

"*Project de Loi de Nouvelle Organisation du Marché de l'Electricité*": Implications for the French Wholesale Market

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Abstract

The "*Project de Loi de Nouvelle Organisation du Marché de l'Electricité*", known as the NOME law, presents a reform of the French electricity market which consists in redistributing part of the incumbent's nuclear generation to its competitors. This paper illustrates the effect of such redistribution in the wholesale electricity market equilibria. We find that the use of NOME's redistributed generation has no procompetitive effect: the system equilibrium price remains at its highest possible value. Profits in the wholesale market of the incumbent firm ceding production are not modified whereas profits of the firm receiving it increase, for every possible equilibrium. We conclude our analysis by defining a simple rule to establish the optimal penalty to avoid firms using NOME's redistribution in the wholesale market.

Key Words: NOME, electricity markets, market design, pricing behavior, stochastic demand.

JEL Classification: C72, L94

1 Introduction

The "*Project de Loi de Nouvelle Organisation du Marché de l'Electricité*", known as NOME law, presents a reform of the French electricity market aiming at increasing retail competition. The reform consists in redistributing part of the incumbent's nuclear generation to its competitors, at a price close to the cost of nuclear development,¹ and to gradually pass from regulated electricity tariffs to competitive retail pricing. The redistributed generation price would be substantially lower than the average wholesale price since the latter of course

¹This is usually referred to as "the regulated sourcing price for baseload power".

reflects the variable costs of supplying electricity in peak hours, as well as the opportunity cost of exporting to neighboring countries with higher generation costs. To avoid entrants to use the redistributed generation in the wholesale market (instead of serving French electricity consumers at a low price), the NOME law foresees a penalty to ensure that the average positive benefits of arbitrage in the wholesale market is returned to the electricity system.

The previous description underlines the importance of some design issues regarding the NOME law. The project discourages the use of the redistributed generation in the wholesale market, but do entrants have real incentives to arbitrage between the retail and the wholesale market? And if so, which is the optimal penalty that would eliminate this behavior? If entrants use the redistributed generation in the wholesale market, one possible penalty is to ask them to repay the "estimated gains" to the incumbent. This principle is based on the reasoning that those gains constituted a profit transfer from the incumbent to the entrant, but is this really the case?

In this paper we answer the previous questions through the assessment of the impact of NOME's generation redistribution in the wholesale market equilibrium. We model generators strategies in the French wholesale electricity market as a two-stage game: in the first stage, firms choose the production capacity they will bid in the wholesale market including the redistributed generation established by the NOME project and, in the second stage, firms choose their bidding prices. Our main result is that the generation redistribution has no pro-competitive effect in the wholesale market from a static perspective but it increases the less efficient firm's profits, without reducing the incumbent's payoff. Moreover we find that, if the incumbent is very large as compared to its competitors before NOME's implementation, but becomes constrained after the generation redistribution, there are strong incentives for those who benefit from NOME's redistribution to use that generation in the wholesale market. The previous analysis allows us to define the optimal penalty to avoid firms using the redistributed generation in the upstream segment of the electricity market.

Among the electricity modelling papers, this work falls into the category defined by Ventosa *et al.* (2005) as "equilibrium models".² In particular, our work is part of the electricity models in which the capacity choice is endogenized. Other works in this category encompass, for instance, von der Fehr and Harbord (1997). The authors model a two-stage game and assume that utilities choose investment in a first stage and price in a second one.³ Le Coq (2002) and Crampes and Creti (2005) follow this type of modelling⁴ and show that electricity generators have incentives to withhold capacity to rise the wholesale system equilibrium price. Both papers, like Kreps and Scheinkman (1983), consider a two-stage game where in a first stage agents bid capacities knowing demand, and in the second one they bid prices. However, differently from the

²See also David and Wen (2000).

³See also Castro-Rodriguez *et al.* (2001) and Reynolds and Wilson (2000) for other examples of the same type of modelling.

⁴See also Crampes and Fabra (2004) for more on this type of wholesale market modelling based on auctions clearing.

previous literature, they model the second stage as a uniform price auction (instead of Bertrand competition) to describe the most common electricity market design. Sanin (2006) adds to Le Coq (2002) and Crampes and Creti (2005) by accounting for a stochastic demand of electricity and comparing price equilibria and collusive incentives under different rationing rules for the auction clearance. Herein, we extend this kind of framework to account for the specificity of the NOME project. In particular, in a first stage firms choose the generation made available without knowing realized demand, in a second stage they choose the bidding price and then the market is cleared by a uniform auction. We analyze how their bidding strategies are modified by the implementation of NOME's law and which should be the penalty to avoid such modifications.

In Section 2 we state the assumptions and timing of the game. In Section 3 we present our assumptions. In Sections 4 and 5 we present our model and in 6 we present the main implications of the NOME law in the wholesale market. In Section 7 we briefly conclude.

2 The context

2.1 The NOME law

NOME's project of law⁵ is set to be applied from January 2011. It was first adopted by the French Senate (*i.e.* the "Assemblée Nationale") the 15th of June 2010 and sent for consultation, both to the Economics, Sustainable Development and Territorial Organization Commission⁶ and to the Finance Commission.⁷ Both commissions have delivered their suggested modifications⁸ which should be studied by the Senate in the following months. NOME law gives regulated access to 100 TWh produced yearly by the historical supplier's nuclear plants⁹ (plus 20 TWh from 2013 on). The conditions for regulated access will be determined by the Energy Ministry and the Economics Ministry, advised by the CRE. In addition, the project includes a financial penalty if the nuclear power received by competitors is not used in serving French retail consumers. However, the amount and the way such penalty will be implemented is not defined yet.

As first suggested by the Champsaur Commission¹⁰ in April 2009, the redistribution of low-cost generation is accompanied by a gradual liberalization of the retail price for electricity. From January the 1st, 2011 industrial consumers will no longer benefit from regulated tariffs and from 2015 also consumers with an intensity lower than 36 kVA will stop being served at regulated tariffs. In the

⁵Find it at <http://www.assemblee-nationale.fr/13/ta/ta0486.asp>

⁶"Commission de l'économie, du développement durable et de l'aménagement du territoire".

⁷"Commission des finances".

⁸See Text n° 644 (2009-2010) delivered the 7th of July 2010, which includes all modifications suggested by both commissions at <http://www.senat.fr/leg/plj09-644.html>

⁹Nowadays, EdF's competitors serve approximately 40 TWh.

¹⁰See the Commission's report at http://www.edf.com/fichiers/fckeditor/Commun/Finance/Publications/Annee/2009/090424-RapportChampsaur_vf.pdf

actual landscape, instead, given that most participants in the retail electricity market are vertically integrated, new entrants in the downstream market supply at higher price as compared to the incumbent retailer Electricité de France (EdF), who benefits from lower cost of nuclear generation. Then, EdF's competitors are unable to offer retail prices that can compete with regulated tariffs negotiated exclusively by the historical operator. In this regard, Solier (2010) shows that the mean price of base one-year-ahead future contracts supplied by EdF's competitors is always higher than the regulated tariffs.

As recommended by the senator Sido and the deputy Poignant¹¹ in April 2010, the law also plans to create a capacity obligation scheme to ensure the diffusion of efficient investments in base-load and peak generation (*i.e.* smart grids). Finally, the law includes a reform in the local electricity tax in order to comply with the directive¹² 2003/96/CE.

All in all, the reform would amend the European Commission observations regarding the maintenance of regulated tariffs as state aid to local firms.¹³

2.2 The wholesale electricity market

The French wholesale electricity market represents, in mean, 39% (225 TWh yearly) of all injections/sustractions of electricity in the system (either over the counter or through Powernext or European Energy Exchange). From those 225 TWh only 6% are imported from neighboring countries,¹⁴ the rest being produced mostly by the historical operator.¹⁵ The HHI rises to 8.600 in terms of production and its 7.876 in terms of electricity used to satisfy end users.¹⁶

Since September 2001 (just four month after the registration of the firsts over-the-counter transactions in the french wholesale electricity market) part of the historical producer's nuclear capacity was auctioned. This mechanism is called VPP (which stands for Virtual Power Plant) and allow EdF competitors to reserve nuclear capacity for a certain period varying between 3 and 48 months. Auctions take place, in mean, once every three months and the price paid by each auctioneer depends both on the reservation's duration and on whether it concerns peak or base load.

The capacity reserved by VPP, which represents 7% (16 TWh yearly) of the wholesale market, may be used or not (generally it will depend on the VPP price as compared to the market price for that same load) and its price is defined by an ascending price auction (in multiple rounds).¹⁷ Instead, NOME's redistributed capacity amounts at 100 TWh produced yearly by EdF nuclear

¹¹ See Poignant-Sido report at http://www.lesechos.fr/medias/2010/0402//020459998448_print.pdf

¹² See http://eur-lex.europa.eu/smartapi/cgi/sga_doc?smartapi!celexplus!prod!DocNumber&lg=fr&type_doc=Directive&an_doc=2003&nu_doc=96

¹³ For details see <http://europa.eu/rapid/pressReleasesAction.do?reference=IP/09/376>

¹⁴ See http://www.cre.fr/fr/marches/marche_de_l_electricite/marche_de_gros#a1

¹⁵ See http://www.cre.fr/fr/marches/observatoire_des_marches#a1

¹⁶ Data in CRE (2010).

¹⁷ For more detail on the way those auctions were organized in France as compared to other countries see Ausubel and Crampton (2010).

plants¹⁸ (almost 50% of the wholesale market), plus 20 TWh from 2013 on. Besides the quantitative difference, NOME's law redistribution of nuclear capacity differs from the VPP mechanisms in two important ways: (i) the capacity is not assigned by a market clearing mechanism but as a function of each competitor's market share in the retail market, at a preestablished regulated price; (ii) the capacity received must be used for production and, if that is not the case, either a penalty or a diminution in next period allocation is implemented by the regulator.

As underlined by Finon (2010) the wholesale price will be most of the time superior to the regulated price of nuclear generation at which competitors pay NOME's redistributed generation. In what follows, we study if these price differential gives enough incentives to use the redistributed capacity in the wholesale market and the penalty that should be imposed by the regulator to discourage such behavior.

3 Assumptions and timing of the game

We assume there are two generators G_a and G_b . It is common knowledge that one of them, lets say G_a , has a larger generation capacity ($K_{a \max} > K_{b \max}$) and more efficient so the marginal costs satisfy: $c_a < c_b$.

We also assume there exists a price cap fixed by the regulator denoted by $\hat{p} > c_b$. After the introduction of the NOME law a portion η of G_a 's low cost generation is given to G_b at a price p_η .¹⁹ Maximal generation capacities after the introduction of NOME $K_{a \max, \eta}$ and $K_{b \max, \eta}$ are now $K_{a \max, \eta} = (1 - \eta)K_{a \max}$ and $K_{b \max, \eta} = K_{b \max} + \eta K_{a \max}$.

We model generators strategies as a two-stage game: in the first stage, firms choose the production capacity and, in the second stage, firms bid prices.

We proceed by backward induction so the last stage exposed is the one played first. In the *second stage* of the game, we assume that generators play a capacity constrained game, *i.e.* when bidding a price p_i , $i = a, b$, each generator i knows the capacity chosen by its competitor and the level of inelastic demand for the day-ahead market that in the first stage, when firms chose a quantity to bid K_i , was unknown.

For each market period, the market is cleared by a uniform price auction organized by the System Operator (henceforth, SO). The marginal price for electricity is the one bidded by the high bidder and both generators are paid this price for the amount of electricity they are called for. The low bidder is dispatched first and the higher bidder serves the residual demand. When two generators bid the same price we assume that proportional rationing is used, that is, each generator G_i is dispatched a quantity equal to $K_i / (K_i + K_j)$.

¹⁸Nowadays, EdF's competitors represent approximately 40 TWh of the French retail market.

¹⁹Since both η and p_η are decided by the regulator we consider $\eta p_\eta K_{a \max, \eta}$ as a fixed cost that does not affect short run decisions.

The day is divided into market periods (loads) but the bid price must be held fixed for more than one market period. This means generators don't know the actual level of demand when they decide their level of production. We capture this feature by assuming that in the *first stage*, where generators choose to produce any amount lower than their maximum capacity $K_i \leq K_{i \max, \eta}$, they do not know the level of demand that can be either high (peak) D_p with probability h or low, D_n , with probability $(1-h)$. We assume that only the output effectively produced generates costs and that NOME's redistributed generation that is not used in the wholesale market is used in the retail market (*i.e.* there is no opportunity cost). In addition, we assume that $K_{a \max, \eta} + K_{b \max, \eta} > D_p$, *i.e.* shortage can only be provoked by firms. This is discouraged as we consider that the regulator sets a shortage penalty in the form of a fixed fine S .

4 Second stage: price game

The choice of the quantity K_i that firms wish to make available in the first stage of the game lead them to one of four possible ex-post demand cases: (a) a case in which each generator can serve the demand alone *i.e.* low demand case; (b) a case in which one generator can serve the demand alone whereas the other has bid a quantity lower than realized demand; (c) a case in which no generator has bid enough quantity to serve the demand alone.

Case a) Low demand

If once revealed demand is such that $D < K_i$; $i = a, b$, both generators have bid enough quantity to satisfy all the market by themselves the SO dispatches it.

Proposition 1 *When $K_a > D$ and $K_b + K_{b, \eta} > D$, *i.e.* none of the generators is capacity constrained, the pair (p_a^{br}, p_b^{br}) that are a Nash equilibrium of the price subgame are:*

i) when $K_{b, \eta} < D$

$$\begin{aligned} p_a^{br} &= \max(c_b - \varepsilon, c_a) & \text{which gives} & \quad \Pi_a = (c_b - c_a)D \\ p_b^{br} &= c_b & \text{which gives} & \quad \Pi_b = 0 \end{aligned} \quad (1)$$

where ε tends to 0.

ii) when $K_{b, \eta} > D$

$$\begin{aligned} p_a^{br} &= c_a & \text{which gives} & \quad \Pi_a = 0 \\ p_b^{br} &= c_a & \text{which gives} & \quad \Pi_b = 0 \end{aligned} \quad (2)$$

Proof. See Mathematical Appendix. ■

Since the bigger operator is more efficient ($c_a < c_b$), before NOME redistribution of generation, G_a could win the game through the threat of undercutting its competitor. After NOME's redistribution, this is only the case when $K_{b, \eta} < D$. Instead, when $K_{b, \eta} > D$ both firms have symmetric marginal costs and we find

Bertrand's (1883) price competition result, *i. e.* we have either of the two generators serving total demand at a price c_a .

Summary 2 *When NOME's redistribution is enough high and the quantity obtained is used in the wholesale market so that $K_{b,\eta} > D$, the system equilibrium price is equal to the lowest marginal cost c_a .*

In Section 5 we show that, unfortunately, the previous equilibrium is not a subgame perfect nash equilibrium (SPNE).

Case b) Intermediate demand

If once revealed demand is $K_i < D < K_j$ there exists a continuum of equilibria where the price cap \hat{p} is offered by the firm that bid a higher quantity knowing that the other will offer less to avoid being undercut (See Appendix A). In particular, if the first stage is played such that $K_b + K_{b,\eta} < D < K_a$, it is optimal for G_b to bid a price lower than the price cap and be dispatched first while G_a bids the price cap and is dispatched the residual demand.

If instead the first stage is played such that, even if $K_{a \max,\eta} > K_{b \max,\eta}$, G_a chooses to bid a lower quantity than G_b *i.e.* $K_a^{br} < D < K_b^{br} + K_{b,\eta}^{br}$, it is optimal for G_a to bid a price lower than the price cap and be dispatched first while G_b bids the price cap and is dispatched the residual demand.

Case c) High demand

If once revealed demand is $K_i < K_j < D < K_i + K_j$ we find two sets of equilibria in pure strategies: one in which G_b bids the price cap and serves just residual demand and another one in which G_a is the one that bids the price cap while G_b undercuts this bid and is dispatched first. This is the case as the production of both agents is needed to satisfy demand and therefore both know that they will be called into operation. For a complete characterization of both pure strategy equilibria and the mixed strategy equilibrium, see Crampes-Creti (2005).

5 First stage: capacity game

In the previous section we showed that different price equilibria can be reached depending on the quantities K_a and K_b chosen in this first stage (as compared to the level of demand realized). Herein, we present the case where K_a and K_b are chosen when demand is stochastic.²⁰

Let us define a parameter $\delta = \frac{\hat{p} - c_b}{\hat{p} - c_a} < 1$ that captures the disadvantage of G_b in terms of costs. The condition $\alpha_a > c_b$ on (10) is equivalent to $K_b + K_{b,\eta} < D\delta$ which ensures that bidding the price cap is better than a fight in prices²¹ for G_a .

²⁰For a simplified version where quantity declarations are made for a given demand see Appendix A.

²¹If $K_b + K_{b,\eta} < D\delta$, which is equivalent to $K_b + K_{b,\eta} < D \frac{\hat{p} - c_b}{\hat{p} - c_a}$, multiplying by $(\hat{p} - c_a)$ and subtracting $c_a D$ to both sides we get $(\hat{p} - c_a)(D - K_b - K_{b,\eta}) > (c_b - c_a)D$ which we already showed is equivalent to $\alpha_a > c_b$.

Similarly, the condition $\beta_a > c_b$ on (12) can be rewritten as $K_a \leq \frac{1}{1-\delta}(D - K_b - K_{b,\eta})$ which ensures that $(\hat{p} - c_a)(D - K_b - K_{b,\eta}) \geq (c_b - c_a)K_a$ is satisfied.²²

Considering the general case where none of the generators is a priori capacity constrained, i.e. both could serve demand alone ($D_n < D_p < K_{b\max} + \eta K_{a\max} < (1-\eta)K_{a\max}$) the best response functions K_a^{br} and $K_b^{br} + K_{b,\eta}^{br}$ under stochastic demand obtain for each agent G_a and G_b respectively:

$$K_a^{br}(K_b + K_{b,\eta}) = \begin{cases} K_a > D_p - K_b - K_{b,\eta} & \text{if } K_b + K_{b,\eta} \leq D_n\delta \\ K_a = D - \varepsilon & \text{if } K_b + K_{b,\eta} > D_n\delta \end{cases} \quad (3)$$

$$K_b^{br} + K_{b,\eta}^{br} = \begin{cases} K_b + K_{b,\eta} > D_p - K_a & \text{if } K_a < D_n \\ K_b + K_{b,\eta} = D_n\delta - \varepsilon & \text{if } K_a \geq D_n \end{cases} \quad (4)$$

Then, the intersection between the best replies gives the equilibria described in the following lemma.

Proposition 3 *There are three families of equilibria in the case of ex-ante low stochastic demand:*

$$\begin{aligned} i) (K_a^*, K_b^* + K_{b,\eta}^*) &= \{K_a^*, K_b^* + K_{b,\eta}^* \text{ s.t. } K_a^* < D_n, K_b^* + K_{b,\eta}^* \leq D_n\delta, \quad (5) \\ &K_a^* + K_b^* + K_{b,\eta}^* \geq D_p\} \\ ii) &K_a^* \geq D_n, K_b^* + K_{b,\eta}^* = D_n\delta - \varepsilon \\ iii) &K_a^* = D_n - \varepsilon, K_b^* + K_{b,\eta}^* \geq D_n\delta \end{aligned}$$

Proof. See Mathematical Appendix. ■

Each of these families of equilibria²³ in the first-stage-game lead generators to a different outcome in the next step of the game. Comparing the payoffs for each possible realization of demand we find that:

Corollary 4 *Type ii equilibria is preferred by G_b to type i and iii in that order, and type iii is preferred by G_a to type i and ii in that order:*

$$\begin{aligned} ii &\succ_b i \succ_b iii \\ iii &\succ_a i \succ_a ii \end{aligned} \quad (6)$$

Proof. See Mathematical Appendix. ■

As profits of G_i are decreasing with K_j , in *type i* the case where the restriction on the sum of capacities is satisfied with equality $K_a^* + K_b^* + K_{b,\eta}^* = D_p$ Pareto dominates the case where $K_a^* + K_b^* + K_{b,\eta}^* > D_p$. This is possible when D_p is realized as, on the one hand, $K_a^* < D_n$ and then $K_a^* + K_b^* + K_{b,\eta}^* < (1+\delta)D_n$, and, on the other hand, given the condition on the distance between D_n and D_p

²² $K_a \leq \frac{1}{1-\delta}(D - K_b - K_{b,\eta})$, given the definition of δ , means that $K_a \leq \frac{\hat{p}-c_a}{c_b-c_a}(D - K_b - K_{b,\eta})$ equivalent to $(\hat{p} - c_a)(D - K_b - K_{b,\eta}) \geq (c_b - c_a)K_a$, multiplying both sides by K_a and subtracting c_a to both sides, the previous expression is equivalent to $\beta_a > c_b$.

²³ Note that to be in ii) it must be the case that $\eta K_{a\max} \geq D_n\delta - K_b^{br} - \varepsilon$ and to be in iii) it must be the case that $\eta K_{a\max} \leq D_n\delta - K_b^{br}$.

mentioned in the proof of (5), it is true that: $D_p < (1 + \delta)D_n$. When demand is D_n , in *type i* equilibria the point where $K_a^* + K_b^* + K_{b,\eta}^* = D$ is satisfied cannot be reached since firms always play $K_a^* + K_b^* + K_{b,\eta}^* \geq D_p$ to avoid the shortage penalty S . When demand is low both get lower profits than when demand is high. None of the three families of equilibria in (5) can be discarded by a Pareto dominance criterion.

If the smaller generator is capacity constrained, as long as $D_n\delta < K_{b\max} + \eta K_{a\max} < D_n$, the equilibria described in (5) are still possible. Instead, if the capacity constraint of the smaller generator is so strong that $K_{b\max} + \eta K_{a\max} < D_n\delta$, the only family of equilibria possible is:

Corollary 5 *If the capacity constraint of the smaller generator is binding to the extent that $K_{b\max} + \eta K_{a\max} < D_n\delta$, the only possible equilibrium is:*

$$(K_a^*, K_b^* + K_{b,\eta}^*) = \quad (7)$$

$$\{K_a^*, K_b^* + K_{b,\eta}^* \text{ s.t. } K_a^* \leq D_n, K_b^* + K_{b,\eta}^* \leq K_{b\max} + \eta K_{a\max}, \quad (8)$$

$$K_a^* + K_b^* + K_{b,\eta}^* \geq D_p\}$$

Finally, if both generators are capacity constrained, *i.e.* $(1 - \eta)K_{a\max} < D_n < D_p < K_{a\max} + K_{b\max}$, the equilibria described in (5) are reachable as long as K_a^* and $K_b^* + K_{b,\eta}^*$ are lower than D_p but still higher than D_n . If it is not the case, the only possible equilibria is:

Corollary 6 *If $K_{b\max} + \eta K_{a\max} < (1 - \eta)K_{a\max} < D_n$ the equilibria becomes:*

$$(K_a^*, K_b^*) = \{K_a^*, K_b^* + K_{b,\eta}^* \text{ s.t. } K_a^* \leq (1 - \eta)K_{a\max}, \quad (9)$$

$$K_b^* + K_{b,\eta}^* \leq \min(D_n\delta, K_{b\max} + \eta K_{a\max}),$$

$$K_a^* + K_b^* + K_{b,\eta}^* \geq D_p\}$$

6 The effect of NOME's redistribution

NOME's nuclear generation redistribution could have two effects in the wholesale market, depending on the strength of the redistribution as well as on the situation regarding the relative size of competitors when the redistribution takes place. If before NOME the incumbent's competitor was not only less efficient, but also very small as compared to the incumbent, NOME's redistribution amplifies the typology of subgame perfect equilibria. In fact, the capacity endowment given to the competitor could make it possible to pass from a situation where the only family of equilibria possible is (7) to a situation where all three families of equilibria described in (5) are possible. On the contrary, if the incumbent was not very large as compared to its competitors, NOME redistribution could constraint its capacity to an extent that equilibria are restricted to a situation like the one described in (9).

The case of a capacity constrained competitor is particularly interesting: if the incumbent's competitors are small in relation to the off peak demand D_n ,

they have incentives to use NOME's redistributed generation in the wholesale market. This strategy is all the more profitable when it allows generator G_b to pass from an equilibria *type i* to an equilibria *type ii* where it is dispatched first, making higher profits. However, such behavior does not result in lower wholesale prices.²⁴ Then, the regulator is interested in fixing a penalty P to avoid the use of the redistributed capacity in the wholesale market. It would be enough for that penalty to be effective to equalize, for each play, the profits after the use of NOME's redistributed generation to the profits without using such quantity.

Proposition 7 *The use of NOME's generation in the wholesale market has no (static) procompetitive effect since the system equilibrium price is the price cap \hat{p} .*

Corollary 8 *G_b 's profits increase in all possible equilibria; G_a 's profits are kept unchanged unless $K_a^* = (1 - \eta)K_{a \max}$.*

Corollary 9 *The regulator can avoid the use of NOME's redistributed generation in the wholesale market by fixing a penalty equal to $P = (c_b - c_a) K_{b,\eta}$.*

According to our analysis, the penalty should not be repaid to G_A , since the incumbent still makes the same profits in the wholesale market as before NOME's redistribution, unless the equilibrium where he is capacity constrained realizes (i.e. when $K_a^* = (1 - \eta)K_{a \max}$). Regarding the penalty, it is sufficient to pay the difference between marginal costs to discourage firms to use NOME generation in the wholesale market.

7 Conclusion

In this paper we have shown the effect of using NOME's redistributed generation in the wholesale electricity market as well as the incentives for firms to do so. We find that the use of NOME's redistributed generation has no procompetitive effect: the system equilibrium price is still the highest possible in a context with inelastic demand. Profits in the wholesale market of the incumbent firm ceding part of its generation are not modified whereas profits of the firm receiving NOME generation increase, for every possible equilibrium. In particular, if the redistribution leverages the capacity restrictions of small generators, some families of equilibria that could not be attained before the redistribution are now attainable. Since these equilibria are very favorable in terms of profits for the firm receiving the low-cost generation, we derive a simple rule to establish the optimal penalty to avoid firms using NOME's redistribution in the wholesale market.

²⁴The increase in profits when passing from an equilibria *type i* to an equilibria *type ii* when using NOME's redistributed generation in the wolesale market can be considered as a dynamic procompetitive effect.

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9 Appendix A

9.1 Second-stage: price equilibria

Herein we show the price equilibria for each of the cases mentioned in the main text.

9.1.1 Case b) Intermediate demand

Solution 10 When demand is lower than the largest quantity declared, if G_a (i.e. the most efficient generator) is the one with the capacity advantage, that is $K_b^{br} + K_{b,\eta}^{br} < D < K_a^{br}$, there are two possible families of equilibria for (p_a^{br}, p_b^{br}) :

$$\begin{aligned} p_a^{br} &= \hat{p} \quad \text{which gives} \quad \Pi_a = (\hat{p} - c_a)(D - K_b - K_{b,\eta}) & (10) \\ p_b^{br} &\in [0, \alpha_a] \quad \text{which gives} \quad \Pi_b = (\hat{p} - c_a)K_{b,\eta} + (\hat{p} - c_b)K_b \\ \alpha_a &= c_a + (\hat{p} - c_a) \frac{(D - K_b - K_{b,\eta})}{D} \quad \text{where } \alpha_a > c_b \end{aligned}$$

b) Equilibria defined in Case a if $\alpha_a < c_b$ where α_a is the threshold that ensures G_b to sell all its production when G_a bids at the price cap.

Proof. See Mathematical Appendix. ■

If $K_a^{br} < D < K_b^{br} + K_{b,\eta}^{br}$, instead of the two families of equilibria described in the previous lemma we have only one family of equilibria symmetric to the one described in (10).

Solution 11 When demand is lower than the larger capacity declared, if G_b , the less efficient generator, is the one with the capacity advantage, that is $K_a^{br} < D < K_b^{br} + K_{b,\eta}^{br}$, there is only one possible family of equilibria for (p_a^{br}, p_b^{br}) :

$$\begin{aligned} p_b^{br} &= \hat{p} \quad \text{which gives} \quad \begin{cases} \Pi_b = (\hat{p} - c_a)(D - K_a) & \text{if } D - K_a < K_{b,\eta}^{br} \\ \Pi_b = (\hat{p} - c_a)K_{b,\eta}^{br} + (\hat{p} - c_b)(D - K_a - K_{b,\eta}^{br}) & \text{otherwise} \end{cases} & (11) \\ p_a^{br} &\in [0, \alpha_b] \quad \text{which gives} \quad \Pi_a = (\hat{p} - c_a)K_a \\ \alpha_b &= \hat{p} - \frac{(\hat{p} - c_a)K_a}{D} \end{aligned}$$

Proof. See Mathematical Appendix. ■

9.1.2 Case c) High demand

If once revealed demand is $K_i < K_j < D < K_i + K_j$ we find two sets of equilibria in pure strategies.

Solution 12 If the pair $(K_a^{br}, K_b^{br} + K_{b,\eta}^{br})$ chosen in the first stage determines that $K_i^{br} < K_j^{br} < D$, we find two set of equilibria in pure strategies for the pair (p_a^{br}, p_b^{br}) :

a)

$$\begin{aligned} p_a^{br} &= \hat{p} \quad \text{which gives} \quad \Pi_a = (\hat{p} - c_a)(D - K_b - K_{b,\eta}) & (12) \\ p_b^{br} &\in [0, \beta_a] \quad \text{which gives} \quad \Pi_b = (\hat{p} - c_a)K_{b,\eta} + (\hat{p} - c_b)K_b \\ \beta_a &= c_a + (\hat{p} - c_a) \frac{(D - K_b - K_{b,\eta})}{K_a} \quad \text{where } \beta_a > c_b \end{aligned}$$

where β_a is the threshold for p_b^{br} that ensures that G_a is better off bidding the price cap \hat{p} .

b)

$$p_b^{br} = \hat{p} \quad \text{which gives} \quad \begin{cases} \Pi_b = (\hat{p} - c_a)(D - K_a) & \text{if } D - K_a < K_{b,\eta}^{br} \\ \Pi_b = (\hat{p} - c_a)K_{b,\eta}^{br} + (\hat{p} - c_b)(D - K_a - K_{b,\eta}^{br}) & \text{otherwise} \end{cases} \quad (13)$$

$$\begin{aligned} p_a^{br} &\in [0, \beta_b] \quad \text{which gives} \quad \Pi_a = (\hat{p} - c_a)K_a \\ \text{with} \quad \beta_b &= c_a + (\hat{p} - c_a) \frac{(D - K_a)}{K_{b,\eta}^{br}} \quad \text{and} \quad \frac{(\hat{p} - c_a)}{K_{b,\eta}^{br}} - \frac{(\hat{p} - c_b)}{(K_{b,\eta}^{br} + K_b^{br})} < \frac{c_b - c_a}{(D - K_a)} \end{aligned}$$

Proof. See Mathematical Appendix. ■

If $\beta_a < c_b$, b) is the unique set of equilibria in pure strategies. When $\beta_a > c_b$, none of this two set of equilibria can be discarded by a Pareto dominance criterion as both generators would prefer to be the lower bidder in order to sell all the capacity they made available. Larson and Salant (2003) shows that in the case of uniform auctions where prices can vary continuously²⁵ and all the players know they must be called into operation, is more likely that generators play in mixed strategies. We characterize the equilibrium in mixed strategies in which, for a given strategy of the competitor, each generator is indifferent between all the prices over which it randomizes.

Solution 13 If the pair $(K_a^{br}, K_b^{br} + K_{b,\eta}^{br})$ chosen in the first stage determines that $K_i^{br} < K_j^{br} < D$ and $\beta_a > c_b$, we find the following mixed strategies equilibrium for p_j^{br} :

$$p_j^{br} \sim F_i(p_i) \quad \text{on} \quad [c_j, \hat{p}] \quad (14)$$

where $F_i(p_i)$ describes the cumulative distribution of probabilities for G_i bids in the support $[c_j, \hat{p}]$.

Proof. See Mathematical Appendix. ■

²⁵This implies that infinitesimal undercutting is possible.

9.2 First stage: capacity choice for *ex-ante* low deterministic demand

In this subsection we analyze the case where (i) generators *know* the level of demand before deciding their level of production and, (ii) none of them is a priori capacity constrained, *i.e.* $D < K_{b\max} + \eta K_{a\max} < (1 - \eta)K_{a\max}$.

In this case the best response functions in the capacity game can be obtained as:

Solution 14 *The best response function of G_a in the case of deterministic low demand is:*

$$K_a^{br}(K_b + K_{b,\eta}) = \begin{cases} K_a > D - K_b - K_{b,\eta} & \text{if } K_b + K_{b,\eta} \leq D\delta \\ K_a = D - \varepsilon & \text{if } K_b + K_{b,\eta} > D\delta \end{cases} \quad (15)$$

where $\varepsilon > 0$ tends to zero.

Proof. See Mathematical Appendix. ■

Solution 15 *The best response function of G_b in the case of deterministic low demand is:*

$$K_b^{br} + K_{b,\eta}^{br} = \begin{cases} K_b + K_{b,\eta} > D - K_a & \text{if } K_a < D \\ K_b + K_{b,\eta} = D\delta - \varepsilon & \text{if } K_a \geq D \end{cases} \quad (16)$$

Proof. The best response for G_b is derived computing Π_b in each of the cases in the previous lemma. ■

Then, the equilibria are:

Solution 16 *There are three families of equilibria in the case of deterministic low demand:*

$$\begin{aligned} & i) (K_a, K_b + K_{b,\eta}) = & (17) \\ & \{K_a, K_b + K_{b,\eta} \text{ s.t. } K_a < D, K_b + K_{b,\eta} \leq D\delta, K_a + K_b + K_{b,\eta} \geq D\} \\ & ii) K_a \geq D, K_b + K_{b,\eta} = D\delta - \varepsilon \\ & iii) K_a = D - \varepsilon, K_b + K_{b,\eta} \geq D\delta \end{aligned}$$

Proof. The equilibrium can be easily found by the intersection of the best reply functions. ■

10 Mathematical Appendix

10.1 Proof of Proposition 1

From Bertrand (1883) we know that price competition in a duopoly with symmetric firms gives the competitive outcome. If firms are not symmetric, the one with the lowest marginal costs will undercut the other one's bid for every play.

10.2 Proof of Solution 9

a) The subindex a in α_a is to underline the fact that this threshold is determined comparing the profits for G_a in (3) with the ones he could receive if he undercuts G'_b 's bid and serve all the demand by himself: assume that G_a bids α_a . In this case $\Pi_a^\alpha = (\alpha_a - c_a)D$ that after substituting α_a by its value becomes $\Pi_a^\alpha = (\hat{p} - c_a)(D - K_b - K_{b,\eta})$ that is equal to the profit he would obtain by bidding the price cap. But in order to undercut G'_b 's bid and compete for the full demand he should offer his capacity at a price *lower* than α_a as the interval is closed in this value. This would be a dominated strategy as he would receive less profits than bidding the price cap. That is why the best response of G_a when G_b bids a price between $[0, \alpha_a]$ is to bid the price cap \hat{p} and serve just the residual demand.

b) The threshold α_a is increasing with the residual demand $(D - K_b - K_{b,\eta})$. If this demand is sufficiently low, that is, if $\alpha_a < c_b$ we fall in the equilibria of *Case a*. If G_a observes this relation between α_a and c_b he will undercut G_b 's bid (c_b) as by the definition of α_a , this relation between parameters means: $(\hat{p} - c_a)(D - K_b - K_{b,\eta}) < (c_b - c_a)D$.

The fact that G_b bids a price between $[0, \alpha_a]$ is credible only if that gives him positive profits, which is the case as long as $\alpha_a - c_b > 0$. Otherwise profits of G_b are negative when bidding $[0, \alpha_a]$ making this bidding strategy not credible and equilibria is the one described in Solution 1.

10.3 Proof of Solution 10

If $D - K_a < K_{b,\eta}$ then G_b is indifferent between bidding $\pi_b = (\hat{p} - c_a)(D - K_a)$ and $\pi_b = (\alpha_b^u - c_a)D$ if

$$\alpha_b^u = \hat{p} - \frac{(\hat{p} - c_a)K_a}{D} = (\hat{p} - c_a)\frac{(D - K_a)}{D} + c_a \text{ where } G_b \text{ bids the price cap} \quad (18)$$

Then, for any price bid by G_a lower than α_b^u he will prefer to bid the price cap.

If $D - K_a > K_{b,\eta}$ then G_b is indifferent between bidding

$$\pi_b = (\hat{p} - c_a)K_{b,\eta}^{br} + (\hat{p} - c_b)(D - K_a - K_{b,\eta}^{br}) \text{ and } \pi_b = (\alpha_b - c_a)K_{b,\eta}^{br} + (\alpha_b - c_b)(D - K_{b,\eta}^{br}) \text{ if}$$

$$\alpha_b^d = \hat{p} - \frac{(\hat{p} - c_b)K_a}{D}. \quad (19)$$

For any price bid by G_a lower than α_b^d he will prefer to bid the price cap.

Since $(\hat{p} - c_a) > (\hat{p} - c_b)$, we see that $\alpha_b^u < \alpha_b^d$ and therefore, if G_a bids α_b^u then G_b prefers to bid the price cap.

It is a credible strategy for G_A to bid α_b^u instead of fighting for prices since $(\hat{p} - c_a)K_a - \frac{(\hat{p} - c_a)K_a}{D}K_a \geq (c_b - c_a)D$ and also $(\hat{p} - \frac{(\hat{p} - c_a)K_a}{D} - c_a)K_a \geq 0$ since $1 \geq \frac{K_a}{D}$.

10.4 Proof of Solution 11

a) The proof follows the same line as the previous proposition: if G_a chooses a price lower than β_a he would sell all the capacity declared K_a . In this case he could get at most $\Pi_a^\beta = (\beta_a - c_a)K_a = (\hat{p} - c_a)(D - K_b - K_{b,\eta})$ given the definition of β_a . Otherwise, if $p_b^{br} > \beta_a$, G_a would bid any price below: $p_b^{br} > p_a^{br} > \beta_a$ as in this way he would sell all the capacity declared earning more than bidding the price cap with

$$\beta_b^u = (\hat{p} - c_a) \frac{(D - K_a)}{K_{b,\eta}^{br}} + c_a \text{ when } G_b \text{ bids the price cap} \quad (20)$$

or

$$\beta_b^d = (D - K_a) \frac{(\hat{p} - c_b)}{(K_{b,\eta}^{br} + K_b^{br})} + c_b \quad (21)$$

It is the case that $\beta_b^u < \beta_b^d$ if $(D - K_a) \frac{(\hat{p} - c_a)}{K_{b,\eta}^{br}} + c_a < (D - K_a) \frac{(\hat{p} - c_b)}{(K_{b,\eta}^{br} + K_b^{br})} + c_b$
i. e. $\frac{(\hat{p} - c_a)}{K_{b,\eta}^{br}} - \frac{(\hat{p} - c_b)}{(K_{b,\eta}^{br} + K_b^{br})} < \frac{(c_b - c_a)}{(D - K_a)}$.

Bidding β_b^u is a credible strategy for G_A since $(D - K_a) \frac{(\hat{p} - c_a)}{K_{b,\eta}^{br}} K_a \geq 0$.

10.5 Proof of Solution 12

We proof the statement in relation to G_a as the solution in relation to G_b is symmetric. For any given strategy of G_b , that is, for a distribution of probabilities f_b played by G_b , G_a is indifferent between all the prices over which he randomizes, *i.e.* any price belonging to $[p_{a \min}, \hat{p}]$. To find f_b we maximize the expected profits of G_a with respect to p_a :

$$\begin{aligned} E(\Pi_a(p_a, f_b(p_b))) &= \int_{p_{b \min}}^{\hat{p}} \Pi_a(u) f_b(u) du \quad (22) \\ &= \int_{p_{b \min}}^{p_a} (p_a - c_a) (D - K_b - K_{b,\eta}) f_b(u) du + \int_{p_a}^{\hat{p}} \Pi_a(u) f_b(u) du = \\ &= (p_a - c_a) (D - K_b - K_{b,\eta}) F_b(p_a) + \int_{p_a}^{\hat{p}} \Pi_a(u) f_b(u) du \end{aligned}$$

The F.O.C. is then obtained by deriving:

$$\frac{\partial E(\Pi_a(p_a, f_b(p_b)))}{\partial p_a} = 0 \quad (23)$$

$$\begin{aligned} (D - K_b - K_{b,\eta}) [(p_a - c_a) f_b(p_a) + F_b(p_a)] + (\hat{p} - c_a) K_a f_b(\hat{p}) - (p_a - c_a) K_a f_b(p_a) &= 0 \\ ((D - K_b - K_{b,\eta}) - K_a) (p_a - c_a) f_b(p_a) + (D - K_b - K_{b,\eta}) F_b(p_a) &= 0 \end{aligned}$$

as $f_b(\hat{p}) = 0$ given that *punctual* value of any continuous density function is zero.

The solution of the differential equation obtained gives us the distribution we were looking for²⁶:

$$F_b(p_b) = \left[\frac{p_b - c_a}{A_b} \right]^{\gamma_b} \quad \text{where} \quad \gamma_b = \frac{(D - K_b - K_{b,\eta})}{(K_a + K_b + K_{b,\eta} - D)} \quad (24)$$

where $A_b = \hat{p} - c_a$, the value in (22) of $p_{b \min} = c_a$. It can be determined taking into account that $F_b(p_b)$ accumulates all the probability between $[p_{b \min}, \hat{p}]$ which means that $F_b(\hat{p}) = \left[\frac{\hat{p} - c_a}{A_b} \right]^{\gamma_b} = 1$. Similarly, as the minimum value of $F_b(p_b)$ is zero it must be the case that the minimum value for $p_{b \min} = c_a$. Also, $\gamma_b > 0$ when $K_b < D$. Of course $D < K_a + K_b + K_{b,\eta}$ as the case of rationing is described in the following section.

Then, the expected profits of G_a for any play of G_b can be found:

$$\int_{p_a}^{\hat{p}} \Pi_a(u) f_b(u) du = (\hat{p} - c_a) (D - K_b - K_{b,\eta}) - (p_a - c_a) (D - K_b - K_{b,\eta}) \left[\frac{p_b - c_a}{\hat{p} - c_a} \right]^{\frac{(D - K_b - K_{b,\eta})}{(K_a + K_b + K_{b,\eta} - D)}} \quad (25)$$

That is:

$$E(\Pi_a(p_a, f_b(p_b))) = (\hat{p} - c_a) (D - K_b - K_{b,\eta}) \quad (26)$$

As required, is independent from p_a : this is the expected profits for G_a from playing, in this high demand case, any price belonging to $[c_a, \hat{p}]$.

Using the same reasoning the expected profits that G_b would derive from bidding any price belonging to $[c_b, \hat{p}]$ for any play of G_a would be:

$$E(\Pi_b(p_b, f_a(p_a))) = (\hat{p} - c_b) (D - K_a) \quad (27)$$

Equations (26) and (27) characterize the profit derived from the equilibrium in mixed-strategies when demand is higher than the larger capacity declared available: $K_i < K_j < D < K_i + K_j$.

10.6 Proof of Solution 13, 14 and 15

1. Best reply functions

Fist we derive the best reply functions of G_a for any play of G_b relying on the following graph:

²⁶Is easy to verify that (24) is the solution for the last row of (23). Rearranging to find $\frac{f_b}{F_b}$ in function of all the other parameters we can derive $\frac{\partial F_b(p_b)}{\partial p_b} = \frac{\gamma_b}{A_b} \left[\frac{p_b - c_a}{A_b} \right]^{\gamma_b - 1}$ and substituting its value in the diferencial equation we find $\frac{f_b}{F_b} = \frac{A_b}{A_b} \gamma_b (p_b - c_a)^{-1}$ that is exactly what the third row of (23) describes.

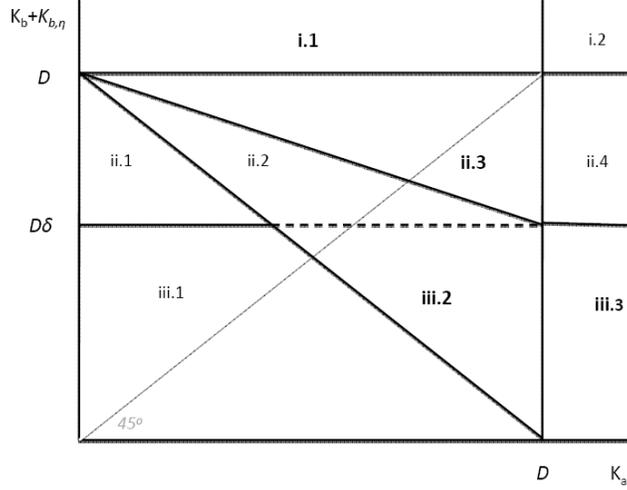


Figure 1. Best reply functions.

We analyze the play of G_a for each play of G_b in each of the zones delimited in Figure 1.

Zone i:

G_a thinks that G_b will make available $D \leq K_b + K_{b,\eta}$. Then G_a could play:

i.1) $K_a < D$ where we would be in the equilibrium defined in *Case b: intermediate demand* when G_b has the advantage in capacity, i.e.:

$$\Pi_a = (\hat{p} - c_a)K_a$$

i.2) $K_a \geq D$ (includes the vertical line) where we would be in *Case a: low demand*.

The best response of G_a in this zone is $K_a < D$.

Zone ii:

G_a thinks that G_b will make available $D\delta < K_b + K_{b,\eta} < D$. Then G_a could play:

ii.1) $K_a < D - K_b - K_{b,\eta}$ where we would be in *Case d: not served demand*.

ii.2) $D - K_b - K_{b,\eta} \leq K_a < \frac{1}{1-\delta}(D - K_b - K_{b,\eta})$ (includes line) where we would be in the equilibrium in mixed strategies defined in *Case c: high demand*.

ii.3) $\frac{1}{1-\delta}(D - K_b - K_{b,\eta}) < K_a < D$ where we would be in *Case b: intermediate demand* where in most of the area $\Pi_a = (\hat{p} - c_a)K_a$ and at worst²⁷ $\Pi_a = (\hat{p} - c_a)(D - K_b - K_{b,\eta})$.

ii.4) $D \leq K_a$ (includes line) where we would be in the equilibrium in *Case a: low demand*.

The best response of G_a in this zone is $\frac{1}{1-\delta}(D - K_b - K_{b,\eta}) < K_a < D$.

Line:

G_a thinks that G_b will make available $D\delta = K_b + K_{b,\eta}$. Then G_a could play:

1) $K_a < D$ where we would be in *Case d: not served demand* like in *ii.1* and *iii.1*. or for K_a that tends to D we get to *Case c: high demand* earning at

²⁷As most of the area is over the 45 degree line drawn in grey.

most²⁸ $\Pi_a = (\hat{p} - c_a)(D - K_b - K_{b,\eta})$. Then, $K_a = D$ belongs to the following case so profits are equal to the ones obtained in zone *ii.2*.

2) $D \leq K_a$ where we would be in *Case a: low demand*.

The best response of G_a in this line is $K_a < D$.

Zone iii:

G_a thinks that G_b will make available $K_b + K_{b,\eta} < \delta D$. Then G_a could play:

iii.1) $K_a < (D - K_b - K_{b,\eta})$ where we would be in *Case d: not served demand*.

iii.2) $(D - K_b - K_{b,\eta}) \leq K_a < D$ where we would be in the equilibrium in mixed strategies defined in *Case c: high demand* where in most of the area $\Pi_a = (\hat{p} - c_a)(D - K_b - K_{b,\eta})$ and at most $\Pi_a = (\hat{p} - c_a)K_a$ could be obtained to the left of the grey line.

iii.3) $D < K_a$ where we would be in the equilibrium in *Case b: intermediate demand* where K_a has the capacity advantage.

The best response of G_a in this zone is $(D - K_b - K_{b,\eta}) < K_a$.

The bold numbers and the thick line in Figure 1 are the best response of G_a for any play of G_b when both players know the level of demand before deciding about capacity.

The best response for G_b is derived computing Π_b for each zone as we just did for G_a .

2. Intersection between best reply functions.

Intersecting both best response functions we find these three families of equilibria that constitute the shaded area in the following figure. This area is the intersection between the fully grey area that represents the best response of G_a and the dotted area that represents the best response of G_b .

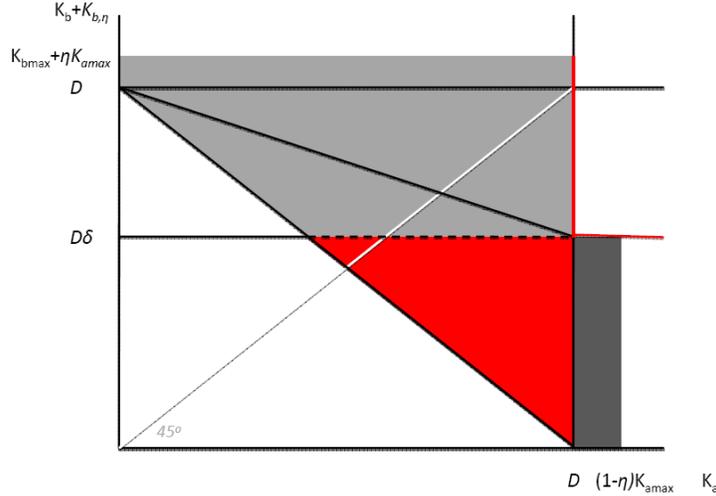


Figure 2. Intersection between best reply functions.

²⁸ As we can see $K_a = D$ belongs to the second case so profits are equal to the ones obtained in zone *ii.2*.

Is worth noting that none of the equilibria Pareto dominates the others, as each generator is better-off when it is dispatched first and then paid the price cap offered by the other generator. We will go back to this argument after discussing the case we are most interested in: the analysis given stochastic demand.

10.7 Proof of equations (3) and (4)

Now that demand can be peak D_p and off peak D_n , with a certain probability h and $(1-h)$ respectively, the payoffs will be the ones delimited by the regions described in Figure 3.

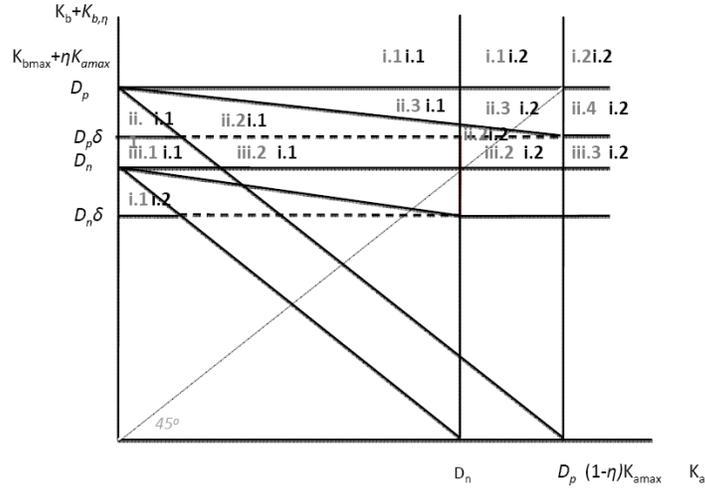


Figure 3. Profits and demand uncertainty.

The complete reasoning is described hereafter.

If G_a thinks that G_b will make available $D_p \leq K_b + K_{b,\eta}$. Then G_a could play:

- a) $K_a > D_p$ where he would earn $\Pi_a = \Pi_a(i.2.)$ for sure, where $\Pi_a(i.2.)$ is the profit he received in Figure 1 from playing in zone $i.2.$
- b) $D_n < K_a < D_p$ where he would earn $E(\Pi_a) = h\Pi_a(i.2) + (1-h)\Pi_a(i.1)$
- c) $K_a < D_n$ where he would earn $\Pi_a = \Pi_a(i.1)$ for sure.

The best response of G_a is $K_a < D_n$.

If G_a thinks that G_b will make available $D_p\delta < K_b + K_{b,\eta} < D_p$. Then G_a could play:

- d) $K_a > D_p$ where he would earn $E(\Pi_a) = h\Pi_a(ii.4) + (1-h)\Pi_a(i.2)$.
- e) $\frac{1}{1-\delta}(D_p - K_b - K_{b,\eta}) < K_a < D_p$ and $D_n < K_a$ where he would earn $E(\Pi_a) = h\Pi_a(ii.3) + (1-h)\Pi_a(i.2)$
- f) $D_n < K_a < \frac{1}{1-\delta}(D_p - K_b - K_{b,\eta})$ where he would earn $E(\Pi_a) = h\Pi_a(ii.2) + (1-h)\Pi_a(i.2)$
- g) $\frac{1}{1-\delta}(D_p - K_b - K_{b,\eta}) < K_a < D_n$ where he would earn $E(\Pi_a) = h\Pi_a(ii.3) + (1-h)\Pi_a(i.1)$

h) $(D_p - K_b - K_{b,\eta}) < K_a < \frac{1}{1-\delta}(D_p - K_b - K_{b,\eta})$ and $K_a < D_n$ where he would earn $E(\Pi_a) = h\Pi_a(ii.2) + (1-h)\Pi_a(i.1)$

i) $K_a < D_p - K_b - K_{b,\eta}$ where he would earn $E(\Pi_a) = h\Pi_a(ii.1) + (1-h)\Pi_a(i.1)$

The best response of G_a in this zone is $\frac{1}{1-\delta}(D_p - K_b - K_{b,\eta}) < K_a < D_n$.

If G_a thinks that G_b will make available $\mathbf{D}_n < \mathbf{K}_b + \mathbf{K}_{b,\eta} < \mathbf{D}_p \delta$. Then G_a could play:

j) $K_a > D_p$ where he would earn $E(\Pi_a) = h\Pi_a(iii.3) + (1-h)\Pi_a(i.2)$.

l) $D_n < K_a < D_p$ where he would earn $E(\Pi_a) = h\Pi_a(iii.2) + (1-h)\Pi_a(i.2)$

m) $D_p - K_b - K_{b,\eta} < K_a < D_n$ where he would earn $E(\Pi_a) = h\Pi_a(iii.2) + (1-h)\Pi_a(i.1)$

n) $K_a < D_p - K_b - K_{b,\eta}$ where he would earn $E(\Pi_a) = h\Pi_a(iii.1) + (1-h)\Pi_a(i.1)$

The best response of G_a in this zone is $D_p - K_b - K_{b,\eta} < K_a < D_n$.

If G_a thinks that G_b will make available $\mathbf{D}_n \delta < \mathbf{K}_b + \mathbf{K}_{b,\eta} < \mathbf{D}_n$. Then G_a could play:

o) $K_a > D_p$ where he would earn $E(\Pi_a) = h\Pi_a(iii.3) + (1-h)\Pi_a(ii.4)$.

p) $D_n < K_a < D_p$ where he would earn $E(\Pi_a) = h\Pi_a(iii.2) + (1-h)\Pi_a(ii.4)$.

q) $D_p - K_b - K_{b,\eta} < K_a < D_n$ and $\frac{1}{1-\delta}(D_n - K_b - K_{b,\eta}) < K_a < D_n$ where he would earn $E(\Pi_a) = h\Pi_a(iii.2) + (1-h)\Pi_a(ii.3)$

r) $D_p - K_b - K_{b,\eta} < K_a < \frac{1}{1-\delta}(D_n - K_b - K_{b,\eta})$ where he would earn $E(\Pi_a) = h\Pi_a(iii.2) + (1-h)\Pi_a(ii.2)$

s) $\frac{1}{1-\delta}(D_n - K_b - K_{b,\eta}) < K_a < D_p - K_b - K_{b,\eta}$ where he would earn $E(\Pi_a) = h\Pi_a(iii.1) + (1-h)\Pi_a(ii.3)$

t) $D_n - K_b - K_{b,\eta} < K_a < \frac{1}{1-\delta}(D_n - K_b - K_{b,\eta})$ and $K_a < D_p - K_b - K_{b,\eta}$ where he would earn $E(\Pi_a) = h\Pi_a(iii.1) + (1-h)\Pi_a(ii.2)$

u) $K_a < D_n - K_b - K_{b,\eta}$ where he would earn $E(\Pi_a) = h\Pi_a(iii.1) + (1-h)\Pi_a(ii.1)$

The best response of G_a in this zone is $D_p - K_b < K_a < D_n$ and $\frac{1}{1-\delta}(D_n - K_b - K_{b,\eta}) < K_a < D_n$

If G_a thinks that G_b will make available $\mathbf{K}_b + \mathbf{K}_{b,\eta} < \mathbf{D}_n \delta$. Then G_a could play:

v) $K_a > D_p$ where he would earn $\Pi_a = \Pi_a(iii.3)$.

w) $D_p - K_b - K_{b,\eta} < K_a < D_p$ where he would earn $E(\Pi_a) = h\Pi_a(iii.2) + (1-h)\Pi_a(iii.3)$

x) $D_p - K_b - K_{b,\eta} < K_a < D_n$ where he would earn $\Pi_a = \Pi_a(iii.2)$

y) $D_n < K_a < D_p - K_b - K_{b,\eta}$ where he would earn $E(\Pi_a) = h\Pi_a(iii.1) + (1-h)\Pi_a(iii.3)$

z) $D_n - K_b - K_{b,\eta} < K_a < D_n$ and $K_a < D_p - K_b - K_{b,\eta}$ where he would earn $E(\Pi_a) = h\Pi_a(iii.1) + (1-h)\Pi_a(iii.2)$

za) $K_a < D_n - K_b - K_{b,\eta}$ where he would earn $\Pi_a = \Pi_a(iii.1)$

The best response of G_a in this zone is $D_p - K_b - K_{b,\eta} < K_a$.

10.8 Proof Proposition 2

Intersecting both best response functions we find the families of subgame perfect equilibria for the case of stochastic demand distributed between D_n and D_p . These equilibria constitute the shaded grey area in the following figure that is the intersection between the black area that represents the best response of G_a and the dotted area that represents the best response of G_b .

To ensure the existence of type i equilibria (the triangular part of the grey area) we assume that the distance between D_n and D_p is such that $(1 + \delta)D_n > D_p$. Otherwise only equilibria ii and iii would be possible as the triangular area in the figure would disappear. It is a strict inequality because if it was the case that $(1 + \delta)D_n = D_p$, the constraint $K_a + K_b \geq D_p$ could never be satisfied as the equilibrium is defined for $K_a < D_n$.

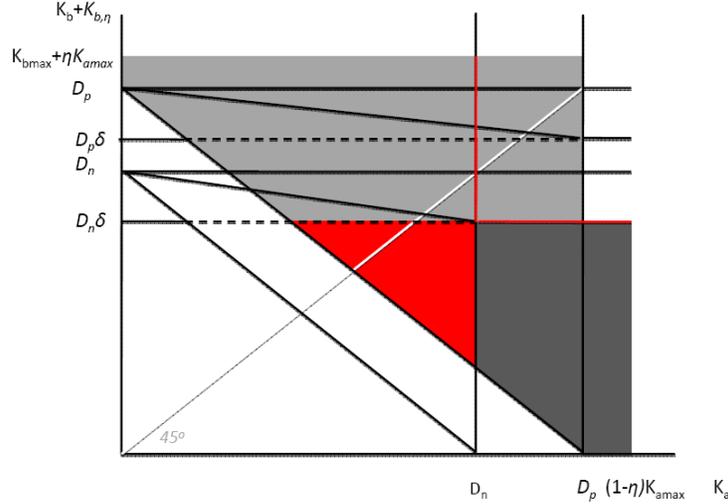


Figure 4. Equilibria of the capacity game.

10.9 Proof of Lemma 12

Type ii equilibria lead us to *Case c* or, if $K_a > D_p$, to *Case b part a*²⁹ where G_a has the capacity advantage. Then, type ii equilibria when demand is D_p implies the following payoffs:

$$\begin{aligned}\Pi_a &= (\hat{p} - c_a)(D_p - \delta D_n) \\ \Pi_b &= (\hat{p} - c_b)D_n\delta\end{aligned}$$

Symmetrically, type iii equilibria lead us to the same *Case c* or *Case b* where G_b has the capacity advantage if $K_b > D_p$:

$$\begin{aligned}\Pi_a &= (\hat{p} - c_a)D_n \\ \Pi_b &= (\hat{p} - c_b)(D_p - D_n) > 0\end{aligned}$$

²⁹We will never be in *part b* as K_b cannot be higher than $D_n\delta$ which is equivalent to say that α_a cannot be lower than c_b .

On the other hand, in type i , if demand is D_p , we fall in *Case c* where $E(\Pi_i) = (\hat{p} - c_i)(D_p - K_j)$.

Using the condition on the distance between D_n and D_p imposed, *i.e.* $(1 + \delta)D_n > D_p$, we can directly compare the payoffs of each player for each equilibria and conclude that:

a) when D_p is realized G_b prefers type ii equilibria to type iii as in the first case he gets $\Pi_b = (\hat{p} - c_b)D_n\delta$ for sure while in the second case he gets something lower that tends to the previous Π_b when $D_p \rightarrow (1 + \delta)D_n$. Moreover, type i is preferred to type iii , as in type i he expects to sell $D_p - K_a$ where $K_a < D_n$.

b) when D_p is realized G_a prefers type iii equilibria to type i equilibria that is preferred to type ii .

10.10 Proof of Corollary 3

Type ii equilibria lead us to *Case c* or, if $K_a > D_p$, to *Case b part a*³⁰ where G_a has the capacity advantage. Then, type ii equilibria when demand is D_p implies the following payoffs:

$$\Pi_a = (\hat{p} - c_a)(D_p - \delta D_n) \text{ and } \Pi_b = (\hat{p} - c_b)D_n\delta$$

Symmetrically, type iii equilibria lead us to the same *Case c* or *Case b* where G_b has the capacity advantage if $K_b > D_p$:

$$\Pi_a = (\hat{p} - c_a)D_n \text{ and } \Pi_b = (\hat{p} - c_b)(D_p - D_n) > 0$$

On the other hand, in type i , if demand is D_p , we fall in *Case c* where $E(\Pi_i) = (\hat{p} - c_i)(D_p - K_j)$.

Using the condition on the distance between D_n and D_p imposed in the proof of equation (5), *i.e.* $(1 + \delta)D_n > D_p$, we can directly compare the payoffs of each player for each equilibria and conclude that:

a) when D_p is realized G_b prefers type ii equilibria to type iii as in the first case it gets $\Pi_b = (\hat{p} - c_b)D_n\delta$ for sure while in the second case it gets something lower that tends to the previous Π_b when $D_p \rightarrow (1 + \delta)D_n$. Moreover, type i is preferred to type iii , as in type i it expects to sell $D_p - K_a$ where $K_a < D_n$.

b) when D_p is realized G_a prefers type iii equilibria to type i equilibria that is preferred to type ii .

A similar reasoning can be applied to derive which family of equilibria is preferred by each agent when D_n is realized.

³⁰We will never be in *part b* as K_b cannot be higher than $D_n\delta$ which is equivalent to say that α_a cannot be lower than c_b .