"Project de Loi de Nouvelle Organisation du Marché de l’Electricité": Implications for the French Wholesale Market

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Abstract
The "Project de Loi de Nouvelle Organisation du Marché de l’Electricité", known as the NOME law, presents a reform of the French electricity market which consists in redistributing part of the incumbent’s nuclear generation to its competitors. This paper illustrates the effect of such redistribution in the wholesale electricity market equilibria. We find that the use of NOME’s redistributed generation has no procompetitive effect: the system equilibrium price remains at its highest possible value. Profits in the wholesale market of the incumbent firm ceding production are not modified whereas profits of the firm receiving it increase, for every possible equilibrium. We conclude our analysis by defining a simple rule to establish the optimal penalty to avoid firms using NOME’s redistribution in the wholesale market.

Key Words: NOME, electricity markets, market design, pricing behavior, stochastic demand.

JEL Classification: C72, L94

1 Introduction
The "Project de Loi de Nouvelle Organisation du Marché de l’Electricité", known as NOME law, presents a reform of the French electricity market aiming at increasing retail competition. The reform consists in redistributing part of the incumbent’s nuclear generation to its competitors, at a price close to the cost of nuclear development,\(^1\) and to gradually pass from regulated electricity tariffs to competitive retail pricing. The redistributed generation price would be substantially lower than the average wholesale price since the latter of course

\(^1\)This is usually referred to as "the regulated sourcing price for baseload power".
reflects the variable costs of supplying electricity in peak hours, as well as the opportunity cost of exporting to neighboring countries with higher generation costs. To avoid entrants to use the redistributed generation in the wholesale market (instead of serving French electricity consumers at a low price), the NOME law foresees a penalty to ensure that the average positive benefits of arbitrage in the wholesale market is returned to the electricity system.

The previous description underlines the importance of some design issues regarding the NOME law. The project disourages the use of the redistributed generation in the wholesale market, but do entrants have real incentives to arbitrage between the retail and the wholesale market? And if so, which is the optimal penalty that would eliminate this behavior? If entrants use the redistributed generation in the wholesale market, one possible penalty is to ask them to repay the "estimated gains" to the incumbent. This principle is based on the reasoning that those gains constituted a profit transfer from the incumbent to the entrant, but is this really the case?

In this paper we answer the previous questions through the assessment of the impact of NOME's generation redistribution in the wholesale market equilibrium. We model generators strategies in the French wholesale electricity market as a two-stage game: in the first stage, firms choose the production capacity they will bid in the wholesale market including the redistributed generation established by the NOME project and, in the second stage, firms choose their bidding prices. Our main result is that the generation redistribution has no pro-competitive effect in the wholesale market from a static perspective but it increases the less efficient firm’s profits, without reducing the incumbent’s payoff. Moreover we find that, if the incumbent is very large as compared to its competitors before NOME’s implementation, but becomes constrained after the generation redistribution, there are strong incentives for those who benefit from NOME’s redistribution to use that generation in the wholesale market. The previous analysis allows us to define the optimal penalty to avoid firms using the redistributed generation in the upstream segment of the electricity market.

Among the electricity modelling papers, this work falls into the category defined by Ventosa et al. (2005) as "equilibrium models." In particular, our work is part of the electricity models in which the capacity choice is endogenized. Other works in this category encompass, for instance, von der Fehr and Harbord (1997). The authors model a two-stage game and assume that utilities choose investment in a first stage and price in a second one. Le Coq (2002) and Crampes and Creti (2005) follow this type of modelling and show that electricity generators have incentives to withhold capacity to rise the wholesale system equilibrium price. Both papers, like Kreps and Scheinkman (1983), consider a two-stage game where in a first stage agents bid capacities knowing demand, and in the second one they bid prices. However, differently from the

\footnote{See also David and Wen (2000).}
\footnote{See also Castro-Rodriguez et al. (2001) and Reynolds and Wilson (2000) for other examples of the same type of modelling.}
\footnote{See also Crampes and Fabra (2004) for more on this type of wholesale market modelling based on auctions clearing.
previous literature, they model the second stage as a uniform price auction (instead of Bertrand competition) to describe the most common electricity market design. Sanin (2006) adds to Le Coq (2002) and Crampes and Creti (2005) by accounting for a stochastic demand of electricity and comparing price equilibria and collusive incentives under different rationing rules for the auction clearance. Herein, we extend this kind of framework to account for the specificity of the NOME project. In particular, in a first stage firms choose the generation made available without knowing realized demand, in a second stage they choose the bidding price and then the market is cleared by a uniform auction. We analyze how their bidding strategies are modified by the implementation of NOME’s law and which should be the penalty to avoid such modifications.

In Section 2 we state the assumptions and timing of the game. In Section 3 we present our assumptions. In Sections 4 and 5 we present our model and in 6 we present the main implications of the NOME law in the wholesale market. In Section 7 we briefly conclude.

2 The context

2.1 The NOME law

NOME’s project of law\(^5\) is set to be applied from January 2011. It was first adopted by the French Senate (i.e. the "Assemblée Nationale") the 15th of June 2010 and sent for consultation, both to the Economics, Sustainable Development and Territorial Organization Commission\(^6\) and to the Finance Commission.\(^7\) Both commissions have delivered their suggested modifications\(^8\) which should be studied by the Senate in the following months. NOME law gives regulated access to 100 TWh produced yearly by the historical supplier’s nuclear plants\(^9\) (plus 20 TWh from 2013 on). The conditions for regulated access will be determined by the Energy Ministry and the Economics Ministry, advised by the CRE. In addition, the project includes a financial penalty if the nuclear power received by competitors is not used in serving French retail consumers. However, the amount and the way such penalty will be implemented is not defined yet.

As first suggested by the Champsaur Commission\(^10\) in April 2009, the redistribution of low-cost generation is accompanied by a gradual liberalization of the retail price for electricity. From January the 1st, 2011 industrial consumers will no longer benefit from regulated tariffs and from 2015 also consumers with an intensity lower than 36 kVA will stop being served at regulated tariffs. In the

\(^5\) Find it at http://www.assemblee-nationale.fr/13/ta/ta0486.asp
\(^6\) "Commission de l’économie, du développement durable et de l’aménagement du territoire".
\(^7\) "Commission des finances".
\(^8\) See Text n° 644 (2009-2010) delivered the 7th of July 2010, which includes all modifications suggested by both commissions at http://www.senat.fr/legal/plj09-644.html
\(^9\) Nowadays, EdF’s competitors serve approximately 40 TWh.
actual landscape, instead, given that most participants in the retail electricity market are vertically integrated, new entrants in the downstream market supply at higher price as compared to the incumbent retailer Electricité de France (EdF), who benefits from lower cost of nuclear generation. Then, EdF’s competitors are unable to offer retail prices that can compete with regulated tariffs negotiated exclusively by the historical operator. In this regard, Solier (2010) shows that the mean price of base one-year-ahead future contracts supplied by EdF’s competitors is always higher than the regulated tariffs.

As recommended by the senator Sido and the deputy Poignant\textsuperscript{11} in April 2010, the law also plans to create a capacity obligation scheme to ensure the diffusion of efficient investments in base-load and peak generation (\textit{i.e.} smart grids). Finally, the law includes a reform in the local electricity tax in order to comply with the directive\textsuperscript{12} 2003/96/CE.

All in all, the reform would amend the European Commission observations regarding the maintenance of regulated tariffs as state aid to local firms.\textsuperscript{13}

### 2.2 The wholesale electricity market

The French wholesale electricity market represents, in mean, 39\% (225 TWh yearly) of all injections/sustractions of electricity in the system (either over the counter or trough Powernext or European Energy Exchange). From those 225 TWh only 6\% are imported from neighboring countries,\textsuperscript{14} the rest being produced mostly by the historical operator.\textsuperscript{15} The HHI rises to 8.600 in terms of production and its 7.876 in terms of electricity used to satisfy end users.\textsuperscript{16}

Since September 2001 (just four month after the registration of the firsts over-the-counter transactions in the french wholesale electricity market) part of the historical producer’s nuclear capacity was auctioned. This mechanism is called VPP (which stands for Virtual Power Plant) and allow EdF competitors to reserve nuclear capacity for a certain period varying between 3 and 48 months. Auctions take place, in mean, once every three months and the price paid by each auctioneer depends both on the reservation’s duration and on whether it concerns peak or base load.

The capacity reserved by VPP, which represents 7\% (16 TWh yearly) of the wholesale market, may be used or not (generally it will depend on the VPP price as compared to the market price for that same load) and its price is defined by an ascending price auction (in multiple rounds).\textsuperscript{17} Instead, NOME’s redistributed capacity amounts at 100 TWh produced yearly by EdF nuclear

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\textsuperscript{13}For details see http://europa.eu/rapid/pressReleasesAction.do?reference=IP/09/376

\textsuperscript{14}See http://www.cre.fr/fr/marches/marche_de_l_electricite/marche_de_gros#a1

\textsuperscript{15}For more detail on the way those auctions were organized in France as compared to other countries see Ausubel and Crampton (2010).
plants\textsuperscript{18} (almost 50\% of the wholesale market), plus 20 TWh from 2013 on. Besides the quantitative difference, NOME’s law redistribution of nuclear capacity differs from the VPP mechanisms in two important ways: (i) the capacity is not assigned by a market clearing mechanism but as a function of each competitor’s market share in the retail market, at a preestablished regulated price; (ii) the capacity received must be used for production and, if that is not the case, either a penalty or a diminution in next period allocation is implemented by the regulator.

As underlined by Finon (2010) the wholesale price will be most of the time superior to the regulated price of nuclear generation at which competitors pay NOME’s redistributed generation. In what follows, we study if these price differential gives enough incentives to use the redistributed capacity in the wholesale market and the penalty that should be imposed by the regulator to discourage such behavior.

3 Assumptions and timing of the game

We assume there are two generators $G_a$ and $G_b$. It is common knowledge that one of them, lets say $G_a$, has a larger generation capacity ($K_{a\text{ max}} > K_{b\text{ max}}$) and more efficient so the marginal costs satisfy: $c_a < c_b$.

We also assume there exists a price cap fixed by the regulator denoted by $\hat{p} > c_b$. After the introduction of the NOME law a portion $\eta$ of $G_a$’s low cost generation is given to $G_b$ at a price $p_\eta$\textsuperscript{19}. Maximal generation capacities after the introduction of NOME $K_{a\text{ max},\eta}$ and $K_{b\text{ max},\eta}$ are now $K_{a\text{ max},\eta} = (1 - \eta)K_{a\text{ max}}$ and $K_{b\text{ max},\eta} = K_{b\text{ max}} + \eta K_{a\text{ max}}$.

We model generators strategies as a two-stage game: in the first stage, firms choose the production capacity and, in the second stage, firms bid prices.

We proceed by backward induction so the last stage exposed is the one played first. In the second stage of the game, we assume that generators play a capacity constrained game, \textit{i.e.} when bidding a price $p_i$, $i = a, b$, each generator $i$ knows the capacity chosen by its competitor and the level of inelastic demand for the day-ahead market that in the first stage, when firms chose a quantity to bid $K_i$, was unknown.

For each market period, the market is cleared by a uniform price auction organized by the System Operator (henceforth, SO). The marginal price for electricity is the one bidded by the high bidder and both generators are paid this price for the amount of electricity they are called for. The low bidder is dispatched first and the higher bidder serves the residual demand. When two generators bid the same price we assume that proportional rationing is used, that is, each generator $G_i$ is dispatched a quantity equal to $K_i/(K_i + K_j)$.

\textsuperscript{18}Nowadays, EdF’s competitors represent approximately 40 TWh of the French retail market.

\textsuperscript{19}Since both $\eta$ and $p_\eta$ are decided by the regulator we consider $\eta p_\eta K_{a\text{ max},\eta}$ as a fixed cost that does not affect short run decisions.
The day is divided into market periods (loads) but the bid price must be held fixed for more than one market period. This means generators don’t know the actual level of demand when they decide their level of production. We capture this feature by assuming that in the first stage, where generators choose to produce any amount lower than their maximum capacity $K_i \leq K_{i, \text{max}, \eta}$, they do not know the level of demand that can be either high (peak) $D_P$ with probability $h$ or low, $D_n$, with probability $(1-h)$. We assume that only the output effectively produced generates costs and that NOME’s redistributed generation that is not used in the wholesale market is used in the retail market (i.e. there is no opportunity cost). In addition, we assume that $K_{a, \text{max}, \eta} + K_{b, \text{max}, \eta} > D_P$, i.e. shortage can only be provoked by firms. This is discouraged as we consider that the regulator sets a shortage penalty in the form of a fixed fine $S$.

4 Second stage: price game

The choice of the quantity $K_i$ that firms wish to make available in the first stage of the game lead them to one of four possible ex-post demand cases: (a) a case in which each generator can serve the demand alone i.e. low demand case; (b) a case in which one generator can serve the demand alone whereas the other has bid a quantity lower than realized demand; (c) a case in which no generator has bid enough quantity to serve the demand alone.

Case a) Low demand

If once revealed demand is such that $D < K_i; i = a, b$, both generators have bid enough quantity to satisfy all the market by themselves the SO dispatches it.

**Proposition 1** When $K_a > D$ and $K_b + K_{b, \eta} > D$, i.e. none of the generators is capacity constrained, the pair $(p_{a, \text{br}}, p_{b, \text{br}})$ that are a Nash equilibrium of the price subgame are:

i) when $K_{b, \eta} < D$

$$p_{a, \text{br}} = \max(c_b - \varepsilon, c_a) \quad \text{which gives} \quad \Pi_a = (c_b - c_a)D$$

ii) when $K_{b, \eta} > D$

$$p_{a, \text{br}} = c_a \quad \text{which gives} \quad \Pi_a = 0$$

Proof. See Mathematical Appendix.

Since the bigger operator is more efficient ($c_a < c_b$), before NOME redistribution of generation, $G_a$ could win the game trough the threat of undercutting its competitor. After NOME’s redistribution, this is only the case when $K_{b, \eta} < D$. Instead, when $K_{b, \eta} > D$ both firms have symmetric marginal costs and we find
Bertrand's (1883) price competition result, i.e. we have either of the two generators serving total demand at a price $c_a$.

Summary 2 When NOME’s redistribution is enough high and the quantity obtained is used in the wholesale market so that $K_{b,\eta} > D$, the system equilibrium price is equal to the lowest marginal cost $c_a$.

In Section 5 we show that, unfortunately, the previous equilibrium is not a subgame perfect nash equilibrium (SPNE).

Case b) Intermediate demand

If once revealed demand is $K_i < D < K_j$ there exists a continuum of equilibria where the price cap $\hat{p}$ is offered by the firm that bid a higher quantity knowing that the other will offer less to avoid being undercut (See Appendix A). In particular, if the first stage is played such that $K_b + K_{b,\eta} < D < K_a$, it is optimal for $G_b$ to bid a price lower than the price cap and be dispatched first while $G_a$ bids the price cap and is dispatched the residual demand.

If instead the first stage is played such that, even if $K_{a,\max,\eta} > K_{b,\max,\eta}$, $G_a$ chooses to bid a lower quantity than $G_b$ i.e. $K_a^{br} < D < K_b^{br} + K_{b,\eta}^{br}$, it is optimal for $G_a$ to bid a price lower than the price cap and be dispatched first while $G_b$ bids the price cap and is dispatched the residual demand.

Case c) High demand

If once revealed demand is $K_i < D < K_j < K_i + K_j$ we find two sets of equilibria in pure strategies: one in which $G_b$ bids the price cap and serves just residual demand and another one in which $G_a$ is the one that bids the price cap while $G_b$ undercuts this bid and is dispatched first. This is the case as the production of both agents is needed to satisfy demand and therefore both know that they will be called into operation. For a complete characterization of both pure strategy equilibria and the mixed strategy equilibrium, see Crampes-Creti (2005).

5 First stage: capacity game

In the previous section we showed that different price equilibria can be reached depending on the quantities $K_a$ and $K_b$ chosen in this first stage (as compared to the level of demand realized). Herein, we present the case where $K_a$ and $K_b$ are chosen when demand is stochastic.\textsuperscript{20}

Let us define a parameter $\delta = \frac{\hat{p} - c_a}{\hat{p} - c_b} < 1$ that captures the disadvantage of $G_b$ in terms of costs. The condition $\alpha_a > c_b$ on (10) is equivalent to $K_b + K_{b,\eta} < D\delta$ which ensures that bidding the price cap is better than a fight in prices\textsuperscript{21} for $G_a$.

\textsuperscript{20}For a simplified version where quantity declarations are made for a given demand see Appendix A.

\textsuperscript{21}If $K_b + K_{b,\eta} < D\delta$, which is equivalent to $K_b + K_{b,\eta} < D\frac{\hat{p} - c_a}{\hat{p} - c_b}$, multiplying by $(\hat{p} - c_a)$ and substracting $c_a D$ to both sides we get $(\hat{p} - c_a)(D - K_b - K_{b,\eta}) > (c_b - c_a)D$ which we already showed is equivalent to $\alpha_a > c_b$. 

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Similarly, the condition $\beta_a > c_b$ on (12) can be rewritten as $K_a \leq \frac{1}{1-c_b}(D - K_b - K_{b,\eta})$ which ensures that $(\bar{p} - c_a)(D - K_b - K_{b,\eta}) \geq (c_b - c_a)K_a$ is satisfied.\footnote{Similarly, this condition is valid for $K_b$.}

Considering the general case where none of the generators is a priori capacity constrained, i.e., both could serve demand alone ($D_n < D_p < K_{b,\max} + \eta K_{a,\max} < (1-\eta)K_{a,\max}$) the best response functions $K^\text{br}_a$ and $K^\text{br}_b + K^\text{br}_{b,\eta}$ under stochastic demand for each agent $G_a$ and $G_b$ respectively:

$$K^\text{br}_a (K_b + K_{b,\eta}) = \begin{cases} 
K_a > D_p - K_b - K_{b,\eta} & \text{if } K_b + K_{b,\eta} \leq D_n \delta \\
K_a = D - \varepsilon & \text{if } K_b + K_{b,\eta} > D_n \delta 
\end{cases}$$

(3)

$$K^\text{br}_b + K^\text{br}_{b,\eta} = \begin{cases} 
K_b + K_{b,\eta} > D_p - K_a & \text{if } K_a < D_n \\
K_b + K_{b,\eta} = D_n \delta - \varepsilon & \text{if } K_a \geq D_n 
\end{cases}$$

(4)

Then, the intersection between the best replies gives the equilibria described in the following lemma.

**Proposition 3** There are three families of equilibria in the case of ex-ante low stochastic demand:

i) $(K^*_a, K^*_b + K^*_b)$ s.t. $K^*_a < D_n, K^*_b + K^*_b, \eta \geq D_n \delta$,

ii) $K^*_a \geq D_n, K^*_b + K^*_b, \eta = D_n \delta - \varepsilon$,

iii) $K^*_a = D_n - \varepsilon, K^*_b + K^*_b, \eta \geq D_n \delta$

**Proof.** See Mathematical Appendix. \qed

Each of these families of equilibria\footnote{Note that to be in ii) it must be the case that $\eta K_{a,\max} \geq D_n \delta - K^\text{br}_b - \varepsilon$ and to be in iii) it must be the case that $\eta K_{a,\max} \leq D_n \delta - K^\text{br}_b$.} in the first-stage-game lead generators to a different outcome in the next step of the game. Comparing the payoffs for each possible realization of demand we find that:

**Corollary 4** Type ii equilibria is preferred by $G_b$ to type i and iii in that order, and type iii is preferred by $G_a$ to type i and ii in that order:

$$ii \succ b \succ i \succ iii$$

(6)

$$iii \succ a \succ i \succ ii$$

**Proof.** See Mathematical Appendix. \qed
mentioned in the proof of (5), it is true that: $D_p < (1 + \delta)D_n$. When demand is $D_n$, in type $i$ equilibria the point where $K_a^* + K_b^* + K_{b,\eta}^* = D$ is satisfied cannot be reached since firms always play $K_a^* + K_b^* + K_{b,\eta}^* \geq D_p$ to avoid the shortage penalty $S$. When demand is low both get lower profits than when demand is high. None of the three families of equilibria in (5) can be discarded by a Pareto dominance criterion.

If the smaller generator is capacity constrained, as long as $D_n < K_b^{max} + K_a^{max} < D_p$, the equilibria described in (5) are still possible. Instead, if the capacity constraint of the smaller generator is so strong that $K_b^{max} + K_a^{max} < D_n$, the only family of equilibria possible is:

**Corollary 5** If the capacity constraint of the smaller generator is binding to the extent that $K_b^{max} + \eta K_a^{max} < D_n, \delta$, the only possible equilibrium is:

$$\begin{align*}
(K_a^*, K_b^* + K_{b,\eta}^*) &= (7) \\
\{K_a^*, K_b^* + K_{b,\eta}^* \text{ s.t. } K_a^* &\leq D_n, K_b^* + K_{b,\eta}^* \leq K_b^{max} + \eta K_a^{max}, (8) \\
K_a^* + K_b^* + K_{b,\eta}^* &\geq D_p\}
\end{align*}$$

Finally, if both generators are capacity constrained, i.e. $(1 - \eta)K_a^{max} < D_n < D_p < K_a^{max} + K_b^{max}$, the equilibria described in (5) are reachable as long as $K_a^*$ and $K_b^* + K_{b,\eta}^*$ are lower than $D_p$ but still higher than $D_n$. If it is not the case, the only possible equilibria is:

**Corollary 6** If $K_b^{max} + \eta K_a^{max} < (1 - \eta)K_a^{max} < D_n$ the equilibria becomes:

$$\begin{align*}
(K_a^*, K_b^*) &= \{K_a^*, K_b^* + K_{b,\eta}^* \text{ s.t. } K_a^* &\leq (1 - \eta)K_a^{max}, (9) \\
K_b^* + K_{b,\eta}^* &\leq \min(D_n, K_b^{max} + \eta K_a^{max}), \\
K_a^* + K_b^* + K_{b,\eta}^* &\geq D_p\}
\end{align*}$$

6 The effect of NOME’s redistribution

NOME’s nuclear generation redistribution could have two effects in the wholesale market, depending on the strength of the redistribution as well as on the situation regarding the relative size of competitors when the redistribution takes place. If before NOME the incumbent’s competitor was not only less efficient, but also very small as compared to the incumbent, NOME’s redistribution amplifies the typology of subgame perfect equilibria. In fact, the capacity endowment given to the competitor could make it possible to pass from a situation where the only family of equilibria possible is (7) to a situation where all three families of equilibria described in (5) are possible. On the contrary, if the incumbent was not very large as compared to its competitors, NOME redistribution could constraint its capacity to an extent that equilibria are restricted to a situation like the one described in (9).

The case of a capacity constrained competitor is particularly interesting: if the incumbent’s competitors are small in relation to the off peak demand $D_n$,
they have incentives to use NOME’s redistributed generation in the wholesale market. This strategy is all the more profitable when it allows generator $G_b$ to pass from an equilibria type $i$ to an equilibria type $ii$ where it is dispatched first, making higher profits. However, such behavior does not result in lower wholesale prices. Then, the regulator is interested in fixing a penalty $P$ to avoid the use of the redistributed capacity in the wholesale market. It would be enough for that penalty to be effective to equalize, for each play, the profits after the use of NOME’s redistributed generation to the profits without using such quantity.

**Proposition 7** The use of NOME’s generation in the wholesale market has no (static) procompetitive effect since the system equilibrium price is the price cap $\hat{p}$.

**Corollary 8** $G_b$’s profits increase in all possible equilibria; $G_a$’s profits are kept unchanged unless $K_a^* = (1 - \eta)K_{a\max}$.

**Corollary 9** The regulator can avoid the use of NOME’s redistributed generation in the wholesale market by fixing a penalty equal to $P = (c_b - c_a)K_{b\eta}$.

According to our analysis, the penalty should not be repaid to $G_A$, since the incumbent still makes the same profits in the wholesale market as before NOME’s redistribution, unless the equilibrium where he is capacity constrained realizes (i.e. when $K_a^* = (1 - \eta)K_{a\max}$). Regarding the penalty, it is sufficient to pay the difference between marginal costs to discourage firms to use NOME generation in the wholesale market.

## 7 Conclusion

In this paper we have shown the effect of using NOME’s redistributed generation in the wholesale electricity market as well as the incentives for firms to do so. We find that the use of NOME’s redistributed generation has no procompetitive effect: the system equilibrium price is still the highest possible in a context with inelastic demand. Profits in the wholesale market of the incumbent firm ceding part of its generation are not modified whereas profits of the firm receiving NOME generation increase, for every possible equilibrium. In particular, if the redistribution leverages the capacity restrictions of small generators, some families of equilibria that could not be attained before the redistribution are now attainable. Since these equilibria are very favorable in terms of profits for the firm receiving the low-cost generation, we derive a simple rule to establish the optimal penalty to avoid firms using NOME’s redistribution in the wholesale market.

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24 The increase in profits when passing from an equilibria type $i$ to an equilibria type $ii$ when using NOME’s redistributed generation in the wholesale market can be considered as a dynamic procompetitive effect.
8 References


9 Appendix A

9.1 Second-stage: price equilibria

Herein we show the price equilibria for each of the cases mentioned in the main text.

9.1.1 Case b) Intermediate demand

**Solution 10** When demand is lower than the largest quantity declared, if \( g_a \) (i.e. the most efficient generator) is the one with the capacity advantage, that is \( K_{br} + K_{b,\eta} < D < K_{br} \), there are two possible families of equilibria for \((p_{br}^a, p_{br}^b)\):

\[
\begin{align*}
p_{br}^a &= \hat{p} \quad \text{which gives} \quad \Pi_a = (\hat{p} - c_a)(D - K_b - K_{b,\eta}) \quad (10) \\
p_{br}^b &\in [0, \alpha_a] \quad \text{which gives} \quad \Pi_b = (\hat{p} - c_a)K_{b,\eta} + (\hat{p} - c_b)K_b \\
\alpha_a &= c_a + (\hat{p} - c_a)\frac{(D - K_b - K_{b,\eta})}{D} \quad \text{where } \alpha_a > c_b
\end{align*}
\]

b) Equilibria defined in Case a if \( \alpha_a < c_b \) where \( \alpha_a \) is the threshold that ensures \( g_b \) to sell all its production when \( g_a \) bids at the price cap.

**Proof.** See Mathematical Appendix.

If \( K_{br} < D < K_{br} + K_{b,\eta} \), instead of the two families of equilibria described in the previous lemma we have only one family of equilibria symmetric to the one described in (10).

**Solution 11** When demand is lower than the larger capacity declared, if \( g_b \), the less efficient generator, is the one with the capacity advantage, that is \( K_{br} < D < K_{br} + K_{b,\eta} \), there is only one possible family of equilibria for \((p_{br}^a, p_{br}^b)\):

\[
\begin{align*}
p_{br}^b &= \hat{p} \quad \text{which gives} \quad \Pi_b = (\hat{p} - c_a)(D - K_a) \quad \text{if } D - K_a < K_{br} \\
p_{br}^b &= (\hat{p} - c_a)K_{b,\eta} + (\hat{p} - c_b)(D - K_a - K_{br}) \quad \text{otherwise} \quad (11) \\
p_{br}^a &\in [0, \alpha_b] \quad \text{which gives} \quad \Pi_a = (\hat{p} - c_a)K_a \\
\alpha_b &= (\hat{p} - c_a)\frac{K_a}{D}
\end{align*}
\]

**Proof.** See Mathematical Appendix.

9.1.2 Case c) High demand

If once revealed demand is \( K_i < K_j < D < K_i + K_j \) we find two sets of equilibria in pure strategies.
Solution 12: If the pair \((K_a^{br}, K_b^{br} + K_{b,\eta}^{br})\) chosen in the first stage determines that \(K_a^{br} < K_j^{br} < D\), we find two set of equilibria in pure strategies for the pair \((p_a^{br}, p_b^{br})\):

\[
\begin{align*}
p_a^{br} &= \hat{p} \quad \text{which gives} \quad \Pi_a = (\hat{p} - c_a)(D - K_b - K_{b,\eta}) \\
p_b^{br} &\in [0, \beta_a] \quad \text{which gives} \quad \Pi_b = (\hat{p} - c_a)K_{b,\eta} + (\hat{p} - c_b)K_b \\
\beta_a &= c_a + (\hat{p} - c_a)\frac{(D - K_b - K_{b,\eta})}{K_a} \quad \text{where} \ \beta_a > c_b
\end{align*}
\]

where \(\beta_a\) is the threshold for \(p_b^{br}\) that ensures that \(G_a\) is better off bidding the price cap \(\hat{p}\).

\[
\begin{align*}
p_b^{br} &= \hat{p} \quad \text{which gives} \quad \Pi_b = (\hat{p} - c_a)(D - K_a) \quad \text{if} \quad D - K_a < K_{b,\eta}^{br} \\
p_b^{br} &= (\hat{p} - c_a)K_{b,\eta} + (\hat{p} - c_b)(D - K_a - K_{b,\eta}^{br}) \quad \text{otherwise}
\end{align*}
\]

with \(\beta_b = c_a + (\hat{p} - c_a)\frac{(D - K_a)}{K_{b,\eta}^{br}}\) and \(\frac{(\hat{p} - c_b)}{K_{b,\eta}^{br}} < \frac{c_b - c_a}{(D - K_a)}\)

Proof. See Mathematical Appendix. ■

If \(\beta_a < c_b\), \(b)\) is the unique set of equilibria in pure strategies. When \(\beta_a > c_b\), none of this two set of equilibria can be discarded by a Pareto dominance criterion as both generators would prefer to be the lower bidder in order to sell all the capacity they made available. Larson and Salant (2003) shows that in the case of uniform auctions where prices can vary continuously\(^{25}\) and all the players know they must be called into operation, is more likely that generators play in mixed strategies. We characterize the equilibrium in mixed strategies in which, for a given strategy of the competitor, each generator is indifferent between all the prices over which it randomizes.

Solution 13: If the pair \((K_a^{br}, K_b^{br} + K_{b,\eta}^{br})\) chosen in the first stage determines that \(K_i^{br} < K_j^{br} < D\) and \(\beta_a > c_b\), we find the following mixed strategies equilibrium for \(p_j^{br}\):

\[
p_j^{br} \sim F_i(p_i) \quad \text{on} \quad [c_j, \hat{p}] \tag{14}
\]

where \(F_i(p_i)\) describes the cumulative distribution of probabilities for \(G_i\) bids in the support \([c_j, \hat{p}]\)

Proof. See Mathematical Appendix. ■

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\(^{25}\)This implies that infinitesimal undercutting is possible.
9.2 First stage: capacity choice for ex-ante low deterministic demand

In this subsection we analyze the case where (i) generators know the level of demand before deciding their level of production and, (ii) none of them is a priory capacity constrained, i.e. \( D < K_{b,\max} + \eta K_{a,\max} < (1-\eta)K_{a,\max} \).

In this case the best response functions in the capacity game can be obtained as:

**Solution 14** The best response function of \( G_a \) in the case of deterministic low demand is:

\[
K_a^{br}(K_b + K_{b,\eta}) = \begin{cases} 
K_a > D - K_b - K_{b,\eta} & \text{if } K_b + K_{b,\eta} \leq D \delta \\
K_a = D - \varepsilon & \text{if } K_b + K_{b,\eta} > D \delta
\end{cases}
\]

where \( \varepsilon > 0 \) tends to zero.

**Proof.** See Mathematical Appendix.

**Solution 15** The best response function of \( G_b \) in the case of deterministic low demand is:

\[
K_b^{br} + K_{b,\eta}^{br} = \begin{cases} 
K_b + K_{b,\eta} > D - K_a & \text{if } K_a < D \\
K_b + K_{b,\eta} = D \delta - \varepsilon & \text{if } K_a \geq D
\end{cases}
\]

**Proof.** The best response for \( G_b \) is derived computing \( \Pi_b \) in each of the cases in the previous lemma.

Then, the equilibria are:

**Solution 16** There are three families of equilibria in the case of deterministic low demand:

i) \((K_a, K_b + K_{b,\eta}) = \)

\[\{K_a, K_b + K_{b,\eta} \ s.t. \ K_a < D, K_b + K_{b,\eta} \leq D \delta, K_a + K_b + K_{b,\eta} \geq D\}\]

ii) \(K_a \geq D, K_b + K_{b,\eta} = D \delta - \varepsilon\)

iii) \(K_a = D - \varepsilon, K_b + K_{b,\eta} \geq D \delta\)

**Proof.** The equilibrium can be easily found by the intersection of the best reply functions.

10 Mathematical Appendix

10.1 Proof of Proposition 1

From Bertrand (1883) we know that price competition in a duopoly with symmetric firms gives the competitive outcome. If firms are not symmetric, the one with the lowest marginal costs will undercut the other one’s bid for every play.
10.2 Proof of Solution 9

a) The subindex $a$ in $\alpha_a$ is to underline the fact that this threshold is determined comparing the profits for $G_a$ in (3) with the ones he could receive if he undercuts $G_b$’s bid and serve all the demand by himself: assume that $G_a$ bids $\alpha_a$. In this case $\Pi^a_\alpha = (\alpha_a - c_a) D$ that after substituting $\alpha_a$ by its value becomes $\Pi^a_\alpha = (\hat{p} - c_a)(D - K_b - K_{b,\eta})$ that is equal to the profit he would obtain by bidding the price cap. But in order to undercut $G_b$’s bid and compete for the full demand he should offer his capacity at a price lower than $\alpha_a$ as the interval is closed in this value. This would be a dominated strategy as he would receive less profits than bidding the price cap. That is why the best response of $G_a$ when $G_b$ bids a price between $[0, \alpha_a]$ is to bid the price cap $\hat{p}$ and serve just the residual demand.

b) The threshold $\alpha_a$ is increasing with the residual demand $(D - K_b - K_{b,\eta})$. If this demand is sufficiently low, that is, if $\alpha_a < c_b$ we fall in the equilibria of Case a. If $G_a$ observes this relation between $\alpha_a$ and $c_b$ he will undercut $G_b$’s bid $(c_b)$ as by the definition of $\alpha_a$, this relation between parameters means: $(\hat{p} - c_a)(D - K_b - K_{b,\eta}) < (c_b - c_a) D$.

The fact that $G_b$ bids a price between $[0, \alpha_a]$ is credible only if that gives him positive profits, which is the case as long as $\alpha_a - c_b > 0$. Otherwise profits of $G_b$ are negative when bidding $[0, \alpha_a]$ making this bidding strategy not credible and equilibria is the one described in Solution 1.

10.3 Proof of Solution 10

If $D - K_a < K_{b,\eta}$ then $G_b$ is indifferent between bidding $\pi_b = (\hat{p} - c_a)(D - K_a)$ and $\pi_b = (\alpha^u_b - c_a) D$ if

$$\alpha^u_b = \hat{p} - \frac{(\hat{p} - c_a) K_a}{D} = (\hat{p} - c_a) \frac{(D - K_a)}{D} + c_a$$

where $G_b$ bids the price cap \hspace{1cm} (18)

Then, for any price bid by $G_a$ lower than $\alpha^u_b$ he will prefer to bid the price cap.

If $D - K_a > K_{b,\eta}$ then $G_b$ is indifferent between bidding $\pi_b = (\hat{p} - c_a) K_{b,\eta} + (\hat{p} - c_b)(D - K_a - K_{b,\eta})$ and $\pi_b = (\alpha_b - c_a) K_{b,\eta} + (\alpha_b - c_b)(D - K_{b,\eta})$ if

$$\alpha^d_b = \hat{p} - \frac{(\hat{p} - c_b) K_a}{D}.$$ \hspace{1cm} (19)

For any price bid by $G_a$ lower than $\alpha^d_b$ he will prefer to bid the price cap.

Since $(\hat{p} - c_a) > (\hat{p} - c_b)$, we see that $\alpha^u_b < \alpha^d_b$ and therefore, if $G_a$ bids $\alpha^u_b$ then $G_b$ prefers to bid the price cap.

It is a credible strategy for $G_A$ to bid $\alpha^u_b$ instead of fighting for prices since $(\hat{p} - c_a) K_a - \frac{(\hat{p} - c_a) K_a}{D} K_a \geq (c_b - c_a) D$ and also $\left(\hat{p} - \frac{(\hat{p} - c_a) K_a}{D} - c_a\right) K_a \geq 0$ since $1 \geq \frac{K_a}{\hat{p}}$. 

15
10.4 Proof of Solution 11

a) The proof follows the same line as the previous proposition: if \( G_a \) chooses a price lower than \( \beta_a \) he would sell all the capacity declared \( K_a \). In this case he could get at most \( \Pi_a^b = (\beta_a - c_a)K_a = (\bar{p} - c_a)(D - K_b - K_{b,n}) \) given the definition of \( \beta_a \). Otherwise, if \( p_{br}^b > \beta_a \), \( G_a \) would bid any price below: \( p_{br}^b > p_{br}^c > \beta_a \) as in this way he would sell all the capacity declared earning more than bidding the price cap with

\[
\beta_b^u = (\bar{p} - c_a)\frac{(D - K_a)}{K_{br,b,n}} + c_a \quad \text{when } G_b \text{ bids the price cap} \quad (20)
\]

or

\[
\beta_b^d = (D - K_a)\frac{(\bar{p} - c_b)}{K_{br,b,n}} + c_b \quad (21)
\]

It is the case that \( \beta_b^u < \beta_b^d \) if \( (D - K_a)(\bar{p} - c_a) + c_a < (D - K_a)\frac{(\bar{p} - c_b)}{K_{br,b,n}} + c_b \)

i. e. \( \frac{(\bar{p} - c_a)}{K_{br,b,n}} - \frac{(\bar{p} - c_b)}{K_{br,b,n} + K_b} < \frac{(\bar{p} - c_a)}{(D - K_a)} \).

Bidding \( \beta_b^u \) is a credible strategy for \( G_A \) since \( (D - K_a)\frac{(\bar{p} - c_a)}{K_{br,b,n}} K_a \geq 0 \).

10.5 Proof of Solution 12

We proof the statement in relation to \( G_a \) as the solution in relation to \( G_b \) is symmetric. For any given strategy of \( G_b \), that is, for a distribution of probabilities \( f_b \) played by \( G_b \), \( G_a \) is indifferent between all the prices over which he randomizes, i.e. any price belonging to \( [p_{a_{\min}},\bar{p}] \). To find \( f_b \) we maximize the expected profits of \( G_A \) with respect to \( p_a \):

\[
E(\Pi_a(p_a, f_b(p_b))) = \int_{p_{a_{\min}}}^{\bar{p}} \Pi_a(u)f_b(u)du \quad (22)
\]

\[
\int_{p_{a_{\min}}}^{p_a} (p_a - c_a)(D - K_b - K_{b,n}) f_b(u)du + \int_{p_a}^{\bar{p}} \Pi_a(u)f_b(u)du =
\]

\[
= (p_a - c_a)(D - K_b - K_{b,n}) F_b(p_a) + \int_{p_a}^{\bar{p}} \Pi_a(u)f_b(u)du
\]

The F.O.C. is then obtained by deriving:

\[
\frac{\partial E(\Pi_a(p_a, f_b(p_b)))}{\partial p_a} = 0 \quad (23)
\]

\[
(D - K_b - K_{b,n}) [(p_a - c_a)f_b(p_a) + F_b(p_a)] + (\bar{p} - c_a)K_a f_b(\bar{p}) - (p_a - c_a)K_a f_b(p_a) = 0
\]

\[
((D - K_b - K_{b,n}) - K_a)(p_a - c_a)f_b(p_a) + (D - K_b - K_{b,n}) F_b(p_a) = 0
\]

as \( f_b(\bar{p}) = 0 \) given that punctual value of any continuous density function is zero.
The solution of the differential equation obtained gives us the distribution we were looking for:

\[ F_b(p_b) = \left[ \frac{p_b - c_a}{A_b} \right]^{\gamma_b} \quad \text{where} \quad \gamma_b = \frac{(D - K_b - K_{b,\eta})}{(K_a + K_b + K_{b,\eta} - D)} \]  

where \( A_b = \hat{p} - c_a \), the value in (22) of \( p_{b,\min} = c_a \). It can be determined taking into account that \( F_b(p_b) \) accumulates all the probability between \([p_{b,\min}, \hat{p}]\) which means that \( F_b(\hat{p}) = \left( \frac{\hat{p} - c_a}{A_b} \right)^{\gamma_b} = 1 \). Similarly, as the minimum value of \( F_b(p_b) \) is zero it must be the case that the minimum value for \( p_{b,\min} = c_a \). Also, \( \gamma_b > 0 \) when \( K_b < D \). Of course \( D < K_a + K_b + K_{b,\eta} \) as the case of rationing is described in the following section.

Then, the expected profits of \( G_a \) for any play of \( G_b \) can be found:

\[
\int_{p_a}^{\hat{p}} \Pi_a(u)f_b(u)du = (\hat{p} - c_a)(D - K_b - K_{b,\eta}) - (p_a - c_a)(D - K_b - K_{b,\eta}) \left[ \frac{p_b - c_a}{\hat{p} - c_a} \right]^{\left( \frac{D - K_b - K_{b,\eta}}{K_a + K_b + K_{b,\eta} - D} \right)}
\]

That is:

\[
E(\Pi_a(p_a, f_b(p_b))) = (\hat{p} - c_a)(D - K_b - K_{b,\eta})
\]

As required, is independent from \( p_a \); this is the expected profits for \( G_a \) from playing, in this high demand case, any price belonging to \([c_a, \hat{p}]\).

Using the same reasoning the expected profits that \( G_b \) would derive from bidding any price belonging to \([c_b, \hat{p}]\) for any play of \( G_a \) would be:

\[
E(\Pi_b(p_b, f_a(p_a))) = (\hat{p} - c_b)(D - K_a)
\]

Equations (26) and (27) characterize the profit derived from the equilibrium in mixed-strategies when demand is higher than the larger capacity declared available: \( K_i < K_j < D < K_i + K_j \).

### 10.6 Proof of Solution 13, 14 and 15

1. Best reply functions

First we derive the best reply functions of \( G_a \) for any play of \( G_b \) relying on the following graph:

---

26Is easy to verify that (24) is the solution for the last row of (23). Rearranging to find \( \frac{\partial F_b(p_b)}{\partial p_b} \) in function of all the other parameters we can derive \( \frac{\partial F_b(p_b)}{\partial p_b} = \frac{\gamma_b}{A_b} \left( \frac{p_b - c_a}{A_b} \right)^{\gamma_b - 1} \) and substituting its value in the differential equation we find \( \frac{\partial F_b(p_b)}{\partial p_b} = \frac{A_b}{A_b} \gamma_b (p_b - c_a)^{-1} \) that is exactly what the third row of (23) describes.
We analyze the play of $G_a$ for each play of $G_b$ in each of the zones delimited in Figure 1.

**Zone i:**
$G_a$ thinks that $G_b$ will make available $D \leq K_b + K_{b,\eta}$. Then $G_a$ could play:

i.1) $K_a < D$ where we would be in the equilibrium defined in Case b: intermediate demand when $G_b$ has the advantage in capacity, i.e.:
\[ \Pi_a = (\hat{p} - c_a)K_a \]

i.2) $K_a \geq D$ (includes the vertical line) where we would be in Case a: low demand.

The best response of $G_a$ in this zone is $K_a < D$.

**Zone ii:**
$G_a$ thinks that $G_b$ will make available $D \delta < K_b + K_{b,\eta} < D$. Then $G_a$ could play:

ii.1) $K_a < D - K_b - K_{b,\eta}$ where we would be in Case d: not served demand.

ii.2) $D - K_b - K_{b,\eta} \leq K_a < \frac{1}{1-\delta}(D - K_b - K_{b,\eta})$ (includes line) where we would be in the equilibrium in mixed strategies defined in Case c: high demand.

ii.3) $\frac{1}{1-\delta}(D - K_b - K_{b,\eta}) < K_a < D$ where we would be in Case b: intermediate demand where in most of the area $\Pi_a = (\hat{p} - c_a)K_a$ and at worst\[^{27}\]
\[ \Pi_a = (\hat{p} - c_a)(D - K_b - K_{b,\eta}). \]

ii.4) $D \leq K_a$ (includes line) where we would be in the equilibrium in Case a: low demand.

The best response of $G_a$ in this zone is $\frac{1}{1-\delta}(D - K_b - K_{b,\eta}) < K_a < D$.

Line:
$G_a$ thinks that $G_b$ will make available $D \delta = K_b + K_{b,\eta}$. Then $G_a$ could play:

1) $K_a < D$ where we would be in Case d: not served demand like in ii.1 and iii.1. or for $K_a$ that tends to $D$ we get to Case c: high demand earning at

\[^{27}\]As most of the area is over the 45 degree line drawn in grey.
most \( \Pi_a = (\hat{\rho} - c_a)(D - K_b - K_{b,\eta}) \). Then, \( K_a = D \) belongs to the following case so profits are equal to the ones obtained in zone ii.2.

2) \( D \leq K_a \) where we would be in Case a: low demand.

The best response of \( G_a \) in this line is \( K_a < D \).

Zone iii:

\( G_a \) thinks that \( G_b \) will make available \( K_{b,\eta} < D \). Then \( G_a \) could play:

iii.1) \( K_a < (D - K_b - K_{b,\eta}) \) where we would be in Case d: not served demand.

iii.2) \( (D - K_b - K_{b,\eta}) \leq K_a < D \) where we would be in the equilibrium in mixed strategies defined in Case c: high demand where in most of the area \( \Pi_a = (\hat{\rho} - c_a)(D - K_b - K_{b,\eta}) \) and at most \( \Pi_a = (\hat{\rho} - c_a)K_a \) could be obtained to the left of the grey line.

iii.3) \( D < K_a \) where we would be in the equilibrium in Case b: intermediate demand where \( K_a \) has the capacity advantage.

The best response of \( G_a \) in this zone is \( (D - K_b - K_{b,\eta}) < K_a \).

The bold numbers and the thick line in Figure 1 are the best response of \( G_a \) for any play of \( G_b \) when both players know the level of demand before deciding about capacity.

The best response for \( G_b \) is derived computing \( \Pi_b \) for each zone as we just did for \( G_a \).

2. Intersection between best reply functions.

Intersecting both best response functions we find these three families of equilibria that constitute the shaded area in the following figure. This area is the intersection between the fully grey area that represents the best response of \( G_a \) and the dotted area that represents the best response of \( G_b \).

![Figure 2. Intersection between best reply functions.](image)

\(^{28}\)As we can see \( K_a = D \) belongs to the second case so profits are equal to the ones obtained in zone ii.2.
Is worth noting that none of the equilibria Pareto dominates the others, as each generator is better-off when it is dispatched first and then paid the price cap offered by the other generator. We will go back to this argument after discussing the case we are most interested in: the analysis given stochastic demand.

10.7 Proof of equations (3) and (4)

Now that demand can be peak $D_p$ and off peak $D_n$, with a certain probability $h$ and $(1 - h)$ respectively, the payoffs will be the ones delimited by the regions described in Figure 3.

![Figure 3. Profits and demand uncertainty.](image)

The complete reasoning is described hereafter.

If $G_a$ thinks that $G_b$ will make available $D_p < K_b + K_{b,n}$. Then $G_a$ could play:

a) $K_a > D_p$ where he would earn $\Pi_a = \Pi_a(i.2)$ for sure, where $\Pi_a(i.2)$ is the profit he received in Figure 1 from playing in zone $i.2$.

b) $D_n < K_a < D_p$ where he would earn $E(\Pi_a) = h\Pi_a(i.2) + (1 - h)\Pi_a(i.1)$

c) $K_a < D_n$ where he would earn $\Pi_a = \Pi_a(i.1)$ for sure.

The best response of $G_a$ is $K_a < D_n$.

If $G_a$ thinks that $G_b$ will make available $D_p, D_p < K_b + K_{b,n} < D_p$. Then $G_a$ could play:

d) $K_a > D_p$ where he would earn $E(\Pi_a) = h\Pi_a(i.4) + (1 - h)\Pi_a(i.2)$.

e) $\frac{1}{1-h}(D_p - K_a - K_{b,n}) < K_a < D_p$ and $D_n < K_a$ where he would earn $E(\Pi_a) = h\Pi_a(i.3) + (1 - h)\Pi_a(i.2)$

f) $D_n < K_a < \frac{1}{1-h}(D_p - K_b - K_{b,n})$ where he would earn $E(\Pi_a) = h\Pi_a(i.2) + (1 - h)\Pi_a(i.1)$

g) $\frac{1}{1-h}(D_p - K_b - K_{b,n}) < K_a < D_n$ where he would earn $E(\Pi_a) = h\Pi_a(i.3) + (1 - h)\Pi_a(i.1)$

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h) \((D_p - K_b - K_{b,\eta}) < K_a < \frac{1}{1-h}(D_p - K_b - K_{b,\eta})\) and \(K_a < D_n\) where he would earn \(E(\Pi_a) = h\Pi_a(i.2) + (1-h)\Pi_a(i.1)\)

i) \(K_a < D_p - K_b - K_{n,\eta}\) where he would earn \(E(\Pi_a) = h\Pi_a(ii.1) + (1-h)\Pi_a(ii.1)\)

The best response of \(G_b\) in this zone is \(\frac{1}{1-h}(D_p - K_b - K_{b,\eta}) < K_a < D_n\).

If \(G_a\) thinks that \(G_b\) will make available \(D_n < K_b + K_{b,\eta} < D_p\). Then \(G_a\) could play:

j) \(K_a > D_p\) where he would earn \(E(\Pi_a) = h\Pi_a(iii.3) + (1-h)\Pi_a(i.2)\).

k) \(D_n < K_a < D_p\) where he would earn \(E(\Pi_a) = h\Pi_a(iii.2) + (1-h)\Pi_a(ii.4)\).

l) \(D_p - K_b - K_{n,\eta} < K_a < D_n\) where he would earn \(E(\Pi_a) = h\Pi_a(iii.2) + (1-h)\Pi_a(ii.3)\)

m) \(K_a < D_p - K_b - K_{n,\eta}\) where he would earn \(E(\Pi_a) = h\Pi_a(iii.1) + (1-h)\Pi_a(ii.1)\)

The best response of \(G_b\) in this zone is \(D_p - K_b - K_{b,\eta} < K_a < D_n\).

If \(G_a\) thinks that \(G_b\) will make available \(\Pi_a \Pi_a(iii.3)\). Then \(G_a\) could play:

v) \(K_a > D_p\) where he would earn \(\Pi_a = \Pi_a(iii.3)\).

w) \(D_p - K_b - K_{b,\eta} < K_a < D_p\) where he would earn \(E(\Pi_a) = h\Pi_a(ii.2) + (1-h)\Pi_a(iii.3)\)

x) \(D_p - K_b - K_{n,\eta} < K_a < D_n\) where he would earn \(\Pi_a = \Pi_a(ii.2)\)

y) \(D_n < K_a < D_p - K_b - K_{n,\eta}\) where he would earn \(E(\Pi_a) = h\Pi_a(iii.1) + (1-h)\Pi_a(iii.3)\)

z) \(D_n - K_b - K_{n,\eta} < K_a < D_a\) and \(K_a < D_p - K_b - K_{b,\eta}\) where he would earn \(E(\Pi_a) = h\Pi_a(iii.1) + (1-h)\Pi_a(ii.3)\)

The best response of \(G_a\) in this zone is \(D_p - K_b - K_{b,\eta} < K_a\).
10.8 Proof Proposition 2

Intersecting both best response functions we find the families of subgame perfect equilibria for the case of stochastic demand distributed between $D_n$ and $D_p$. These equilibria constitute the shaded grey area in the following figure that is the intersection between the black area that represents the best response of $G_a$ and the dotted area that represents the best response of $G_b$.

To ensure the existence of type $i$ equilibria (the triangular part of the grey area) we assume that the distance between $D_n$ and $D_p$ is such that $(1 + \delta)D_n > D_p$. Otherwise only equilibria $ii$ and $iii$ would be possible as the triangular area in the figure would disappear. It is a strict inequality because if it was the case that $(1 + \delta)D_n = D_p$, the constraint $K_a + K_b \geq D_p$ could never be satisfied as the equilibrium is defined for $K_a < D_n$.

10.9 Proof of Lemma 12

Type $ii$ equilibria lead us to Case $c$ or, if $K_a > D_p$, to Case $b$ part a\(^29\) where $G_a$ has the capacity advantage. Then, type $ii$ equilibria when demand is $D_p$ implies the following payoffs:

$$\Pi_a = (\hat{p} - c_a)(D_p - \delta D_n)$$
$$\Pi_b = (\hat{p} - c_b)D_n \delta$$

Symmetrically, type $iii$ equilibria lead us to the same Case $c$ or Case $b$ where $G_b$ has the capacity advantage if $K_b > D_p$:

$$\Pi_a = (\hat{p} - c_a)D_n$$
$$\Pi_b = (\hat{p} - c_b)(D_p - D_n) > 0$$

\(^29\)We will never be in part $b$ as $K_b$ cannot be higher than $D_n \delta$ which is equivalent to say that $\alpha_a$ cannot be lower than $c_b$. 

Figure 4. Equilibria of the capacity game.
On the other hand, in type $i$, if demand is $D_p$, we fall in Case $c$ where
\[ E(i) = (\hat{p} - c_i)(D_p - K_j). \]

Using the condition on the distance between $D_n$ and $D_p$ imposed, i.e. $(1 + \delta)D_n > D_p$, we can directly compare the payoffs of each player for each equilibrium and conclude that:

a) when $D_p$ is realized $G_b$ prefers type $ii$ equilibria to type $iii$ as in the first case it gets $\Pi_b = (\hat{p} - c_b)D_n\delta$ for sure while in the second case it gets something lower that tends to the previous $\Pi_b$ when $D_p \to (1 + \delta)D_n$. Moreover, type $i$ is preferred to type $iii$, as in type $i$ he expects to sell $D_p - K_a$ where $K_a < D_n$.

b) when $D_p$ is realized $G_a$ prefers type $iii$ equilibria to type $i$ equilibria that is preferred to type $ii$.

10.10 Proof of Corollary 3

Type $ii$ equilibria lead us to Case $c$ or, if $K_a > D_p$, to Case $b$ part a\textsuperscript{30} where $G_a$ has the capacity advantage. Then, type $ii$ equilibria when demand is $D_p$ implies the following payoffs:

\[ \Pi_a = (\hat{p} - c_a)(D_p - \delta D_n) \quad \text{and} \quad \Pi_b = (\hat{p} - c_b)D_n\delta \]

Symmetrically, type $iii$ equilibria lead us to the same Case $c$ or Case $b$ where $G_b$ has the capacity advantage if $K_b > D_p$:

\[ \Pi_a = (\hat{p} - c_a)D_n \quad \text{and} \quad \Pi_b = (\hat{p} - c_b)(D_p - D_n) > 0 \]

On the other hand, in type $i$, if demand is $D_p$, we fall in Case $c$ where $E(i) = (\hat{p} - c_i)(D_p - K_j)$.

Using the condition on the distance between $D_n$ and $D_p$ imposed in the proof of equation (5), i.e. $(1 + \delta)D_n > D_p$, we can directly compare the payoffs of each player for each equilibrium and conclude that:

a) when $D_p$ is realized $G_b$ prefers type $ii$ equilibria to type $iii$ as in the first case it gets $\Pi_b = (\hat{p} - c_b)D_n\delta$ for sure while in the second case it gets something lower that tends to the previous $\Pi_b$ when $D_p \to (1 + \delta)D_n$. Moreover, type $i$ is preferred to type $iii$, as in type $i$ it expects to sell $D_p - K_a$ where $K_a < D_n$.

b) when $D_p$ is realized $G_a$ prefers type $iii$ equilibria to type $i$ equilibria that is preferred to type $ii$.

A similar reasoning can be applied to derive which family of equilibria is preferred by each agent when $D_n$ is realized.

\textsuperscript{30}We will never be in part $b$ as $K_b$ cannot be higher than $D_n\delta$ which is equivalent to say that $\alpha_a$ cannot be lower than $c_b$. 

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