Below-Cost Pricing in Multiproduct Competition*

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Abstract

Firms rarely supply only single product. Multiproduct firms are able to cross-subsidize among product lines which could lead to below-cost pricing on part of products. This paper develops a new model to analyze below-cost pricing by multiproduct firms. We show that below-cost pricing arises in equilibrium in the absence of any efficiency justifications stemming from asymmetric information on prices, and it serves as a discriminatory mechanism for screening consumers with heterogeneous shopping costs. In all equilibria firms earn positive profits from multi-stop shoppers who incur lower shopping costs, in spite that pure price competition dissipates its profit from one-stop shoppers. We show also that banning below-cost pricing could improve social welfare but may lead to higher prices for consumers.

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1 Introduction

There is an overwhelmingly dominant literature of price competition based on single product firms, however firms rarely supply only single product. Price competition with multiproduct firms does not share the same properties as that with single product firms, it therefore deserves innovative studies to understand the distinguishing features of multiproduct price competition.

Multiproduct firms often involve non-linear pricing and bundling, and they are also able to cross-subsidize among their product lines which could lead to below-cost pricing on part of product lines, so their pricing behaviors distinguish from single product firms. The rationale for competitive nonlinear pricing and bundling and their impact on social welfare are widely discussed in the economic literature, by contrast only a relatively small body of literature devoted to the study of cross subsidizing and below-cost pricing in multiproduct competition.

The practices of below-cost pricing are allegeable to antitrust enforcement. For instance, in the US, 22 states are equipped with general sales-below-cost laws, and 16 additional states prohibit below-cost sales on motor fuel. In the EU, below-cost resale is banned in Belgium, France, Ireland, Luxembourg, Portugal, and Spain, and is restricted in other countries including Austria, Denmark, Germany, Greece, Italy, Sweden and Switzerland. Whereas, practitioners face a dilemma in assessing such practices conducted by multiproduct firms, and in the absence of any regulations they tend to tackling them with predatory pricing approaches. But, below-cost pricing could form an optimal cross-subsidizing strategy for multiproduct firms and could thus be a persistent pricing strategy, in which case courts and competition authorities are unlikely to show the feasibility that the "predator" could recoup the losses incurred during the predation phase by raising the prices after driving the rival out of the market. For instance, in its 1997

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1 Such below-cost pricing strategy is well-known as loss leading in retail markets. For instance, in its recent report of the investigation on grocery markets, the UK Competition Commission (2008) notes that most large retailers in the UK engage in loss leading, and it finds that the sales of loss leader products represent up to 6% of a retailer’s total sales.

2 For instance, Matutes and Regibeau (1992) investigate bundling with product-specific preferences and with complementarity of products. Thanassoulis (2007) studies the competitive mixed bundling and its impact on consumer surplus, while Armstrong and Vickers (2010) examine competitive non-linear pricing, in a model in which consumers have heterogeneous and elastic demands and can buy from more than one suppliers, and compare the welfare with linear pricing.

3 See e.g., Bolton, Brodley and Riordan (2000) and Eckert and West (2003) for detailed discussions of how predatory-pricing tests should be designed.

4 The feasibility of recoupment is often a necessary condition for a case of predation; in the U.S., for example,
report, the UK Office of Fair Trading argued that, in the analysis of alleged predation in retailing cases, a price-cost comparison is of little use, since pricing below cost on individual items may be profitable without being predatory.\(^5\)

If below-cost pricing by multiproduct firms is not predatory, then what is the rationale for using such practices? This is the primary question for the theoretical research on price competition with multiproduct firms, and understanding firms’ incentives is the key to the evaluation of welfare effect for below-cost pricing.

It is a widely accepted view in the economic and marketing literature that below-cost pricing for multiproduct firms serves as an advertising strategy adopted to attract consumers facing imperfect information of prices;\(^6\) below-cost pricing may then compensate consumers for their imperfect information and thereby improve consumer surplus.\(^7\) While this viewpoint may provide a reasonable explanation for firms’ pricing behavior when facing consumers with imperfect price information, the theoretical foundation supporting this point of view is far from general and robust.\(^8\) Moreover, there are also many market environments where consumers (or customers) are (almost) perfectly informed on prices due to regular and repeated purchases, which are most likely to occur in the grocery retailing and other markets involving regular purchases.

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\(^5\)It is also argued by the UK Competition Commission in its report (2008): "We find that the pattern of below-cost selling that we observed by large grocery retailers does not represent behavior that was predatory in relation to other grocery retailers." (See Competition Commission (2008) at p. 98).

\(^6\)Lal and Matutes (1994), for example, consider a situation where multi-product firms compete for consumers who are initially unaware of prices, and find that in equilibrium firms may indeed choose to advertise a few loss leaders in order to increase store traffic. Ellison (2005) develops the model to analyze add-on pricing, and shows that loss leading can be optimal when firms advertise base goods while add-on prices are unobserved.

\(^7\)Walsh and Whelan (1999) show that, in the presence of imperfect information, loss leading can generate the same long-run equilibrium outcomes as those observed under a laissez-faire full information scenario.

\(^8\)Lah and Matutes (1994) established their results in a rather stylized setting and show that in equilibrium both stores advertise the same products and consumers only visit one store; this appears to be inconsistent with the observations that retail stores often choose different advertising goods and also some consumers are engaged in visiting multi-stores for the mix-and-match of their baskets.
ing,9 and the rationale for below-cost pricing in these markets are not well understood.10 In particular, resolving this open question allows us to assess the welfare impact for such practices in the absence of any efficiency justifications stemming from imperfect information.

This paper aims at filling this gap. We develop a new model of price competition between two multiproduct firms where both firms offer the same product lines. We abstract away from the above-mentioned efficiency justifications by assuming that consumers are perfectly informed of all prices. Our key modelling feature is to account for the heterogeneity in consumers’ shopping costs incurred for using a supplier:11 some consumers face higher shopping costs, e.g., because of tighter time constraints or lower taste for shopping, and thus have a strong preference for one-stop shopping, whereas others have lower shopping costs and can therefore benefit from multi-stop shopping.

We first present the main insights in the baseline setting where two firms supply the same (two) product lines with perfect substitution. To capture the fact that firms are often specialized in some products and may thus enjoy a comparative advantage in these markets against the rival (but it is unlikely that one firm owns comparative advantages in all product markets),12 we consider the case where each firm possesses a comparative advantage in one product stemming from its lower unit cost than the rival, and thus the comparative disadvantage in another product. For the simplicity of analysis, we assume that a firm’s comparative advantage in one product exactly offsets its comparative disadvantage in another market so that both firms indeed offer the same social value for their assortments.13 As the products of two firms are perfectly substitute, tough price competition for one-stop shoppers leads to a standard Bertrand competition outcome for the assortment: each firm charges the total price for the assortment at cost and earns no profit from one-stop shoppers. However, since each firm incurs a lower unit cost in one product

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9 See, for instance, Dobson (2002).

10 Ambrus and Weinstein (2008) study Bertrand competition among symmetric firms competing for one-stop shoppers. They first show that below-cost pricing cannot arise when consumers have inelastic demand. When demand is elastic, pricing below cost can occur but only under rather specific forms of demand complementarity; in particular, below-cost pricing cannot arise when consumer demand is sufficiently diverse. The scope for below-cost pricing in these settings, as well as its impact on consumers and welfare, still needs to be assessed.

11 Following Klemperer (1992), we will refer to consumers’ real or perceived costs of using a supplier as shopping costs. This modelling feature is widely adopted in the literature of multiproduct competition, including Klemperer (1997) and Armstrong and Vickers (2010).

12 The comparative advantage for some product may stem from a lower marginal (unit) cost or higher quality.

13 No insight will change substantially when this symmetry assumption is dropped.
than its rival, it is able to charge a price of the advantaged product lower than the rival’s unit cost but still above its own cost, at the same time it could reduce its price on the disadvantaged product to below cost level so as to keep the total price for the assortment at the total cost. This cross-subsidizing strategy, consisting of pricing below cost for the disadvantaged product and pricing above cost the advantaged product, could attract consumers who incur lower shopping costs to source two suppliers and pick each advantaged product at a lower price, and each firm then makes a positive profit from these multi-stop shoppers who only purchase the advantaged product.

Therefore, below-cost pricing serves here as a discriminatory mechanism for screening multi-stop shoppers, who incur lower shopping costs, from one-stop shoppers with higher shopping costs, which allows firms to make positive profits from multi-stop shoppers, in contrast to zero profit from one-stop shoppers due to pure price competition. We show in the baseline setting that such below-cost pricing arises as a unique (symmetric) Nash equilibrium when consumers are sufficiently heterogeneous in their shopping costs.14 When instead shopping costs are not sufficiently heterogeneous among consumers (so that firms cannot screen consumers according to their shopping costs successfully), all consumers prefer multi-stop shopping; there are multiple equilibria and in all equilibria firms involve in below-cost pricing.

We then evaluate the welfare effect of below-cross pricing in the baseline setting. To this end, we compare the social welfare under two regimes regarding on whether below-cost pricing is banned or not.15 We find that, in all equilibria involving below-cost pricing, banning such practice would force each firm to charge a price of the disadvantaged product at cost which may lead to a reduction of the price for the advantaged product. As a result, more consumers will prefer multi-stop shopping to pick both advantaged products. This indeed improves the efficiency of distribution and thus increases the total social welfare.

To examine the robustness of the analysis, we extend the basic setting to incorporate the horizontal differentiation between firms where consumers have heterogeneous firm-specific preferences. We show that when horizontal differentiation is relatively weak (so that competition is tough), then cross-subsidizing still leads to below-cost pricing for the disadvantaged product in the symmetric equilibria, and moreover the equilibrium prices converge to these under the equilibrium prices converge to these under the

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14 Since cross-subsidizing allows firms to make additional profit by discriminating consumers, they have no incentive to adopt pure and mixed bundling instead.

15 Unfortunately, there is no pure-strategy Nash equilibrium when below-cost pricing is banned, in which case we resort to the mixed strategy equilibrium.
baseline setting.

This paper is a complementary piece to Chen and Rey (2010) which sheds a first light on the exploitative use of loss leading in retail competition, in a model of asymmetric competition between large retailers and small retailers where large retailers offer broader product lines than smaller rivals. By contrast, this paper analyzes symmetric competition between two multiproduct firms (offering same product lines) and finds that adopting below-cost pricing allows firms to discriminate consumers and extract more rents from multi-stop shoppers, with no predatory motivation. This finding provides a new rationale for the analysis of below-cost pricing in the absence of any efficiency justification for imperfect information and enriches the literature of price competition with multiproduct firms.16

The rest of the paper is organized as follows. Section 2 presents the baseline model of price competition with two multiproduct firms, and Section 3 shows that below-cost pricing arises in a unique Nash equilibrium. We then conduct a welfare analysis regarding on a ban of below-cost pricing in section 4 and also check the robustness of the result by extending the baseline model in section 5. Finally we conclude in section 6.

2 The model

Two firms, 1 and 2, compete in a local market and both offer the same product lines with perfect substitution. For the simplicity of exposition, we consider only two products A and B.17 Supplying product A incurs a unit cost c for firm 2 whereas a lower unit cost \( c - \delta \geq 0 \) (with \( \delta > 0 \)) for firm 1, so firm 1 indeed possesses a comparative advantage in product A; by contrast, firm 2 enjoys a comparative advantage in product B, which comes from its lower unit cost \( c - \delta \) versus firm 1’s unit cost c.18 The assumption of relative comparative advantage in different products reflects the fact that firms usually specialize in different product markets from which they involve

16 Lah and Matutes (1989) analyzed the pricing strategy of multiproduct duopolies in a modified Hotelling model. They find that, under some conditions, firms jointly price discriminate between the two types of consumers (rich and poor) and may even achieve the same level of profits as if they maximized joint profits; while for another set of conditions, the unique equilibrium of the pricing strategy resembles that in a standard Hotelling model with one good. In both cases, there is no cross-subsidizing and thus below-cost pricing never arises in equilibria.

17 While both firms offer two products with perfect substitution, we will denote by \( A_i \) (respectively \( B_i \)) the product A (resp B) offered by firm \( i, i = 1, 2 \), for the ease of exposition.

18 Assuming the same unit costs \( c_A = c_B = c \) simplifies the exposition, while the analysis applies to more general case with different unit cost for two products: \( c_A \neq c_B \).
lower supplying costs than rivals, but it is unlikely that the same firm could specialize in all relevant product lines. For instance, a large retailer may have long-run relationship with meat suppliers from which it obtains a better discount than its competitors, by contrast other retailers may obtain a better discount from the sea food suppliers. The assumption that two firms enjoy the same degree of comparative advantage but in different products is mainly for the simplicity of exposition, in which case two firms indeed offer the same social value for the assortment $AB$ since products are perfectly substitute.

Consumers are assumed to be continuously distributed with the total population normalized to 1. Each consumer desires at most one unit of $A$ and $B$, and consuming product $A$ and $B$ delivers a utility $u_A$ and $u_B$ respectively while consuming both $A$ and $B$ yields $u_{AB}$. We assume these values are homogeneous across consumers.

Consumers incur a shopping cost for sourcing a supplier. This shopping cost may reflect the opportunity cost of the time involved in visiting one store, or it might measure a consumer’s perceived cost of dealing with a firm. To highlight the fact that consumers may be more or less time-constrained, or value their shopping experience in different ways, we assume that the shopping cost, denoted by $s$, varies across consumers and is distributed in $[0, S]$ according to a cumulative distribution function $F(\cdot)$, with density function $f(\cdot)$, and the inverse hazard rate, $h(\cdot) \equiv F(\cdot)/f(\cdot)$, is strictly increasing. We assume that the upper bound of shopping costs does not exceed the social value, that is, $S \leq w_{AB} \equiv u_{AB} - 2c + \delta$, so that it is socially desirable to serve all consumers. Finally, we focus on the cases where it is socially desirable for each firm to offer products $A$ and $B$, as well as the assortment $AB$, that is, $u_A, u_B > c$ and $u_{AB} - u_A > c$, $u_{AB} - u_B > c$.

We model price competition as follows: (i) Firm $i, i = 1, 2$, simultaneously set their prices, respectively $(p_i, q_i)$, where $p_i$ stands for the price of firm $i$’s "good" product with comparative advantage while $q_i$ denotes the price for firm $i$’s "bad" product with comparative disadvantage, so $p_1 (p_2)$ denotes the price for product $A_1 (B_2)$ and $q_1 (q_2)$ denotes the price for $B_1 (A_2)$ respectively. (ii) Consumers then observe all prices and make their shopping decisions. When making these

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19 We allow for partial substitution and complementarity of products $A$ and $B$, that is, $u_{AB}$ could be either lower or higher than $u_A + u_B$.

20 We will check the robustness of the analysis in the extension where consumers have heterogeneous firm-specific preferences.

21 This assumption ensures that profit functions are single-peaked.

22 We first consider stand-alone prices, and show later that allowing for bundled discounts cannot increase firms’ profits; see the remark in section 3.
decisions, consumers are fully aware of all prices and take into account the value of the proposed assortments as well as their shopping costs.

3 Equilibrium below-cost pricing

We focus on the setting where each firm supplies both products and offers a positive value for consumers: \( v_i \equiv u_{AB} - p_i - q_i > 0 \, ^{23} \) A consumer is willing to buy both products \( A_i \) and \( B_i \) from firm \( i \) if the value for the assortment offsets the shopping cost: \( v_i \geq s \). Moreover, since both products are perfectly substitute, a consumer will patronize firm \( i \) if it offers a better value than its rival, firm \( j \), that is

\[
v_i = u_{AB} - p_i - q_i > v_j = u_{AB} - p_j - q_j,\]

or equivalently the former charges less overall prices than the latter

\[
p_i + q_i < p_j + q_j.
\]

It appears that the pure price competition for one-stop shoppers leads to the same equilibrium prices for the assortment that cover the total cost exactly

\[
p_i + q_i = p_j + q_j = 2c - \delta,
\]

and as a result firms make no profit from selling the assortment to one-stop shoppers.

One candidate pricing strategy is the simple replication of the standard Bertrand competition result in each separate product market, that is, the price of each product is equal to its unit cost: \( p_i = c - \delta \) and \( q_i = c \), for \( i = 1, 2 \). However, this standard Bertrand pricing cannot form an equilibrium in this multiproduct setting, due to the heterogeneity of consumer’s shopping cost. Suppose both firms charge \( p_i = c - \delta \) and \( q_i = c \), then the price for product \( A_1 \) is lower than that of \( A_2 \): \( p_1 = c - \delta < q_2 = c \) (recall that firm 1 possesses a comparative advantage in product \( A \)), and by contrast the price for \( B_2 \) is strictly lower than that of \( B_1 \): \( p_2 = c - \delta < q_1 = c \).

For consumers who would buy the assortment \( A_2B_2 \) from firm 2 could now benefit from buying instead \( A_1 \) from firm 1 while still keep buying \( B_2 \) from firm 2. Such multi-stop shopping brings an extra value \( \delta \), at an additional shopping cost \( s \), and consumers with lower shopping cost such that \( s \leq \delta \) are indeed willing to do so. Firm 1 can now benefit from the following cross-subsidizing strategy: raising its price on \( A_1 \) slightly by \( \varepsilon \) (so \( p_1 = c - \delta + \varepsilon \) ) while reducing

\[ ^{23} \text{We check in the appendix that other cases cannot arise in equilibria.} \]
its price on $B_1$ (i.e., $q_1 = c - \varepsilon$) by the same amount so as to keep the overall price $p_1 + q_1$ unchanged. Doing so brings no impact on one-stop shoppers as the total price is not changed but yields a positive profit from selling additional $A_1$ to multi-stop shoppers.

Therefore, in equilibria, both firms must make positive profits from multi-stop shoppers. Notice that multi-stop shoppers will buy $A_1$ from firm 1 and $B_2$ from firm 2, which yields a gross value

$$v_{12} \equiv u_{AB} - p_1 - p_2,$$

at an extra shopping cost $s$, so they prefer multi-stop shopping to patronizing only firm 1 if

$$s \leq \tau_1 \equiv v_{12} - v_1 = q_1 - p_2,$$

where $\tau_1$ denotes the threshold of shopping cost for which consumers are indifferent between multi-stop shopping and one-stop shopping in firm 1; it indeed reflects the price gap between the disadvantaged product $B_1$ and the advantaged product $B_2$. Similarly, consumers prefer multi-stop shopping than patronizing only firm 2 if

$$s \leq \tau_2 \equiv v_{12} - v_2 = q_2 - p_1,$$

where $\tau_2$ is the price gap between product $A_1$ and $A_2$. Therefore, the demand of multi-stop shoppers is $F(\tau)$, where $\tau = \min\{\tau_1, \tau_2\}$.

Firm $i$ then faces additional demand for its advantaged product from multi-stop shoppers, which yields a profit

$$\Pi_i = (p_i - c + \delta) F(\tau).$$

We now solve for the equilibrium prices. By (1), we must have $\tau_1 = \tau_2 = 2c - \delta - p_1 - p_2$. Then the profit maximization program pins down to:

$$\max_{p_i} (p_i - c + \delta) F(2c - \delta - p_i - p_j),$$

and the best responses can be characterized by the first-order conditions

$$p_i - c + \delta = h(2c - \delta - p_i - p_j),$$

for $i = 1, 2$.

The equilibrium prices for advantaged products are then derived by solving these two equations$^{25}$, as given by

$$p_1^* = p_2^* = p^* = c - \delta + h(\tau^*) = c - \delta + h(2c - \delta - 2p^*).$$

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$^{24}$We show in the appendix that mix-and-match for $A_2$ and $B_1$ will never arise in equilibrium.

$^{25}$The profit functions are strictly quasi-concave since $h(\cdot)$ is strictly increasing by assumption, so the best responses $p_i - c + \delta = h(2c - \delta - p_i - p_j)$, $i = 1, 2$, are strictly decreasing in $p_i$, thus the equilibrium prices derived from the intersection of these two best responses must be unique.
The equilibrium threshold $\tau^*$ satisfies

$$\tau^*_1 = \tau^*_2 = \tau^* = 2c - \delta - 2p^* = \delta - 2h(\tau^*)$$

and is determined by the following equation

$$\tau^* + 2h(\tau^*) = \delta,$$

which amounts to

$$\tau^* = j^{-1}(\delta),$$

where the function $j(x) = x + 2h(x)$ is strictly increasing in $x$.

The equilibrium prices for the disadvantaged products can be further solved by using (1), which yields

$$q^*_1 = q^*_2 = q^* = 2c - \delta - p^* = c - h(\tau^*);$$

it appears that each firm prices its disadvantaged product below cost: $q^* < c$.

Therefore, below-cost pricing arises in equilibrium as an optimal cross-subsidizing strategy where each firm charges a price below cost for its disadvantaged product meanwhile charges a price above cost for its advantaged product. The total price for the assortment is still at the level of total cost due to tough price competition for one-stop shoppers, but each firm earns a strictly positive profit from multi-stop shoppers:

$$\pi^* = (p^* - c + \delta) F(\tau^*) = h(\tau^*) F(\tau^*) > 0.$$ 

Since $h(\tau^*) < \delta$ and $F(\tau^*) \leq 1$, it follows that the equilibrium profit level is strictly lower than the efficiency gain: $\pi^* < \delta$; this is because competition for one-stop shoppers dissipates its profit for consumers with $s > \tau^*$.

Notice that, the above equilibrium arises only when $\tau^* \leq S$, that is, consumers are sufficiently heterogeneous in their shopping cost. Summarizing the above analysis leads to

**Proposition 1** When consumers’ shopping costs are sufficiently heterogeneous (i.e., $S \geq \tau^*$), there exists a unique Nash equilibrium where multiproduct firms compete for one-stop shoppers as well as multi-stop shoppers, and each firm involves pricing below-cost for its disadvantaged product while keeping the price for the assortment at the total cost; in this equilibrium, each firm earns a positive profit from multi-stop shoppers.
Proof: See Appendix A.

Single product vs. Multiple products

One may argue that, since each firm incurs a loss by selling its product with the comparative disadvantage, it would be better to drop that product and specialize only in the advantaged product. However, unless both firms can coordinate and commit not to expand its product line, each firm has a strong incentive to increase its product line unilaterally.

To see this, let’s start with a benchmark where each firm specializes in its advantaged product only, that is, firm 1 provides $A_1$ and firm 2 provides $B_2$ only. In this case, each firm is the monopolist in its market and charges the a monopoly price $p^m$. When the cost advantage is not too large such that the monopoly price is above the rival’s cost, that is, $p^m > c$, then a firm can always benefit from extending its product line unilaterally. Suppose firm 1 introduces product $B_1$ unilaterally, which brings an additional social value equal to $u_{AB} - u_A - c > 0$. Firm 1 can still make the monopoly profit from selling $A_1$, in addition it can make a positive profit by undercutting the rival in market $B$, with an additional profit margin up to $p^m - c$. Therefore, specializing in one market cannot form a Nash equilibrium. On the other hand, suppose firm 1 deviates unilaterally from the equilibrium by dropping the product line $B_1$ (say, charging a price $q_1 = +\infty$), while keeping the price of $A_1$ unchanged. Doing so will not affect its profit from multi-stop shoppers, as both the price $p^*$ and the threshold $\tau = \tau_2 = 2c - \delta - 2p^*$ are not affected, however firm 1 would lose all one-stop shoppers. Thus, unilateral dropping of the product line is not profitable.

Bundling

Firms may be engaged in pure bundling, that is, tying both products together physically such that consumers cannot unbundle it. While pure bundling as a practice of market foreclosure is widely discussed in the literature, in this setting, adopting pure bundling cannot improve firm’s profit. Notice that, if one firm commits to the pure bundling strategy, the other firm has no choices but to tie also its products, and both firms will adopt pure bundling in equilibrium. In this case there is no multi-stop shoppers, and the pure Bertrand competition for one-stop shoppers results to zero profit for each firm, so this pure bundling equilibrium is strictly dominated.

Firms may also adopt mixed bundling, in which case each firm provides a bundle of two products as well as two separate products, and accordingly offers three prices: one for advantaged product, one for disadvantaged product, and one for the bundle. When the price for the bundle is even lower than the price for the advantaged product (that is, firms charge a negative price
for the disadvantaged products), consumers would have incentives to be engaged in arbitrage: buy the bundles from both firms and then drop the disadvantaged products while mix-and-match the remaining advantaged products. This problem does not arise in our setting, since in equilibrium the price for the disadvantaged product is still positive: \( \tau^* > 0 \) implies \( h(\tau^*) < \delta \), so \( q^* = c - h(\tau^*) > c - \delta \geq 0 \), and the price for the bundle is strictly higher than that for the advantaged product only.

Moreover, since one-stop shoppers only purchase the assortment while multi-stop shoppers only buy the advantaged product, no consumers will pick the stand-alone disadvantaged product. So only two prices matter here: the total price \( y_i \equiv p_i + q_i \) for the bundle and the price \( p_i \) for the advantaged product. Alternatively, these prices can be implemented through stand-alone prices, \( p_i \) for advantaged product and \( q_i = y_i - p_i \) for the disadvantaged product. Therefore, offering an additional bundled discount based on two stand-alone prices could not improve each firm’s profit.

Remark: In the basic setting, each firm’s comparative advantage in one product stems from its lower unit cost than its rival. Our analysis also goes through when the comparative advantage comes from the higher quality, i.e., \( u_{A_1} > u_{A_2} \) and \( u_{B_2} > u_{B_1} \), such that the advantaged product generates a better social value than the disadvantaged one: \( u_{A_1} - c_{A_1} > u_{A_2} - c_{A_2} \), with \((u_{A_1} - c_{A_1}) - (u_{A_2} - c_{A_2}) = \delta \), where \( c_{A_1} \) (\( c_{A_2} \)) stands for the cost of product \( A_1 \) (\( A_2 \)), and similarly \( u_{B_2} - c_{B_2} > u_{B_1} - c_{B_1} \). But we need to assume that the unit cost for the disadvantaged product is higher enough to avoid a negative price in equilibrium.

Equilibria with only multi-stop shoppers

When consumers’ shopping costs are not sufficiently heterogeneous such that \( S < \tau^* \), the above-mentioned equilibrium cannot be supported. In this case, each firm will be better off to serve multi-stop shoppers only, so in any candidate equilibria we must have \( \tau_i = \tau_j = S \). There are multi-equilibria, with prices characterized by the following constraints:

\[
\begin{align*}
p_i & \geq c - \delta + h(S) \quad \text{and} \quad q_i \leq c - h(S), \\
q_i - p_j & = S \quad \text{and} \quad p_i + q_i = p_j + q_j.
\end{align*}
\]

In all equilibria, firms still charge the prices below-cost for the disadvantaged product, although consumers will never buy that product. Below-cost pricing for the disadvantaged product thus places a competitive pressure to the rival’s advantage product. Each firm earns a profit \( \pi_i = p_i - c + \delta \), and the maximum profit can be achieved at the equilibrium prices \( q_i = c - h(S) \) and \( p_i = c - S - h(S) \), which yields \( \hat{\pi}(S) \equiv \delta - S - h(S) \). So firm’s profit increases when \( S \) decreases.
In particular, if consumers incur no shopping costs (i.e., $S = 0$), then each firm could realize its efficiency gain due to cost advantage: $\hat{\pi}(0) = \delta$.

We then conclude:

**Proposition 2** When consumers’ shopping costs are not sufficiently heterogeneous (i.e., $S < \tau^*$), there are multi-equilibria where firms serve only multi-stop shoppers and charge a price below-cost for the disadvantaged product.

*Proof:* See Appendix B.

### 4 Banning below-cost pricing: welfare analysis

Below-cost pricing is allegeable to antitrust enforcement. The above analysis shows that below-cost pricing arises in equilibrium as a cross-subsidizing strategy, without any efficiency justification stemming from incomplete information. We now compare the social welfare under two regimes regarding on below-cost pricing is banned or not.

To investigate the welfare effect of such below-cost pricing, we need to analyze the equilibrium if below-cost pricing were banned.

When consumers’ shopping costs are sufficiently heterogeneous, namely, $S > \tau^* = j^{-1}(\delta)$, in which case there exists a unique Nash equilibrium were below-cost pricing allowed. Consider now a ban on below-cost pricing which forces firms to charge $q_i = c$. Since the pure price competition for one-stop shoppers leads to pricing at cost for the assortment: $p_i + q_i = 2c - \delta$, we must have $p_i \leq c - \delta$ in any pure-strategy equilibria. However, such pricing strategy cannot form a Nash equilibrium: The demand of multi-stop shoppers changes continuously with respect to the prices, increasing $p_i$ above cost yields positive profit.

Indeed, there exists no pure-strategy equilibrium when below-cost pricing is banned in this case. Suppose there exist such a (pure-strategy) equilibrium, then price competition for one-stop shoppers must lead to $p + q = 2c - \delta$. Since it is prohibited to charge $q < c$, so any equilibrium must involve $p = c - \delta$, a contradiction.

However, there do exist a mixed-strategy equilibrium where the firm’s minimax profit can be achieved when the rival prices both goods at cost. For the simplicity of exposition, from now on we will use the margins instead of the prices as the strategy variables. Let $r_i \equiv p_i + q_i - 2c + \delta$ and $\rho_i \equiv p_i - c + \delta$ denote the total margin for the assortment and the margin for the advantaged

26 Due to quasi-concavity of the profit function, charging any prices $q_i > c$ is never optimal.
product for firm $i$ respectively; suppose the rival prices both goods at cost, that is, $r_j = \rho_j = 0$, and firm $i$ charges prices such that $r_i = \rho_i = r > 0$ (that is, firm $i$ sells the "bad" product at cost). In this case, firm $j$ attracts all one-stop shoppers while firm $i$ can make a profit from selling the advantaged product to multi-stop shoppers, who prefer multi-stop shopping rather than patronizing firm $j$ if $s \leq \tau_j = q_j - p_i = \delta - r$. The minimax profit can thus be written as

$$\hat{\pi} = \max_r r F(\delta - r) = \hat{r} F(\delta - \hat{r}),$$

where $\hat{r}$ is uniquely characterized by:

$$\hat{r} = h(\delta - \hat{r}),$$

and the threshold for multi-stop shopping is given by

$$\hat{\tau} = l^{-1}(\delta),$$

where $l(x) \equiv x + h(x)$ is strictly increasing in $x$.

Note that this minimax profit cannot be supported by any pure strategy Nash equilibrium, as the rival would have an incentive to raise its price for its advantaged product. Consider now a candidate mixed-strategy equilibrium such that $\rho_i = r_i$, where $r_i$ is distributed according to the same distribution $K(\hat{r})$ (that is, the price distributions of the two firms are symmetric) over some interval $\hat{r} \in [\underline{r}, \overline{r}]$, with a continuous density $k(\hat{r})$ over $[\underline{r}, \overline{r}]$. Without loss of generality we can assume $\underline{r} \leq \delta (< w_{AB} = u_{AB} - 2c + \delta)$. Suppose that firm $i$ adopts $\rho_i = r_i = r$, and consider consumers’ response, as a function of the realization $\rho_j = r_j = \hat{r}$ of the rival’s price:

- consumers buy both goods from firm $i$ if:

$$v_i = u_{AB} - p_i - c = w_{AB} - r \geq 0, \quad v_i \geq v_j = w_{AB} - \hat{r},$$

and

$$v_i \geq v_{ij} = w_{AB} + \delta - r - \hat{r} - s,$$

which amounts to (since $r \leq \delta < w_{AB}$):

$$\hat{r} \geq r \text{ and } s \geq \delta - \hat{r}.$$ 

- consumers engage in multi-stop shopping if:

$$v_{ij} \geq v_i, \quad v_j \geq 0,$$

and

$$v_{ij} \geq v_j = w_{AB} - \hat{r} - \hat{\tau},$$

where $\hat{\tau} \equiv \tau_j = q_j - p_i = \delta - r$. 

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which boils down to (using the fact that from $r \leq \delta < w_{AB}$, it follows that $s \leq \delta - \hat{r}$ implies $s \leq w_{AB} + \delta - r - \hat{r}$):

$$s \leq \delta - r \text{ and } s \leq \delta - \hat{r}.$$  

Figure 1 depicts consumers’ response.

![Figure 1](image)

Firm $i$’s expected profit is equal to:

$$\pi_i (r) = rE [OSS_i + MSS],$$

where $OSS_i$ is the number of one-stop shoppers going to firm $i$ and $MSS$ is the mass of multi-stop shoppers, as a function of the realized prices of the rival, $\rho_j = r_j = \tilde{r}$, which can be further written as

$$\pi_i (r) = r [1 - K (r) + K (r) F (\delta - r)] = r [1 - K (r) (1 - F (\delta - r))] .$$

Given the equilibrium expected profit $\pi^*$, we must have:

$$r [1 - K (r) (1 - F (\delta - r))] = \pi^*.$$  

We show in the Appendix C that there exists such an equilibrium where $\pi = \tilde{\pi}$ and $\pi^* = \hat{\pi}$; the above equilibrium condition then yields:

$$K (r) = \frac{1 - \hat{\pi}}{1 - F (\delta - r)} = \frac{1 - \hat{\pi} F (\delta - \hat{r})}{1 - F (\delta - r)}. \quad (4)$$
This mix-strategy equilibrium yields a strictly positive expected profit $\bar{\pi} > 0$ for each firm. We now compare the social welfare in two regimes regarding on whether below-cost pricing is allowed or not. Notice that, the social welfare absent banning can be expressed by

$$W = w_{AB} + \int_{0}^{\tau} (\delta - s) f(s) ds,$$

where the second term is additional social welfare generated from multi-stop shopping; while the social welfare under banning is

$$W^b = w_{AB} + \int_{\tilde{\tau}}^{\hat{\tau}} \left( \int_{0}^{\delta-r} (\delta - s) f(s) ds \right) k(r) dr,$$

which is decreasing with $r$. So the net gain of social welfare is equal to

$$\Delta W = W^b - W = \int_{\tilde{\tau}}^{\hat{\tau}} \left( \int_{0}^{\delta-r} (\delta - s) f(s) ds \right) k(r) dr - \int_{0}^{\tau^*} (\delta - s) f(s) ds.$$

Since $\tilde{\tau} = \delta - \tilde{\tau} = j^{-1}(\delta) > \tau^* = j^{-1}(\delta)$, we then have

$$\Delta W \geq \int_{0}^{\delta-\hat{\tau}} (\delta - s) f(s) ds - \int_{0}^{\tau^*} (\delta - s) f(s) ds > 0.$$  

It follows that banning below-cost pricing increases the total social welfare as more multi-stop shoppers are served.

Summarizing the above analysis yields:

**Proposition 3** Suppose below-cost pricing is banned. When consumers are sufficiently heterogeneous in shopping costs (i.e., $S > \tau^*$), there exists a symmetric mixed-strategy equilibrium in which firms charge prices at cost for the disadvantaged products, and the total margin $r$ is distributed over $[\tilde{\tau}, \hat{\tau}]$ according to a c.d.f. $\tilde{K}(.)$ that has a continuous density $\tilde{k}(.)$ over that range and is derived by modifying the function $K(.)$ such that it is increasing and concave in the range. Moreover, total social welfare is higher than that in the absence of a ban.

**Equilibria with multi-stop shoppers only**

When consumers’ shopping costs are not sufficiently diversified, i.e., $S < \tau^*$, there are multi-equilibria with multi-stop shoppers only. Banning below-cost pricing would force each firm to price its disadvantaged product at cost: $q_i^b = q_j^b = c$, and thus in candidate equilibria we must have $p_i^b = p_j^b = c - S$ (where the superscript $b$ stands for banning here). Moreover, firm $i$ cannot benefit from unilateral increasing of $p_i$ if $p_i = c - S$ maximizes $(p_i - c + \delta) F(q_j - p_i)$, which amounts to $S + h(S) \leq \delta$, or $S \leq l^{-1}(\delta)$, where $l(x) \equiv x + h(x)$. In this equilibrium, banning below-cost pricing would increase the price for the advantaged product as $p_i = q_j - S$
increases with \( q_j \). However, as all consumers are served as multi-stop shoppers in this regime, banning below-cost pricing does not affect the social welfare, it simply transfers some surplus from consumers to firms. Notice that the total social welfare generated from multi-stop shopping can be expressed as

\[
W = \int_0^\tau (w_{AB} + \delta - s) f(s) ds,
\]

which remains unchanged as \( \tau = S \) in both regimes.

When \( j^{-1}(\delta) \leq S \leq t^{-1}(\delta) \), banning below-cost pricing still leads to the equilibrium with \( q_i^b = q_j^b = c \) and \( p_i^b = p_j^b = c - S \), in which case all consumers are multi-stop shoppers. Comparing with the equilibria prices absent banning, we have \( p_i^* - p_i^b = S - \delta + h(\tau^*) < h(\tau^*) - h(S) \leq 0 \), so the prices for both products increase. The net welfare gain is then

\[
\Delta W = \int_0^S (\delta - s) f(s) ds - \int_0^\tau (\delta - s) f(s) ds,
\]

which is positive since \( S \geq \tau \).

To summarize:

**Proposition 4** Suppose below-cost pricing is banned. When consumers are not sufficiently heterogeneous in shopping costs (i.e., \( S < t^{-1}(\delta) \)), there exists a unique equilibrium in which all consumers are multi-stop shoppers. Moreover, consumer surplus is lower while total social welfare is higher than that absent banning.

## 5 Extension: horizontal differentiation

Firms are often differentiated horizontally to avoid pure price competition. To check the robustness of the analysis in the baseline setting, we incorporate the heterogeneity of consumer’s utility from shopping experience. Namely, a consumer incurs a preference shock \( -\frac{\tau}{2} \) from purchasing one good in firm 1 while incurs a utility shock \( -\frac{\tau}{2} (1 - x) \) from buying one good in firm 2. Therefore, consumers for one-stop shopping obtain a value \( u_{AB} - tx - p_1 - q_1 = v_1 - tx \) from buying \( A_1B_1 \) in firm 1 and a value \( v_2 - t(1 - x) \) from patronizing firm 2, while multi-stop shoppers obtain a value \( v_{12} - \frac{\tau}{2} \) from buying the assortment \( A_1B_2 \).\(^{27}\) We assume that \( t \) is bounded above

\(^{27}\)We assume that consumer’s preference shock is firm-specific rather than product-specific, which leads to the fact that the preference shock for multi-stop shoppers is independent of \( x \). This assumption is mainly for the simplicity of exposition, and allowing preference shocks to be product-specific brings no substantial change in the analysis.
such that $w_{AB} - \frac{t}{2} > 0$ to ensure that it is socially desirable to serve the marginal consumer with $x = 1/2$.

The parameter $t$ reflects the elasticity of demand: the lower $t$, the faster consumers drop in case of a price increase. In particular, this setting converges to the case of pure price competition with perfect substitutes when $t$ goes to zero. The parameter $x \in [0, 1]$ can be interpreted as the "distance" between the consumer’s ideal variety and that proposed by firm 1, and is distributed according to a cumulative distribution function $G(\cdot)$, with density $g(\cdot)$, which allows for quite general demand functions. However, we will restrict attention to symmetric distribution (that is, the density $g(\cdot)$ satisfies $g(x) = g(1 - x)$) and assume also that the inverse hazard rate of the distribution is strictly increasing.

As firms are horizontally differentiated, competition for one-stop shoppers will not lead to zero margin for assortments. One-stop shoppers are willing to patronize firm 1 if $v_1 - tx \geq s$, and they prefer that to patronizing firm 2 if

$$v_1 - tx \geq v_2 - t(1 - x),$$

that is, if

$$x \leq \hat{x} \equiv \max\{0, \frac{1}{2} - \frac{1}{2t}(p_1 + q_1 - p_2 - q_2)\}.$$  

Moreover, consumers prefer to multi-stop shopping than patronizing only firm 1 if

$$v_{12} - \frac{t}{2} - s \geq v_1 - tx,$$

which amounts to

$$s \leq \lambda_1(x) \equiv \max\{0, \tau_1 - t(\frac{1}{2} - x)\}.$$  

Similarly, consumers prefer multi-stop shopping rather than patronizing only firm 2 if

$$s \leq \lambda_2(x) \equiv \max\{0, \tau_2 - t(x - \frac{1}{2})\}.$$  

Therefore, for consumers with $x \leq \hat{x}$, they prefer multi-stop shopping rather than patronizing only firm 1 if $s \leq \lambda_1(x)$, while for consumers with $x \geq \hat{x}$, they prefer multi-stop shopping rather than visiting only firm 2 if $s \leq \lambda_2(x)$. Therefore the demand for the assortment $A_1B_1$ can be expressed as

$$OSS_1 \equiv \int_0^{\hat{x}} [1 - F(\lambda_1(x))] g(x) dx,$$

and similarly for the assortment $A_2B_2$

$$OSS_2 \equiv \int_{\hat{x}}^1 [1 - F(\lambda_2(x))] g(x) dx;$$
while the demand for the assortment $A_1B_2$ can be written as

$$MSS \equiv \int_0^{\hat{x}} F(\lambda_1(x))g(x)dx + \int_{\hat{x}}^1 F(\lambda_2(x))g(x)dx.$$ 

Therefore, the profit of firm $i, i = 1, 2$, is given by

$$\Pi_i = (p_i + q_i - 2c + \delta) OSS_i + (p_i - c + \delta) MSS.$$ 

The above characterization is demonstrated in Figure 2.

![Figure 2](image)

To characterize the equilibrium prices, we consider the impact of a small change on $q_1$ and $p_1$ respectively and then evaluate it at the symmetric candidate equilibrium with $p_1 + q_1 = p_2 + q_2$ (we have $\hat{x} = 1/2$, $\lambda_1(\hat{x}) = \lambda_2(\hat{x})$, and $\int_0^{1/2} f(\lambda_1(x))g(x)dx = \int_{1/2}^1 f(\lambda_2(x))g(x)dx$ in the symmetric candidate equilibrium). Consider first a modification of $p_1$ by $dr$ while a change of $q_1$ by $-dr$ such that the total price for the assortment remains unchanged. Such a change does not affect the behavior of marginal one-stop shoppers who are indifferent between patronizing firm 1 and 2: $\hat{x} = \frac{1}{2} - \frac{1}{2\Pi}(p_1 + q_1 - p_2 - q_2)$ is not affected. However (see Figure 2):

- For $x \in [1/2, 1]$, marginal consumers with $s = \lambda_2(x)$, who were indifferent between buying $A_1B_2$ or patronizing firm 2, now prefer one-stop shopping in firm 2, therefore firm 1 loses $f(\lambda_2(x))dr$ consumers, on which it no longer earns the margin $p_1 - c + \delta$. This brings an overall impact on firm 1’s profit

$$-(p_1 - c + \delta) \int_{1/2}^1 f(\lambda_2(x))g(x)dxdr.$$
In addition, it alters the choices between one-stop shopping in firm 1 and multi-stop shopping: for \( x \in [0, 1/2] \), marginal consumers with \( s = \lambda_1(x) \), who were indifferent between multi-stop shopping or patronizing firm 1, now prefer patronizing firm 1 and buy also \( B_1 \), therefore firm 1 gains from an increase of demand for \( B_1 \) with \( f(\lambda_1(x))dx \), on which it earns a margin \( q_1 - c \), and the total gain equals to

\[
(q_1 - c) \int_0^{1/2} f(\lambda_1(x))g(x)dx.
\]

On the other hand, firm 1 increases its price \( p_1 \) by \( \delta \) on the mass \( MSS \) of consumers that buy \( A_1 \).

The overall impact on firm 1’s profit by such an adjustment must be offset in equilibrium, which yields

\[
(q_1 - c - (p_1 - c + \delta)) \int_0^{1/2} f(\lambda_1(x))g(x)dx + MSS = 0.
\] (5)

Consider now a small change on \( q_1 \) by \( \delta \) while keeping \( p_1 \) constant. It does not affect the consumer’s choices between multi-stop shopping or patronizing firm 2. However:

For \( x \in [0, 1/2] \), marginal consumers with \( s = \lambda_1(x) \), who were indifferent between multi-stop shopping and patronizing only firm 1, now prefer multi-stop shopping, and firm 1 loses \( f(\lambda_1(x))dx \) consumers who now stop buying \( B_1 \), which leads to a loss

\[
-(q_1 - c) \int_0^{1/2} f(\lambda_1(x))g(x)dx.
\]

On the other hand, it alters the decision of marginal consumers (one-stop shoppers) with \( x = \hat{x} = 1/2 \), who were indifferent between buying from firm 1 or 2, now are attracted by firm 2, on which firm 1 incurs a loss

\[
-(p_1 + q_1 - 2c + \delta) \frac{g(1/2)}{2t} \left(1 - F(\lambda_1(1/2))\right)\delta;
\]

Finally, firm 1 gains from the increase of its price \( q_1 \) by \( \delta \) on the mass \( OSS_1 \) of consumers.

In equilibrium, these effects must be cancelled out and we must have

\[
(q_1 - c) \int_0^{1/2} f(\lambda_1(x))g(x)dx + \frac{(p_1 + q_1 - 2c + \delta)}{2t} g(1/2) \left(1 - F(\lambda_1(1/2))\right) = OSS_1.
\] (6)

First-order conditions for \( q_2 \) and \( p_2 \) can be characterized similarly.
We show now that below-cost pricing still arises in equilibrium when competition becomes very tough. Indeed, from (6), it is straightforward to see that the total price \( p_1 + q_1 \) must converge to \( 2c - \delta \) when \( t \) goes to zero, otherwise the equation cannot hold. But then from (5) we have

\[
(2 (q_1 - c) - (p_1 + q_1 - 2c + \delta)) \int_0^{1/2} f(\lambda_1(x))g(x)dx + MSS = 0,
\]

and it follows that \( q_1 < c \) when \( t \) tends to zero. In particular we have

\[
\lim_{t \to 0} q_1 = q^* = c - h(\tau^*),
\]
\[
\lim_{t \to 0} p_1 = p^* = c - \delta + h(\tau^*).
\]

That is, the equilibrium prices converge to that under pure price competition, so below-cost pricing arises in equilibrium for \( t \) sufficiently small, that is, there exists a value \( t^0 \) such that \( q^* < c \) for \( t < t^0 \), in which case each firm charges a price below-cost for its disadvantaged product.

**Proposition 5** Suppose two firms are horizontally differentiated and each firm possesses a comparative advantage in one product market. When competition becomes fierce (i.e., for \( t < t^0 \)), there exists a symmetric equilibrium where each firm prices below-cost for its disadvantaged product and earns a positive profit from multi-stop shoppers.

**Proof:** See Appendix D.

**6 Conclusions**

Firms rarely supply only single product. Multiproduct firms are able to cross-subsidize among product lines which could lead to below-cost pricing on part of products. This paper develops a new model to analyze below-cost pricing by multiproduct firms. We show that below-cost pricing arises in equilibrium in the absence of any efficiency justifications stemming from asymmetric information on prices, and it serves as a discriminatory mechanism for screening consumers with heterogeneous shopping costs. In all equilibria firms earn positive profits from multi-stop shoppers who incur lower shopping costs, in spite that pure price competition dissipates its profit from one-stop shoppers. We show also that banning below-cost pricing could improve social welfare but may lead to higher prices for consumers.

Our analysis sheds a new light on the rationale of below-cost pricing by multiproduct firms and identifies the key factors underlying it: asymmetry in comparative advantages and heterogeneity in consumers’ shopping costs, and the insights are robust to variations with product
differentiation among firms. However, policy measures against below-cost pricing should also take into account potential efficiency justifications, and empirical studies are needed to assess the resulting balance.

We have furthermore restricted attention to individual unit demands, as this appears reasonable for grocery retailing and other regular purchases, and also neglected any correlation between consumers’ valuations for the goods and their shopping costs; whether our insights apply to market environments where consumers’ individual demands are elastic, or underlying characteristics (e.g., wealth) affect both shopping costs and willingness to pay, is left to future research.

Appendix A: Proof of proposition 1

Recall that \( v_i = u_{AB} - p_i - q_i, \) \( i = 1, 2, \) and \( v_{12} = u_{AB} - p_1 - p_2, \) and denote by \( \hat{v}_{12} = u_{AB} - q_1 - q_2 \) the value from multi-stop shopping when consumers pick the disadvantaged product from each firm. For the simplicity of exposition, we denote by MSS multi-stop shoppers while denote by OSS\( _i \) the one-stop shoppers who patronizing firm \( i. \) While there are several possible regimes depending on the existence of multi-stop shoppers and/or one-stop shoppers, we show first that only two of them are relevant in equilibria.

**Lemma 1** Only two regimes are relevant in equilibria: Regime 1 where there exist both multi-stop and one-stop shoppers and regime 2 in which there exist only multi-stop shoppers; moreover, multi-stop shoppers pick both advantaged products.

**Proof:** This result is established following three claims:

**Claim 1:** At least one firm must offer \( v_i > 0 \) for some \( i. \)

**Proof:** Suppose \( \max\{v_i, v_j\} \leq 0 \) and thus \( p_i + q_i \geq u_{AB}. \) Since there are no OSS in this case, to keep attracting MSS, the prices for the advantaged product must be bounded: \( p_1 < u_{AB} - u_B \) and \( p_2 < u_{AB} - u_A. \) Then for firm 1, keeping \( p_1 \) unchanged while reducing the price \( q_1 \) to \( q'_1 = u_{AB} - p_1 - \varepsilon \) can attract some one-stop shoppers as now \( v'_1 = \varepsilon > 0, \) which increases its profit by switching some multi-stop shoppers into one-stop shoppers from which it earns an extra profit since \( q'_1 = u_{AB} - p_1 - \varepsilon > u_A - \varepsilon > c. \) So \( \max\{v_i, v_j\} \leq 0 \) cannot arise in equilibria.

**Claim 2:** There must exist multi-stop shoppers.

**Proof:** Suppose there exists only one-stop shoppers. This must be the case that \( v = \max\{v_1, v_2\} > 0 \) and \( \max\{v_1, v_2\} \geq \{v_{12}, \hat{v}_{12}\}, \) that is, even consumers with lowest shopping
cost \((s = 0)\) are attracted to one-stop shopping. From \(\max\{v_1, v_2\} \geq v_{12}\), we have \(p_1 + p_2 \geq \min\{p_1 + q_1, p_2 + q_2\}\), meanwhile \(\max\{v_1, v_2\} \geq \tilde{v}_{12}\) is equivalent to \(q_1 + q_2 \geq \min\{p_1 + q_1, p_2 + q_2\}\).

Suppose \(p_i + q_i < p_j + q_j\) and thus \(v_i > v_j\), then firm \(i\) attracts all \(OSS_i\) and \(\pi_j = 0\). Moreover, from the above constraints, we have \(p_j \geq q_i\) and \(q_j \geq p_i\). Consider the following cases:

- If \(p_i + q_i > 2c - \delta\), then firm \(j\) can benefit from undercutting by decreasing its total price to be slightly lower than \(p_i + q_i\) to attract \(OSS_j\) and make a profit;

- If \(p_i + q_i < 2c - \delta\), then \(\pi_i < 0\); in this case firm \(i\) will be better off to avoid a loss by increasing its prices;

- If \(p_i + q_i = 2c - \delta\) and \(q_j > p_i\), then increasing \(p_i\) by \(\varepsilon\) can make additional profit from \(OSS_i\) while still keep attracting all of them (they will not turn to \(MSS\) as long as \(q_j \geq p_i + \varepsilon\));

- If \(p_i + q_i = 2c - \delta\) and \(p_j > q_i\), then increasing \(q_i\) by \(\varepsilon\) can make additional profit from \(OSS_i\) while still keep attracting all of them.

Suppose now firms charge the same prices, then:

- If \(p_i + q_i = p_j + q_j > 2c - \delta\), then at least one firm can benefit from undercutting the rival.

- If \(p_i + q_i = p_j + q_j < 2c - \delta\), then raising its price slightly can avoid a loss.

- If \(p_i + q_i = p_j + q_j = 2c - \delta\), then we must have \(p_1 + p_2 \geq p_i + q_i = 2c - \delta\), and at least some firm charges \(p_1 \geq c - \delta/2 > c - \delta\) and thus \(q_i < c\). It is then profitable to transform some one-stop shoppers into multi-stop shoppers by raising \(p_i\) slightly.

**Claim 3:** Multi-stop shoppers only pick the advantaged products.

**Proof:** Suppose multi-stop shoppers pick two disadvantaged products; this must be the case that each firm charges a price for its disadvantaged product lower than its rival’s product (who has comparative advantage), that is, \(q_i < p_j\) and \(q_j < p_i\). However, each firm must charge the product a price above cost: \(q_i \geq c\), otherwise it will run a loss as they make zero profit from one-stop shoppers. This implies that \(p_i > q_j \geq c\) and \(p_j > q_i \geq c\). Consider now firm \(i\)’s undercutting such that \(p_i' = q_j - \varepsilon\), then:

- it transforms all \(MSS\) into \(OSS_i\): consumers who originally buy disadvantaged product from firm \(j\) would now buy instead the advantaged product from firm \(i\), while they keep
buying its disadvantaged product since \( q_i < p_j \); firm \( i \) then gains from this undercutting as \( p'_i = q_j - \varepsilon \geq c > c - \delta \);

- it may attract additional \( OSS_i \) from the rival since it offers more value than before \( v'_i > v_i \);
- in particular, consumers will not turn to \( MSS \) who buy both advantaged products since \( p_j > q_i \).

Therefore at least one firm can always benefit from such a deviation if each firm offers its disadvantaged product a price lower than the rival’s advantaged product. It follows that in any equilibria, each firm must offer its advantaged product cheaper than the rival’s disadvantaged product and as a result multi-stop shoppers will pick only advantaged products.

Thanks to Lemma 1, we need to focus only on two regimes. Consider the first regime where there are both \( OSS \) and \( MSS \). The following lemma shows that competition for one-stop shoppers will lead to at-cost pricing for each firm’s assortment.

**Lemma 2** Suppose there exist both one-stop and multi-stop shoppers, then price competition for one-stop shoppers leads to pricing at cost for the assortment, that is: \( p_i + q_i = p_j + q_j = 2c - \delta \).

*Proof:* We must have \( \max\{v_1, v_2\} > 0 \), otherwise there is no \( OSS \); we must also have \( q_j > p_i \) and \( q_i > p_j \), otherwise there is no \( MSS \). Suppose \( v_j > v_i \) and thus \( \max\{v_i, v_j\} = v_j > 0 \), then \( \tau = \tau_j < \tau_i \)

- If \( p_j + q_j > 2c - \delta \), then decreasing \( q_i \) slightly such that \( v'_i = v_j + \varepsilon \) allows firm \( i \) to attract all \( OSS_i \) and make a non-trivial profit, which is sufficient to offsets a trivial loss due to a reduction of the \( MSS \) population (\( \tau \) may decrease by \( \varepsilon \));
- If \( p_j + q_j \leq 2c - \delta \), increasing slightly \( p_j \) such that \( v'_j \geq v_i \) could increase its profit from \( OSS_j \) and may also increase its profit from \( MSS \);

Suppose now \( v_j = v_i \) and thus \( \tau_j = \tau_i \), then:

- If \( p_j + q_j = p_i + q_i < 2c - \delta \), then at least firm \( i \) has an incentive to raise \( q_i \) to avoid a loss from \( OSS_i \), without affecting the profit from \( MSS \);
- If \( p_j + q_j = p_i + q_i > 2c - \delta \), then at least one firm can benefit from undercutting the rival.
So we must have \( p_j + q_j = p_i + q_i = 2c - \delta \) in any candidate equilibria, in which case no firm can benefit from a unilateral deviation: undercutting the rival would incur a loss while raising the price would lose all one-stop shoppers. Moreover, when \( S > j^{-1}(\delta) \), the candidate equilibrium \((p^*, q^*)\) is characterized by equation (2) and (3). To see that \((p^*, q^*)\) indeed forms a Nash equilibrium, consider a unilateral deviation of firm \( i \) to any prices \((p_i, q_i)\) which yields a total profit:

\[
\pi_i = (p_i - c + \delta)F(\tau) + (p_i + q_i - 2c + \delta)OSS_i,
\]

where \( OSS_i \) stands for the demand of \( OSS_i \) (with a slightly abuse of notation). Notice that we must have \( p_i + q_i \leq p^* + q^* = 2c - \delta \), as otherwise firm \( i \) can attract all \( OSS_i \). Moreover, since \( \tau = \min\{q^* - p_i, q_i - p^*\} \), we have

\[
(p_i - c + \delta)F(\tau) \leq (p_i - c + \delta)F(q^* - p_i) \leq \pi^*_i,
\]

where the second inequality comes from the fact that the profit function \((p_i - c + \delta)F(q^* - p_i)\) is quasi-concave and is maximized for \( p_i = p^* \). It follows that \( \pi_i \leq \pi^*_i \) and no firm can benefit from a unilateral deviation.

**Appendix B: Proof of Proposition 2**

We now turn to the candidate equilibria where there are only \( MSS \). Recall that consumers are willing to visit two firms only if \( s \geq v_{12}/2 \), and they prefer \( MSS \) to \( OSS_i \) if \( s \leq \tau_i = v_{12} - v_i \). Two cases need to be taken into account:

**Case 1:** If \( v_{12}/2 \leq S \), then we must have \( \max\{v_1, v_2\} \leq v_{12}/2 \). Suppose \( v_i > v_{12}/2 \), then \( \tau_i = v_{12} - v_i < v_{12}/2 < \{v_i, S\} \), therefore consumers with \( s > \tau_i \) are willing to go one-stop shopping since \( v_i - s > 0 \).

In this case, since \( \tau_i = v_{12} - v_i > v_{12}/2 \), the total population for \( MSS \) is \( F(v_{12}/2) \), and firm \( i \) earns a profit \( \pi_i = (p_i - c + \delta)F(v_{12}/2) \), with \( p_i \geq c - \delta \). Suppose \((p_i, q_i)\) and \((p_j, q_j)\) form a candidate equilibrium with \( p_j \geq p_i \). Consider a deviation of \( q_i \) such that \( v_j' = v_{12}/2 + \varepsilon \), then \( q_j' = \frac{1}{2}(u_{AB} + p_j - p_i - 2\varepsilon) \geq \frac{1}{2}(u_{AB} - 2\varepsilon) > c \). Such a deviation transforms some \( MSS \) into \( OSS_i \), and firm \( i \) makes an extra profit from selling disadvantaged product \((F(v_j') - F(\tau_i'))(q_j' - c)\), as well as additional profit from selling more advantaged product \((F(v_j') - F(v_{12}/2))(p_i - c + \delta)\). Therefore no equilibrium can arise in this regime.

**Case 2:** If \( v_{12}/2 > S \), then it must be \( S \leq \tau_i, \tau_j \), as otherwise we would have \( S > \tau_i = v_{12} - v_i > 2S - v_i \), which implies \( v_i > S > \tau_i \) and some consumers prefer to be \( OSS_i \). In this case, all consumers are served as multi-stop shoppers, and each firm makes a profit \( \pi_i = p_i - c + \delta \).

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Thus, there exist candidate equilibria with only MSS if $S \leq \tau_i$, $\tau_j$. Since firm’s profit increases with $p_i$, the constraints $\tau_j = q_j - p_i \geq S$, $j = 1, 2$, must be binding, which yields $p_i = q_j - S$ (and also $p_i \geq c - \delta$). Moreover, in any candidate equilibria, each firm must charge $q_i \leq c$. Suppose $q_i > c$ and consider firm $j$’s undercutting: $q_j' = q_i - \varepsilon$ such that $\tau_j' = S - \varepsilon$: doing so yields an extra profit for firm $j$ up to $(q_j' - c)(1 - F(S - \varepsilon)) > 0$.

Denote by $(p_i^0, q_i^0)$ and $(p_j^0, q_j^0)$ the prices supporting a candidate equilibrium. The equilibrium conditions can be characterized by checking the feasible deviations:

- It is obvious that firm $i$ can never benefit from transforming MSS into OSS$_j$, in which case it can make only zero profit. Moreover, if firm $i$ deviates to $\tau_i = q_i - p_j^0 \leq 0$ and transforms all MSS into OSS$_i$, it then makes a profit

$$
\pi_i = p_i + q_i - 2c + \delta \leq p_i^0 + q_i^0 - 2c + \delta \leq p_i^0 - c + \delta = \pi_i^0,
$$

where the first inequality comes from the relation $p_i + q_i \leq p_j^0 + q_j^0 = p_i^0 + q_i^0$, and the second inequality comes from the fact that $q_i^0 \leq c$, therefore such deviation is never profitable.

- Consider now firm $i$’s deviation by transforming some MSS into OSS$_i$, which can be achieved by reducing $\tau_i = q_i - p_j^0$ such that $\tau_i \leq \{S, \tau_j\}$. Firm $i$ then chooses $p_i$ and $q_i$ to maximize

$$
\pi_i = (p_i - c + \delta)F(\tau_i) + (p_i + q_i - 2c + \delta)(1 - F(\tau_i)),
$$

subject to the constraints $p_i + q_i \leq p_j^0 + q_j^0 = p_i^0 + q_i^0$ and $q_i \leq p_j^0 + S$. The optimal deviation must then involve $p_i = p_j^0 + q_j^0 - q_i = p_i^0 + q_i^0 - q_i$, so we can rewrite the above profit function as

$$
\pi_i = (p_i^0 + q_i^0 - q_i - c + \delta)F(\tau_i) + (p_i^0 + q_i^0 - 2c + \delta)(1 - F(\tau_i))
= (p_i^0 - c + \delta) + (q_i^0 - q_i)F(\tau_i) + (q_i^0 - c)(1 - F(\tau_i))
= (p_i^0 - c + \delta) + (q_i^0 - c) + (c - q_i)F(q_i - p_j^0).
$$

To rule out such deviation, it must be

$$
q_i^0 = \arg\max_{q_i \leq q_i^0} (c - q_i) F(q_i - p_j^0).
$$

It follows that the first-order derivative as evaluated at $q_i = q_i^0$ (thus $q_i^0 - p_j^0 = S$) must be non-negative: $(c - q_i^0) f(S) - F(S) \geq 0$, which is equivalent to

$$
q_i^0 \leq c - h(S).
$$
• By contrast, firm $i$ may want to transform some MSS into OSS$_j$, in which case we must have $\tau_j = q_j^0 - p_i \leq \{S, \tau_i\}$. Firm $i$ then chooses $p_i$ and $q_i$ to maximize

$$\pi_i = (p_i - c + \delta)F (q_j^0 - p_i).$$

To form an equilibrium $p_i^0$ must solve the maximization program

$$\max_{p_i \leq p_i^0}(p_i - c + \delta)F (q_j^0 - p_i),$$

which implies that the first-order derivative as evaluated at $p_i = p_i^0$ must be non-negative and thus

$$p_i^0 \geq c - \delta + h(S).$$

To summarize, the candidate equilibria must satisfy $p_i^0 \geq c - \delta + h(S)$, $q_i^0 \leq c - h(S)$, and $q_j^0 = p_j^0 + S$, in which case we must have $c - h(S) \geq q_i^0 = p_j^0 + S \geq c - \delta + h(S) + S$ and thus $S \leq j^{-1}(\delta)$. 

**Appendix C: Mixed strategy equilibrium**

Consider a candidate equilibrium such that $\rho_i = r_i$, where $r_i$ is distributed according to the same distribution $K(\tilde{r})$ over some interval $\tilde{r} \in [\underline{r}, \bar{r}]$, with a continuous density $k(\tilde{r})$ over $[\underline{r}, \bar{r}]$. Suppose first that $\tau > \delta$. Adopting $\tau$ would then yield zero profit:

• it would attract no one-stop shopper (since the rival undercuts with probability 1)
• and no multi-stop shopper either, since the relevant threshold is then $\tau = \delta - \tau < 0$.

But adopting instead $r$ slightly below $\delta$ would attract some (multi-stop and one-stop) shoppers and yield a positive profit, a contradiction. Therefore, without loss of generality we can assume $\tau \leq \delta$ ($< w_{AB}$).

We show now that there exists a symmetric mixed strategy equilibrium in which each firm adopts $\rho_i = r_i = r$, which is distributed over $[\tilde{r}, \bar{r}]$ according to a c.d.f. $\hat{K}(\cdot)$, which has a continuous density $\hat{k}(\cdot)$ over that range and is such that:

• if the function $K(\cdot)$, defined by (4), satisfies $K'(r) > 0$ for $r \in [\tilde{r}, \bar{r}]$, then $\hat{K}(\cdot) = K(\cdot)$;
• otherwise, $\hat{K}(\cdot)$ is constructed as follows:
let \( \bar{r} \) denote the lowest \( r \) for which \( K' (r) = 0 \), \( r_1 \) denote the lowest \( r > \bar{r} \) such that
\( K (r_1) = K (\bar{r}) \), and denote by \( K_1 (.) \) the function such that \( K_1 (r) = K (r_1) \) and \( k_1 (r) = 0 \) over the interval \([\bar{r}, r_1]\) and coincides with \( K (.) \) otherwise;

- if \( K'_1 (.) > 0 \) for \( r > r_1 \), \( \hat{K} (.) = K_1 (.) \); otherwise, denote by \( r_2 \) the lowest \( r > r_1 \) for which \( K' (r) = 0 \), \( r_2 \) denote the lowest \( r > r_1 \) such that \( K (r_2) = K (r_1) \), and \( K_2 (.) \) denote the function such that \( K_2 (r) = K_1 (r_2) \) and \( k_2 (r) = 0 \) over the interval \([r_2, r_2]\), and coincides with \( K_1 (.) \) otherwise;

- if \( K'_2 (.) > 0 \) for \( r > r_2 \), \( \hat{K} (.) = K_2 (.) \); otherwise, repeat the same steps until the function so defined is everywhere weakly increasing.

We first characterize some properties of the function \( K \) defined by (4). By construction, we have:

\[
K (\hat{r}) = \frac{1 - \frac{\hat{r} F (\delta - \hat{r})}{\hat{r}}}{1 - F (\delta - \hat{r})} = 1.
\]

Conversely, for \( r = \hat{r} \):

\[
K (\hat{r}) = \frac{1 - \frac{\hat{r}}{\hat{r}}}{1 - F (\delta - \hat{r})} = 0.
\]

In addition, for any \( r < \hat{r} \) we have \( K (r) < 1 \); indeed \( K (r) \geq 1 \) would imply:

\[
r [1 - K (r) (1 - F (\delta - r))] < r [1 - (1 - F (\delta - r))] = r F (\delta - r) < \hat{r} F (\delta - \hat{r}),
\]

contradicting the equilibrium condition.

Furthermore:

\[
K' (r) = \frac{(1 - F (\delta - r)) \frac{\hat{r}}{\hat{r}} - (1 - \frac{\hat{r}}{\hat{r}}) f (\delta - r)}{(1 - F (\delta - r))^2}.
\]

Therefore:

- For \( r = \hat{r} \), the numerator boils down to:

\[
(1 - F (\delta - \hat{r})) \frac{\hat{r} F (\delta - \hat{r})}{\hat{r}^2} - \left(1 - \frac{\hat{r} F (\delta - \hat{r})}{\hat{r}}\right) f (\delta - \hat{r}) = (1 - F (\delta - \hat{r})) \left(\frac{F (\delta - \hat{r})}{\hat{r}} - f (\delta - \hat{r})\right),
\]

which, using the first-order condition \( F (\delta - \hat{r}) = \hat{r} f (\delta - \hat{r}) \), implies \( K' (\hat{r}) = 0 \). In addi-
tion, using the assumption $h'(.) > 0$ and $F(\delta - \hat{r}) = \hat{r} f(\delta - \hat{r})$, we have:

$$K''(\hat{r}) = \frac{((1 - F(\delta - r)) \frac{d^2}{dx^2} - (1 - \frac{d}{dx}) f(\delta - r))|_{x=\hat{r}}}{(1 - F(\delta - \hat{r}))^2} - \frac{2K'(\hat{r})}{1 - F(\delta - \hat{r}) f(\hat{r})}$$

$$= \frac{\left( f(\delta - r) \frac{d^2}{dx^2} - 2(1 - F(\delta - r)) \frac{d}{dx} f(\hat{r}) + (1 - \frac{d}{dx}) f'(\delta - r) \right)|_{x=\hat{r}}}{(1 - F(\delta - \hat{r}))^2}$$

$$= \frac{f'(\delta - r) - 2 \frac{dF(\delta - \hat{r})}{dx}}{1 - F(\delta - r)}$$

$$< \frac{f(\delta - r) - 2 \frac{d}{dx} f(\delta - \hat{r})}{1 - F(\delta - r)} = - \frac{f(\delta - \hat{r})}{1 - F(\delta - r)} < 0.$$

where the first equality uses $K'(\hat{r}) = 0$, the inequality stems from $h'(.) > 0$, which implies $f'(s) < f^2(s)/F(s)$, and the following equality follows from the first-order condition $F(\delta - \hat{r}) = \hat{r} f F(\delta - \hat{r})$. Since $K''(\hat{r}) < 0$ and $K'(\hat{r}) = 0$, it follows that $K'(r) > 0$ for $r$ close to (and below) $\hat{r}$.

- For $r = \hat{\pi}$, the numerator of $K'(.)$ boils down to:

  $$(1 - F(\delta - \hat{\pi})) \frac{1}{\hat{\pi}} - \left( 1 - \frac{\hat{\pi}}{\hat{\pi}} \right) f(\delta - \hat{\pi}) = \frac{(1 - F(\delta - \hat{\pi}))}{\hat{\pi}} > 0,$$

  implying $K'(\hat{\pi}) > 0$.

Therefore, the function $K(.)$ defined over $[\hat{\pi}, \hat{r}]$ by (4) and its density $k(.) = K'(.)$ satisfy:

- $K(\hat{r}) = 1$ and $k(\hat{r}) = 0$;
- $K(r) < 1$ for $r < \hat{r}$ and $k(r) > 0$ for $r$ slightly below $\hat{r}$;
- $K(\hat{\pi}) = 0$ and $k(\hat{\pi}) > 0$;
- $K(r) > 0$ and $k(r) > 0$ for $r$ slightly above $\hat{\pi}$.

We now show that the above algorithm defines a unique, weakly increasing function $\hat{K}$. Figure 3 illustrates the algorithm.
Note first that the function $K$, defined by (4), is continuously differentiable. Furthermore, in the range $[\hat{\pi}, \hat{r}]$, its derivative $k$ is positive for $r = \hat{\pi}$ as well as in the neighborhood of $\hat{r}$; therefore, it is either positive everywhere in this range (in which case the algorithm yields $\hat{K} = K$) or there exists some $r < \hat{r}$ for which $K'(r) = 0$. Let $r_1$ denote the lowest such $r$; by construction, $r_1 < \hat{r}$ and thus $K(r_1) = K(\hat{r}) = 1$. Therefore, there exists $r < \hat{r}$ such that $K_1(r) = K(r_1)$; let $r_1$ denote the lowest of such $r$. By construction, the function $K_1$ defined by the algorithm is continuous, weakly increasing on $[\hat{\pi}, r_1]$ and coincides with $K$ on $[r_1, \hat{r}]$. If $K$ is weakly increasing over $[r_1, \hat{r}]$, the algorithm stops and $\hat{K} = K_1$, otherwise the algorithm is applied to the restriction of $K$ to $[r_1, \hat{r}]$, and so forth.

The function $\hat{K}$ that is so obtained has the following properties in the range $[\hat{\pi}, \hat{r}]$:

- $\hat{K}(r)$ is continuous and weakly increases from $\hat{K}(\hat{\pi}) = 0$ to $\hat{K}(\hat{r}) = 1$;
- $\hat{K}(r) \geq K(r)$;
- the density $\hat{k}(\cdot)$ either coincides with $k(\cdot)$, or is equal to 0.

We now show that the function $\hat{K}$ supports a symmetric mixed strategy equilibrium. By construction, in this equilibrium each firm $i$ adopts a pricing policy such that $\rho_i = r_i = r$, distributed according to the c.d.f. $\hat{K}$; in particular:

- no firm chooses a price below $\hat{\pi}$, above $\hat{r}$, or in any interval $[r_i, \tau_i]$.
• in the rest of the range \([\tilde{\pi}, \tilde{\rho}]\), the firms adopt a price \(r\) with a density distribution that coincides with \(k = K'\).

To show that this is indeed an equilibrium, let us suppose that firm \(j\) adopts this strategy and check that its rival \(i\) cannot benefit from deviating. Note that, any deviation such that \(\rho_i > r_i\) is not allowed since it involves below-cost pricing for the disadvantaged product. Suppose first that firm \(i\) deviates and adopts prices such that \(\rho_i < r_i\). Adopting \(r_i > \tilde{r}\) would attract no one-stop shoppers and would thus yield an expected profit equal to \(\rho F (\delta - \rho) \leq \tilde{\pi}\); thus, without loss of generality we can restrict attention to deviations such that \(\rho_i < r_i \leq \tilde{r}\).

Consider first consumers’ response, as a function of the realization \(\rho_j = r_j = \tilde{r}\) of the rival’s price:

• consumers buy both goods from firm \(i\) if:

\[
v_i = w_{AB} - r_i \geq 0, \quad v_i \geq v_j, \quad \text{and} \quad v_i \geq v_{12} = w_{AB} + \delta - \rho - \tilde{r} - s,
\]

that is \((r \leq w_{AB} \text{ and})\):

\[
\tilde{r} \geq r \quad \text{and} \quad s \geq \delta + r - \rho - \tilde{r}.
\]

• consumers engage in multi-stop shopping if:

\[
v_{12} \geq v_i, \quad v_j \geq 0,
\]

that is,

\[
s \leq \delta - \rho \quad \text{and} \quad s \leq \delta + r - \rho - \tilde{r}.
\]
Figure 4 depicts consumers’ response.

Consider now an increase in $\rho_i$:

- it increases the profit achieved on multi-stop shoppers (region $MSS$);
- it transforms some multi-stop shoppers into own one-stop shoppers (transfer from $MSS$ to $OSS_i$), which yields an additional $r - \rho$ on these consumers;
- however, it also transforms some multi-stop shoppers into one-stop shoppers going to the rival (transfer from $MSS$ to $OSS_j$), which yields a loss $\rho$ on these consumers.

The overall impact on firm $i$’s expected profit is thus equal to:

$$
\frac{\partial \pi_i}{\partial \rho_i} = MSS + (r - \rho) \frac{\partial OSS_i}{\partial \rho_i} - \rho \frac{\partial OSS_j}{\partial \rho_i}
$$

$$
= K(r) F(\delta - \rho) + \int_\delta^\tilde{\delta} k(\tilde{r}) F(\delta + r - \rho - \tilde{r}) \, d\tilde{r}
$$

$$
+ (r - \rho) \int_\delta^\tilde{\delta} k(\tilde{r}) f(\delta + r - \rho - \tilde{r}) \, d\tilde{r} - \rho K(r) f(\delta - \rho)
$$

$$
> K(r) (F(\delta - \rho) - \rho f(\delta - \rho)),
$$

which is positive for any $\rho < \tilde{\rho}$, a contradiction.

Consider now a deviation that of the type $\rho_i = r_i$, where $r_i$ is outside the equilibrium range.
• choosing \( r_i < \hat{r} \) yields an expected profit equal to \( r_i \), which is thus lower than \( \pi_i(\hat{r}) = \hat{\pi} \);

• choosing \( r_i > \hat{r} \) attracts no one-stop shoppers; the expected profit thus cannot exceed \( r_i F(\delta - r_i) < \hat{r} F(\delta - \hat{r}) = \max_r r F(\delta - r) \);

• choosing \( r_i \) in one of the intervals \((r_\ell, r_i)\) yields an expected profit (since then \( \hat{K} > K \)):

\[
r \left[ 1 - \hat{K}(r)(1 - F(\delta - r)) \right] < r \left[ 1 - K(r)(1 - F(\delta - r)) \right] = \hat{\pi}.
\]

There is thus no profitable deviation.

**Appendix D: Proof of Proposition 4**

Consider the symmetric equilibrium with \( p_1 = p_2 = p \) and \( q_1 = q_2 = q \), then we have \( \hat{x} = 1/2 \) and \( \lambda_1(\hat{x}) = \tau_1 = \lambda_2(\hat{x}) = \tau_2 = \tau \). Solving for \( p \) and \( q \) yields

\[
q - c = \frac{A \times OSS_1 - C \times MSS}{A(A + 2C)}, \quad \text{and} \quad p - c + \delta = \frac{(A + C) \times MSS + A \times OSS_1}{A(A + 2C)},
\]

where

\[
A \equiv \int_0^{1/2} f(\lambda_1(x))g(x)dx,
\]

\[
C \equiv \frac{1}{2t}g\left(\frac{1}{2}\right)(1 - F(\tau)),
\]

\[
MSS = 2\int_0^{1/2} F(\lambda_1(x))g(x)dx,
\]

\[
OSS_1 = \int_0^{1/2} [1 - F(\lambda_1(x))]g(x)dx.
\]

Notice that \( C \to +\infty \) and \( \lambda_1(x) \to \tau \) when \( t \to 0 \), therefore we obtain

\[
\lim_{t \to 0} q = q^* = c - h(\tau^*),
\]

\[
\lim_{t \to 0} p = p^* = c - \delta + h(\tau^*).
\]

Since \( p \) and \( q \) vary continuously in \( t \), it follows that there exists some \( t^0 > 0 \) such that \( q < c \) for \( t < t^0 \).
References


