

# TIMES OF BROKEN PROMISES. FINANCIAL FRAGILITY VS. SECURITIZATION\*

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## Abstract

We consider an infinitely repeated game with imperfect monitoring in an environment with moral hazard. We identify efficient equilibria of this game with securitization and show that the model can qualitatively account for the rise in securitization volume before the 2007 financial crisis and its drop afterwards. We provide an alternative rationale for securitization that does not rely on risk sharing. We also show that under certain conditions securitization is superior to the use of financial fragility as a disciplining device, as advocated by Calomiris and Kahn (1991) and Diamond and Rajan (2001a,b).

**JEL classification:** G21, C73, L14

**Keywords:** bank runs, securitization, imperfect monitoring, repeated games

## 1 Introduction

Securitization markets were at the root of the financial crises which started in the United States in 2007 (Brunnermeier, 2009; Gorton, 2008). The run-up to the crisis in the United States had two stylized facts:

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1. Extremely low actual and expected short term and long term interest rates (Boeri and Guiso, 2008; Mizen, 2008, pp. 533–534; Brunnermeier, 2009, p. 77).
2. A steep rise in the volume of securitization. According to SIFMA, the stock of outstanding mortgage backed securities in the US rose from USD 5.93 trillion in 2004 to USD 9.14 trillion in 2007. The stock of outstanding asset backed securities (excluding mortgages) rose from USD 1.09 trillion in 2000 to USD 1.82 trillion in 2004 and USD 2.47 trillion in 2007.<sup>1</sup>

Once the financial crisis erupted, doubts about the financial institutions' incentives to check the quality of the assets underlying the securitized instruments surfaced. In this paper we present a simple theory of how the expected interest rate is related to the incentives of financial intermediaries to perform a costly action such as, for example, monitoring loans. The theory is not new, as it is a simple application of results well-known in the repeated games literature to financial contracting, an issue taken up before us by, for example, Boot and Thakor (1994). Our approach does, however shed light on the desirability of using financially fragile structure as a disciplining mechanism as proposed by Diamond and Rajan (2001b). This is where we think the main contribution of our paper lies.

There are a multitude of stories that can be told which are consistent with the the relationship between interest rates and securitization volume. After the financial crisis, the predominant view has been that the growth in securitization has been excessive, meaning inefficiently high. For example, one very popular account using the mechanics described in James (1988) has the link between interest rates and bank profits and

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<sup>1</sup>The source for these values is <http://www.sifma.org/uploadedFiles/Research/Statistics/StatisticsFiles/SF-US-Mortgage-Related-Outstanding-SIFMA.xls> and <http://www.sifma.org/uploadedFiles/Research/Statistics/StatisticsFiles/SF-US-ABS-Outstanding-SIFMA.xls>. Accessed on 2011-05-04.

regulatory arbitrage as its main ingredients. In a very stylized exposition, the story goes as follows. Low interest rates imply lower per-loan margins for financial intermediaries. This loss can be partially made up by increasing the number of loans. Since the increase in loans is limited by regulatory ratios, there is an incentive to take them off the bank's balance sheet through securitization. Since the motivation to increase the degree of securitization is aimed at circumventing regulatory constraints, which arguably were designed with efficiency in mind, they most probably lead to an inefficient outcome.

In this paper we offer a story of how the interest rate is related to securitization issuance in which the increase in securitization is a constrained efficient outcome to a moral-hazard problem in the presence of long-lived financial institutions. The environment we consider is an infinite repetition of the environment considered by Calomiris and Kahn (1991) and Diamond and Rajan (2001a,b). In those models it is assumed that due to environmental and contracting constraints the payment promised to a lender can still be affected by an action of the borrower. The borrower cannot commit to behave: there is a moral hazard problem.

Only if there is an external way of punishing the lender, good behavior can be enforced. The solution according to Calomiris and Kahn (1991) is to let the borrower become a banker by setting up a fragile structure. The possibility of a bank run as a punishment keeps the borrower in check and works as a disciplining mechanism. Diamond and Rajan (2001a,b) use a similar framework to focus on the provision of liquidity by banks. In their model, when a lender has a special skill in collecting repayments from debtors, debt contracts are illiquid since not all their market value can be reaped by the lender. The creation of a bank offers a solution to this problem via deposit contracts. The bank resists the incentive to extract value from investors because the deposit contract gives lenders the option to withdraw their funds before the termination date. Then, it is the

threat of a run that disciplines the banker and creates liquidity. In that context, the financial fragility inherent to the banking system is what is required in order to satisfy the demand for liquidity.

The models of Calomiris and Kahn (1991) and Diamond and Rajan (2001a,b) amount to one-shot games in intertemporal punishments are ruled out by construction. In this paper we argue that an alternative to punishments through the use of a financially fragile structure is to rely on the intertemporal incentives that arise when the stage game is repeated, implying that the borrower experiences the need for financing repeatedly. The need for a bank and its financially fragile structure to resolve the moral hazard problem disappears. A simpler contract which we interpret as the securitization of loans is enough to provide incentives. Our interpretation of the optimal contract gives a new rationale for the existence of securitization as an arrangement which solves a moral hazard problem.

In an environment in which an agent is long-lived, the availability of intertemporal punishments will matter and, in consequence, the discount rate plays an important role as well. The Folk Theorems in the literature on infinitely repeated games tell us that allocations on the Pareto frontier can be attained provided that the players do not discount the future too heavily. Further, the psychological discount rate of a player is tied to the interest rate. Although the interest rate is, in principle, a market price which is external to preferences, the marginal conditions of an intertemporal optimization problem link the market interest rate to the discount rate, a preference parameter.<sup>2</sup> This implies that our result will be able to relate the interest rate to the contract chosen to solve the moral hazard problem, which is the issuance of securitized assets in our case.<sup>3</sup>

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<sup>2</sup>The link between the discount rate and the interest rate is ubiquitous in economics and finance. It is, for example, why in present value calculations a stream of future income is rendered into its present value equivalent by using the interest rate as the discount factor.

<sup>3</sup>Strictly speaking, since we deal with forward looking behavior, it is the expected interest rate, not the current interest rate, which is of importance. The two are, of course, closely tied together by

Our model also provides an additional rationale for securitization beyond risk sharing. To understand what we mean by this consider what securitization entails. Securitization involves the pooling and repackaging of cash flow producing financial assets into securities that are then sold to investors. Securitization therefore consists of two activities: asset pooling and “credit enhancing”. “Credit enhancing” means that their credit quality is increased above that of the originator’s unsecured debt or underlying asset pool. The events of the financial crisis which started in 2007 cast a shadow of doubt over “credit enhancing” function (Brunnermeier, 2009) and have given securitization a bad press. One contribution of our paper is to show that there is a positive role for securitization which is unrelated to its asset pooling and “credit enhancing” functions. It may be the arrangement which provides the incentives to solve a moral-hazard friction. Incidentally, the function performed by securitization in our model implies that the assumption of risk aversion on the side of investors as in Greenbaum and Thakor (1987) is unnecessary as well and we dispense with it in our model.

## 1.1 Review of previous literature

The idea that securitization may be a solution to moral hazard has been visited several times in previous literature and is too extensive to review in its entirety. Benveniste and Berger (1987) show how securitization with recourse offers a lender protection against moral hazard. Doherty and Richter (2002) consider an insurance market and show that moral-hazard can be reduced by combining a securitized asset with an index hedge in a model with mean-variance preferences. Plantin (2011) develops a model in which securitization solves a moral hazard problem and which also tries to explain low interest

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arbitrage relations. In this paper we do not explicitly set up a complete model in which the interest rate and its arbitrage relations are determined endogenously, since we want to focus on the incentive problem of interest.

rates and increased securitization as the outcomes of a constrained-optimal arrangement. In all these examples, the ongoing repeated interaction between a financial intermediary and its lenders is not addressed. Also, to our knowledge, our paper is the first to realize that a securitization contract is the natural outgrowth of the environment considered by Calomiris and Kahn (1991); Diamond and Rajan (2001b) when it is realized that interactions do not have a certain stopping point.

Our result is arguably more relevant for the banking and bank runs started by the seminal work of Bryant (1980) and Diamond and Dybvig (1983). One of the key insights in that literature is that banks are financially fragile, meaning that demandable deposits are unmatched by assets which are liquid enough to repay the full amount of demand deposits at any moment in time. Financial fragility may serve a useful purpose. For example, Wallace (1996) shows that without a fragile structure, the banking system cannot do better than autarkic consumption. Ennis and Keister (2003) construct a model in which banks choosing a lower level of liquidity increases the rate of long-term economic growth. The strand of the literature we relate to in this paper (Calomiris and Kahn, 1991; Diamond and Rajan, 2001b) argues that the useful role played by fragility is to discipline the banker. By setting up a financially fragile structure, the banker is at risk of bank runs which serve as a punishment if the banker misbehaves. Our results show that the rationale for intentionally setting up a financially fragile structure is weak when repeated interaction is allowed.

The effect of access to a financial market on incentives of borrowers has been studied before. Gorton and Pennacchi (1995) is an example of this literature which is close to our paper. They construct a model in which bank loans should not be marketable because of moral hazard problems (costly monitoring of a loan, in their case) and find that loan sales may be partially efficient. In contrast with our model, the main reason

to sell loans on the market is a differential funding cost for the bank's own equity. Also, their setting is not an infinitely repeated game and incentives to monitor loans arise from a static cost-benefit analysis.

The model using an infinitely repeated game setup most closely related to ours is Boot and Thakor (1994). Like us, they consider an infinitely repeated stage game with a moral hazard problem. However, the way they relate their infinitely repeated game to the literature on banking is different. The borrower in their case is a long-lived entrepreneur while the lender is a bank. In our case, since we want to relate to Calomiris and Kahn (1991) and Diamond and Rajan (2001b), the identity of the player's is the opposite: the lender is an investor, who in our model may be short-lived, and the long-lived borrower is a financial institution. In addition, there are differences in the way they model the environment of the stage game, which lead to slightly different results. For example, in their model the welfare loss from moral hazard only disappears after the first favorable (imperfect) signal on the lender's action.

Finally, there is a literature which studies how reputation formation and the evolution over time of the incentive effects of reputation to mitigate conflicts of interest between borrowers and lenders. An example of this literature is Diamond (1989). In his model borrowers use the proceeds of their loans to fund projects. In the absence of reputation effects, borrowers have incentives to select excessively risky projects. This literature is a bit farther from our model, since it deals with models of asymmetric information, where the type of the borrower needs to be learned. In order to delineate the difference between hidden types and hidden actions, we follow the suggestion of Cabral (2005) and prefer the use of the word "trust" rather than "reputation" in this paper. That is also why we talk of promises and broken promises in our title.

## 2 Basic setup

We consider an infinitely repeated stage game. The stage game embeds the environments of Calomiris and Kahn (1991) and Diamond and Rajan (2001a) as special cases. This is shown in an Appendix.

### 2.1 Description of the stage game

There are two dates within the stage game: date 1 and date 2, and two types of agent: an entrepreneur and an investor. Both types are risk-neutral. The cashless entrepreneur has access to a project which requires one unit of investment at date 1 and returns  $R > 1$  at date 2. The investor is in reality a continuum (of mass one) of investors who have an endowment of one dollar at date 1. Both types of agent have access to a storage technology (which has a gross return of 1).

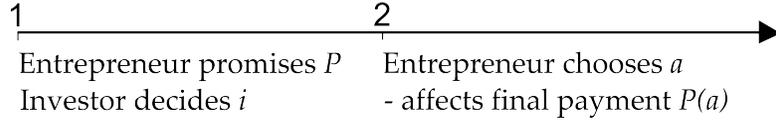
In order to produce, the entrepreneur needs to borrow from the investor. The entrepreneur proposes to borrow 1 and to repay  $P$ . The investor either accepts or rejects to lend under these terms. There is an unmodeled imperfection which implies that the entrepreneur still has some discretion left over the final payment. We model this by letting  $P$  depend on  $a$ , an action by the entrepreneur.<sup>4</sup>

The entrepreneur takes action  $a$  at date 2. It affects the payoff the investor obtains:  $P(a)$ . All we assume is that  $a \in A$  where  $A$  is a compact subset of the real line. The promised repayment  $P$  can be thought of as the joint promise of the function  $P(a)$  and a specific action  $\hat{a}$  such that  $P = P(\hat{a})$ .

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<sup>4</sup>In this paper we are completely agnostic about the origin of this imperfection. It can be argued that using legally unenforceable, discretionary financial contracts in circumstances where legally enforceable contracts are feasible may be efficient in some cases (e.g. Boot, Greenbaum, and Thakor, 1993).

By letting the repayment depend on an action by the entrepreneur our model is able to fit several different types of moral hazard environments and does not impose any particular way in which the surplus is distributed between the entrepreneur and the investor. For example, the choice variable  $a$  may be interpreted as absconding by the entrepreneur (as in Calomiris and Kahn, 1991) or as ex-post renegotiation, once the investor has handed over the cash (as in Diamond and Rajan, 2001a,b).<sup>5</sup>



**Figure 1:** *Time line of the stage game.*

Let  $i \in \{0, 1\}$  stand for the decision of the investor to lend to the entrepreneur. Then the entrepreneur's payoff is

$$P(a)i + 1(1 - i) = 1 + [P(a) - 1]i \quad (1)$$

while the investor obtains

$$[R - P(a)]i. \quad (2)$$

Let  $a^*$  be the (not necessarily unique) choice of  $a$  that maximizes the entrepreneurs payoff:

$$a^* \equiv \arg \max_a [R - P(a)]i. \quad (3)$$

From the inspection of the payoff functions, and given the timing of actions, it is immediate that, in a subgame perfect equilibrium, the investor will only provide funding when  $P(a^*) \geq 1$ . On the other hand, funding is always efficient since  $R > 1$ . This

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<sup>5</sup>Appendix A shows how our environment embeds that of Calomiris and Kahn (1991) and Diamond and Rajan (2001a,b).

implies that sometimes funding does not occur even though it is efficient.

We define the set of actions by the entrepreneur that are consistent with the investor desiring to offer funding  $\tilde{A} \in \{a \in A : P(a) \geq 1\}$ . Any  $\tilde{a} \in \tilde{A}$  allows the project to be funded. An inefficiency arises whenever a project is not funded. In our notation, this means whenever  $a^* \notin \tilde{A}$ .

## 2.2 The dynamic game

The dynamic game consists of the infinite repetition of the stage game. Time transpires discretely  $t = 0, 1, 2, \dots$ . The entrepreneur is assumed to be a long-lived player who faces a sequence of investors. As usual, the long-run player discounts future income by a discount factor. Since the entrepreneur is risk-neutral, the risk-free rate is a good candidate to discount income streams.

We will consider two cases, depending on whether a low or a high risk-free interest rate is expected. Denote possible values for the expected interest rates by  $r_L$  and  $r_H$  where  $r_L \leq r_H$ . For simplicity, we assume that the expectation of the interest rate is constant through time. This very simple process for the interest rate is already enough to generate the result we are after: that variations in the expected interest rate qualitatively account for the evolution of securitization volume. Of course, further complicating the process for the interest rate, although being more realistic, would only add degrees of freedom.<sup>6</sup>

The formula for the present value of an income stream  $\{x_t\}$  for an interest rate  $r$  is

$$PV = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} x_t \quad (4)$$

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<sup>6</sup>For example, a more general framework would be to posit a two-state Markov process for the interest rate. The interest rate starts out either at  $r_L$  or  $r_H$  and is governed by a transition matrix. Letting the probability  $\Pr[r_s|r_s]$  tend to one implies that, in the limit, states are absorbing and, therefore, that the expected interest rate is constant at either  $r_L$  or  $r_H$ .

The entrepreneur's payoff is the present value of the infinite stream  $\{[R - P(a_t)]i_t\}$ , using  $r_s \in \{r_L, r_H\}$ , as the relevant interest rate.

$$V_E = \frac{r_s}{1 + r_s} PV = \frac{r_s}{1 + r_s} \sum_{t=0}^{\infty} \frac{1}{(1 + r_s)^t} [R - P(a_t)]i_t \quad (5)$$

We have used an affine transformation of the present value formula which consists of multiplying by  $\frac{r_s}{1+r_s}$ . As a consequence of this transformation, intertemporal utility of a constant payoff stream does not depend on the particular value of the rate  $r_s$  used to discount future payoffs.

### 3 Overcoming moral hazard

In this section we show that securitization is a way of overcoming a moral hazard problem. Using a repeated game framework to address the moral hazard problem also provides a link between expected interest rates and the amount of securitization performed. A higher expected interest rate implies that the future is discounted more heavily and makes breaking promises less costly relative to keeping them. Times of high interest rates are times of broken promises.

In the model, as argued before, funding does not always occur in equilibrium which, given the assumption that  $R > 1$ , creates an inefficiency. In this section we analyze formally how the moral-hazard problem can be overcome through the securitization of the loan. We also consider the proposal of setting up a bank as an alternative solution. These two solutions turn out to be equivalent, in the sense that they are payoff-equivalent.

### 3.1 Securitization as a commitment device

Suppose that the entrepreneur has access to a financial market at each date  $t$ . The entrepreneur can now choose any “promised action”  $\tilde{a} \in \tilde{A}$  and finance the project by selling a claim with face value  $P(\tilde{a})$  and keeping the remainder of the proceeds  $R - P(\tilde{a})$ . This activity is essentially what securitization is about. The securitized asset  $P(\tilde{a})$  is placed in the asset market and  $R - P(\tilde{a})$  is an equity tranche which is kept on the balance sheet of the entrepreneur.<sup>7</sup> The action  $\tilde{a}$  is the action the entrepreneur promises to perform in order to give the claim a value which is more than one, the outside option of investors.

Formally, we define:

**Definition 1** *For any  $\tilde{a} \in \tilde{A}$ , a securitization contract is a pair  $\{P(\tilde{a}), \tilde{a}\}$ , where  $P(\tilde{a})$  is a face value and  $\tilde{a}$  is the promised action.*

Why do investors believe that the claim is worth  $P(\tilde{a})$  and not  $P(a^*)$ , which is what they would obtain if the entrepreneur chose to perform action  $a^*$ ? The answer is that if the entrepreneur values the future sufficiently enough, then he may be prevented from choosing  $a^*$  by the market reaction.

The effect of the securitization contract on the stage game can be analyzed through a payoff matrix.

	$\tilde{a}$	$a^*$
1	$P(\tilde{a}), R - P(\tilde{a})$	$P(a^*), R - P(a^*)$
0	1, 0	1, 0

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<sup>7</sup>Notice that “equity tranche” is a bit of a misnomer, since  $R$  is certain. In the more general case in which  $R$  is uncertain, this would be a proper equity tranche. Given the assumption of risk neutrality, the analysis remains the same. Notice also that, in our setting, the securitized asset does not consist of a pool of underlying assets. This is without loss of generality, since agents are risk neutral in our setting.

The first payoff in each cell belongs to the investor and the second to the entrepreneur. The investor's action  $i \in \{0, 1\}$  is depicted on rows and the entrepreneur's action  $a$  on columns. The payoff matrix contains has only two columns because, even though there may be an infinite number of actions, given  $\tilde{a}$  and  $a^*$ , these are the only two columns of interest. They represent the promised action and the best possible deviation.<sup>8</sup>

As is clear from the matrix, a commitment problem appears when  $P(a^*) < 1$ . Assume that behavior can be perfectly observed ex-post. Then players can punish a deviating entrepreneur by shutting him out of the market.

**Proposition 1** *Any action  $\tilde{a} \in \tilde{A}$  can be enforced through the use of the securitization contract  $\{P(\tilde{a}), \tilde{a}\}$  provided that*

$$r_s \leq \bar{r} \equiv \frac{R - P(\tilde{a})}{P(\tilde{a}) - P(a^*)}. \quad (6)$$

All proofs that are not in the main text can be found in Appendix B.

The intuition for the result in Proposition 1 is straightforward. Low  $r_s$  implies that future payoffs are discounted lightly and have a greater weight in the decision of whether deviating is profitable. The threshold  $\bar{r}$  is increasing in  $R$ . As  $R$  goes up, the value foregone when being punished becomes greater. Also,  $\bar{r}$  depends on the distance between  $P(\tilde{a})$  and  $P(a^*)$ . If these two values are closer together, then the temptation of deviating becomes smaller.

Consider the comparative statics of moving from a low interest rate  $r_L$  to a high interest rate  $r_H$ . If the condition in (6) held with a low interest rate, it may fail to hold for a higher interest rate since the value from keeping the promised action  $\tilde{a}$  is discounted

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<sup>8</sup>The payoff matrix is not the normal form of the stage game. Our  $2 \times 2$  matrix is constructed for actions, not for strategies.

more heavily while the value from deviating accrues in the present, and therefore does not depend on the interest rate. Our model identifies a period of high expected interest rates with broken promises.

Through the lens of our model, the period of low interest rates leading up to the surge in securitization volume may be interpreted as an increase in the incentives to keep promises. If the low interest rate is expected to persist for an extended period (infinitely in our model), investors rationally decide to lend, trusting that the entrepreneur will not choose to deviate. Once the expectation of a low interest rate is not validated, incentives from keeping promises fall and investors decide that trusting the entrepreneur is no longer warranted.

The liquidity squeeze at the start of the financial crisis implied that inter-bank lending rates suddenly jumped to levels which were rarely seen before. This rise in interest rates reduces the present value of streams of future income. Realizing this, it is rational by lenders to withdraw from the markets of asset backed securities. Our model is consistent with the drying-up of these markets in the early stages of the crisis. It provides another reason for this phenomenon in addition to what considered probably the most important reason for the drying-up: uncertainty about the ultimate identity of the holders of troubled assets.

The change from keeping the promise to breaking it in our model is very abrupt. We have modeled only one entrepreneur and two values for the expected interest rate. A richer model, with more detail on the side of interest rate expectations and heterogeneity on the side of the entrepreneurs would possibly generate a smoother reaction to a change in interest rate expectations.

### 3.2 Financial fragility as a commitment device

The traditional approach in the literature has been to look for a commitment device that works in the static setting. Calomiris and Kahn (1991) and Diamond and Rajan (2001a) identify demand deposits as the key institutional arrangement that solves the commitment problem.

Demand deposits solve the problem as follows. Suppose that  $a^* \notin \tilde{A}$ . Absent any commitment device, the lender is not willing to lend. The entrepreneur sets up a bank that issues demand deposits with face value  $P(\tilde{a})$  for some  $\tilde{a} \in \tilde{A}$ . The bank therefore implicitly promises to take action  $\tilde{a}$ . If it does not hold this promise and chooses  $a^*$ , then depositors run on the bank, completely wiping out profits. This occurs in the *same* period. In a bank run, the borrower has to liquidate prematurely and is able to produce only  $b < 1$  (resources are destroyed). If the bank run occurs, the entrepreneur/banker obtains 0. In the case of no bank run, payoffs are  $R - P(\tilde{a})$  for the entrepreneur and  $P(\tilde{a})$  for investors.

This description of the bank setup is a reduced form of a standard bank run model in which depositors face a collective action problem. This model has multiple equilibria. The bank run equilibrium is Pareto-dominated by the equilibrium without a bank run.

Considering the game including the investment decision, there are two Nash equilibria. One in which the efficient action  $\tilde{a}$  is taken and there is no bank run, and the other one in which the project is not financed because  $a^*$  is anticipated. The subgame perfect equilibrium is the efficient one. Therefore, the commitment problem is solved.

The game theoretical literature on banking has modeled demand deposits in a static setting, i.e. not as an infinitely repeated stage game.<sup>9</sup> Suppose that the stage game is

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<sup>9</sup>As, for example, in Diamond and Dybvig (1983) and the literature thereafter.

infinitely repeated, meaning that the bank is a long run player that faces a sequence of short run investors. As is well known from the literature on repeated games, playing the subgame perfect equilibrium of the stage game each period is a subgame perfect equilibrium of the entire infinitely repeated game.

### 3.3 Equivalence result

In the special case of perfect monitoring, using banks as a commitment device or using a securitization contract is in a sense equivalent. The reason is that on the equilibrium path punishments are never used and, therefore, payoffs consist of  $(P(\tilde{a}), R - P(\tilde{a}))$  in both cases. The following proposition expresses exactly what we mean by this.

**Proposition 2** *For  $r \leq \bar{r}$ , the same maximal payoffs  $(P(\tilde{a}), R - P(\tilde{a}))$  are attained either by using a financially fragile structure or by using securitization as a commitment device .*

**Proof.** The result follows directly from Proposition 1 and from Lemma 2 in Diamond and Rajan (2001b). Q.E.D.

### 3.4 Discussion of the equivalence result

The standard bank-run model (Diamond and Dybvig, 1983) is static in the sense that it is a finite game. Thus, intertemporal incentives that arise in infinitely repeated games are ruled out. Calomiris and Kahn (1991) are the first to realize that the multiplicity of equilibria may be useful to provide incentives. Bank runs are a direct consequence of the financially fragile structure of banks. Bank runs prove a useful as punishment, especially if they are off-equilibrium-path threats so that their costs must not be borne by those

who punish. But the idea that financial intermediaries (and banks in particular) may not enjoy the benefits of trust arising from repeated interaction seems artificial. Our model removes the assumption of a final period and finds that the case in favor of financial fragility as an incentive provision mechanism is weakened. Further, with securitization, punishments work through asset markets. Thus, our findings are related in spirit to those of Jacklin (1987). We find that a market mechanism achieves the same allocations as a bank but without being prone to bank runs.

While in Calomiris and Kahn (1991) depositors are required to undertake costly monitoring of the bank to anticipate future problems and Diamond and Rajan (2001b) assume public monitoring. If information provision is weakened and the more realistic case of imperfect public monitoring is considered, then the equivalence result proved in this section breaks down. Imperfect monitoring puts punishments on the equilibrium path. In case of a financially fragile structure the especially destructive nature of bank runs is helpful in obtaining efficiency since it implies punishments which the long run player wants to avoid at all costs. On the other hand, if it is necessary to sometimes punish on the equilibrium path, then bank runs lose their appeal and the alternative of securitization gains in attractiveness. The next section studies what happens to our equivalence result with imperfect monitoring.

## 4 Imperfect monitoring

Now we generalize the environment to the case on which players have only noisy information about past play. The game theory literature knows this environment as games with imperfect public monitoring. In this type of models actions are not directly observed, but players' actions affect the distribution of signals. Intertemporal incentives can be re-

stored by using punishments based on these signals. Since deviations are not perfectly observable, punishments occur on the equilibrium path (Fudenberg and Levine, 1994).

## 4.1 Changes to the basic setup

In the context of our model, the investor does not observe the action of the banker/claim issuer perfectly, but only receives an imperfect signal of it. As is usual in the literature, we restrict to public strategies and use the concept of perfect public equilibrium (PPE) as our equilibrium concept.

We consider the smallest set of signals necessary to generate incentives  $Y = \{\underline{y}, \bar{y}\}$ . Signals are produced by action profiles according to the following law:

$$\rho(\bar{y}|a) = \begin{cases} 1 - \varepsilon & \text{if } (i, a) \in \{(1, \tilde{a}), (0, a^*)\} \\ \varepsilon & \text{otherwise} \end{cases} \quad (7)$$

The rank condition requires  $\varepsilon \neq \frac{1}{2}$ .<sup>10</sup> Without loss of generality we assume  $\varepsilon < \frac{1}{2}$ .

Since the objective of this section is to compare the relative merits of a financially fragile structure and securitization for the provision of incentives, we can forget about different interest rate expectations. In what follows we set  $r = r_L = r_H$ .

## 4.2 Securitization as a commitment device

In the case of imperfect monitoring, given that punishments occur on the equilibrium path, the specific details of how punishments are used have a direct impact on the player's maximal intertemporal payoffs. Further, the details of which strategies deliver

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<sup>10</sup>See Fudenberg, Levine, and Maskin (1994) for the necessity and intuition of the rank condition.

highest payoffs depend on the interest rate. The following Lemma presents the maximum payments that can be achieved in a pure-strategy PPE. If players are allowed to choose whether to punish using a mixed strategy with a correlation device, then still higher payoffs may be possible (Mailath and Samuelson, 2006, Chapter 7). We disregard this possibility. On the one hand, because it is questionable whether agents are able to coordinate in anonymous asset markets. On the other hand, because, even if it were possible, it would only strengthen the results in this section.

**Lemma 1** *For sufficiently small  $\varepsilon$  there exists  $\bar{r}_{sc} < \bar{r}$  such that, if  $r \leq \bar{r}_{sc}$ , the following payoffs are attainable in a PPE of the infinitely repeated game with imperfect monitoring:*

$$\begin{aligned} v_E^{sec} &= (1 - \varepsilon)[R - P(\tilde{a})] \\ v_I^{sec} &= (1 - \varepsilon)P(\tilde{a}) + \varepsilon \end{aligned} \tag{8}$$

The payoffs calculated in Lemma 1 are generated by using stick-and-carrot strategies. Grim trigger strategies, as the ones used in the proof of 1 are an extreme case in the sense that punishments are very harsh. Given that punishments happen on the equilibrium path due to imperfect monitoring, this lowers the payoff that can be attained by the long-run player. Typically, a more efficient punishment mechanism has a stick and carrot regime. If it can be sustained by the interest rate, the strategy yielding the highest payoffs has the investors punishing for one period when  $\underline{y}$  is observed and then going back to playing the cooperative stage game strategy profile. Payoffs for strategies that punish for more than one period will be somewhere in between this case and the grim trigger case.<sup>11</sup>

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<sup>11</sup>For completeness, Appendix C derives the payoffs for the grim trigger strategy.

### 4.3 Financial fragility as a commitment device

When monitoring is imperfect, the bank run can be used as a punishment as well. The bank run occurs if the signal  $\underline{y}$  is observed and therefore bank runs occur on the equilibrium path.<sup>12</sup> If the bank run is used to punish, then the present value of the discounted payoff for the bank depends on the assumption of what happens to the bank the next period. If after a run the bank can start anew in the following period, the normalized discounted value of bank profits is given by

$$\begin{aligned} v_E^{bank} &= \frac{r}{1+r} \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} (1-\varepsilon)[R - P(\tilde{a})] \\ &= (1-\varepsilon)[R - P(\tilde{a})] \end{aligned} \quad (9)$$

This value represents an upper bound of bank profits. On the other hand, the lower bound is given by the case in which after a run, the bank is not allowed to reopen.

$$\begin{aligned} v_E^{bank} &= \frac{r}{1+r} \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} (1-\varepsilon)^{t+1}[R - P(\tilde{a})] \\ &= (1-\varepsilon) \frac{r}{r+\varepsilon} [R - P(\tilde{a})] \end{aligned} \quad (10)$$

Therefore, the maximal intertemporal value for entrepreneur/banker is somewhere in the interval

$$v_E^{bank} \in \left[ (1-\varepsilon) \frac{r}{r+\varepsilon} [R - P(\tilde{a})], (1-\varepsilon)[R - P(\tilde{a})] \right], \quad (11)$$

depending on how easy it is for the bank to recover from a bank run.

Investors are short-lived players of the game. For the case when the bank closes permanently after a bank run, the maximal payoff for the short-lived investor is given by

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<sup>12</sup>In the version of the model presented in this section, bank runs occur in equilibrium such as in Peck and Shell (2003), although bank runs in that model do not serve a punishment purpose.

$$v_I^{bank} = (1 - \varepsilon)^{t+1}P(\tilde{a}) + (1 - \varepsilon)^t\varepsilon b + [1 - (1 - \varepsilon)^t]. \quad (12)$$

To understand the meaning of this equation, notice that the period  $t$  short run player faces a bank which is still alive with probability  $(1 - \varepsilon)^t$  and observes a favorable signal with probability  $1 - \varepsilon$ . Therefore, with probability  $(1 - \varepsilon)^t$  he obtains  $P(\tilde{a})$ . With probability  $(1 - \varepsilon)^t\varepsilon$  a bad signal is observed for a bank that is still alive and therefore a bank run occurs. In the remainder of the cases a bank run has already occurred before  $t$ . The investor only has access to the storage technology.

On the other hand, for the case when the banks reappears after the period when a bank run occurs, the utility of the investor does not depend on the period. The utility in this case is

$$v_I^{bank} = (1 - \varepsilon)P(\tilde{a}) + \varepsilon b. \quad (13)$$

We observe that in the case of the short-lived player there is a utility cost whenever  $\underline{y}$  because in order to discipline the behavior of the entrepreneur/bank a bank run is necessary. However, this run is costly for the investor given that  $b < 1$ .

#### 4.4 Securitization vs. financial fragility

Now we can compare the utility for both long run and short run players of the banking and securitization arrangements. As is apparent from the previous sections, differing strategies yield different maximal payoffs for participants. To make the comparison which is most favorable to banking, we use the upper bound for the payoffs of players in a banking arrangement. This corresponds to the case in which the bank is allowed

to reopen immediately after having suffered a bank run. The main theoretical result of the paper is stated in the following proposition.

**Proposition 3** *Whenever  $r \leq \bar{r}_{sc}$  (one-period stick and carrot strategies are an equilibrium), securitization delivers maximal payoffs which are a Pareto-improvement over those enforced by the threat of a bank-run.*

**Proof.** In both arrangements the maximal payoff for the entrepreneur coincides

$$v_E = (1 - \varepsilon)[R - P(\tilde{a})] \quad (14)$$

For the investor, on the other hand,

$$v_I^{bank} = (1 - \varepsilon)P(\tilde{a}) + \varepsilon b < (1 - \varepsilon)P(\tilde{a}) + \varepsilon 1 = v_I^{sec} \quad (15)$$

since  $b < 1$ .

Q.E.D.

This result may seem paradoxical at first sight to someone not familiar with imperfect monitoring games. With perfect monitoring, worse punishments generally lead to higher equilibrium payoffs. Thus, given that the bank run is a more onerous punishment, payoffs should be higher. With imperfect monitoring, punishments are on the equilibrium path implying that there is a tradeoff between providing the adequate incentives and increasing the payoff to the long run player. In our case, the stick and carrot strategy is already providing maximal incentives. Therefore, going to the more onerous punishment does not help.

Notice that the result is not an artifact of the assumed timing. If, for example, the bank run is delayed one period, there could not be a higher payoff for the long-run player

since  $(1 - \varepsilon)[R - P(\tilde{a})]$  is already the highest payoff that can be achieved if there is a probability  $\varepsilon$  of observing the wrong signal.

## 4.5 Discussion

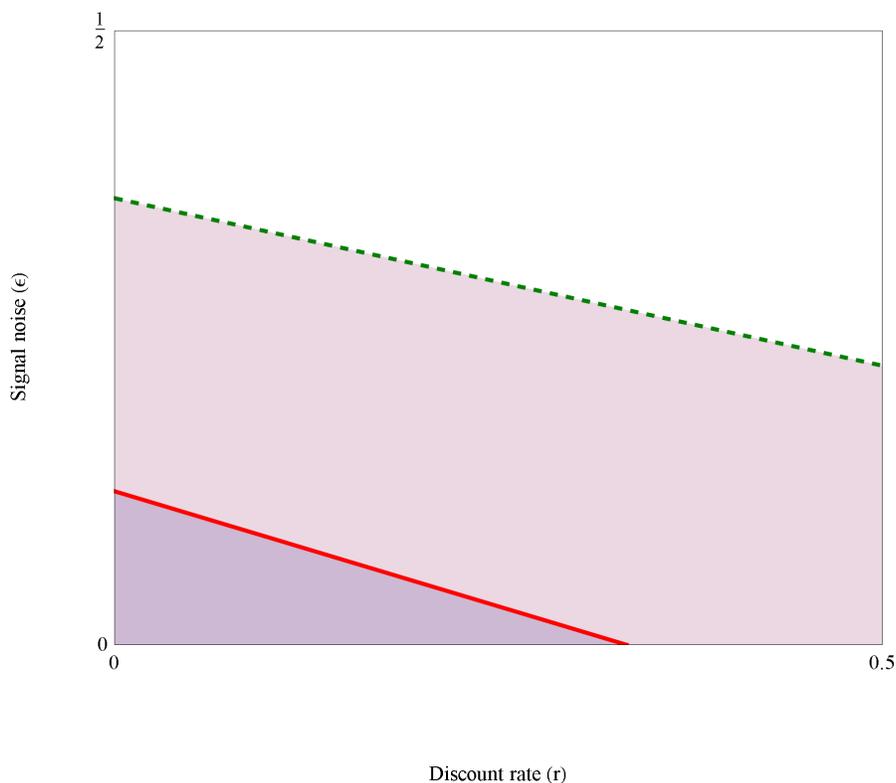
Securitization is a Pareto-improvement over a the provision of incentives through the threat of bank-runs. However, this result depends on the values of the discount rate  $r$  and the signal noise  $\varepsilon$ . For example, a large  $r$  implies that the value of deviating from keeping the promise of  $\tilde{a}$  is large relative to the present value of punishments. Therefore, it may not be possible to provide the necessary intertemporal incentives. Figure 2 exemplifies the alternative arrangements that are possible for each combination of both parameters.<sup>13</sup>

There are three distinct regions in the figure. The white area on the upper-right corner corresponds to the combinations of  $r$  and  $\varepsilon$  for which an equilibrium with securitization is not possible. In this region, the only incentives that work are those provided by the threat of a bank-run. The dashed line corresponds to the limiting combinations of  $r$  and  $\varepsilon$  for which it is possible to sustain cooperation through grim trigger strategies. Therefore, in the area below the dashed line (but above the solid line) both securitization and bank-run threats are feasible. Securitization with grim-trigger strategies is superior to bank-run threats if the bank is closed permanently after a bank run. If the bank is allowed to reopen, then the comparison for this area is not clear and depends on the welfare cost of the bank run, which is modeled by the parameter  $b$ .<sup>14</sup>

The area below the solid line corresponds to combinations of parameters for which securitization achieves the highest payoffs (a one-period stick and carrot strategy is an

<sup>13</sup>To construct the figure the rest of the parameters have been set to those implied by Diamond and Rajan (2001a). These values are  $R = 1.5$ ,  $P(a^*) = 0.8$  and  $P(\tilde{a}) = 1.1$ .

<sup>14</sup>This discussion is based on the equilibrium payoffs derived in Appendix C.



**Figure 2:** *Regions in which securitization is feasible.*

equilibrium). As shown in Proposition 3, the securitization agreement dominates bank-run threats in this case. The figure illustrates that securitization arrangements will dominate and therefore be preferred in an environment of low interest rates and when the quality of monitoring is better.

We have already discussed that the low interest rates in the period leading up to the crisis in 2007 provide an adequate environment in which keeping promises can be sustained by intertemporal incentives. The version of the model with imperfect information adds the quality of monitoring to the mix. One of the features early in the crisis was the increase in confusion and uncertainty in financial markets. The parameter  $\varepsilon$  measures how well the actions of a financial institution are mapped into observable signals. It may be that the turbulent environment at the start of the crisis decreased the accuracy of signals

that market participants were accustomed to. Thus, the drop in securitization volume in 2007 may be explained not only by a rise in the interest rate, but also by increased uncertainty in financial markets.

## 5 Conclusion

In this paper we consider an infinitely repeated game in an environment with moral hazard. We identify efficient equilibria of this game with securitization and show that the model can qualitatively account for the rise in securitization volume before the 2007 financial crisis and its drop afterwards.

The model we consider provides an alternative rationale for securitization that does not rely on risk sharing. Repeated access to markets generates intertemporal incentives which are sufficient to remove the costs of moral hazard that appear in a static environment.

We also show that under certain conditions securitization is superior to the use of financial fragility as a disciplining device. The use of financial fragility for disciplining starts with the work by Calomiris and Kahn (1991) and Diamond and Rajan (2001a,b). These articles have been influential in turning the ability to commit stemming financial fragility into a powerful idea in the banking literature.

A contribution of this paper is to show that the use of repeated games with imperfect public monitoring can change received wisdom in the financial intermediation literature. We show that the departure from the standard one-shot game environment can provide new insights and overturn previous results. In particular, the argument in favor of financial fragility of Calomiris and Kahn (1991) and Diamond and Rajan (2001a,b) is extremely dependent on a static environment.

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## 6 Appendix

### A. Relationship to Calomiris and Kahn (1991) and Diamond and Rajan (2001a)

As already argued, the environment of the stage game is quite general in that it fits different situations with moral hazard. For example, the environment of Calomiris and Kahn (1991) can be obtained by interpreting  $a$  as absconding. Letting  $A = \{0, 1\}$ ,  $P(a) = Pa$  with  $1 \leq P \leq R$  yields their environment. In this case,  $a^* = 0$  and  $P(a^*) = 0$ . The project is never financed.

Diamond and Rajan (2001a) consider a case in which the contract which promises repayment  $P(a)$  includes the possibility of liquidation. If the investor liquidates, then he obtains  $L < 1$  at either date 1 or date 2. Set  $A = \{0, 1\}$ ,  $R > 1$ . For any  $\rho > 0$ , specify  $P(a) = (1 + \rho)a + L(1 - a) = L + (1 - L + \rho)a$ . This is the function in Diamond and Rajan (2001a). Therefore,  $a^* = 0$  and  $P(a^*) = L < 1$ . Both the entrepreneur and the investor would be better off if the entrepreneur could commit to  $\tilde{a} = 1$  which would imply a repayment of  $P(\tilde{a}) = 1 + \rho$ .

### B. Proofs

**Proof of Proposition 1.** Consider the grim-trigger punishment consisting of shutting a deviator out of the market forever. For fixed  $r_s$  the relevant incentive compatibility constraint for the long run player is

$$R - P(\tilde{a}) \geq \frac{r_s}{1 + r_s} [R - P(a^*)] \quad (16)$$

The left hand side is the value of the constant stream  $\{R - P(\tilde{a})\}$  according to (5). The right hand side is the value of obtaining  $R - P(a^*)$  once and zero ever after. Solving this inequality for  $r_s$  yields (6). Since in this case Nash reversion is the worst possible punishment, the threshold  $\bar{r}$  is the highest interest rate for which  $\tilde{a}$  can be enforced. Q.E.D.

#### Proof of Lemma 1.

We use  $v_1$  for the continuation value in the case that  $\bar{y}$  is observed and  $v_0$  for the continuation value in the case that  $\underline{y}$  is observed. We can obtain  $v_0$  and  $v_1$  solving the following system of equations:

$$v_1 = \frac{r}{1 + r} [R - P(\tilde{a})] + \frac{1}{1 + r} [(1 - \varepsilon)v_1 + \varepsilon v_0] \quad (17)$$

$$v_0 = \frac{1}{1 + r} [(1 - \varepsilon)v_1 + \varepsilon v_0] \quad (18)$$

Solving the system we get:

$$v_0 = \frac{1 - \varepsilon}{1 + r} [R - P(\tilde{a})] \quad (19)$$

$$v_1 = \left(1 - \frac{\varepsilon}{1 + r}\right) [R - P(\tilde{a})] \quad (20)$$

The incentive constraint for the long-run player (entrepreneur/banker) is given by the following equation:

$$\frac{r}{1 + r} [R - P(\tilde{a})] + \frac{1}{1 + r} ((1 - \varepsilon)v_1 + \varepsilon v_0) \geq \frac{r}{1 + r} [R - P(a^*)] + \frac{1}{1 + r} (\varepsilon v_1 + (1 - \varepsilon)v_0) \quad (21)$$

The left hand side is the payoff of keeping the promise while the right hand side is the value of choosing  $a^*$  and enduring the punishment. Replacing  $v_0$  and  $v_1$  in the incentive constraint and solving for  $r$  yields

$$r \leq \bar{r}_{sc} \equiv \bar{r} - \varepsilon \frac{[R - P(\tilde{a})] + [R - P(a^*)]}{P(\tilde{a}) - P(a^*)} - (1 + \varepsilon) \quad (22)$$

Notice that there is an upper bound on  $\varepsilon$  such that this equilibrium exists. By checking  $\bar{r}_{sc} > 0$  we can solve for the range of  $\varepsilon$  for which the one-period stick and carrot is an equilibrium.

$$\varepsilon < \bar{\varepsilon}_{sc} \equiv \frac{1}{2} \left(1 - \frac{P(\tilde{a}) - P(a^*)}{R - P(\tilde{a})}\right) \quad (23)$$

Then the expected payoff for the long-run player is given by

$$\begin{aligned} v_E^{secsc} &= (1 - \varepsilon)v_1 + \varepsilon v_0 \\ &= (1 - \varepsilon)[R - P(\tilde{a})] \end{aligned} \quad (24)$$

The payoff of the sequence of short-run players does not depend on the period  $t$ . In equilibrium, the entrepreneur always chooses  $\tilde{a}$ , so that short run players only play  $i = 0$  in the case that they have observed  $\underline{y}$  by mistake, which happens with probability  $\varepsilon$ .

$$v_I^{sec} = (1 - \varepsilon)P(\tilde{a}) + \varepsilon 1 \quad (25)$$

For any  $\varepsilon < \bar{\varepsilon}_{sc}$ , if  $r \leq \bar{r}_{sc}$ , there exists a Perfect Public Equilibrium generating the expected payoffs given in (24) and (25).

We have solved only for the equilibrium which after a perceived deviation punishes for one period and returns to the efficient outcome afterwards. In the case in which the punishment stage is longer than one period, the difference between  $v_1$  and  $v_0$  will widen, thus reducing the expected value of the intertemporal payoff, but at the same time increasing the range of discount rates for which the cooperative equilibrium exists. In any case, stick and carrot payoffs for arbitrary punishment lengths will be in the interval defined by the grim trigger and one-period stick and carrot payoffs.

Given that at least one period of punishment is needed for providing incentives, the one-period stick and carrot equilibrium is also the best possible equilibrium of the repeated game. Q.E.D.

### C. Securitization with imperfect monitoring and grim trigger strategies

Grim trigger strategies correspond to playing  $(1, \tilde{a})$  as long as  $\bar{y}$  is observed and  $(0, a^*)$  forever starting when  $\underline{y}$  is observed for the first time.

We use  $v_1$  for the continuation value in the case that  $\bar{y}$  is observed and  $v_0$  for the continuation value in the case that  $\underline{y}$  is observed. We can obtain  $v_0$  and  $v_1$  solving the following system of equations:

$$v_1 = \frac{r}{1+r}[R - P(\tilde{a})] + \frac{1}{1+r}[(1-\varepsilon)v_1 + \varepsilon v_0] \quad (26)$$

$$v_0 = \frac{r}{1+r}0 + \frac{1}{1+r}v_0 \quad (27)$$

Solving the system we get:

$$v_0 = 0 \quad (28)$$

$$v_1 = \frac{r}{r+\varepsilon}[R - P(\tilde{a})] \quad (29)$$

The relevant incentive constraint for the long-run player can be expressed in the following way

$$v_1 \geq \frac{r}{1+r}[R - P(a^*)] + (1-\varepsilon)v_0 + \varepsilon v_1 \quad (30)$$

The left hand side is the payoff of cooperating while the right hand side is the value of choosing  $a^*$  and enduring the punishment. Replacing  $v_0$  and  $v_1$  in the incentive constraint and solving for  $r$  we obtain

$$r \leq \bar{r}_{gt} \equiv \bar{r} - \varepsilon \frac{[R - P(\tilde{a})] + [R - P(a^*)]}{P(\tilde{a}) - P(a^*)} \quad (31)$$

The maximal payoff for the long run player (entrepreneur) when a grim trigger strategy is played is given by

$$\begin{aligned} v_E^{sec_{gt}} &= (1-\varepsilon)v_1 + \varepsilon v_0 \\ &= (1-\varepsilon)\frac{r}{r+\varepsilon}[R - P(\tilde{a})] \end{aligned} \quad (32)$$

Notice that in this case, the expected payoff for the short-lived investors depends on the period in which they are playing:

$$v_I^{sec_{gt}} = (1-\varepsilon)^t P(\tilde{a}) + [1 - (1-\varepsilon)^t] \quad (33)$$

This equation is analogous to equation (12). Notice that the exponent is  $t$  (instead of  $t+1$ ) in this equation because punishment is delayed one period while in the case of a bank it is immediate.