Input choice under carbon constraint

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Abstract

This paper assesses the impact of emission trading on short-term input demand as well as on long-term production decisions, tacking into account uncertainty in the polluting input price and abatement by input substitution. We find that firms decisions depend on the interplay between three effects. First, the "average cost effect", due to the carbon price, causes a decrease in the input capacity with respect to a reference case where the permits market does not exist. Second, the "marginal variability effect" or the impact of price variability, which instead leads to an expansion of the installed equipment. Third, the "technology effect", i.e. the extent of substitution between polluting and clean inputs. Model simulations show that this interaction can result in weak emission reductions.

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1 Introduction

Despite a large body of literature on environmental policy and cleaner technology innovation and diffusion,[1] less is known on the interaction between

[1] Some papers consider the incentives for research and innovation while others study the diffusion of the new technology (Downing and White, 1986; Fisher et al. 1998, Jung

short-term (or end-of-pipe) abatement decisions and long term investment in production capacity. However, when a market for permits like the European Trading Scheme (henceforth, EU ETS) is introduced, in the short term, only marginal changes in the production technology are feasible, and abating by reducing the output scale can be considered as the exception rather than the rule. Thus regulated firms face a basic choice between buying (or selling) allowances, and altering the production process to reduce emissions.[2] For instance, the largest affected sector that received the lower amount of initial permits, i.e. the fuel-burn energy producers, can abate by fuel-switching. This process involves the replacement of high-carbon fuels with low-carbon alternatives. Still, to make the fuel-switching feasible, a sufficient amount of low-carbon capacity has to be installed.

The objective of this paper is to assess the impact of carbon constraints on short-term input demand as well as on capacity choice, in a simple two stage model. In the first stage, a competitive and risk neutral representative firm invest in capacity and in the second how to use production factors, according to a given production technology which determines the extent of substitution between the carbon-free input and the carbon-intensive one. Environmental regulation is achieved through a market for pollution permits. The unit cost of the carbon-intensive fuel thus includes the permits' price multiplied by its emission intensity and the input price which is assumed random.

In setting our simple two stage model we extend De Villemeur et al. (2006),[3] sharing similar motivations with the literature on investment (Abel et al.,1996, Dixit and Pindyck 1994) and on firms' behavior under uncertainty (Sandmo 1971, Wolak and Kolstad 1991). In fact, uncertainty on the input

et al., 1996; Milliman et al. 1989). Other works focus on the timing of environmental policy and show that, in some cases, this timing influences the ranking of the alternative instruments (Jaffe et al., 2002 provide an extensive survey on technological change and environmental problems). Another strand of the literature is concerned with the impact of technological change on marginal abatement cost, in a macroeconomic perspective (for a survey, see Baker et al, 2007).

[2]Large firms, in order to accomplish the already existing severe European environmental regulations, have mostly reached high environmental standards either in production processes or in reducing the offending gas released as a by-product in the air. See Szabo et al. (2006) for a more comprehensive discussion.

[3]In particular, we borrow from this work the structure of the two-stage production decision model. However, De Villemeur et al. (2006) focuse on the separation of investment decisions, which are set consistently with expected state of natures, from short-term inputs' demand, to provides micro-foundations for short-term cost inefficiencies.

price plays a key role in the capacity choice, even though we do not consider the problem of the optimal timing for investment.

To understand the determinants of capacity choice, we compare the results of our model to a conterfactual scenario without environmental regulation and/or without uncertainty. We unveil three driving forces. First, the "average cost effect", due to the carbon price, causes a decrease in the input capacity with respect to a "business as usual" scenario without environmental regulation. Second, the "marginal variability effect" or the impact of price variability, which instead leads to an expansion of the installed capacity. As a consequence of the investment irreversibility, firms react to uncertainty by leaving some capacity unused but available if the uncertainty becomes high. This effect is amplified if the carbon price is itself random. Third, these decisions interact with the "technology effect": the optimal input capacity first increase and then decrease as the substitution elasticity increases.

To illustrate the interplay beween these three effects, we simulate our model under both deterministic and random CO_2 price, assuming normal distributions. Optimal capacity increases with respect to the input price standard deviation. Moreover, the higher the elasticity of substitution, the stronger the sensitivity to price variability. Thus optimal capacity is constant for the case of strict complementarity as inputs must be used in fixed proportion. On the contrary, in the case of perfect substitution, the optimal capacity is either 0 or strictly positive, given that input demand is necessarily a corner solution. As for pollution, for the set of parameters used in the simulation, we calculate that emissions reduction amounts at around 10 percent with respect to a case where there is no carbon price, under the assumption of a Cobb Douglas production function. When the technology effect allows stronger substitutability, larger emission reduction obtains. Moreover the emission saving is larger when the price of CO_2 is known compared to the case where it is a random variable. As uncertainty in CO_2 price increases the input price variability, the marginal variability effect dominates and optimal capacity becomes larger. This in turn leads to an increase of input demand and thus emissions.

The paper is organized as follows. Related literature which has been recently developed in the context of the EU ETS is summarized in Section 2. The two stages of the producer's behavior, that is short-run input demand and long-run capacity choice under risk neutrality is developed in Section 3. The role of uncertainty, environmental regulation and production technology is discussed in Section 4. Model simulations illustrate solutions for optimal capacities, input demand and CO_2 emissions (Section 5). We briefly conclude by evoking some policy implications and directions for further research.

2 Related literature in the context of the European carbon market

There is empirical evidence that during the EU ETS Phase I (i.e. from 2005 to 2007), carbon price has induced some emissions abatement in the electricity sector, in the form of intra-fuel substitution (brown to hard coal) in Germany and improved CO_2 efficiency in the UK (Convery et al, 2008). This evidence is confirmed by Delarue et al. (2008) for the overall European market during the summer season in 2005. Similarly, Denny *et al.* (2009) show that abatement depends not only on the price of allowances, but also on the load level of the electric system and the ratio between natural gas and coal prices. At a more aggregate level, Considine *et al.* (2009) examine the demand for carbon permits, carbon based fuels, and carbon-free energy for 12 European countries after the introduction of the EU ETS. A short-run restricted cost function is estimated in which carbon permits, high-carbon fuels, and low-carbon fuels are variable inputs, conditional on quasi-fixed carbon-free energy production from nuclear, hydro, and renewable energy capacity. The results indicate that prices for permits and fuels affect the composition of inputs in a statistically significant way. The estimates suggest that for every 10 percent rise in carbon and fuel prices, the marginal cost of electric power generation increases by 8 percent in the short-run.

Several empirical papers on carbon price drivers have tested whether the switching price that makes an electricity producer indifferent in using coal or gas[4] can be considered a significant regressor (for an extensive survey on this topic, see Bonacina et al. 2009). Despite some mixed results on the significativity of the switching variable, fuel price models form an intrinsic part of carbon price description (Fehr et al., 2006).

[4]Formally the switching price (*switch* in €/ton CO_2) is a result of the following relationship: $e_g \times switch + eff_g \times gas = e_c \times switch + eff_c \times coal$, where e_g,e_c measure the emission intensity of gas and coal respectively and eff_g, eff_c the efficiency rate of gas-fired and coal plants. In the above relationship, the LHS measures the marginal cost of producing electricity with a gas-fired plant in a carbon-constrained framework and the RHS is the same for a coal-fired unit.

We believe that the models dealing with the adjustment of the production process to a carbon constraint have three main drawbacks.

First, existing approaches disregard the effect of emission allowances on inputs' demand. A partial exception can be found in Jouvet et al. (2005, 2007), who endogenize the technological dimension and analyze inter-industry redistribution of production and inputs (capital and labour). However the authors, by opting for a macroeconomic approach, let perfectly mobile input demand, therefore neglecting the impact of potential capacity constraints. A more micro-economic approach can be found in Newbery (2008), who demonstrates that in the EU ETS, fixing the quantity rather than the price of carbon reduces the price elasticity of demand for gas, amplifying both the market power of gas suppliers, and the impact of gas price increases on the electricity price. This contribution shows important short-term effects, but does not analyze how in turn a modified demand for gas can affect firms investment or equipment decisions in using such less polluting fuel.

Second, there has been no attempt to consider the effect of both fuel and permit price uncertainty on firms' production decisions in terms of input demand and capacity choice. In fact, the literature on short term abatement assumes perfect information.[5]

Third, imperfect substitutability of inputs is ignored. If the hypothesis of perfect substitution can be acceptable when referred to electrical utilities that switch from coal to gas, this simplification seems unsatisfactory, notably when we enlarge the analysis to the combination of energy and non-energy inputs in the production function. Technological constraints influence the extent of substitution possibilities.

Our analysis is a first attempt to encompass some issues actually neglected by the relevant literature on firms' production decisions and environmental regulation. We thus study input demand and capacity choice under carbon constraint, by analyzing the role of input price uncertainty and substitutability. However, how to induce long-term technological change is beyond the scope of this paper, which addresses only one aspect of the firm-level response to a CO_2 price.

[5] Indeed uncertainty has been introduced in other domains related to environmental policies. Uncertainty ranges from agency problems with asymmetric information due to time and/or regulatory ambiguity (Laffont and Tirole, 1996; Farzin and Kort, 2000), to welfare maximization issues with uncertain expected benefits/damages (Baker et al., 2006; Baker, 2007), encompassing endogenous or exogenously driven technological uncertainty.

3 The model for producer's behavior

In the first stage, a representative risk neutral firm invests in capacity and in the second it decides how to use inputs, according to a specific technology. In what follows, we analyze the cost minimization problem, at given capacity, and then we endogenize the capacity choice.

We consider the simple case of a production technology with two inputs where input 1 requires some equipment whose size, denoted by $\overline{x_1}$, represents a capacity constraint ($x_1 \leq \overline{x_1}$). To simplify the model, we assume that the input x_2 is unconstrained. Input usage and input capacity are expressed in the same unit [6]. The production function $y = f(x_1, x_2)$ behaves according to standard regularity conditions and is concave.

We assume first that the cost of the equipment depends only on its size and second, that the marginal cost of capacity x_1 is equal to c_1 and constant.[7] Input 1 is a polluting input. Total emissions are equal to ex_1, where e is the per unit emission of 1. The allowed quotas Q is assumed to be such that $Q < e\overline{x_1}$. The price of a permit is denoted by Π. The total unit cost of input 1 is therefore $P_1 + e\Pi$, which we denote p_1 from now on. Input 2 is a clean input and its price P_2 is normalized to 1.

3.1 The short-run input demand function

In the *ex post* program corresponding to the second period, the firm minimize short-run costs given input capacity constraint and the realized price for input 1. At this stage, the model describes the firm behavior under an exogenous input quantity constraint, as in Squires (1994). The *ex post* cost minimization program is:

[6] For example, electric power is produced by combining fuel and equipement, and the range of available equipement can be fully defined by the equipement's size (KW capacity) and the equipement's fuel requirements. Ignoring labor requirements, maintenance requirements and the fact that heat rate varies nonlinearly with the percent of plant capacity used at any instant, the ex post production function for a plant can be defined by a fixed coefficient prodution function. See Stewart (1979) for a more general discussion on this point.

[7] This is the final simplification. Firms could choose simultaneously the quantity and the quality of capital good. See Muller (2000) and their extension of the putty-clay model of capital and energy of Atkeson and Kehoe (1999).

$$
\begin{aligned}
&\underset{x_1,x_2}{Min} \quad p_1 x_1 + x_2 + c_1 \overline{x_1} = \\
&\text{With} \quad p_1 = P_1 + e\Pi \\
&\text{Subject to } y = f(x_1, x_2),\ x_1 \leq \overline{x_1} \text{ and } Q < e\overline{x_1}.
\end{aligned} \tag{1}
$$

Following the virtual prices approach by Neary and Roberts (1980), the optimal input demand will be

$$
\begin{cases} \overline{x_1},\ \underline{x_2} & \text{if } p_1 \leq \eta_1, \\ x_1^*(p_1, y),\ x_2^*(p_1, y) & \text{if } p_1 > \eta_1. \end{cases} \tag{2}
$$

The variable η_1 is the virtual price of input 1 at which the unconstrained demand for input 1 is exactly equal to $\overline{x_1}$. The input quantity $\underline{x_2}$ follows directly from the production function constraint $y = f(\overline{x_1}, \underline{x_2})$.[8] A complete characterization of the virtual price can be found in Lee and Pitt (1987) in particular it could be written as a function $\eta_1(\overline{x_1}, y)$.

The following figure illustrates the optimal input demand. In the short term the maximum amount of input 1 the firm can use is limited by $\overline{x_1}$ and the isoquant is an arc with an extrema points $\left(\overline{x_1}, \underline{x_2}\right)$. For a given capacity level, the ability to switch between the two inputs depends on the realized price and is limited.

We distinguish substitution possibilities and switching capacity simply to keep in mind that the marginal rate of technical substitution is a local measure while the switching capacity is a global one and represents the extent of substitution possibilities in the short-run.

[8] A formal presentation is $\underline{x_2} = x_2(\overline{x_1})$ such that $y \equiv f(\overline{x_1}, \underline{x_2})$.

Optimal Input Demand

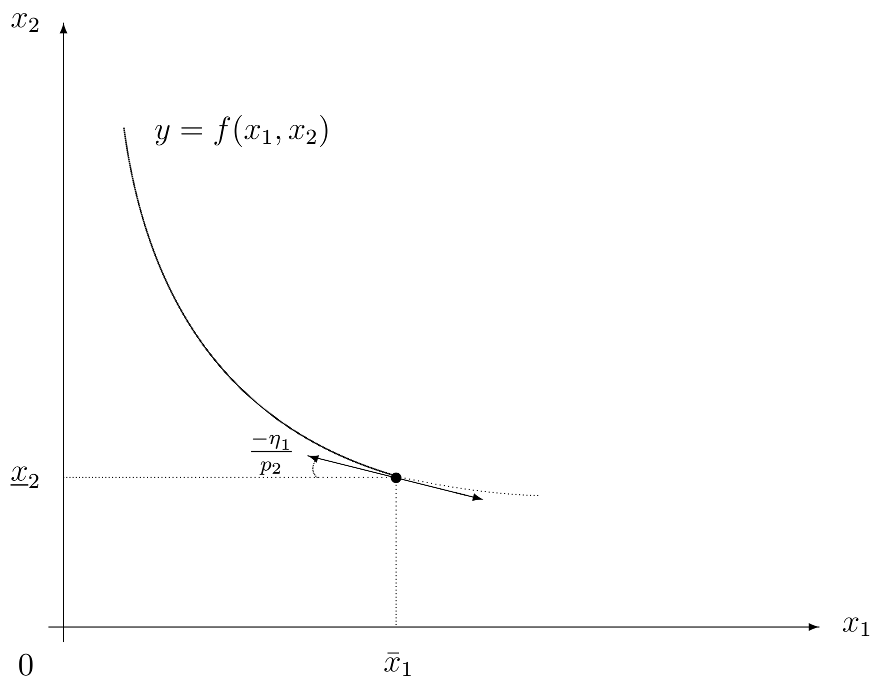

Figure 1.

3.2 The long-run optimal capacity program under risk neutrality

Recall that input 1 only needs some equipment with fixed size; the price $p_1 = P_1 + e\Pi$ is random. Notice that for the moment we assume that the permits price is deterministic. Considering p_1 as a random variable with density function $\phi(p_1)$, cumulative density function $\Phi(p_1)$ and with $p_1 \in [0, +\infty[$, the first stage program including environmental cost is

$$\min_{\overline{x_1}} TC = \min_{\overline{x_1}} \int_0^{\eta_1} \left[(P_1\overline{x_1} + \underline{x_2}) + \Pi(e\overline{x_1} - Q) \right] \phi(p_1)dp_1 + \int_{\eta_1}^{+\infty} \left[(P_1x_1^* + x_2^*) + \Pi(ex_1^* - Q) \right] \phi(p_1)dp_1 + c_1\overline{x_1}. \tag{3}$$

Let us remark that depending on the value of input price p_1 and hence the optimal level of input use x_1^*, the firm will sell or buy permits depending on the effective emission level. If $ex_1^* > Q$ the firm needs to buy $\Pi(ex_1^* - Q)$. On the contrary, if $ex_1^* > Q$ the firm can sell $\Pi(Q - ex_1^*) = -\Pi(ex_1^* - Q)$. In fact in all the case the situation is such that the firm perceive a fixed revenue ΠQ and pay for effective emission Πex_1^*. The only aspect that changes with respect of a cost minimization program without environmental regulation is that the price of input 1 is $P_1 + \Pi e$ instead of simply p_1. Notice that with respect to a pigouvian tax, the equilibrium level of costs (and profits as well, being the output fixed) will be different as the tradable allowances could either generate a deficit or a surplus.

For each possible capacity level the firm faces two cases depending on the *ex post* realized price. If the relative price of input 1 is low enough the firm would use a huge amount of this input but the firm may be constrained by the installed input capacity level. Otherwise, if the price is high, there is in some sense a reserve capacity, the optimal level of input demand being lower than the capacity constraint.

The firm's trade-off is simple, by choosing a large capacity $\overline{x_1}$ the firm is able to reach allocative efficiency for a large range of possible prices p_1 but this capacity has a cost. The first order condition for capacity choice is as follows:

$$\frac{\partial TC}{\partial \overline{x_1}} = \int_0^{\eta_1} (p_1 + \frac{\partial \underline{\underline{x_2}}}{\partial \overline{x_1}})\phi(p_1)dp_1 + c_1 = 0. \tag{4}$$

Simple manipulation of the first order condition leads to the following equation which provides the price threshold η_1 such that the capacity choice is optimal:

$$\Phi(\eta_1)\left[\eta_1 - \mathbb{E}(p_1\ /p_1 \leq \eta_1)\right] = c_1. \tag{5}$$

The optimal level of capacity is such that the expected marginal gain in allocative efficiency is equal to the marginal cost in capacity. Equation (5) can be solved first in η_1 and, in a second step $\overline{x_1}$ can be determined as a function of the virtual price. This means that the first order condition can be solved in $\overline{x_1}$ *regardless the production function specification.*

The second order condition is:

$$\frac{\partial \eta_1}{\partial \overline{x_1}}\phi(p_1) \leq 0 \tag{6}$$

This condition is satisfied since we assume a concave production function which implies that $\frac{\partial \eta_1}{\partial \overline{x_1}} \leq 0$. As a consequence, the expected marginal gain associated to a marginal increase in the capacity level is always decreasing.

4 Uncertainty, carbon markets, technology: contrasting forces

We now investigate comparative statics which uncover the impact of uncertainty, environmental regulation and production technology. As we will see, the analysis reveals some effects that create contrasting forces in determining short-term and long-term firm's decisions.

4.1 The role of uncertainty

In this section, we show how price uncertainty affects the choice of the input capacity level. To this end, let $\overline{x_1}^c$ denote the optimal input capacity level under certainty. Suppose that the price is known and equal to μ. The cost minimization program of the firm could be simplified in this case in one step only:

$$\begin{gathered} \underset{x_1, x_2, \overline{x_1}}{Min} \quad \mu x_1 + x_2 + c_1 \overline{x_1}, \\ \text{Subject to } y = f(x_1, x_2),\ x_1 \leq \overline{x_1}. \end{gathered} \tag{7}$$

Clearly the input capacity constraint will be binding because there is no gain to hold reserve capacity. As a result the optimal solution for both demand and capacity for input 1 are the same and equal to:

$$\overline{x_1}^c = x_1^*(\mu + c_1, y). \tag{8}$$

Denote by $\overline{x_1}^*$ the input capacity chosen by a risk neutral firm subject to environmental regulation, when the price of input 1 is random. The impact of uncertainty on capacity choice can be easily analyzed.

Proposition 1 *(Uncertainty effect) Uncertainty of the polluting input price increases the optimal input capacity $\overline{x_1}^*$ above the level chosen under certainty $\overline{x_1}^c$.*

Proof. Suppose that while the price $p_1 = P_1 + \Pi e$ is random with $\mathbb{E}(p_1) = \mu$, the firm decides to fix the input capacity at $\overline{x_1}^c$. The marginal gain to increase marginally the capacity $\overline{x_1}^c$ is:

$$(\mu + c_1) \int_0^{\mu + c_1} \phi(p_1) dp_1 - \int_0^{\mu + c_1} p_1 \phi(p_1) dp_1. \tag{9}$$

Note that, for the assumed input capacity constraint $\overline{x_1}^c$, we obtain here a particular value of the gain associated to a marginal increase of capacity when the virtual price is $\mu + c_1$ (see Equation **5**)**.** This marginal gain can be rewritten as:

$$\begin{aligned} & (\mu + c_1) \left[1 - \int_{\mu + c_1}^{+\infty} \phi(p_1) dp_1\right] - \mu + \int_{\mu + c_1}^{+\infty} p_1 \phi(p_1) dp_1 \qquad (10) \\ = \quad & c_1 + \int_{\mu + c_1}^{+\infty} (p_1 - (\mu + c_1)) \phi(p_1) dp_1. \end{aligned}$$

Provided that the distribution probability is not degenerated over $[\mu + c_1, +\infty[$ which means that with a non zero probability the realized price could be greater that $\mu + c_1$, then the integral in the marginal gain is positive and so the marginal gain greater than the marginal cost of the capacity c_1. Since concavity of the production function implies that the marginal gain is decreasing with respect to the capacity, it follows that the firm increases the capacity level above $\overline{x_1}^c$. ■

The intuition behind Proposition 4.1 is that when faced to a random price of input, the risk-neutral firm keeps the option to use input 1 up to the capacity constraint, in order to benefit from low production costs during low input price. Comparing our results with those of models with incremental investment (as for instance Pindyck, 1988), we see here a different investment strategy. Instead of reducing the amount of installed capacity with respect to the optimal input capacity level under certainty, uncertainty leads to "oversize" the equipment. This effect is a consequence of the once and for all decision and resembles the one obtained by Hartl and Kort (1996) and Dangl (1999), where uncertainty in future demand leads to an increase in optimal installed capacity.

As a complement to the result of Proposition 1, it is interesting to investigate how a marginal increase in uncertainty affects firms' decisions. We thus consider a modified cost minimization program of the firm, where the price of input 1 is now equal to $\gamma p_1 + \theta$. It is easy to show that *ex post* optimal demand for inputs are:

$$\begin{cases} \overline{x_1}^*,\ \underline{x_2}^* & \text{if } p_1 \leq \frac{\eta_1 - \theta}{\gamma}, \\ x_1^*(\gamma p_1 + \theta, y),\ x_2^*(\gamma p_1 + \theta, y) & \text{if } p_1 > \frac{\eta_1 - \theta}{\gamma}. \end{cases} \tag{11}$$

The first order condition associated to the ex ante cost minimization with respect to the input capacity could now be written as:

$$\Phi(\frac{\eta_1 - \theta}{\gamma}) \left[\eta_1 - \mathbb{E}(\gamma p_1 + \theta \ / p_1 \leq \frac{\eta_1 - \theta}{\gamma}) \right] = c_1. \tag{12}$$

We study the effect of an increase in the variability of the density function of the price in terms of a mean preserving spread, that is:

$$d\mathbb{E}(\gamma p_1 + \theta) = \mu d\gamma + d\theta = 0 \Leftrightarrow \frac{d\theta}{d\gamma} = -\mu. \tag{13}$$

Differentiating the first order condition in equation (12) with respect to γ and using condition (13), we obtain

$$\frac{\partial \eta_1}{\partial \overline{x_1}^*} \frac{\partial \overline{x_1}^*}{\partial \gamma} d\gamma = \frac{\int_0^{\frac{\eta_1 - \theta}{\gamma}} (p_1 - \mu)\phi(p_1)dp_1}{\int_0^{\frac{\eta_1 - \theta}{\gamma}} \phi(p_1)dp_1} \tag{14}$$

The impact of input price variability can be summarized as follows.

Proposition 2 *(Marginal variability effect). A marginal increase in uncertainty increases the optimal capacity $\overline{x_1}^*$.*

Proof. We need to prove the negative sign of the integral in the numerator of the RHS in equation (14), as the denominator is always positive. A consequence of Proposition 1 is that η_1 solution of (12) is lower than $\gamma\mu + \theta + c_1$. It is easy to show that the numerator of the RHS in equation (14) is negative when $\eta_1 = \gamma\mu + \theta + c_1$.

$$\int_0^{\mu + \frac{c_1}{\gamma}} (p_1 - \mu)\phi(p_1)dp_1 = -\int_{\mu + \frac{c_1}{\gamma}}^{+\infty} (p_1 - \mu)\phi(p_1)dp_1. \tag{15}$$

which is clearly negative.

The result follows as, $\int_0^{\frac{\eta_1-\theta}{\gamma}}(p_1-\mu)\phi(p_1)dp_1$, which is an increasing function with respect to η_1,is negative when $\eta_1 = \gamma\mu + \theta + c_1$, this integral is necessarily negative when $\eta_1 \leq \gamma\mu + \theta + c_1$. ■

Propositions 1 and 2 both go into the same direction of accruing the optimal capacity with respect to a case without uncertainty. However, the unused capacity entails an environmental cost. This limits the incentive to hold reserve capacity, as the following Section shows.

4.2 The role of carbon markets

The polluting input price does depend on environmental regulation. The following result sheds light on the interaction of this additional cost with firms' capacity choice under uncertainty.

Proposition 3 *(Average cost effect) The optimal input capacity $\overline{x_1}^*$ is smaller than the one a risk neutral firm would choose absent environmental regulation.*

Proof. The carbon market price, shifting the distribution of the price P_1 by a constant Πe, represents an increase of the average input price: $\mathbb{E}(P_1) < \mathbb{E}(p_1)$.To measure the impact of such increase on the optimal capacity, we take the first order condition (12) as an identity at $\overline{x_1}^*$, and we differentiate it with respect to θ, which gives:

$$\frac{\partial\eta_1}{\partial\overline{x_1}^*}\frac{\partial\overline{x_1}^*}{\partial\theta}d\theta = \int_0^{\frac{\eta_1-\theta}{\gamma}}\phi(p_1)dp_1. \tag{16}$$

Provided that $\eta_1 - \theta$ is positive the RHS is positive. The concavity of the production function ensures $\frac{\partial\eta_1}{\partial\overline{x_1}} \leq 0$. These two condition allow to calculate $\frac{\partial\overline{x_1}^*}{\partial\theta} < 0$, and prove the result. ■

Proposition 3 shows what we call the "average cost effect" of carbon markets prices increases the price η_1. This reduces the oversizing effect and the optimal installed capacity shrinks.

An interesting issue is to consider a stochastic permits price as it determines an additional source of input price variability. Therefore a random CO_2 price could balance the average and the marginal variability effect, as Proposition 4 shows.

Proposition 4 *When the permits price is random, the optimal input capacity $\overline{x_1}^{\pi}$ is above $\overline{x_1}^{*}$ only if the variability of the permits price is high.*

Proof. Denote the expected CO_2 price as $\mathbb{E}(\Pi) = \mu^{\pi}$ and the corresponding optimal capacity level as $\overline{x_1}^{\pi}$.The expected input price when Π is random is $\mathbb{E}(p_1) = \mu + e\mu^{\pi} > \mu$, and the associated threshold price is $\eta_1^{\pi} = \gamma\left(\mu + e\mu^{\pi}\right) + \theta + c_1$. From Proposition 3, it follows that:

$$\frac{\partial \eta_1}{\partial \overline{x_1}^{*}} \frac{\partial \overline{x_1}^{*}}{\partial \theta} d\theta > \frac{\partial \eta_1^{\pi}}{\partial \overline{x_1}^{\pi}} \frac{\partial \overline{x_1}^{\pi}}{\partial \theta} d\theta. \tag{17}$$

We will consider now the effect of an increase in the variability of the density function in terms of a mean preserving spread which is now:

$$d\mathbb{E}(\gamma p_1 + \theta) = \left(\mu + e\mu^{\pi}\right) d\gamma + d\theta = 0 \Leftrightarrow \frac{d\theta}{d\gamma} = -\left(\mu + e\mu^{\pi}\right). \tag{18}$$

From Proposition 2, evaluating the numerator of the RHS in equation (14) at $\eta_1^{\pi} = \gamma\left(\mu + e\mu^{\pi}\right) + \theta + c_1$, leads to:

$$-\int_{(\mu+e\mu^{\pi})+\frac{c_1}{\gamma}}^{+\infty} (p_1 - \mu)\phi(p_1)dp_1 > -\int_{\mu+\frac{c_1}{\gamma}}^{+\infty} (p_1 - \mu)\phi(p_1)dp_1. \tag{19}$$

Therefore

$$\frac{\partial \eta_1}{\partial \overline{x_1}^{*}} \frac{\partial \overline{x_1}^{*}}{\partial \gamma} d\gamma < \frac{\partial \eta_1^{\pi}}{\partial \overline{x_1}^{\pi}} \frac{\partial \overline{x_1}^{\pi}}{\partial \gamma} d\gamma. \tag{20}$$

As a consequence of the increase in the expected input price (equation (17)) and the increase in the marginal variability (equation (20)), $\overline{x_1}^{\pi}$ exceeds $\overline{x_1}^{*}$ only if the variability of the permits price is high, or:

$$\overline{x_1}^{\pi} > \overline{x_1}^{*} \Leftrightarrow \frac{\partial \eta_1^{\pi}}{\partial \overline{x_1}^{\pi}} \frac{\partial \overline{x_1}^{\pi}}{\partial \gamma} d\gamma > \frac{\partial \eta_1^{\pi}}{\partial \overline{x_1}^{\pi}} \frac{\partial \overline{x_1}^{\pi}}{\partial \theta} d\theta. \tag{21}$$

■

As a corollary to the analysis of the role of carbon markets, it is clear that if environmental regulation would be achieved by imposing a pigouvian tax, the optimal input capacity would always be smaller than $\overline{x_1}^{*}$, provided that the tax rate is deterministic.

4.3 The role of technology

To study the role of technology, we consider a CES production function:

$$y = \gamma \left[\delta x_1^{\rho} + (1-\delta) x_2^{\rho}\right]^{1/\rho}, \tag{22}$$

where y represents output, and x_i for $i = 1, 2$ represents input usage. The CES production function is defined for $\rho \in]-\infty, 1]$, and $0 \leq \delta \leq 1$. Moreover we know that the CES production function leads to the Leontieff production function when $\rho \to -\infty$, the Cobb-Douglas production function when $\rho = 0$, and the linear production function, as long as $\rho = 1$. We denote by $\sigma = \frac{1}{1-\rho}$ the substitution elasticity between the two inputs.

For the CES technology, the virtual price associated to the input capacity constraint is

$$\eta_1 = \left[\left(\frac{y}{\gamma \overline{x_1}}\right)^{\rho} - \delta\right]^{\frac{1-\rho}{\rho}} (1-\delta)^{\frac{-1}{\rho}} \delta. \tag{23}$$

Denoting by η_1^* the solution of equation 5, the technology plays now a role in the optimal capacity level which is determined by

$$\overline{x_1}^* = \frac{y}{\gamma} \left[\delta + (1-\delta) \left(\frac{(1-\delta)\eta_1^*}{\delta}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{-1}{\rho}}. \tag{24}$$

Equation (24) shows that the optimal capacity could be expressed as the demand for input 1 evaluated at a particular relative price η_1^* which does not depend on the elasticity of substitution. As a consequence, comparing optimal capacities levels which follow from different CES functions distinguished by the values of the elasticity of substitution σ only, is rather simple. The optimal capacity satisfies the standard equality between the MRTS at the point $\left(\overline{x_1}, \underline{x_2}\right)$ and the relative price η_1^*, i.e.:

$$MRTS = \frac{d\underline{x_2}(\overline{x_1})}{d\overline{x_1}} = -\eta_1^*. \tag{25}$$

The key point is to study the relationship between the optimal capacity, which reflects the price uncertainty of the polluting input, and the technology. The following result applies.

Proposition 5 *(Technology effect).The optimal input capacity first increase and then decrease as the substitution elasticity increases.*

Proof. The optimal capacity $\overline{x_1}$ can be expressed as a function of σ.Deriving $\overline{x_1}^*$ with respect to the substitution elasticity σ leads to the following non-linear expression:

$$\frac{\partial \overline{x_1}^*}{\partial \sigma} = \frac{\overline{x_1}^*}{1-\sigma}\left[\frac{1}{\sigma}\ln(\frac{\gamma \overline{x_1}^*}{y}) - \sigma \ln(\frac{(1-\delta)\eta_1^*}{\delta})(1-\delta(\frac{\gamma \overline{x_1}^*}{y})^{\frac{\sigma-1}{\sigma}}\right]. \tag{26}$$

It is easy to show that if $\frac{(1-\delta)\eta_1^*}{\delta} = 1$, then $\overline{x_1}^* = \frac{y}{\gamma}$ which is independent of σ. Moreover we know that optimal input capacity is $\overline{x_1}^* = \frac{y}{\gamma\delta}$ when the production function is linear ($\sigma \to \infty$) and $\eta_1^* < \frac{\delta}{1-\delta}$. Similarly we have $\overline{x_1}^* = \frac{y}{\gamma}$ when the production function is Leontieff ($\sigma = 0$). To understand the shape of the optimal input capacity as a function of σ, it is convenient to find, depending on σ, the value of the MRTS leading to a fixed capacity $\overline{x_1}^* = \frac{y}{\gamma\delta}$ (solution when goods are perfects substitutes and $\eta_1^* < \frac{\delta}{1-\delta}$). We must find $MRTS(\sigma)$ such that:

$$\frac{y}{\gamma}\left[\delta + (1-\delta)\left(\frac{1-\delta}{\delta}MRTS(\sigma)\right)^{\sigma-1}\right]^{\frac{\sigma}{1-\sigma}} = \frac{y}{\gamma\delta}. \tag{27}$$

Solving the previous equation gives:

$$MRTS(\sigma) = \frac{\delta}{1-\delta}\left[\frac{\delta\left(-1+\delta^{-\frac{1}{\sigma}}\right)}{1-\delta}\right]^{\frac{1}{\sigma-1}}. \tag{28}$$

Since we have:

$$\begin{aligned} lim_{\sigma \to 0} MRTS(\sigma) &= 0, \\ lim_{\sigma \to +\infty} MRTS(\sigma) &= \frac{\delta}{1-\delta}, \\ MRTS'(\sigma) &> 0, \end{aligned}$$

then $\exists\ \widetilde{\sigma} \in]0;+\infty)$ such that $MRTS(\widetilde{\sigma}) = \frac{\eta_1^*}{P_2}$, with $P_2 = 1$.Given η_1^*, and considering the case where $\eta_1^* < \frac{\delta}{1-\delta}$, then the capacity $\overline{x_1} = \frac{y}{\gamma\delta}$ is optimal for a particular value of the substitution elasticity $\widetilde{\sigma}$ corresponding to a CES production function, which represents an intermediate situation in between complementarity and substitutability. As a consequence, the optimal input capacity first increase and then decrease as the substitution elasticity increases. ∎

Figure 2 illustrates results from Proposition 5 where the heavy curve represents the optimal capacity $\overline{x_1}$ for different values of the substitution elasticity. In this figure we consider the case $\eta_1^* < \frac{\delta}{1-\delta}$, the other one could be obtained by symmetry.

Let us remark that for the Cobb-Douglas case, the optimal input capacity is not necessarily larger than $\frac{y}{\gamma\delta}$. For $\sigma = 1$,

$$\overline{x_1}^*_{\sigma=1} = \left(\frac{1-\delta}{\delta}\eta_1^*\right)^{\delta-1}.$$

The optimal capacity is such that as $\eta_1^* \to 0$ then $\overline{x_1}^*_{\sigma=1} \to +\infty$ and as $\eta_1^* \to \frac{\delta}{1-\delta}$ then $\overline{x_1}^*_{\sigma=1} \to \frac{y}{\gamma}$.

Optimal Capacity and MRTS

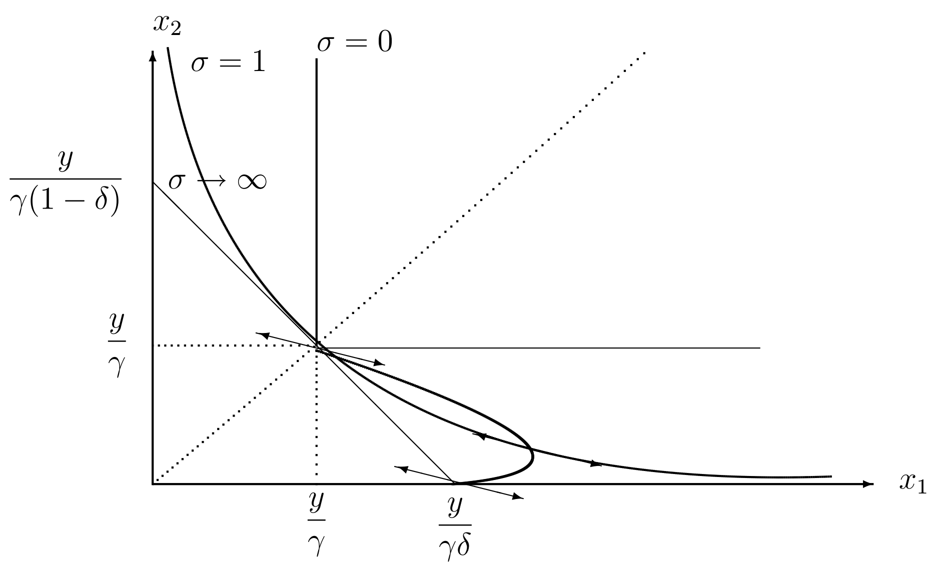

Figure 2.

Comparative statics effects point out that the firm's decisions depend on the complex interplay between several effects. First, the "average cost effect", due to the carbon price, causes a decrease in the input capacity with respect to a reference case where the permits market (or a pigouvian tax) does not exist. Second, the "marginal variability effect" or the impact of both input and eventually carbon price variability, which instead leads to an expansion of the installed equipment. Third, "technology effect", i.e. the substitution between the polluting and clean inputs. To discuss the implication of these results, we simulate our model in the following Section 5.

5 Model simulation

We simulate our model in the case of a CES production function (see equation 22) and use specific values for the substitution elasticity to represent more or less flexible production technologies.

5.1 Optimal virtual price

To find the optimal virtual price, we solve:

$$\mathbb{P}(p_1 \leq \eta_1)\left[\eta_1 - \mathbb{E}(p_1 \;/ p_1 \leq \eta_1)\right] = c_{1,}. \tag{29}$$

when p_1 follows a normal distribution $N(m, s^2)$.

Denoting respectively by ϕ the probability density function for normal distribution and by Φ its cumulative distribution function, we have:

$$\mathbb{E}(p_1 \;/ p_1 \leq \eta_1) = m - s\frac{\phi(\frac{\eta_1 - m}{s})}{\Phi(\frac{\eta_1 - m}{s})}, \tag{30}$$

for the expected price associated to the truncated normal distribution from above with threshold η_1.After simplifications, we obtain:

$$(\eta_1 - m)\Phi(\frac{\eta_1 - m}{s}) + s\phi(\frac{\eta_1 - m}{s}) = c_1. \tag{31}$$

The set of parameters is $c_1 = 0.1$, $m \in [1; 1.5]$ and $s \in [0; 0.5]$.

Figure 3 illustrates the different solutions obtained numerically. Notice that η_1^* is increasing with respect to m and decreasing with respect to s, coherently with the results of Propositions 2 and 3.

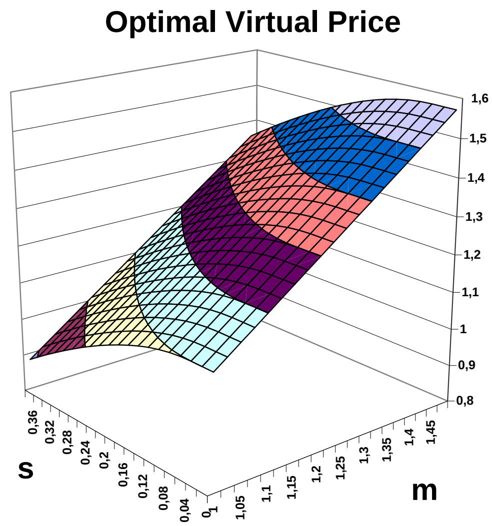

Figure 3.

5.2 Optimal Capacity

To illustrate the results of Proposition 5, we represent the optimal capacity obtained under different assumptions about the input substitution elasticity. The Cobb-Douglas case corresponds to $\sigma = 1$,complementarity to $\sigma = 0.1$, called complementarity, and substitutability to $\sigma = 5$.The input price follows a Normal distribution with the mean fixed at $m = 1.25$.

Optimal capacity is increasing with respect to the input price standard deviation, as Figure 4 illustrates. For the set of parameters used, the sensitivity to input price standard deviation increases with the elasticity of substitution. The optimal capacity is constant for the case of strict complementarity as inputs must be used in fixed proportion. On the contrary, in the case of perfect substitution, the optimal capacity is either 0 or strictly positive, given that input demand is necessarily a corner solution.

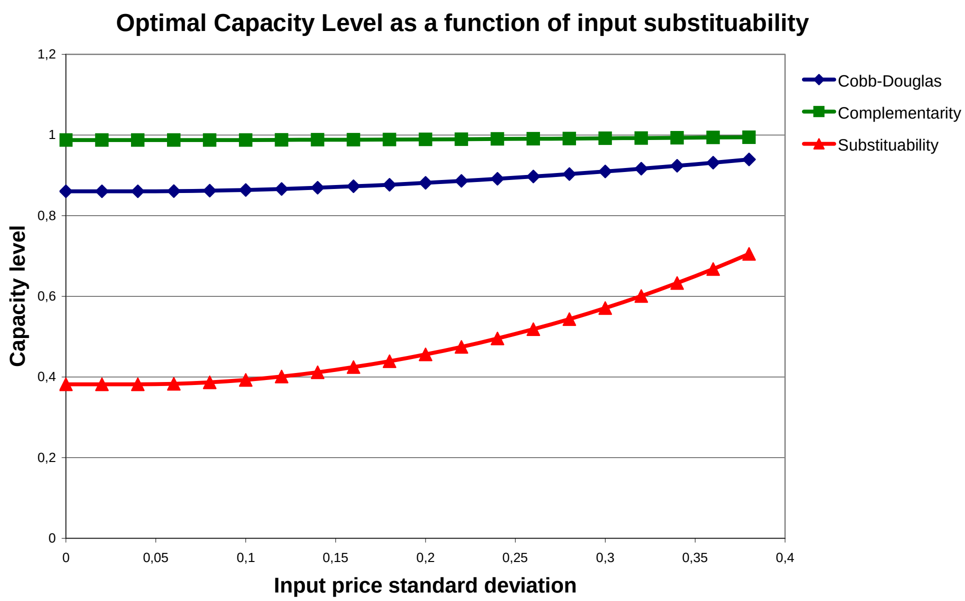

Figure 4.

Let us remark that in this model the expected short term rate of use of the capacity is less than 1 at the optimum. The usage rate of capacity is corresponds to the ratio of expected optimal input demand and the optimal capacity, that is:

$$\frac{\mathbb{E}(x_1^*)}{\overline{x_1}^*} = \frac{\mathbb{P}(p_1 \leq \eta_1)\overline{x_1}^* + \mathbb{P}(p_1 > \eta_1)\mathbb{E}(x_1^* \,/ p_1 > \eta_1)}{\overline{x_1}^*}, \tag{32}$$

As input demands are not constrained by capacity in the short-run, the rate of use of capacity is less than 1 at the optimum, except for very low input price standard deviations. Figure 5 represents $\mathbb{E}(x_1^*)/\overline{x_1}^*$ for the three case considered.

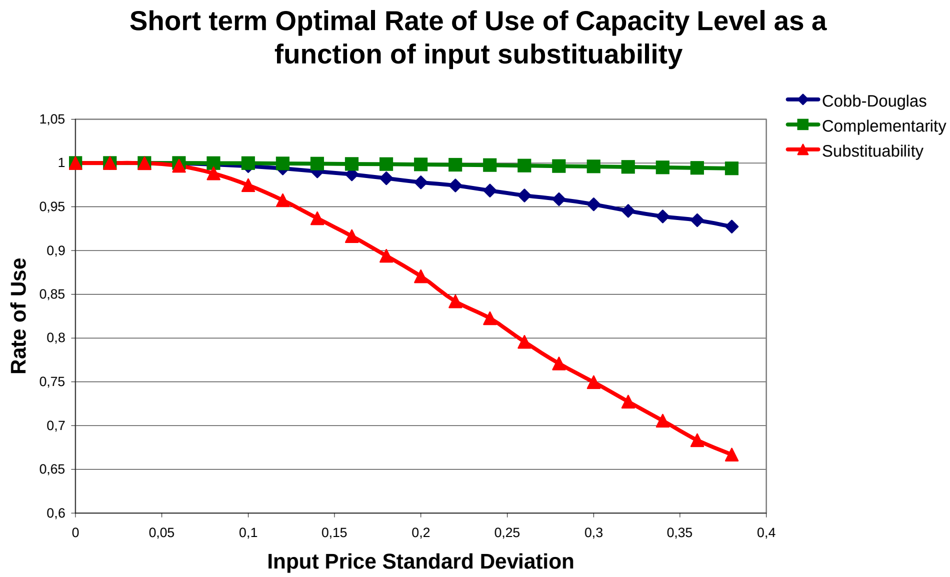

Figure 5.

Notice that the optimal rate of use of capacity decreases with respect to input price standard deviation. This is a direct consequence of the marginal effect of uncertainty that decreases the virtual price η_1 and thus increases the optimal capacity. Of course in the short-term the emission level depends on input demand, as the following Section illustrates.

5.3 CO_2 emissions

As last step of the model simulation, we take into account the emissions level. We consider the case of a fixed and known price for CO_2 $\Pi = 0.4$, and the case where the price for CO_2 is also random, with $\Pi = 0.4 + 0.2\varepsilon$, where ε follows a $N(0,1)$. Following the same steps as before we determine successively the optimal virtual price, the optimal capacities and expected input demand to determine carbon emissions. We normalize to 1 the emission level obtained under the business as usual scenario (which corresponds to $\Pi = 0$), under the assumption that the emission intensity e is equal to 1.

We represent hereafter relative emissions for a fixed and random CO_2 price and compare them to the free emission case, thus obtaining the relative percentage of emission reduction.

Of course in the complementarity case (Figure 6a), as capacities and input demand are imposed by the technology, there is almost no possibility to constrain emissions.

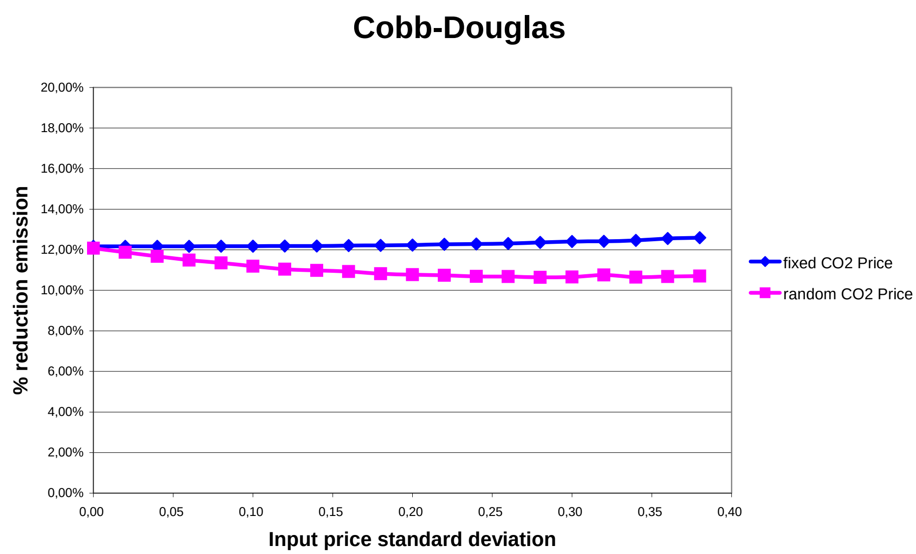

Figure 6a.

For the set of parameters used in the simulation, we obtain that emissions reduction amounts at around 10% with respect to the emissions-free scenario, if the technology is Cobb-Douglas, while CO_2 saving could be higher for larger elasticity of substitution between the polluting and the clean inputs.

Moreover emissions reduction is larger when the price of CO_2 is known compared to the case where it is random. As predicted by Proposition 4, when uncertainty in CO_2 price increases the input price variability, optimal capacities becomes larger. This leads to an increase of input demand and thus emissions (Figures 6b and 6c).

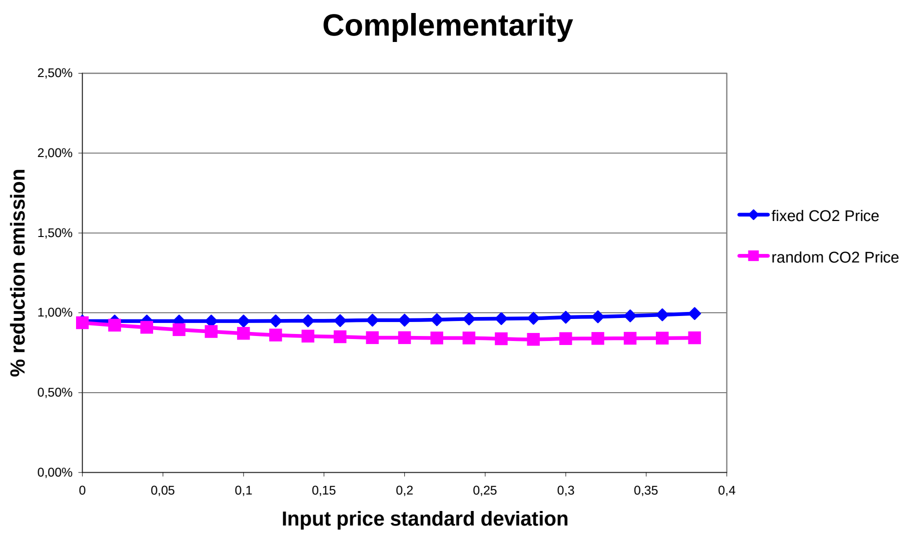

Figure 6b.

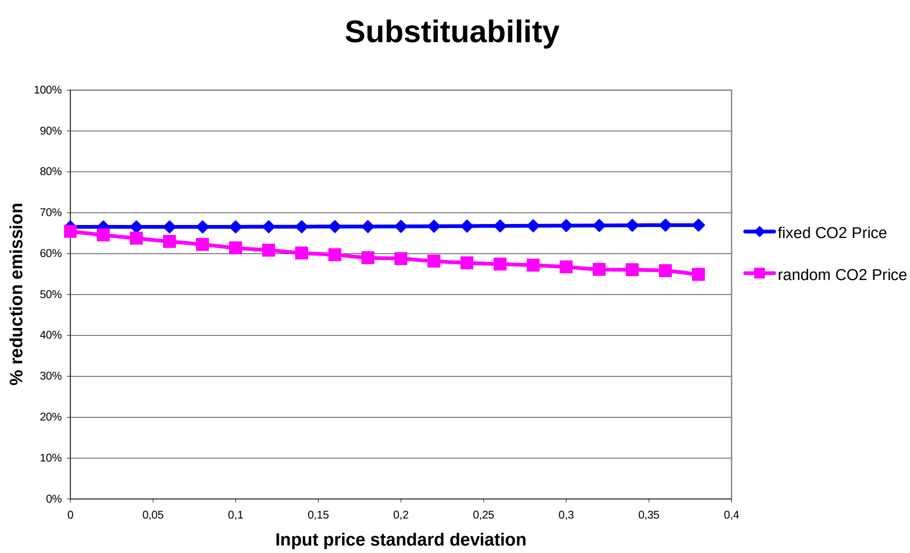

Figure 6c.

6 Conclusions

Our model shows that the interaction between production decisions and carbon constraints is very complex, as long as both short-term and long-term strategies are involved. Taking into account input substitutability, our approach clarifies role of short-term abatement in the form of input substitution, which in theoretical and empirical models is almost undetectable. From an environmental policy perspective, our results show that imposing specific targets for emission reductions without intervening on capacity choices (as for instance subsidies or R&D incentives to clean technologies) and relying on carbon markets only can lead to inefficiencies. Moreover, if the permit price is too volatile, as it can be the case when uncertainty in environmental policy is high, these inefficiencies amplify and emission reduction is weaker. To better understand this latter effect, further research will be devoted to endogenize the equilibrium permits price, by introducing some degree of firms heterogeneity.

7 References

Abel, A.B, and J. Eberly (1996), Optimal Investment with Costly Reversibility, Review of Economic Studies, 63, 581-593.

Atkeson, A., and P. J. Kehoe (1999), Models of Energy Use: Putty-Putty Versus Putty-Clay, American Economic Review, Vol.89, N 4, 1028-1043.

Baker, E. Clarke L., Keisler J., Shittu E. (2007), Uncertainty, Technical Change and Policy Models, Working Paper, UMass Boston, College of Management.

Baumol, W. J. and W. E. Oates (1988). The Theory of Environmental Policy, second edition, Cambridge University Press.

Benz E. and S. Trück (2009), Modeling the price dynamics of CO2 emission allowances, Energy Econ 31 (1), pp. 4–15.

Bonacina M., and C. Cozialpi (2009), Carbon allowances as inputs or financial assets: lesson learned from the Pilot Phase of the EU-ETS, IEFE Working Paper n. 19.

Convery, F.J., Ellerman, D. and De Perthuis, C. (2008) The European carbon market in action: Lessons from the first trading period. Interim Report. Report 162: Cambridge, Massachusetts Institute of Technology, Center Energy and Environmental Policy Resources.

Dangl, T. (1996) Investment and capacity choice under uncertain demand, European Journal of Operational Research 117, 415-428.

Daskalakis G. and R.N. Markellos (2009), Are electricity risk premia affected by emission allowance prices? Evidence from the EEX, Nord Pool and Powernext, Energy Policy 37 (7), pp. 2594–2604.

Delarue E.D, A. D.Ellerman and W. D. D'haeseleer (2008), Short-term CO2 Abatement in the European Power Sector. MIT CEEPR Working Paper 008.

Denny E. and M. O'Malley (2009), The impact of carbon prices on generation-cycling costs, Energy Policy 37 (4) (2009), pp. 1204–1212.

Dixit, A., and R.S, Pindyck (1994), Investment under Uncertainty, Princeton University Press, Princeton, NJ.

Downing, P. B. and White, L. J. (1986). Innovation in pollution control, Journal of Environmental Economics and Management, vol. 13, pp. 18-29.

Fehr, M. and Hinz, J. (2006). A quantitative approach to carbon price risk modeling. Institute of Operations Research, ETH Zurich

Farzin H.E., Kort P. (2000), Pollution Abatement Investment When Environmental Regulation is Uncertain Journal of Public Economic Theory, 2(2), pp. 183-212.

Hartl, R.F. and P.M. Kort (1999), Optimal Input substitution of a firm facing an environmental Constraint, Euroepan Journal of Operational Research, 99, 336-352.

Jaffe A.B., Newell R.G., Stavins R.N. (2002), Environmental Policy and Technological Change, Environmental and Resource Economics, vol. 22, pp. 41-70.

Jouvet, P.A., Michel P. and R. Rotillon (2005) Equilibrium with Pollution Permits, Research in Economics 59.

Jouvet, P.A., (2007). Firms' Characteristic And Equilibrium With Tradeable Permits Market, Australian Economic Papers, Blackwell Publishing, 46(2), 109-121.

Kort P.M. (1995), Optimal investment policies for a polluting firm in an uncertain environment, European Journal of Operational Research 85, 82-96

Lee, L.F. and M.M. Pitt (1987), Microeconometric Models of Rationing,Imperfect Markets, and Non-Negativity Constraints, Journal of Econometrics, 36, 89-110.

Milliman, Scott R., and Raymond Prince (1989), Firm Incentives to Promote Technological Change in Pollution Control. Journal of Environmental Economics and Management 17, 247–265.

Muller, T. (2000), Quality of Capital Goods and Investment Demand: a putty-clay model, Cahiers du Département d'économétrie N 2000.09, Université de Genève.

Newbery D. M. (2008), Climate Change Policy and Its Effect on Market Power in the Gas Market, Journal of the European Economic Association, 6(4), 727-751.

Neary, J. P. and K.W.S, Roberts (1980), The theory of household behavior under rationing, European Economic Review, 13, 25-42.

Sandmo, A. (1971), On the Theory of the Competitive Firm Under Price Uncertainty, American Economic Review, 78(5), 65-73.

Squires, D. (1994), Firm Behavior Under Input Rationing, Journal of Econometrics, 61, 235-257.

Stewart, J. F.(1979), Plant Size, Plant Factor, and the Shape of the Average Cost Function in Electric Power Generation : A Nonhomogeneous Capital Approach, The Bell Journal of Economics, Vol.10, N .2, 549-565.

Szabo, L., Hidalgo, I., Ciscar, J. C., and Soria, A. (2006). CO2 Emission Trading within the European Union and Annex b Countries: the Cement Industry Case. Energy Policy, 34, 72–87.

de Villemeur, E.B., Bousquet, A. and Ladoux, N. (2006), Short-term inefficiencies from long-term optimality in an uncertain environment, working paper IDEI, number 327.

Wolak F. A., and C. D., Kolstad (1991), A Model of Homogeneous Input Demand Under Price Uncertainty American Economic Review, 81 (3), 515-538.