normalize the payoff from the status quo to 0. Let u_i be the value of the proposal for i.

There is an individualized cost c of organizing the second round that has to be paid by each committee member if the second round is reached. We place no restriction on this cost which can be positive or negative. If it's negative, this cost can be thought of as the cost of processing information for each member of the committee but without any possibility to free-ride. Alternatively, it could be the cost of organizing the second round election split among committee members, or an opportunity cost. The cost may be negative if the committee members value the second round, because for example they are eager to learn more about the

issue at stake.

Information about the proposal is incomplete at the outset, so that i only knows a signal x_i about her value. When an issue is selected and added to the agenda, more information becomes available to the voters enabling them to learn u_i. We consider a private value framework. Signals are private as well.

Formally, each pair (u_i, x_i) is drawn independently according to the same joint distribution measure $p(.)$ on $U \times X$, where $U \subseteq \mathbb{R}$ and X is a real interval. We can work with either of the following assumptions: (i) U is an interval and $p(.)$ can be described by a density function, or (ii) U is finite. Then we can use the conditional distribution that we will write $p(u|x)$ and interpret as a probability density function in the first case or a probability mass function in the second case. The proofs are written for case (i) but can be easily adapted to case (ii). Similarly, our assumptions implies that we can characterize the marginal distribution on U by a function $p_U(.)$ which is either a probability density function or a probability mass function, and the marginal distribution on X by a probability density function $p_X(.)$. We make the following additional assumptions about the distribution.

Assumption 1 (Preference Uncertainty). $U \cap \mathbb{R}_+^* \neq \emptyset$ and $U \cap \mathbb{R}_-^* \neq \emptyset$.

Preference Uncertainty means that a voter may be uncertain about whether she will eventually favor the proposal or the status quo.

Assumption 2 (No Indifference). $p(.)$ has no atom at $(0, x)$, for every x.

The No Indifference assumption implies that a voter is almost surely not indifferent between the proposal and the status quo.

Assumption 3 (Full Support). $p(.)$ has full support on $U \times X$.

Full support implies that a voter with private information x may have a value of the proposal anywhere on U. Together with the Preference Uncertainty assumption, it implies that a voter can never be sure whether she will prefer the proposal or the status quo. The full support

assumption makes the discussion easier but is not a necessary assumption and we explain how to relax it in Section 6

Assumption 4 (Compactness). *X is a compact interval $[\underline{x}, \overline{x}]$*

The compactness assumption is a useful technical assumption. We need it to prove the comparative statics results with positive costs, and the existence of an equilibrium. The comparative statics results for $c \leq 0$ hold without the compactness assumption.

We also make the natural assumption that the voters always have a finite expected payoff.

Assumption 5 (Integrability). *$E\big(|u|\,\big|\,x\big) < \infty$, for every x.*

We assume that (u, x) is strictly affiliated. This assumption is key to all our results. Given our assumptions affiliation can be defined by the following monotone likelihood ratio property.

Assumption 6 (Strict Affiliation). *$p(u|x)$ is strictly log-supermodular in (u, x). Equivalently, for every $x' > x$, the ratio $p\big(u|x'\big)/p\big(u|x\big)$ is strictly increasing in u.*

As usual, the interpretation of affiliation is that higher signals mean better news about the value of the proposal. Note that the independence from one another of the pairs (u_i, x_i) implies that the vector $(u_1, \cdots, u_n, x_1, \cdots, x_n)$ is affiliated.

We provide some examples of specifications satisfying our assumptions at the end of Section 3.

Strategies and Pivotal Equilibria. A selection strategy of voter i is a function $\sigma_i : X \to \{0, 1\}$ mapping a type x_i to a ballot, where 1 means that i votes in favor of selecting the proposal. For notational simplicity, we do not consider mixed strategies. This is without loss of generality since we show below that all the best replies feature essentially pure strategies. We say that a selection strategy σ is a threshold strategy if there exists a threshold $t \in X$ such that $\sigma(x) = 1$ for every signal $x > t$ and $\sigma(x) = 0$ for every $x < t$.

We consider sequential equilibria in weakly undominated strategies so as to avoid equilibria in which voters vote against their preferences in the second round. Taking this as given, we can

focus on the first stage game and look at it as a static game. In the first round, any selection strategy of voter i is a best reply to a strategy profile of other voters such that i is almost surely never pivotal. But even though the probability that i is pivotal is 0, the pivotal event is still the only one in which her vote may matter. This suggests the following selection criterion for equilibria. We say that a selection strategy σ_i that is a best reply to σ_{-i} is a *pivotal best reply* if it remains a best reply when conditioning on the event that i is pivotal given σ_{-i}. Of course the set of best replies and the set of pivotal best replies to a profile σ_{-i} coincide if the probability that i is pivotal given σ_{-i} is positive. We will say that a strategy profile σ is a *pivotal equilibrium* if for every i, σ_i is a pivotal best reply to σ_{-i}. In what follows we will focus on symmetric pivotal equilibria.

3   Equilibrium Analysis

We start by recalling the following result implied by affiliation. The proof is standard except in the details. For this reason, and to make the paper self-contained, we give a proof in Appendix A. The calculations are based on Milgrom (2004).

Lemma 1. *If $g(u)$ is nondecreasing and not constant, and $E\big(g(u)|a \leq x \leq b\big)$ exists for some values of a and b, then it is strictly increasing in a and b.*

Decision Stage. Since we ruled out weakly dominated strategies, no matter what observations a player is allowed to make between rounds, she votes for her preferred policy at the decision stage. Therefore we can take this sincere voting behavior as given and proceed to analyze the first-stage game.

Selection Stage. To a profile $\mathbf{x} = (x_1, \ldots, x_n)$ of signals, we can associate a profile $(p_1, \cdots, p_n)$ of probabilities to prefer the proposal for each voter, defined by $p_i = p(u > 0|x_i)$. A voter i knowing the full profile $\mathbf{x}$ would expect the following utility if the issue were to be selected in

the first stage[8]

$$U_i = E\big(u\mathbb{1}_{u>0}|x_i\big) \sum_{\substack{C \subseteq N_i \\ \#C \geq n_d-1}} \prod_{j \in C} p_j \prod_{l \in N_i \smallsetminus C} (1-p_l) + E\big(u\mathbb{1}_{u<0}|x_i\big) \sum_{\substack{C \subseteq N_i \\ \#C \geq n_d}} \prod_{j \in C} p_j \prod_{l \in N_i \smallsetminus C} (1-p_l) \quad (1)$$

where $N_i = N \smallsetminus \{i\}$ is the set of all voters except i. Indeed, i will vote for the proposal in the second stage whenever $u > 0$, winning if a coalition C of at least $n_d - 1$ other players (sincerely) vote likewise. If $u < 0$, she will not support the proposal, and incur a loss if a coalition of at least n_d other voters concur against the status quo. If the issue is not selected, the status quo prevails and the expected utility of a voter is 0. We can write $U_i = U(x_i, \mathbf{x}_{-i})$. By affiliation and Lemma 1, $U(.)$ is strictly increasing in a voter's own type x_i.

Even though the values of the policies for the voters are private and independent as well as their informational types, the two-round process links a voter's value of selecting an issue to the types of other voters so that the first round has the analytical features of a common value election. In particular, the first round of this procedure can aggregate some information. This information is not about the quality of the proposal or the status quo, or any other factor that affects the values of the voters for these outcomes. It is about the number of voters likely to vote for the proposal at the decision stage.

When making her first-stage voting decision, the voter only knows her signal x_i of favoring the final proposal, and must therefore compute the expected value of (1). If she is rational, she conditions her computation on the event $\mathcal{E}_i \equiv \left\{\sum_{j \in N_i} \sigma_j(x_j) = n_s - 1\right\}$ that her vote is pivotal, and compares it to the null payoff that she obtains if the issue is not selected. Because the expression in (1) is strictly increasing in x_i, voters use threshold strategies[9]. From now on, we identify strategies with the corresponding threshold.

In order to understand the information conveyed by the pivotal information we introduce

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[8]Note that this function does not satisfy the information smallness assumption of Gerardi and Yariv (2007), hence allowing for deliberation does not necessarily make different selection rules equivalent as to the sets of sequential equilibria in weakly undominated strategies that they generate.

[9]Other strategies are dominated. The prescription of the strategy when $x_i = t_i$, which is an event of measure 0 because $p_X(.)$ is atomless, is essentially irrelevant for the analysis.

some notations. Define $\bar{p}(t) \equiv p\big(u > 0 | x \geq t\big)$, the probability of eventually favoring the proposal conditionally on having received a signal x above t, and $\underline{p}(t) \equiv p\big(u > 0 | x \leq t\big)$, the probability of eventually favoring the proposal conditionally on having received a signal x below t. Under strict affiliation, Lemma 1 implies that these functions are strictly increasing in t. Furthermore, they satisfy that for every t in the interior of X, $0 < \underline{p}(t) < \tilde{p} < \bar{p}(t) < 1$. Let $\bar{Y}(t)$ be a generic Bernoulli random variable that takes the value 1 with probability $\bar{p}(t)$. We denote by $\bar{Y}_1, \bar{Y}_2, \cdots, \bar{Y}_k$ an i.i.d. sample of size k of this random variable. Similarly, $\underline{Y}(t)$ is a generic Bernoulli random variable with parameter $\underline{p}(t)$.

Now, suppose voters other than i use a threshold $t \in X$. Conditional on her vote being pivotal, voter i knows that exactly $n_s - 1$ of the other $n - 1$ voters have a private signal that lies above t. Therefore she estimates that the tally of votes that will be ultimately cast in favor of the proposal if the issue is selected is given by the random variable

$$S(t) = \bar{Y}_1(t) + \cdots + \bar{Y}_{n_s - 1}(t) + \underline{Y}_1(t) + \cdots + \underline{Y}_{n - n_s}(t).$$

Hence the expected gain from selection for a voter of type x who is pivotal is given by

$$\pi(x, t) \equiv E\big(u \mathbb{1}_{u > 0} | x\big) \Pr\big(S(t) \geq n_d - 1\big) + E\big(u \mathbb{1}_{u < 0} | x\big) \Pr\big(S(t) \geq n_d\big) - c.$$

By affiliation, $\pi(x, t)$ is strictly increasing in x. The best reply of a player of type x to a threshold $t \in X$ is therefore to use the threshold $\beta(t)$ equal to the unique x at which the expected payoff from validation $\pi(x, t)$ changes sign. If there is no such x, either this expected payoff is always negative and then $\beta(t) = \underline{x}$ or it is always positive and then $\beta(t) = \bar{x}$.

When $t \in \{\underline{x}, \bar{x}\}$, the probability that the voter is pivotal is almost surely 0, and therefore any strategy is a best reply. But the function $\beta(.)$ is always defined at $\underline{x}$ and $\bar{x}$, and it uniquely defines pivotal best replies at these points. Elsewhere, best replies and pivotal best replies coincide.

Proposition 1 (Equilibrium Characterization). *Every selection strategy that is not a threshold*

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strategy is never a pivotal best reply. The symmetric pivotal equilibria are exactly the fixed points of $\beta(.)$.

Proof. See Appendix A. □

In particular, equilibrium strategies are essentially pure strategies in the sense that voters may be mixing at the threshold but nowhere else. We can find examples with multiple equilibria in $(\underline{x}, \overline{x})$. In the rest of the paper, we denote by $T^*(n_s, n_d, c)$ the set of fixed points of $\beta(.)$ for given voting rules n_s and n_d.

Example 1 (Binary Preferences). *Suppose that the utility from the proposal can take only two values $-u^- < 0$ and $u^+ > 0$. By a simple transformation, we can reinterpret the signal x_i of a voter i as the probability p_i that she will eventually prefer the proposal. Then, from a transformation of the marginal $p_X(.)$, we can obtain the distribution $F(.)$ of p_i which is supported on a compact interval of $(0, 1)$. With this reinterpretation, $\overline{p}(t) = E_F(p_i | p_i \geq t)$ and $\underline{p}(t) = E_F(p_i | p_i \leq t)$. When other voters use a threshold $t \in [0, 1]$, the expected payoff voter i obtains by voting for selection when she is pivotal is*

$$p_i u^+ \Pr(S(t) \geq n_d - 1) - (1 - p_i) u^- \Pr(S(t) \geq n_d) - c.$$

And her best-reply threshold is then given by

$$\beta(t) = \min \left\{ \left(\frac{c + u^- \Pr(S(t) \geq n_d)}{u^+ \Pr(S(t) \geq n_d - 1) + u^- \Pr(S(t) \geq n_d)} \right)^+, 1 \right\}.$$

Example 2 (Normal Preferences). *Suppose that the preferences of an individual with signal x follow the normal distribution $\mathcal{N}(x, \sigma^2)$ and that the marginal distribution of the signals is given by a distribution $F(.)$ on the interval $[-\chi, \chi]$. Then all our assumptions are satisfied. We have $p(u > 0 | x) = \Phi(\sigma x)$, $\overline{p}(t) = \int_t^\chi \Phi(\sigma x) dF(x) / (1 - F(t))$ and $\underline{p}(t) = \int_{-\chi}^t \Phi(\sigma x) dF(x) / F(t)$.*

In Lemma 4, Appendix B, we prove that the best reply function $\beta(t)$ is continuous, and

therefore the existence of a fixed point follows from Brouwer fixed point theorem. The compactness of X is important for this result.

Proposition 2 (Existence). *There exists a symmetric pivotal equilibrium.*

We say that an equilibrium is responsive if agents react to their private information. If $\beta(.)$ has a fixed point at $\underline{x}$, we have an irresponsive equilibrium in which all voters always vote for selection in the first stage. If there is a fixed point at $\overline{x}$, it constitutes an irresponsive equilibrium in which voters always vote against selection. The next result determines when such equilibria arise and is an immediate corollary of Proposition 1.

Corollary 1. *There exists an irresponsive equilibrium in which all voters vote against selection at the selection stage if and only if*

$$\pi(\overline{x}, \overline{x}) \leq 0.$$

There exists an irresponsive equilibrium in which all voters vote in favor of selection at the selection stage if and only if

$$\pi(\underline{x}, \underline{x}) \geq 0$$

4   Effects of the Rules

The properties of the equilibria are tied to the ratio which measures the contribution of a voter to the probability that the proposal prevails in the second round, conditional on her being pivotal in the first round

$$R(t, n_s, n_d) \equiv \frac{\Pr(S(t) \geq n_d - 1)}{\Pr(S(t) \geq n_d)},$$

where the dependence on n_s comes from the definition of $S(t)$. This link can be seen by noting that the expected payoff $\pi(x, t)$ has the same sign as the expression

$$\hat{\pi}(x, t) = E(u \mathbb{1}_{u>0}|x) R(t, n_s, n_d) + E(u \mathbb{1}_{u<0}|x) - \frac{c}{\Pr(S(t) \geq n_d)}$$

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To study $R(t, n_s, n_d)$, we rely on the following general property of sums of Bernoulli random variables which we prove in Appendix B.

Lemma 2. *Let $\{Y_k\}_{k=1}^{\infty}$ be a sequence of independently distributed Bernoulli random variables that take value 1 with probability p_k, and let $\Sigma \equiv \sum_{k=1}^{K} Y_k$ for some integer K. Then for any nonnegative integer $q \leq K$, the probability $\Pr(\Sigma \geq q)$ is strictly increasing in p_k for any $k \leq K$ and strictly decreasing in q; and the ratio*

$$\rho = \frac{\Pr(\Sigma \geq q - 1)}{\Pr(\Sigma \geq q)}$$

is strictly decreasing in p_k for any $k \leq K$ and strictly increasing in q.

Another way of formulating the first point about ρ is to say that the function $g(p_1, \cdots, p_n; q) \equiv \Pr(\Sigma \geq q)$ is strictly log-supermodular in (p_k, q) on $[0, 1] \times \{0, \cdots, K\}$. The second point about ρ says that the affine extension of $g(q)$ to $[0, K]$ is strictly log-concave.

Lemma 2 has three implications for $R(.)$. First, $R(.)$ is strictly increasing in n_d. Second, it is strictly decreasing in n_s. Indeed, increasing n_s to $n_s' > n_s$ amounts to switching $n_s' - n_s$ of the Bernoulli random variables of the type $\underline{Y}(t)$ in $S(t)$ to random variables of the type $\overline{Y}(t)$, and we know that $\underline{p}(t) < \overline{p}(t)$. Finally, $R(.)$ is strictly decreasing in t since by Lemma 1 $\overline{p}(t)$ and $\underline{p}(t)$ are both strictly increasing in t.

The sense of variation of $\hat{\pi}(x, t)$, and hence of $\beta(t)$, with respect to t, n_s or n_d is easy to pin down if $c \leq 0$ since $R(t, n_s, n_d)$ and $1/\Pr(S(t) \geq n_d)$ move in the same direction. With a positive cost, the two terms $R(t, n_s, n_d)$ and $1/\Pr(S(t) \geq n_d)$ move in opposite directions in response to a change in n_s or n_d. Intuitively, the effect of R should dominate if the cost c is small. This is true, but the proof requires some work to show that it is possible to find a strictly positive upper bound on the cost that will work for any change in t, n_s or n_d.

The fact that $\beta(.)$ is nondecreasing in t when $c \leq 0$ is a form of strategic complementarity: as all other voters symmetrically increase their common selection threshold, a voter wants to increase her own threshold.

This complementarity may disappear as c becomes positive for the following reason. Suppose other players are using a very low threshold t and the selection rule n_s is low, one-vote for example. Then conditional on being pivotal, a voter expects all other voters to have a very low probability of preferring the proposal, $\underline{p}(t)$. But then the probability that the proposal is eventually accepted is very low and does not justify the cost of the second round except if the voter has a very high signal about her own valuation of the proposal. More formally, if n_s is low the term $c/\Pr\big(S(t) \geq n_d\big)$ increases faster than the ratio $R(t)$ as $\underline{p}(t)$ goes down to 0. Hence the best reply to a low threshold of other voters is a high threshold.

Proposition 3 (Best-Reply Comparative Statics).

(i) *The best reply function $\beta(t)$ is nondecreasing in c.*

(ii) *There exists an upper bound $\overline{c} > 0$ such that for every $c < \overline{c}$, the best reply $\beta(t)$ is nondecreasing in n_s, and nonincreasing in n_d.*

(iii) *If $c \leq 0$, the best reply $\beta(t)$ is nondecreasing in t.*

Any variation of β due to a variation in one of these parameters is strict if either one of the initial or the final value of β is in $(\underline{x}, \overline{x})$.

Proof. See Appendix B. □

To study the effects of the voting rules on equilibrium thresholds, we use the following partial order on subsets of X: for every $S, T \subset [0,1]$, $S \leq T$ if and only if $\inf S \leq \inf T$ and $\sup S \leq \sup T$, with at least one of the inequalities holding strictly. If $T = \{t\}$ is a singleton, we will write $S \leq t$ to simplify notations. The following proposition is a corollary of Proposition 3 which says how the best-reply function varies with the rules. We look at the set of equilibria as the image of a function $T^*(n_s, n_d, c)$ from the set of voting rules to the subsets of X. Given the sense of variation of $\beta(.)$ with respect to n_s, n_d and c, the comparative statics on equilibrium thresholds follows from an application of Milgrom and Roberts (1994, Corollary 1).

Proposition 4 (Effects of the Rules). *$T^*(n_s, n_d, c)$ is nondecreasing in c. If $c < \bar{c}$, then $T^*(n_s, n_d, c)$ is also nondecreasing in the selection rule n_s and nonincreasing in the decision rule n_d.*

Hence equilibrium selection thresholds increase with the selection rule and decrease with the decision rule. The latter is not very surprising: the harder it is for the proposal to pass the second round, the more willing voters are to bring the issue to the ballot. The first result may seem more surprising: the more difficult the institution makes it for an issue to be selected, the more selective the voters become. In other words, they fail to offset the effect of the selection rule, and accentuate it instead. Suppose for example that the voters always play according to the maximal stable equilibrium threshold $t^* = \sup T^*$. Then the ex ante probability that any given vote is cast in favor of selection $p_X(x < t^*)$ decreases as n_s increases.

Note that conditional on being pivotal, selecting an issue when the selection rule is more stringent means that the proposal is more likely to pass. However, because voters keep the option of voting against the proposal in the second round, the driving force is more subtle. What matters to a voter is the ratio between the probabilities that the proposal eventually passes conditionally on being pivotal at the selection stage, whether she eventually supports it or not. What we showed is that a more stringent selection rule makes it relatively more likely that the issue passes when the voter eventually doesn't support it compared to when she does. In order to compensate for that, the voter becomes more selective. Remarkably, even though the intuition for this result is not obvious at first, our proof shows that it holds very generally since it does not depend on the distribution of preferences and signals or on the size of the committee, and it resists the introduction of a small cost of moving to the second round.

High Cost. Our comparative statics results are for negative or sufficiently small positive costs. Intuition suggests that the comparative statics is reversed when costs are high. Indeed, the cost term in the modified payoff function $\hat{\pi}(x, t)$ becomes dominant and should drive the comparative statics. As the cost rises, however, voters are less and less inclined to select, and they may completely stop to do so before the comparative statics is reversed.

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5 Democratizing the Agenda

In this section, we compare our results to situations in which the decisions are partially or completely dictatorial. The decision is fully dictatorial if a given member of the committee decides for the committee in both rounds. It is partially dictatorial if a given member of the committee, the agenda setter, decides on selection, but the final decision is democratic. This allows us to understand the effect of democratizing the decision process.

We consider first a decision process entirely controlled by a dictator. Then she knows that she will adopt the proposal in the second stage if and only if she ends up preferring the proposal. Therefore her payoff from selecting the proposal in the first stage is given by

$$\pi^{dict}(x) \equiv E\big(u \mathbb{1}_u > 0 | x\big) - c,$$

and she will opt to select whenever this payoff is positive, and refuse to select whenever it is negative. Because $\pi(x)$ is strictly increasing in x, there exists a unique threshold $t^{dict}(c) \in X$ such that the full dictator adopts the corresponding threshold strategy. It is easy to show that $t^{dict}(c)$ is nondecreasing in c. Because for every t and every x, $\pi(x,t) < \pi^{dict}(x)$, any best reply threshold $\beta(t)$ must be greater than the fully dictatorial threshold t^{dict}. This implies that regardless of the voting rule, the full dictator is less conservative than any given committee member in the democratic process. Therefore sufficiently stringent selection rules will be less likely to select an issue than full dictatorship, but we cannot conclude for milder selection rules.

Proposition 5. *For any voting rule (n_s, n_d), any cost c, we have*

$$T^*(n_s, n_d, c) \geq t^{dict}(c).$$

Furthermore, for any $t^* \in T^*(n_s, n_d, c)$ which is in the interior of X, we have $t^* > t^{dict}(c)$. Fixing n_d and c, there exists a cutoff $N_s \leq n$ such that for every $n_s \geq N_s$ the probability that the issue is selected is higher in the dictatorial process than in any equilibrium of the democratic

process with voting rule (n_s, n_d).

Proof. See Appendix C □

Next we consider the case of a partial dictator, the agenda setter, who only controls the selection process. The voting rule at the selection stage is still denoted n_d. This may be more relevant for policy considerations as it is often the case that the chairman of a committee controls its agenda. To study the behavior of the agenda setter, let Z be a binomial random variable with parameters $\tilde{p}$, and $n - 1$. Recall that $\tilde{p}$ is the probability that a random voter eventually favors the proposal in the absence of additional information. Z is the random variable the agenda setter would use at the selection stage to estimate the tally of votes in favor of the proposal at the decision stage in addition to her own. Then the payoff of the agenda setter if she opts for selection is given by

$$\pi^{as}(x) = E\big(u\mathbb{1}_{u>0}|x\big)\Pr\left(Z \geq n_s - 1\right) + E\big(u\mathbb{1}_{u<0}|x\big)\Pr\left(Z \geq n_s\right) - c.$$

As before, Lemma 1 implies that $\pi^{as}(x)$ is strictly increasing in x so that the agenda setter will use a threshold strategy with a threshold $t^{as}(n_d, c)$. It is easy to adapt our proofs in Section 4 to show that $t^{as}(n_d, c)$ is nondecreasing in c and nonincreasing in n_d for every negative or positive but sufficiently small cost. We can also show that regardless of the decision rule, the agenda setter is more conservative than a committee member under the one-vote selection rule, but less conservative than a committee member under the unanimous selection rule. In particular, the probability that an issue is selected under the one-vote selection rule is always greater than the probability that it is selected by the agenda setter which is itself greater than the probability of selection under unanimous selection.

Proposition 6. *There exists a strictly positive bound $\hat{c} \leq \bar{c}$ such that for every $c < \hat{c}$ and every fixed decision rule n_d, we can find two thresholds $1 \leq N_s^1 \leq N_s^2 \leq n$ such that*

$$T^*\big(N_s^1, n_d, c\big) \leq t^{as}(n_d, c) \leq T^*\big(N_s^2, n_d, c\big).$$

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Furthermore, for every $n_s \leq N_s^1$, the probability of selection is lower under partial dictatorship than under the selection rule n_s. There exists $N_s^3 \geq N_s^2$ such that for every $n_s \geq N_s^3$, the probability of selection under partial dictatorship is greater than under the rule n_s.

Proof. See Appendix C $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ $\square$

Irrational Voters. The partial and the full dictatorship thresholds can also be interpreted as heuristics for the behavior of voters that are not fully rational. Another plausible heuristics is that irrational voters opt for selection whenever their expected payoff from the proposal is higher than the cost of moving to the second round. Such voters do not take into account the existence of a second round of voting. Then they act as if their payoff from selection was $E(u|x) - c$. By Lemma 1, this payoff is strictly increasing in x, and hence naive voters use a threshold strategy with threshold $t^{naive}(c)$. They are more selective than any other voters because they ignore the option value afforded by the existence of a second round.

Proposition 7. *There exists a strictly positive bound $\tilde{c} \leq \bar{c}$ such that for every $c < \tilde{c}$, and every voting rule (n_s, n_d),*

$$t^{naive}(c) \geq T^*(n_s, n_d, c)$$

Proof. See Appendix C $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ $\square$

Welfare. We examine welfare from the point of view of a utilitarian social planner taking democratic decision making as a constraint, that is we fix a certain decision voting rule n_d, and our benchmark is the optimal selection policy of a utilitarian social planner who possesses all the ex interim information (the vector $\mathbf{x}$ of private signals) at the selection stage. The population average expected payoff of the social planner with information $\mathbf{x}$ is then given by

$$\pi_{n_d}^{sp}(\mathbf{x}) = \frac{1}{n} E\left(\mathbb{1}_{N^+(\mathbf{u}) \geq n_d} \sum_{i=1}^{n} u_i \,\Big|\, \mathbf{x} \right) - c,$$

where $N^+(\mathbf{u}) = \# \{j | u_j > 0\}$. The next calculation shows that the structure of this payoff is surprisingly close to that of the pivotal voter's expected payoff.

$$
\begin{aligned}
\pi_{n_d}^{sp}(\mathbf{x}) &= \frac{1}{n}\sum_{i=1}^{n} E\big(u_i \mathbb{1}_{N^+(\mathbf{u})\geq n_d}\big|\,\mathbf{x}\big) - c \\
&= \frac{1}{n}\sum_{i=1}^{n} E\Big(u_i \mathbb{1}_{u_i>0}\mathbb{1}_{N^+(\mathbf{u}_{-i})\geq n_d-1} + u_i \mathbb{1}_{u_i<0}\mathbb{1}_{N^+(\mathbf{u}_{-i})\geq n_d}\Big|\,\mathbf{x}\Big) - c \\
&= \frac{1}{n}\sum_{i=1}^{n}\Big\{ E\big(u_i \mathbb{1}_{u_i>0}\big|\,x_i\big)E\big(\mathbb{1}_{N^+(\mathbf{u}_{-i})\geq n_d-1}\big|\,\mathbf{x}_{-i}\big) + E\big(u_i \mathbb{1}_{u_i<0}\big|\,x_i\big)E\big(\mathbb{1}_{N^+(\mathbf{u}_{-i})\geq n_d}\big|\mathbf{x}_{-i}\big)\Big\} - c \\
&= \frac{1}{n}\sum_{i=1}^{n}\Big\{ E\big(u_i \mathbb{1}_{u_i>0}\big|\,x_i\big)\Pr\big(Z(\mathbf{x}_{-i})\geq n_d-1\big) + E\big(u_i \mathbb{1}_{u_i<0}\big|\,x_i\big)\Pr\big(Z(\mathbf{x}_{-i})\geq n_d\big)\Big\} - c,
\end{aligned}
$$

$$(2)$$

where the first equality is by linearity of the expectation, the second equality is a decomposition between disjoint events, the third equality is by independence across agents, and in the final expression, $Z(\mathbf{x}_{-i})$ is the sum of $n-1$ independent Bernoulli random variables that take value one with respective probability $\hat{p}(x_j) = p\big(u > 0\,|\,x_j\big)$, for $j \neq i$.

By Lemma 1 and Lemma 2, this function is strictly increasing in each x_i. Then we know that there exist a manifold of dimension $n-1$ that can be described by the equation $x_i = H(\mathbf{x}_{-i})$, where the choice of i is irrelevant and $H : \mathbb{R}^{n-1} \to \mathbb{R}$ is nonincreasing in each of its terms, and such that the optimal policy is to opt for selection whenever $x_i > H(\mathbf{x}_{-i})$ and to reject whenever $x_i < H(\mathbf{x}_{-i})$ (the social planner is indifferent when $\mathbf{x}$ is on the manifold). The fact that the same functions $H(.)$ can be used regardless of the choice of i is a consequence of the symmetry assumed in our framework. The case of a two-voter committee is illustrated in panel (d) of Figure 1 (the exact shape of the curve in the figure is fictitious, the only restriction is that it must be nonincreasing and symmetric).

As the figure suggests, a first step in order to compare the outcome of democratic agenda setting to the utilitarian optimal policy is to compare their outcomes on the diagonal. Suppose then that the social planner has received the same informative signal x for each voter. Then,
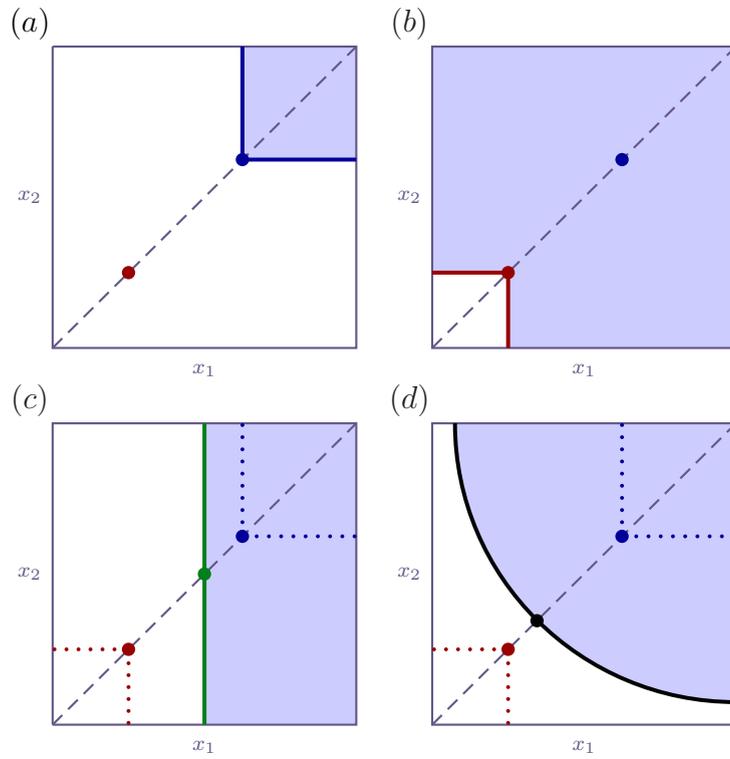
Figure 1: **Welfare in a two-voter committee with fixed decision rule.** *The blue zone represents the set of signals that lead to a second round if (a) the selection rule is unanimity, (b) the selection rule is one-vote, (c) voter 1 is the agenda setter, (d) the agenda setter is a utilitarian social planner with all the ex interim information (the exact shape of the curve is fictitious, we only know that it is nonincreasing and symmetric).*

with a slight abuse of notation, we can write her expected payoff from selection as

$$\pi_{n_d}^{sp}(x) = \frac{1}{n}E\left(\mathbb{1}_{N^+(\mathbf{u})\geq n_d}\sum_{i=1}^{n}u_i\,\bigg|\,x_1 = \cdots = x_n = x\right) - c.$$

Then applying (2), and letting $Z(x)$ denote the sum of $n-1$ independent Bernoulli random variables taking value 1 with probability $\hat{p}(x) = p(u > 0\,|\,x)$, we obtain by symmetry

$$\pi_{n_d}^{sp}(x) = E\big(u\mathbb{1}_{u>0}|x\big)\Pr\big(Z(x) \geq n_d - 1\big) + E\big(u\mathbb{1}_{u<0}|x\big)\Pr\big(Z(x) \geq n_d\big) - c.$$

And the social planner will opt for selection whenever $x > t_{n_d}^{sp}$ and for rejection whenever $x < t_{n_d}^{sp}$, where $t_{n_d}^{sp}$ is the unique value of x at which the expression

$$\hat{\pi}_{n_d}^{sp}(x) = E\big(u\mathbb{1}_{u>0}|x\big)\frac{\Pr\big(Z(x) \geq n_d - 1\big)}{\Pr\big(Z(x) \geq n_d\big)} + E\big(u\mathbb{1}_{u<0}|x\big) - \frac{c}{\Pr\big(Z(x) \geq n_d\big)}$$

crosses 0 ($\underline{x}$ if it stays above 0, $\overline{x}$ if it stays below 0).

Lemma 1 implies that for every t in the interior of X, we have $\underline{p}(t) < \hat{p}(t) < \overline{p}(t)$. Then, by applying Lemma 2, we can show that the socially optimal threshold on the diagonal is between the one-vote threshold and the unanimity threshold. More precisely:

Proposition 8 (Welfare). *There exists $c^{sp} > 0$ such that for every $c < c^{sp}$, and every n_d we can find two selection rules $N_s^1 \leq N_s^2$ such that*

$$T^*\big(N_s^1, n_d, c\big) \leq t_{n_d}^{sp}(c) \leq T^*\big(N_s^2, n_d, c\big).$$

In particular, any equilibrium of the one-vote selection rule always selects weakly too often and any equilibrium of the unanimity selection rule weakly too seldom.

Proof. See Appendix C □

The intuition for the last point of the proposition can be most clearly understood by looking at panel (d) of Figure 1: if the limit of the selection region is given by a nonincreasing function

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that cuts the diagonal between the one-vote and the unanimity thresholds, the optimal selection region must be a subset of the lowest one-vote equilibrium selection region and a superset of the highest unanimous equilibrium selection region.

We now go back to Example 1 where agents have homogeneous values conditional on whether they prefer the proposal to the status quo. In this case there may be a conflict of interest in the second period, but there is no issue of preference intensity. Then the decision rule $n_s^* = \lceil nu^-/(u^- + u^+) \rceil$ is socially optimal upon reaching the second round. In this case, it is natural to consider the problem of the social planner where the second round rule is fixed to n_s^*. But since the second round rule selects the proposal whenever the sum of utilities across agents is positive and we can rewrite the utility of the social planner with signal vector $\mathbf{x}$ as

$$\pi_{n_d^*}^{sp}(\mathbf{x}) = \frac{1}{n} E \left(\mathbb{1}_{\sum u_i > 0} \sum_{i=1}^{n} u_i \Big| \mathbf{x} \right) - c.$$

But then it is clear that for $c \leq 0$, the socially optimal choice is to always select. And we have the following corollary of Proposition 8.

Corollary 2. *If preferences are binary and the decision rule is n_d^*, then if $c \leq 0$ the one-vote selection rule achieves the social optimum which is to always select.*

If agents do not have homogeneous values, however, there is no decision rule that implements the social optimum at the second stage. This is the usual remark that voting rules fail to account for preference intensities. In this case, even if the cost c is 0, the first stage of voting may be worthwhile because it provides the social planner with an additional control for optimization.

6 Alternative Rules and Other Extensions

Sequential Procedures. Actual selection procedures often do not have the structure of our basic simultaneous game. For example, in the case of petitions, the process of gathering signatures is usually sequential. Dekel and Piccione (2000) showed that in symmetric binary

elections, the informative symmetric equilibria of the simultaneous voting game are also sequential equilibria of any sequential voting structure in a certain class. The selection stage of our game is a symmetric binary election that falls in the scope of applications of the first theorem[10] of Dekel and Piccione (2000). Therefore our equilibrium analysis of the simultaneous selection game applies to any sequential selection procedure in this class, which consists of all the games with $T < \infty$ periods such that each voter is called to vote in some period, and voting may be simultaneous in some periods.

Subcommittees. In some committees such as the United States Congress, issues are selected within a subgroup of the voters. To describe this procedure we let $n_S \geq n_s$ be the number of voters in the subcommittee. Making the same assumptions about preferences and information, and considering the voting decision of a member of the selecting committee, it is clear that, conditional on being pivotal, and provided other players are using a threshold t, the random variable that describes the belief of a player about the tally of votes that will finally be cast in favor of the proposal is

$$\tilde{S}(t) = \overline{Y}_1(t) + \cdots + \overline{Y}_{n_s-1}(t) + \underline{Y}_1(t) + \cdots + \underline{Y}_{n_S-n_s}(t) + \tilde{Y}_1 + \cdots + \tilde{Y}_{n-n_S},$$

where $\tilde{Y}$ is a generic Bernoulli random variable that takes the value 1 with probability $\tilde{p}$,

The best reply of a voter to all other players playing with a common threshold t is the value of the signal x at which her expected payoff from selection conditional on being pivotal,

$$\tilde{\pi}(x,t) \equiv E\big(u\mathbb{1}_{u>0}|x\big)\Pr\big(\tilde{S}(t) \geq n_d - 1\big) + E\big(u\mathbb{1}_{u<0}|x\big)\Pr\big(\tilde{S}(t) \geq n_d\big) - c,$$

changes sign.

The equilibria exist and can be characterized as in Proposition 1 by the fixed points of $\tilde{\beta}(.)$. The comparative statics with respect to n_s and n_d work as before. Another question that can be

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[10]That is up to the following detail: for notational convenience, Dekel and Piccione (2000) show their result for a finite type space whereas our type space is the unit interval. The extension of their proof to this case is immediate however.

considered here is the effect of increasing the size of the subcommittee. If the size is increased by 1, then the result depends on whether n_s is also increased by 1 or not. In the first case, one random variable of the type $\tilde{Y}$ is replaced by a random variable of the type $\overline{Y}(t)$ in $\tilde{S}(t)$ and since $\overline{p} \geq \tilde{p}$ for all t, the increased size leads to an increase of the set $\tilde{T}^*$ of fixed points of $\tilde{\beta}(.)$. In the second case, one random variable of the type $\tilde{Y}$ is replaced by a random variable of the type $\underline{Y}(t)$ in $\tilde{S}(t)$ and since $\underline{p} \leq \tilde{p}$ for all t, the increased size leads to a decrease of the set $\tilde{T}^*$ of fixed points of $\tilde{\beta}(.)$.

Relaxing Full Support. If the full support assumption is not satisfied, there may exist signals that make their receiver certain that she will prefer the status quo, and others the proposal. Since these signals can be ordered by the affiliation assumption, let $(\check{x}, \hat{x})$ be the interval of signals such that the voter is uncertain about her preferences, with $\check{x} < \hat{x}$. Then for any signal $x \leq \check{x}$, $p(u > 0|x) = 0$ and for any $x \geq \hat{x}$, $p(u > 0|x) = 1$. Then it is clear that the pivotal best-reply to any threshold $t \leq \check{x}$ should be the same, and similarly for $t \geq \hat{x}$. So we only need to define the pivotal best-reply to $\check{x}$ and $\hat{x}$. This is never a problem for $\hat{x}$, but there may be a problem at $\check{x}$, because $\underline{p}(\check{x}) = 0$. Indeed, if $n_s \leq n_d$ then in the pivotal event, there are at least $n - n_s$ individuals with a null probability of preferring the proposal, implying that the proposal stands no chance in the second round. If $c < 0$, the best reply is clearly to always select, that is $\beta(t) = \check{x}$ for every $t \leq \check{x}$. If $c > 0$, the best reply is to never select $\beta(t) = \hat{x}$ for every $t \leq \check{x}$ (note that this argument proves that there is no hope of extending the result that $\beta(.)$ is nondecreasing in t to the case $x > 0$). The only difficulty is if $c = 0$. Then any threshold is a best reply. A natural way to proceed, however, is to select the limit of $\beta(.)$ as $t \to \check{x}$ from above. This can be done because in this case we know that $\beta(.)$ is continuous and nondecreasing on $(\check{x}, \hat{x})$ and therefore can be extended by continuity to $[\check{x}, \hat{x}]$.

7 Conclusion

We have developed a model of issue selection in committees which predicts that voters are more conservative when the selection rule is more stringent. The decision rule has the opposite effect. Our analysis has been conducted in a pure value framework. It would be interesting to allow for correlations in preferences, but it is hard to derive any conclusions in the general case.

Our model is well suited for experimental testing, and we are currently designing experiments. These experiments could also serve as a test of pivotal voting in a context that differs significantly from the literature. In particular, voting behavior should not depend on the selection rule unless voters are trying to infer something from the outcome of the first stage, and hence are conditioning on a particular event. Our results could also be tested directly on institutional data. But the rules of institutions such as the Supreme Court rarely change, if at all, making identification potentially difficult.

Finally, our model may have normative applications. Our welfare analysis sheds some light on how to choose an agenda-setting procedure for emerging or established institutions that have no explicit rules, such as the Committee of Permanent representatives of the European Union. Our results can also be applied citizens' initiatives, a procedure that has recently been introduced or extended in several European countries. The wide variety of committees that use, or could use, selection rules calls for a better understanding of their effect, and offers several potential applications for this research.

A   Equilibrium Characterization

Proof of Lemma 1: Affiliation. We prove the result for a change in b, and it is easy to adapt the proofs to a change in a. We start by proving that $p(u|a \leq x \leq b)$ is strictly

log-supermodular in (u, b). To this effect, let $b' > b$ and $u' > u$ and note that

$$\frac{p(u'|a \leq x \leq b')}{p(u'|a \leq x \leq b)} = \frac{\int_a^{b'} p(u'|x)dx}{\int_a^b p(u'|x)dx} \frac{p_X(a \leq x \leq b)}{p_X(a \leq x \leq b')} = \left(1 + \frac{\int_b^{b'} \frac{p(u'|x)}{p(u'|b)}dx}{\int_a^b \frac{p(u'|x)}{p(u'|b)}dx}\right) \frac{p_X(a \leq x \leq b)}{p_X(a \leq x \leq b')}$$

$$> \left(1 + \frac{\int_b^{b'} \frac{p(u|x)}{p(u|b)}dx}{\int_a^b \frac{p(u|x)}{p(u|b)}dx}\right) \frac{p_X(a \leq x \leq b)}{p_X(a \leq x \leq b')} = \frac{p(u|a \leq x \leq b')}{p(u|a \leq x \leq b)}.$$

The inequality follows from the affiliation inequality for the terms under the integral sign. Next we show that the cumulative density function $p(u \leq v|a \leq x \leq b)$ is strictly log-supermodular in (v, b). To that effect, let $b' > b$ and $v' > v$

$$\frac{p(u \leq v'|a \leq x \leq b')}{p(u \leq v|a \leq x \leq b')} = \frac{\int_{-\infty}^{v'} p(u|a \leq x \leq b')du}{\int_{-\infty}^{v} p(u|a \leq x \leq b')du} = 1 + \frac{\int_v^{v'} \frac{p(u|a \leq x \leq b')}{p(v|a \leq x \leq b')}du}{\int_{-\infty}^{v} \frac{p(u|a \leq x \leq b')}{p(v|a \leq x \leq b')}du}$$

$$> 1 + \frac{\int_v^{v'} \frac{p(u|a \leq x \leq b)}{p(v|a \leq x \leq b)}du}{\int_{-\infty}^{v} \frac{p(u|a \leq x \leq b)}{p(v|a \leq x \leq b)}du} = \frac{p(u \leq v'|a \leq x \leq b)}{p(u \leq v|a \leq x \leq b)}.$$

This implies that $p(u|a \leq x \leq b)$ is nondecreasing in b in the first-order stochastic dominance order as for every v such that $\inf U < v < \sup U$

$$\frac{p(u \leq v|a \leq x \leq b')}{p(u \leq v|a \leq x \leq b)} < \lim_{v' \to \infty} \frac{p(u \leq v'|a \leq x \leq b')}{p(u \leq v'|a \leq x \leq b)} = 1.$$

Let $k \in K_g$ index the set of points $u_k \in U$ where g is not continuously differentiable. By monotonicity, K_g is countable. Fixing some $v \in U$, we can write that for every $u \in U$,

$$g(u) = g(v) + \sum_{k \in K_g} \delta(u_k)\mathbb{1}_{u \geq u_k}\mathbb{1}_{u_k \geq v} - \sum_{k \in K_g} \delta(u_k)\mathbb{1}_{u \leq u_k}\mathbb{1}_{u_k \leq v}$$

$$+ \int_v^{\sup U} g'(t)\mathbb{1}_{u \geq t}dt - \int_{\inf U}^{v} g'(t)\mathbb{1}_{u \leq t}dt,$$

where $\delta(u_k)$ is the size of the positive jump in the value of g at u_k. Then by Fubini's theorem,

we can write that

$$E\big(g(u)|a \le x \le b\big) = g(v) + \sum_{k \in K_g} \delta(u_k)p\big(u \ge u_k|a \le x \le b\big)\mathbb{1}_{u_k \ge v}$$

$$- \sum_{k \in K_g} \delta(u_k)p\big(u \le u_k|a \le x \le b\big)\mathbb{1}_{u_k \le v}$$

$$+ \int_v^{\sup U} g'(t)p\big(u \ge t|a \le x \le b\big)dt - \int_{\inf U}^v g'(t)p\big(u \le t|a \le x \le b\big)dt,$$

and this expression is nondecreasing in b by the first-order stochastic dominance result that we just proved. If g is not constant, then either $g'(t)$ is strictly positive on a subset of U with positive measure, or $\delta(u_k)$ is strictly positive for some $k \in K_g$. In either case, we can invoke the full support assumption to conclude that

$$E\big(g(u)|a \le x \le b\big)$$

is strictly increasing in b. $\qquad\square$

Proof of Proposition 1: Equilibrium Characterization. The expected utility of voter i if the issue is selected, conditional on the event $\mathcal{E}_i$ that her vote is pivotal, is given by $E\left(U_i|\mathcal{E}_i\right)$. The event that a voter is pivotal does not contain any additional information about her own valuation of the proposal so $E\left(U_i|\mathcal{E}_i\right)$ is equal to

$$E\left(u\mathbb{1}_{u>0}E\Big(\sum_{\substack{C \subseteq N_i \\ \#C \ge n_d-1}} \prod_{j \in C} p_j \prod_{l \in N_i \smallsetminus C}(1-p_l)\Big|\mathcal{E}_i\Big) + u\mathbb{1}_{u>0}E\Big(\sum_{\substack{C \subseteq N_i \\ \#C \ge n_d}} \prod_{j \in C} p_j \prod_{l \in N_i \smallsetminus C}(1-p_l)\Big|\mathcal{E}_i\Big)\middle| x_i\right) - c.$$

The function under the expectation is nondecreasing in u. It is not constant either because the full support assumption implies that every p_i is in $(0,1)$ and therefore the expectations conditioned on the pivotal event are strictly positive. Hence Lemma 1 implies that the expected payoff of validation is strictly increasing in the private signal. Hence any best-reply to a profile

of strategies σ_{-i} such that the pivotal event $\mathcal{E}_i$ has a positive probability is a threshold strategy. For profiles σ_{-i} such that the probability of the pivotal event is null, every strategy is a best reply.

In a symmetric equilibrium, all the voters use the same threshold t, and $\mathcal{E}_i$ is the event that exactly $n_s - 1$ voters in N_i have a type p above t. The expected value of their type is then $\overline{p}(t)$, while for the $n - n_s$ other voters in N_i, it is $\underline{p}(t)$. Because the types are independent, the expected payoff from validation conditional on being pivotal is given by $\pi(x, t)$. The rest of the argument is in the main discussion. $\qquad\square$

B   Comparative Statics and Existence

We start this section with a useful lemma, then we proceed to the proofs of the results in the text.

Lemma 3. *Let $\{Y_k\}_{k=1}^{\infty}$ be a sequence of independently distributed Bernoulli random variables that take value 1 with probability p_k, and let $\Sigma \equiv \sum_{k=1}^{K} Y_k$ for some integer K. Then for any integer q,*

$$\Pr\left(\Sigma \geq q\right)^2 \geq \Pr\left(\Sigma \geq q - 1\right)\Pr\left(\Sigma \geq q + 1\right),$$

and the inequality is strict if $0 \leq q \leq K$.

Proof. The proof works by induction on K. For $K = 1$, we have

$$\Pr\left(\Sigma \geq q\right)^2 = \begin{cases} 1 \text{ if } q \leq 0 \\ p_1^2 \text{ if } q = 1 \\ 0 \text{ if } q \geq 2 \end{cases},$$

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and

$$\Pr\left(\Sigma \geq q-1\right)\Pr\left(\Sigma \geq q+1\right) = \begin{cases} 1 \text{ if } q \leq -1 \\ p_1 \text{ if } q = 0 \\ 0 \text{ if } q \geq 1 \end{cases},$$

hence the claim is satisfied.

Suppose that the claim holds at some $K \geq 1$ and let $\Sigma' = \Sigma + Y$ where Y is a Bernoulli random variable that is independent from $(Y_1, \cdots, Y_K)$ and takes value 1 with probability p. First note that if $q \geq K+1$ or $q \leq 0$, the claim holds trivially for Σ'. Hence we assume that $1 \leq q \leq K$. We have

$$\begin{aligned} \Pr\left(\Sigma' \geq q\right)^2 &= \Big(p\Pr\left(\Sigma \geq q-1\right) + (1-p)\Pr\left(\Sigma \geq q\right)\Big)^2 \\ &= p^2 \Pr\left(\Sigma \geq q-1\right)^2 + 2p(1-p)\Pr\left(\Sigma \geq q-1\right)\Pr\left(\Sigma \geq q\right) + (1-p)^2\Pr\left(\Sigma \geq q\right)^2 \\ &> p^2 \Pr\left(\Sigma \geq q-2\right)\Pr\left(\Sigma \geq q\right) + 2p(1-p)\Pr\left(\Sigma \geq q-1\right)\Pr\left(\Sigma \geq q\right) \\ &\quad + (1-p)^2\Pr\left(\Sigma \geq q-1\right)\Pr\left(\Sigma \geq q+1\right), \end{aligned}$$

where the inequality holds by the induction hypothesis. $\Pr\left(\Sigma' \geq q-1\right)\Pr\left(\Sigma' \geq q+1\right)$ is equal to

$$\begin{aligned} &\Big(p\Pr\left(\Sigma \geq q-2\right) + (1-p)\Pr\left(\Sigma \geq q-1\right)\Big)\Big(p\Pr\left(\Sigma \geq q\right) + (1-p)\Pr\left(\Sigma \geq q+1\right)\Big) \\ &= p^2 \Pr\left(\Sigma \geq q-2\right)\Pr\left(\Sigma \geq q\right) + p(1-p)\Pr\left(\Sigma \geq q-1\right)\Pr\left(\Sigma \geq q\right) \\ &\quad + p(1-p)\Pr\left(\Sigma \geq q-2\right)\Pr\left(\Sigma \geq q+1\right) + (1-p)^2\Pr\left(\Sigma \geq q-1\right)\Pr\left(\Sigma \geq q+1\right). \end{aligned}$$

Therefore Σ' satisfies the claim whenever

$$\Pr\left(\Sigma \geq q-1\right)\Pr\left(\Sigma \geq q\right) \geq \Pr\left(\Sigma \geq q-2\right)\Pr\left(\Sigma \geq q+1\right).$$

Multiplying both sides by $\Pr\left(\Sigma \geq q-1\right)\Pr\left(\Sigma \geq q\right) > 0$, we obtain an inequality that holds by

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the induction hypothesis, and this concludes the proof. $\qquad\square$

Proof of Lemma 2. It is clear that $\Pr\big(\Sigma \geq q\big)$ is strictly decreasing in $q \leq K$. For the sense of variation with respect to p_k, we let $\Sigma' = \Sigma - Y_k$ and write that

$$\Pr\big(\Sigma \geq q\big) = p_k \Pr\big(\Sigma' \geq q - 1\big) + (1 - p_k)\Pr\big(\Sigma' \geq q\big),$$

where the two probabilities on the right-hand side do not depend on p_k and are strictly positive.

The sense of variation of the ratio with respect to q is a direct consequence of Lemma 3. For the sense of variation with respect to p_k, we can write

$$\rho = \frac{p_k \Pr\left(\Sigma' \geq q - 2\right) + (1 - p_k)\Pr\left(\Sigma' \geq q - 1\right)}{p_k \Pr\left(\Sigma' \geq q - 1\right) + (1 - p_k)\Pr\left(\Sigma' \geq q\right)},$$

where the probability functions do not depend on p_k. A little algebra shows that the derivative of ρ with respect to p_k has the same sign as

$$\Pr\left(\Sigma' \geq q - 2\right)\Pr\left(\Sigma' \geq q\right) - \Pr\left(\Sigma' \geq q - 1\right)^2,$$

which is strictly negative by Lemma 3. $\qquad\square$

As a preliminary to the proof of the best-reply comparative statics, we prove the following lemma.

Lemma 4 (Continuity).

(i) *The functions $\bar{p}(t)$ and $\underline{p}(t)$ are continuous in t.*

(ii) *The function*

$$\hat{\pi}(x, t, c) = R(t)E\big(u\mathbb{1}_{u>0}|x\big) + E\big(u\mathbb{1}_{u<0}|x\big) - \frac{c}{\Pr\big(S(t) \geq n_d\big)}$$

is continuous in t and c.

(iii) The best reply function $\beta(.)$ is continuous in t and c.

Proof. We write the proof of point (i) for $\overline{p}(t)$ and it can easily be adapted to $\underline{p}(t)$. By defintion[11],

$$\overline{p}(t) = p\big(u > 0 | x \geq t\big) = \frac{p(u > 0, x \geq t)}{p_X(x > t)},$$

where

$$p\big(u > 0, x \geq t\big) = \int_U \int_X p(u, x) \mathbb{1}_{u>0} \mathbb{1}_{x \geq t} dx du,$$

and

$$p_X\big(x > t\big) = \int_X p_X(x) \mathbb{1}_{x \geq t} dx.$$

For every t, the functions $p_X(x) \mathbb{1}_{x \geq t}$ and $p(u, x) \mathbb{1}_{u>0} \mathbb{1}_{x \geq t}$ are dominated respectively by $p_X(x)$ and $p(u, x)$ which are integrable. Hence the dominated convergence theorem implies the continuity in t of these two integrals. Since $p_X(x \geq t)$ is strictly positive for every t, this proves the continuity of $\overline{p}(t)$.

$\hat{\pi}(x, t)$ is clearly continuous in c, so we only need to prove the continuity in t. First note that $\Pr\big(S(t) \geq n_d - 1\big)$ and $\Pr\big(S(t) \geq n_d\big)$ are clearly continuous in $\overline{p}(t)$ and $\underline{p}(t)$ and always strictly positive. But then (ii) is a direct consequence of (i).

For (iii), we need to show that the continuity of $\hat{\pi}(x, t)$ in t implies the continuity of $\beta(t)$. Suppose by contradiction that $\beta(t)$ is not continuous. Then we can find a sequence (t_n) converging to a point t^* and a closed neighborhood $[a, b]$ of $\beta(t^*)$ such that for every t_n, $\beta(t_n) \notin [a, b]$. Either infinitely many $\beta(t_n)$ are strictly greater than $\beta(t^*)$ or infinitely many $\beta(t_n)$ are strictly lower than $\beta(t^*)$. Suppose the former is true (the proof is symmetric in the other case). Then we can extract from (t_n) a new sequence (t'_n) that converges to t^* such that for every n, $\beta(t'_n) > b > \beta(t^*)$. By definition of $\beta(.)$ and because $\hat{\pi}(x, t)$ is strictly increasing in x, we have $\hat{\pi}(b, t^*) > 0$ and $\hat{\pi}(b, t'_n) < 0$ for every n. But since (t'_n) converges to t, the continuity of $\hat{\pi}(b, t)$ with respect to t implies that $\hat{\pi}(b, t^*) \leq 0$, a contradiction. The proof of the continuity of $\beta(.)$

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[11] We can write these formulas with weak or strict inequalities given the assumption that the distribution is atomless on X.

with respect to c is identical. $\qquad\square$

Proof of Proposition 3: Best-Reply Comparative Statics. Point (i) is straightforward, and the argument for the comparative statics when $c \leq 0$ is given in the text, and results directly from Lemma 2. This proves point (iii). The difficulty is to extend the comparative statics of point (ii) to small positive values of c.

Under the continuity assumption, $\hat{\pi}(x, t)$ is continuous in t by Lemma 4 and strictly increasing in x by Lemma 1. By definition, $\beta(t, \theta, c)$ is the unique x at which the function

$$\hat{\pi}(x, t, \theta, c) = E\big(u\mathbb{1}_{u>0}|x\big)R(t, \theta) + E\big(u\mathbb{1}_{u<0}|x\big) - \frac{c}{\Psi(t, \theta)}$$

crosses 0 ($\underline{x}$ if this function stays above 0, $\overline{x}$ if the function stays below c), where θ is the vector of parameters (n_s, n_d) and $\Psi(t, \theta) = \Pr\big(S_n(t) \geq n_d\big)$. Consider a change from θ to $\theta' \neq \theta$ such that θ and θ' differ in one dimension only. We know from Lemma 2 that any such change results in strict variations of R and Ψ in opposite directions. Suppose $R(t, \theta') > R(t, \theta)$ and $\Psi(t, \theta') < \Psi(t, \theta)$, hence the change is either an increase in n_d or a decrease in n_s. Let Θ be the set of such pairs (θ, θ') of voting rules.

To study the effect of one-dimensional changes of the voting rule, define the function

$$\Delta(x, t, c, \theta, \theta') \equiv \hat{\pi}(x, t, \theta') - \hat{\pi}(x, t, \theta') = E\big(u\mathbb{1}_{u>0}|x\big)\underbrace{\big(R(t, \theta') - R(t, \theta)\big)}_{>0} - c\underbrace{\big(1/\Psi(t, \theta') - 1/\Psi(t, \theta)\big)}_{>0},$$

which is strictly increasing in x by Lemma 1, continuous in t and c by Lemma 4. Because $\Delta(.)$ is strictly increasing in x, we have $\Delta(x, t, c, \theta, \theta') \geq \Delta(\underline{x}, t, c, \theta, \theta')$. Whenever $c \leq 0$ the change from θ to θ' strictly increases $\hat{\pi}$, that is for every $c \leq 0$, $t \in X$ and $(\theta, \theta') \in \Theta$, $\Delta(\underline{x}, t, c, \theta, \theta') > 0$. $\Delta(\underline{x}, .)$ is continuous in t and X is compact, and Θ is finite, therefore it attains its minimum value on $X \times \Theta$. Therefore, for every $c \leq 0$

$$\tilde{\Delta}(c) \equiv \min_{(t, \theta, \theta') \in X \times \Theta} \Delta(\underline{x}, t, c, \theta, \theta') > 0,$$

and furthermore, by Berge maximum theorem, this function is continuous in c. But then there must exist some $\bar{c} > 0$ such that for every $c < \bar{c}$ we have $\tilde{\Delta}(c) > 0$. Therefore, for every $c < \bar{c}$, $x \in X$, $t \in T$ and every $(\theta, \theta') \in \Theta$,

$$\Delta(x, t, c, \theta, \theta') \geq \Delta(\underline{x}, t, c, \theta, \theta') \geq \tilde{\Delta}(c) > 0.$$

This implies that any change of rule in Θ leads to the same comparative statics when $c \leq 0$ and when $c < \bar{c}$ $\qquad\square$

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Proof of Proposition 5. The argument showing that $T^*(n_s, n_d, c) \geq t_{dict}$ is given in the text. This implies that for any $t \in T^*(n_s, n_d, c)$, we have $p_X(x > t) \leq p_x(x > t_{dict})$. But for a rule n_s the probability of selection is not equal to $p_X(x > t)$. However we know that the probability of selection is nondecreasing in n_s. And if $n_s = n$ (the unanimity rule), the probability of selection corresponding to $t \in T^*(n, n_s, c)$ is $p_X(x > t^*)^n \leq p_x(x > t_{dict})$. The rest of the proposition follows from the monotonicity of the probability of selection with respect to n_s. $\qquad\square$

Proof of Proposition 6. Consider the difference between the random variables Z and $S_n(t)$. In the latter, $n_s - 1$ of the $n - 1$ Bernoulli random variables composing the sum have increased in probability of being equal to 1, from $\tilde{p}$ to $\bar{p}(t)$, while the other have decreased from $\tilde{p}$ to $\underline{p}(t)$. Under the unanimous selection rule, therefore, they have all increased, whereas under the one-vote selection rule, they have all decreased. Given Lemma 2, we know that this implies that for $c \leq 0$ the result of the proposition holds. To prove it for sufficiently small positive c, we just need to mimic the proof of Proposition 3 where we add one value $n_s = as$ in the selection dimension of the parameter space. $\qquad\square$

Proof of Proposition 7. The naive voter weighs $E(u\mathbb{1}_{u>0}|x)$ and $E(u\mathbb{1}_{u<0}|x)$ equally with the unit weight. Now for every t, we have $\Pr\big(S_n(t) \geq n_d\big), \Pr\big(S_n(t) \geq n_d - 1\big) < 1$, and $R(t, n_s, n_d) >$

1. Therefore, the naive threshold is always above the rational threshold if $c \leq 0$. For sufficiently small positive costs, we mimic the proof in Proposition 3 where we add a point in Θ for each change from a given voting rule to the naive voter. $\qquad\square$

Proof of Proposition 8. We define the following function from $X \times X \to \mathbb{R}$

$$\hat{\pi}^{sp}_{n_d}(x,t) = E\big(u\mathbb{1}_{u>0}|x\big)\frac{\Pr\big(Z(t) \geq n_d - 1\big)}{\Pr\big(Z(t) \geq n_d\big)} + E\big(u\mathbb{1}_{u<0}|x\big) - \frac{c}{\Pr\big(Z(t) \geq n_d\big)},$$

and denote by $\hat{\pi}^{n_s}_{n_d}(x,t)$ the modified expected payoff function of the pivotal voter as before, where we introduced the indexes n_s and n_d to make the dependence on the voting rule explicit. The fact that $\underline{p}(t) \leq \hat{p}(t) \leq \overline{p}(t)$ and Lemma 2 together imply the following inequalities for any $c \leq 0$, that we can extend to every $c < c^{sp}$ where the existence of $c^{sp} > 0$ is proved by using the same technique as in the proof or Proposition 3:

$$\hat{\pi}^{n_s=n}_{n_d}(x,t) \leq \hat{\pi}^{sp}_{n_d}(x,t) \leq \hat{\pi}^{n_s=1}_{n_d}(x,t),$$

for every x and t. Denoting by $\beta^{sp}_{n_d}(t)$ the x at which $\hat{\pi}^{sp}_{n_d}(x,t)$ crosses 0 ($\underline{x}$ if it stays above 0, $\overline{x}$ if it stays above 0), these inequalities imply the following inequalities

$$\beta^{n_s=n}_{n_d}(t) \geq \beta^{sp}_{n_d}(t) \geq \beta^{n_s=1}_{n_d}(t).$$

Now note that the interval $T^{sp}_{n_d}$ of values at which the function $\hat{\pi}^{sp}_{n_d}(x)$ crosses 0 must also be the set of fixed points of $\beta^{sp}_{n_d}(t)$. But then we can use Milgrom and Roberts (1994, Corollary 1) as in Proposition 4 to conclude that

$$T^*(n_s = 1, n_d, c) \leq t^{sp}_{n_d}(c) \leq T^*(n_s = n, n_d, c).$$

The statement of the proposition then results from the monotonicity of $T^*(n_s, n_d, c)$ in n_s which was proved in Proposition 4.

The fact that $H(.)$ is nonincreasing then implies that for $t \in T^*(1, n_d, c)$ and $t' \in T^*(n, n_d, c)$, the selection region corresponding to t for the one-vote selection rule is a superset of the selection region corresponding to the social planner, and the selection region corresponding to the equilibrium t' for the unanimous selection rule is a subset of the selection region corresponding to the social planner, which leads to the second point of the proposition. $\qquad\square$

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