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Pareto optimality of free trade in case of unemployment[1]

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Résumé: Un modèle de commerce international en équilibre général est présenté, avec deux zones, deux biens et deux facteurs de production, où, du fait de rigidités salariales, le libre échange génère du chomage et par conséquent une situation moins bonne que l'autarcie du point de vue d'une des zones et de certains agents.
S'il existe une mobilité entre les deux facteurs, qui sont du travail qualifié et non qualifié, des théorèmes sont cependant établis montrant que, donnée une dynamique d'ouverture de l'économie, si la vitesse de mobilité du travail est supérieure à certaines limites, où si la vitesse d'ouverture de l'économie est inférieure à d'autres limites, le libre échange peut être obtenu à revenu croissant pour chaque zone. S'il existe en plus la possibilité de redistribuer les revenus entre les agents, des théorèmes de même nature montrent que le libre échange peut être obtenu avec une satisfaction croissante pour chaque agent.

Abstract: A general equilibrium model of international trade is presented, with two zones, two goods and two factors of production, where, due to some rigidities, free trade leads to unemployment and thus to a situation worse than autarky from the point of vue of some zone and of some agents. However, if there is a possible mobility between the two factors, which are unskilled and skilled labour, theorems are proven showing that, given a dynamics of opening of the economy, if the speed of the mobility of labour is above some limits, or if the speed of opening of the economy is below some other limits, then free trade can be obtained through increasing incomes for each zone. If in addition there is a possibility of income redistribution between the agents, then similar theorems show that free trade can be obtained through Pareto improving situations

Mots clés : Commerce international, Pareto optimality, Chomage.

Key Words : International trade, Pareto optimality, Unemployment.

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Abstract

A general equilibrium model of international trade is presented, with two zones, two goods and two factors of production, where, due to wage rigidities, free trade leads to unemployment and thus to a situation worse than autarky from the point of view of some zone and of some agents.

However if there is a possible mobility between the two factors, which are unskilled and skilled labour, theorems are proven showing that, given a dynamics of opening of the economy, if the speed of the mobility of labour is above some limits, or if the speed of opening of the economy is below some other limits, then free trade can be obtained through increasing incomes for each zone.If in addition there is a possibility of income redistribution between the agents, then similar theorems show that free trade can be obtained through Pareto improving situations.

Introduction

The optimality of free trade is usually discussed, and proved, in the framework of general equilibrium models where all markets, including the labour market, are balanced(see for instance Krugman[1993]). A general equilibrium model is presented here where, due to some social rigidities, in fact wage rigidities, free trade between two zones generates unemployment. The situation may then appear to be worse than autarky from the point of view of some zone and of some agents. However, if there is a possible mobility between the two factors of production that we consider -namely unskilled and skilled labour- then, according to the degree of this mobility, the level of the income of each zone can be saved. Moreover, if there is also a possibility of redistribution of income between the agents, their level of utility can be preserved too. More specifically, if one considers precise dynamics of opening of the economy, relations between the speed of opening and the mobility of labour appear, giving conditions for the income of each zone, or the utility level of each agent, to be saved or increased at each stage. Then, from the point of view of the zone or of the agents, each stage, including the final free trade situation, appears to be Pareto superior to the previous one, including the autarkic situation.

Section I presents the model : two zones, two goods, two factors which are two types of labour, the trade between the two zones being regulated by quotas. Section II introduces an equilibrium concept, which can lead to different precise equilibria according to the assumptions made for the mobility of factors. Section III considers trajectories of opening of the economy, which allow to give a precise definition of the mobility of labour, and also explicit dynamics of opening. Section IV then proves a first proposition linking mobility and evolution of the income of the zones ; two theorems are given stating that if the speed of the mobility of labour is high enough, or the speed of opening low enough, then the income of each or both zones can be saved or increased at each stage. Section V proves a second proposition linking mobility and the utility level of agents, supposing there exists a possibility of redistribution of income ; then two additional theorems are given, stating that if the speed of the mobility of labour is high enough again, or the speed of opening low enough again, then the utility level of each agent can be kept or increased at each stage, which is thus Pareto equivalent or superior to the previous one. Section VI last concludes with some comments regarding the respective advantages of "big bang" and progressive transitions to free trade, the model obviously giving support to the second type of scenario.

1. The model

The model presented here, which has already been used in Fuchs[1997a and b], considers exchanges between two zones, North and South, both consuming and producing a good number 1 named textile just for the image, while a good number 2, more sophisticated, is consumed in the two zones but only produced in the North. Goods 1 and 2 are produced from labour only, through constant returns to scale technologies with coefficients k_1 and k_2, identical for textile in N and S[1]. An essential feature of the model is then the existence of two types of labour : an unskilled labour available in quantity ℓ_1 and ℓ_1' in N and S respectively, for the production of textile ; and a skilled labour, available in N only in quantity ℓ_2, for the production of good 2. Let $w_i (i = 1, 2)$ and w_1' be the corresponding wages in N and S respectively, that we shall suppose to be measured in an internationally accepted numéraire. We shall suppose logically that :

$$w_2 > w_1 > 0 \tag{1}$$

but also that:

$$w_1 > w_1' > 0 \tag{2}$$

just because unskilled labour is plentiful in S and considered not to be mobile. Production of goods 1 and 2 will thus be bounded between 0 and :

$$y_i = k_i \ell_i \quad \text{and} \quad y_1' = k_1 \ell_1' \tag{3}$$

and will take place if their local prices are related to wages though:

$$p_i = w_i / k_i \qquad p_1' = w_1' / k_1 \tag{4}$$

with, from (2) :

$$p_1 > p_1' \tag{5}$$

Then we shall suppose that ℓ_1 and ℓ_2 are in fact the number of unskilled and skilled workers of N, who are endowed with identical utility functions u of the form

$$u(c_i) = (c_1)^{\mu} (c_2)^{1-\mu} \tag{6}$$

where c_i is the individual consumption of good i and $0 < \mu < 1$. We suppose that workers are wage earners trying to sell a unit of labour, i.e. they look for the maximal level of u under a budget constraint where the only income is wage and, possibly, positive or negative transfer revenues. The situation in S, where a high level of unemployment and a large informal sector are permanently supposed to exist, will not be described in detail. It will only be supposed that the income S can earn by possibly selling textile to N is totally used to buy good 2 from N.

Last, we shall specify the relations between N and S by the volume $q \geq 0$ of textile authorized to be introduced in N at price p_1' [2]. Because of (5), consumers of N (who are the producers) will try to buy imported textile before "local" textile. For small values of q (the word "small" will receive a precise definition later on) demand for imported textile will be higher than supply so that we have to consider a "rationing scheme" (see Benassy[1982]). For

[1] This identity is only considered here for the sake of comparison with H.O.S models and is not a necessary characteristic.

[2] Quotas could also be seen however as the result of a policy of voluntary export restriction by S.

the sake of simplicity, we shall consider a scheme proportional to demands i.e. we suppose that, given q, a worker of type j ($j = 1$ or 2 according to the fact that he is unskilled or skilled) can buy a quantity c_q^j of textile at price p_1' with

$$(i) \qquad c_q^j = \alpha(q) c_1(p_1', p_2, w_j)$$

where $\alpha(.)$ is a continuously increasing function of q with $\alpha(0) = 0$ and where $c_i(p_1', p_2, w_j)$ is the solution of the program :

$$max \quad u(c_1, c_2)$$

under

$$p_1' c_1 + p_2 c_2 \leq w_j$$

which gives

$$c_1(p_1', w_j) = \frac{\mu w_j}{p_1'} \qquad c_2(p_2, w_j) = \frac{(1-\mu) w_j}{p_2} \tag{7}$$

For the c_q^j to define a rationing scheme we then must impose in addition :

$$(ii) \qquad \lambda_1 c_q^1 + \lambda_2 c_q^2 = q$$

where $\lambda_j \geq 0$ is the number of active workers of type j in N for a a given q (so long there are no transfer revenues, possibly unemployed wage earners of N can buy nothing ; we shall in what follows forget about the integer character of the λ_j and consider them as percentages of the ℓ_j). Given (i) and (ii) one then gets easily using (7):

$$c_q^j = w_j \frac{q}{W(q)} \tag{8}$$

where

$$W(q) = \lambda_1 w_1 + \lambda_2 w_2 \tag{9}$$

is nothing but the total wages, and thus income, in N given q : the share of q that a worker of type j can buy is just his share in total income.

Let us then consider the behaviour of an active worker of type j in N. He will choose a consumption bundle $c_i(p_i, p_1', q, w_j)$ which maximises u given the facts that :

- he can buy textile at price p_1' up to the quantity c_q^j ;
- if his income w_j is high enough he can go on buying textile at price p_1.

Clearly then, according to the value of q there are differents situations.

Let q_2 be the value of q where rationing stops ie, from $(i), \alpha(q_2) = 1$ or:

$$c_{q_2}^j = c_1(p_1', w_j) \tag{10}$$

From (7) and (8), q_2 is independant of j and given by:

$$q_2 / W(q_2) = \mu / p_1' \tag{11}$$

Then obviously, if $q \geq q_2$ all employed workers can buy all the imported textile they wish at price p_1' :

$$\begin{aligned} c_1(p_i, p_i', q, w_j) &= c_1(p_1', w_j) \\ c_2(p_i, p_i', q, w_j) &= c_2(p_2, w_j) \end{aligned} \tag{12}$$

and we are in a pure free trade situation.

Next, if $q < q_2$, an active worker of type j will solve the program

$$\max u(c_q^j + \gamma_1, c_2)$$

under

$$p_1\gamma_1 + p_2 c_2 \leq R_j \equiv w_j - p_1' c_q^j \tag{13}$$

$$\gamma_1 \geq 0$$

(note that, from (11), $R_j > 0$). Considering to begin only the first constraint and defining $\pi = p_1'/p_1$, an other traditional calculation gives

$$\begin{aligned} c_1(p_i, p_1', q, w_j) &= c_1(p_1, w_j) + \mu(1-\pi)c_q^j \\ c_2(p_i, p_1', q, w_j) &= c_2(p_2, w_j) + (1-\mu)\frac{p_1 - p_1'}{p_2} c_q^j \end{aligned} \tag{14}$$

The sum of the values of those two consumptions, at price p_1 and p_2 respectively, is easily seen to be $w_j + (p_1 - p_1')c_q^j$, the last term of the sum representing the extra purchasing power resulting for j from the access to c_q^j at a cheaper price than p_1. Considering now in addition the constraint $\gamma_1 \geq 0$ i.e.:

$$c_1(p_i, p_1', q, w_j) - c_q^j \geq 0$$

we get from (14) :

$$c_q^j \leq \frac{1}{\rho} c_1(p_1, w_j)$$

with

$$\rho = 1 - \mu(1-\pi) \tag{15}$$

Let now q_1 be the value of q defined by :

$$c_{q_1}^j = \frac{1}{\rho} c_1(p_1, w_j) \tag{16}$$

From (7) and (8), q_1 is independent of j and given by:

$$q_1/W(q_1) = \mu/\rho p_1 \tag{17}$$

It is the value at which and above which only imported textile is bought in N. Clearly $q_1 < q_2$ (because $p_1' < p_1 - \mu(p_1 - p_1')$). Thus, if $q \leq q_1$, the demand of a worker of type j is indeed given by (14). If $q_1 \leq q \leq q_2$ we have, using to calculate c_2 the budget constraint of (13) :

$$\begin{aligned} c_1(p_i, p_1', q, w_j) &= c_q^j \\ c_2(p_i, p_1', q, w_j) &= c_2(p_2, w_j)/(1-\mu) - p_1' c_q^j / p_2 \end{aligned} \tag{18}$$

Having specified the behaviour of consumers and knowing those of producers, we then have all elements to introduce a concept of equilibrium, equilibrium with rationing but general equilibrium though.

2. Definition of q-equilibria

In general terms, an equilibrium on the good markets must then satisfy the two conditions that:

- consumption of textile in N equals production in N plus the import q,
- production of good 2 in N matches consumption of N plus the demand that S addresses to N using its exports receipts of textile to buy good 2.

Analytically these equalities read :

$$\begin{aligned} k_1\lambda_1 + q &= \lambda_1 c_1(p_i, p_1', q, w_1) + \lambda_2 c_1(p_i, p_1', q, w_2) \\ k_2\lambda_2 &= \lambda_1 c_2(p_i, p_1', q, w_1) + \lambda_2 c_2(p_i, p_1', q, w_2) + p_1' q/p_2 \end{aligned} \tag{19}$$

Using (7),(14) and (18) both lines reduce to:

- for $0 \leq q \leq q_1$

$$(\mu - 1)w_1\lambda_1 + \mu w_2\lambda_2 = \rho p_1 q \tag{20}$$

- for $q_1 \leq q \leq q_2$

$$\lambda_1 = 0$$

The equivalence of both equations (19) just reflects the facts that total wages sum up to the value of total production while external trade is balanced. Note that, from (20), $q = 0$ i.e. the autarkic situation appears to be compatible with full employment iff:

$$(\mu - 1)w_1\ell_1 + \mu w_2\ell_2 = 0 \tag{21}$$

a condition that we shall suppose to be fulfilled in what follows.

In usual models then, starting from autarky with full employment and increasing q remains compatible with equilibrium on all markets of N, including the labour markets, if (see (20) with $\lambda_j = \ell_j$) there is a decrease in w_1 or an increase in w_2 or a suitable combination of these two movements. In a dual way, we shall make here the following assumption:

Assumption 1 *Wages w_1 and w_2 are rigid and remain equal to their autarkic value.*

Considering actual situations, such an assumption appears to be quite realistic. It means that, however high is q, the unskilled workers refuse any lowering of their wages (their wage level could also be thought of as a minimal wage level defined by a social policy in N) and that the owners of firms producing good 2 do not accept any wage increase of skilled workers. We then introduce:

Definition 1 *Given a wage differential $\pi = p_1'/p_1 = w_1'/w_1$, given a quota q with $0 \leq q \leq q_2$, we call q-equilibrium of the economy a pair (λ_1, λ_2) satisfying (19) and, in addition, $\lambda_1 + \lambda_2 \leq \ell_1 + \ell_2$.*

This definition just reflects the facts that, in the absence of wage (or price) adjustments, it is through unemployment that good markets will remain balanced and that, of course, active population cannot be greater than total population.

In the three dimensional space $(\lambda_1, \lambda_2, q)$ the set Q of q-equilibria for all q corresponds in Figure 1 to the union of the triangle OAB in the plane defined by (20) (with $A = (\ell_1, \ell_2, 0)$) and of the triangle OBC in the plane $\lambda_1 = 0$. The line OB, from (20) with $\lambda_1 = 0$ is defined by :

$$q = \mu w_2 \lambda_2 / \rho p_1 \tag{22}$$

which, from (17) corresponds to the definition of q_1; the line OC, from (11) with $\lambda_1 = 0$ too, is defined by :

$$q = \mu w_2 \lambda_2 / p_1' \tag{23}$$

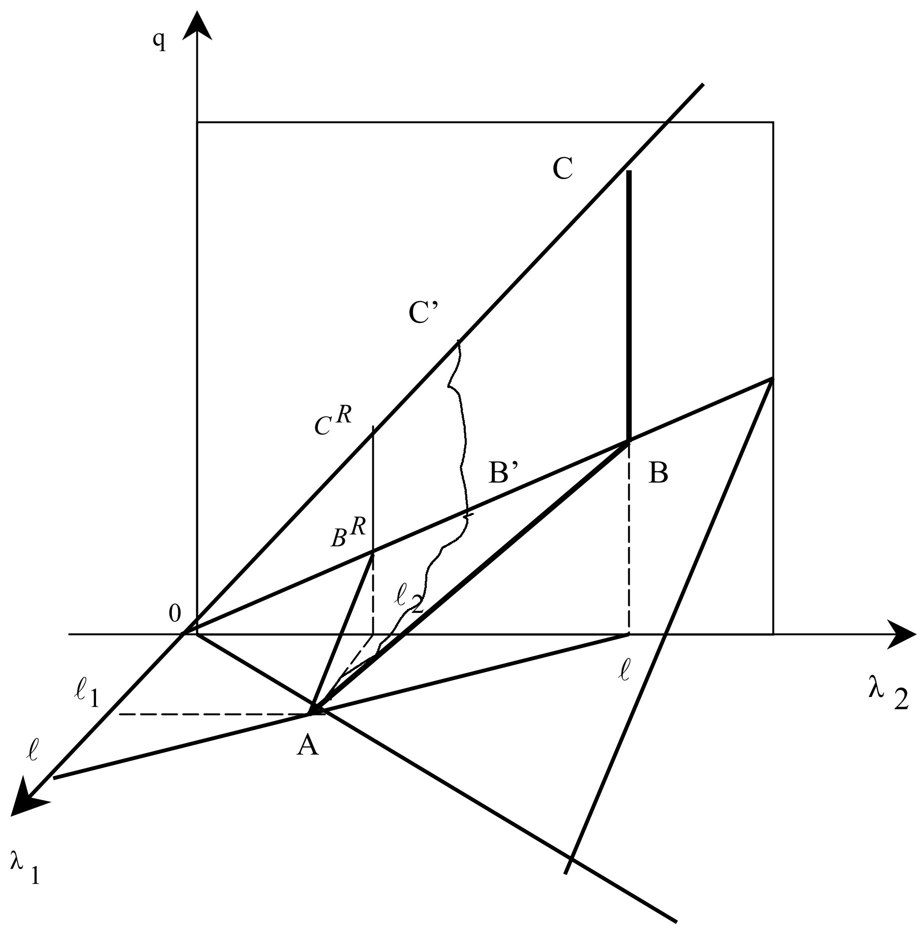

Figure 1

But Definition 1 also allows to consider more precise and very interesting situations. We are first going to specify two extreme sets of particular interest.

Definition 2 *We call* <u>*q-equilibrium with perfect mobility*</u> *a pair* (λ_1, λ_2) *belonging to* Q *and satisfying in addition :*

$$\lambda_1 + \lambda_2 = \ell_1 + \ell_2 = \ell \tag{24}$$

To comment this definition, let us first solve (20) and (24) in λ_1 and λ_2. The determinant associated with the matrix coefficient of the λ_j is easily seen to be $\Delta = (1-\mu)w_1 + \mu w_2 > 0$ so there is a unique solution which is, using (21) :

$$\begin{aligned} \lambda_1^M(q) &= (\mu w_2 \ell - \rho p_1 q)/\Delta &&= \ell_1 - \rho p_1 q/\Delta \\ \lambda_2^M(q) &= [(1-\mu)w_1\ell + \rho p_1 q]/\Delta &&= \ell_2 + \rho p_1 q/\Delta \end{aligned} \tag{25}$$

One can then interpret these λ_j^M as follows : a level q of imports of textile in N creates a level $\ell_1 - \lambda_1^M$ of unemployment among unskilled workers ; but, simultaneously, appears the additional amount of skilled work $\lambda_2^M - \ell_2 = \ell_1 - \lambda_1^M$ for the production of good 2, which is increased due to the purchases of S. This is possible only if unskilled workers can be transformed one to one and costlessly into skilled workers, hence our definition. Of course (25) is only valid for $q \leq q_1$, between q_1 and q_2 one has $\lambda_1^M = 0$ and $\lambda_2^M = \ell$. In Figure 1, the set of q-equilibria with perfect mobility for all q corresponds to the union of the two segments AB and BC.

Definition 3 *We call* <u>*q-equilibrium with perfect rigidity*</u> *a pair* (λ_1, λ_2) *belonging to* Q *and satisfying in addition :*

$$\lambda_2 = \ell_2 \tag{26}$$

Solving (20) in λ_1 then immediately gives, with the use of (21) again :

$$\lambda_1^R(q) = \ell_1 - \rho p_1 q/(1-\mu)w_1 \tag{27}$$

The interpretation now can read as follows : a level q of imports of textile in N creates a level $\ell_1 - \lambda_1^R$ of unemployment among unskilled workers ; but the constancy of the number ℓ_2 of skilled workers means that, this time, there is no mobility between unskilled and skilled manpower, hence our new definition. Again (27) is only valid for $q \leq q_1$: between q_1 and q_2 one has $\lambda_1^R = 0$ and $\lambda_2^R = \ell_2$. In Figure 1, the set of q-equilibria with perfect rigidity for all q corresponds to the union of the two segments AB^R and $B^R\ C^R$.

Note that perfect mobility corresponds to the border of Q with full employment of all workers, while perfect rigidity defines the border of the subset Q' of Q to the left of which both categories of workers know unemployment. Definitions 2 and 3 thus formalize two extreme situations where training either is sufficient to change costlessly the necessary number of unskilled workers into skilled workers or is non existent or inefficient. Within Q' intermediary situations can of course also be considered: this will be the entry in the forthcoming sections.

To complete our definitions however, we are going to calculate the function α of (i) which defines the rationing scheme actually used for the two types of equilibria just introduced. This in turn will allow us to check the coordinates of B and C, B^R and C^R in Figure 1. For q-equilibria with perfect mobility one gets using (25),(10) and(21) :

- for $q \leq q_1$

$$W^M(q) = W(0) + (w_2 - w_1)\rho p_1 q/\Delta \tag{28}$$

and thus :

$$\alpha^M(q) = \frac{p_1' q\Delta}{\mu[w_1 w_2 \ell + (w_2 - w_1)\rho p_1 q]} \tag{29}$$

a short calculation using (17) then gives:

$$q_1^M = \mu w_2 \ell/\rho p_1 \tag{30}$$

and one checks easily that $\lambda_1^M(q_1^M) = 0$;

- for $q_1 \leq q \leq q_2$

$$\alpha^M(q) = p_1' q/\mu w_2 \ell \tag{31}$$

and, from (11) :

$$q_2^M = \mu w_2 \ell/p_1' \tag{32}$$

One checks easily that the λ_j^M are continuous in q_1^M and that α^M is continuous in q_1^M and strictly increasing in $[0, q_2^M[$. For q-equilibria with perfect rigidity one gets similarly using (27),(10)and(21) :

- for $q \leq q_1$

$$W^R(q) = W(0) - \rho p_1 q/(1-\mu) \tag{33}$$

hence:

$$\alpha^R(q) = \frac{p_1' q(1-\mu)}{\mu(\ell_2 w_2 - \rho p_1 q)} \tag{34}$$

$$q_1^R = \mu w_2 \ell_2/\rho p_1 \tag{35}$$

and one checks easily that $\lambda_1^R(q_1^R) = 0$;

- for $q_1 \leq q \leq q_2$

$$\alpha^R(q) = p_1' q/\mu w_2 \ell_2 \tag{36}$$

$$q_2^R = \mu w_2 \ell_2/p_1' \tag{37}$$

Again the λ_j^R are continuous in q_1^R and α^R is continuous in q_1^R and strictly increasing in $[0, q_2^R[$.

Three comments then. First the fact that $\lambda_1(q_1) = 0$ for both equilibria is not a surprise : q_1 is the level of q for which all demands for "local" textile vanish so that all unskilled workers are unemployed then. Next, for the two types of equilibria, q_2 where begins free trade is, without surprise too, equal to the total demand for imported textile of the active skilled workers (in number either ℓ or ℓ_2). Last, the values q_1 and q_2 indeed satisfy equations (22) and (23) and correspond well to the points B^R and B and C^R and C of Figure 1.

3. Trajectories of opening, mobility of labour, dynamics

Keeping in mind Figure 1 and the comments already presented we set :

Definition 4 *We call* <u>*trajectory of opening*</u> *the data of two continuous piecewise differentiable functions* λ_j *defined on an interval* $[0, q_2]$*, with* $0 < q_2 \leq q_2^M$ *and such that :*

- *a pair* $\lambda_1(q), \lambda_2(q)$ *defines a* q*-equilibrium* $\forall q$*,*
- $\lambda_j(0) = \ell_j; \quad \lambda_1(q_2) = 0, \quad \lambda_2(q_2) = p_1' q_2 / \mu w_2$

This is just a precise formulation of a trajectory such as $AB'C'$, starting from autarky with full employment and ending with free trade and unskilled workers either unemployed or skilled (i.e. ending with C' satisfying (22)). Note that the two sets of q-equilibria with perfect mobility and rigidity define two specific extreme trajectories of opening, which are the two extreme possible trajectories on Q'. Along a trajectory, unskilled workers try to get skilled (because it gives access to higher wages) but, due to non explicited rigidities (such as limited amount of training capacities or unability of some workers), they cannot all succeed (except for the case of perfect mobility). We then make more precise what we call mobility through :

Definition 5 *We call* <u>*mobility of labour*</u> *for a given trajectory of opening, the function* m *of* q *given by :*

$$m(q) = [\lambda_2(q) - \ell_2]/\ell_1 \tag{38}$$

Definition 6 *We call* <u>*sensitivity of the mobility of labour*</u>*, for a given trajectory of opening, the function* s *of* q *given by :*

$$s(q) = \frac{d}{dq} m(q) = \frac{1}{\ell_1} \frac{d\lambda_2}{dq} \tag{39}$$

A few observations then :

- the set of q-equilibria with perfect rigidity defines a trajectory of opening for which the corresponding m^R and s^R are constant to zero ;
- the set of q-equilibria with perfect mobility defines a trajectory of opening for which m^M grows from 0 to 1 and, from (25)

$$s^M = \begin{cases} \rho p_1 / \Delta \ell_1 & \text{for } 0 \leq q < q_1^M \\ 0 & \text{for } q_1^M < q \leq q_2^M \end{cases} \tag{40}$$

- for trajectories such as $AB'C'$ in Figure 1, m grows from 0 to a value smaller than 1 and, of course, s is piecewise constant only if AB' and $B'C'$ are segments.

It is then worth to be noted that if a trajectory of opening defines a unique sensitivity of labour, conversely, the data of such a sensitivity defines a unique trajectory of opening. Indeed we have :

Proposition 1 *Given a piecewise continuous function* s *defined on* $[0, q_2^M]$ *and satisfying* $s(q) \leq s^M$ *for all* q*, then can be built from* s *a unique trajectory of opening.*

<u>Proof</u> Let us first define the function.

$$\lambda_2(q) = \ell_2 + \ell_1 \int_0^q s(q')dq' \tag{41}$$

Using (20) we can build

$$\lambda_1^1(q) = [\mu w_2 \lambda_2(q) - \rho p_1 q]/(1-\mu) w_1 \tag{42}$$

which can be written with (21)

$$\lambda_1^1(q) = \ell_1 + [\mu w_2 \ell_1 \int_0^q s(q')dq' - \rho p_1 q]/(1-\mu)w_1$$

Now using the inequality on s and (40), the last quantity between brackets satisfies (using the definition of Δ) :

$$[\quad] \leq \mu w_2 \rho p_1 q/\Delta - \rho p_1 q = -\rho p_1 q(1-\mu)w_1/\Delta$$

inequality which implies that there exists a smaller value $q_1 > 0$ for which $\lambda_1^1(q_1) = 0$ and that $q_1 \leq q_1^M$ (because from (30) $\rho p_1 q_1^M/\Delta = \ell_1$). Similarly then, from (41) :

$$\lambda_2(q) \leq \ell_2 + \rho p_1 q/\Delta$$

so that :

$$\lambda_1^1(q) + \lambda_2(q) \leq \ell \quad \forall q$$

and, in $[0, q_1]$ the triplets $(\lambda_1^1(q), \lambda_2(q), q)$ well correspond to q-equilibria. Then, for $q \geq q_1$, let us consider the function

$$q/w_2\lambda_2(q)$$

We have first from (42) with $\lambda_1^1 = 0$

$$0 < q_1/w_2\lambda_2(q_1) = \mu/\rho p_1 < \mu/p_1'$$

Next, from above, $\lambda_2(q) \leq \ell \quad \forall q$ because in $[0, q_1^M]$ $\lambda_2(q) \leq \ell_2 + \rho p_1 q_1^M/\Delta = \ell$ and, in $[q_1^M, q_2^M], s(q) \leq 0$ so that λ_2 is non increasing. Thus, so long $\lambda_2 \geq 0$.

$$q/w_2\lambda_2(q) \geq q/w_2\ell$$

and there exists a smaller value q_2 for which

$$q_2/w_2\lambda_2(q_2) = \mu/p_1'$$

with, from (32), $q_2 \leq q_2^M$ Defining then

$$\lambda_1(q) = \begin{cases} \lambda_1^1(q) & \text{for } 0 \leq q \leq q_1 \\ 0 & \text{for } q_1 \leq q \leq q_2 \end{cases} \tag{43}$$

it is clear that the pair (λ_1, λ_2) corresponds to a trajectory of opening. ■

The constructions made for the proof of Proposition 1 are easily understood when looking at Figure 1 and at the trajectory $AB'C'$. Note however that it is not excluded here that C' be on the left of C^R since we have put no lower constraint on s.

To give to the two previous definitions all their interest though, we shall now consider an actual process of opening of the economy of N. This will be done by introducing the explicit dynamics :

$$q(t) = Vt \quad t \geq 0 \tag{44}$$

where V will be interpreted as the speed of opening of N i.e. the speed along a trajectory such as $AB'C'$ ($t = 0$ is autarky and t grows until a limit value t_{FT} which corresponds to a point of Figure 1 between 0 and C and thus to free trade) [3] We shall suppose that V is piecewise constant and always positive. In complement of Definition 6 we then introduce :

Definition 7 *We call* *speed of the mobility of labour* *the function* v_L *of* t *given by :*

$$v_L(t) = \frac{d}{dt} m(q(t)) \tag{45}$$

Obviously, using (38), (39) and (44) one just has

$$v_L(t) = V s(q(t)) \tag{46}$$

But first v_L is a quantity that can be calculated from observation since it is just proportional to the speed of increase of the number of skilled workers through time $d\lambda_2/dt$. Next, this definition allows us to introduce a quite reasonable new assumption, namely :

Assumption 2 *There exists in* N *an endogenous maximal speed of the mobility of labour* $v_L^0 > 0$.

In other words the efficiency of the training processes and capacities in N and the existing ability of unskilled workers are such that, whatever the chosen trajectory and speed of opening are, one has:

$$v_L(t) \leq v_L^0 \quad \forall t \tag{47}$$

Given all this material, we are now ready to introduce our key Propositions and Theorems.

4. Pareto optimality for zones

We shall use here as a first criterium for welfare of N and S the level of income of each zone. We shall then make clear that its evolution depends crucially on the degree of mobility of labour. Of course South always wins to the existence of trade since its workers benefit, thanks to the sale of the quota q of textile, from the additional income :

$$\Delta W'(q) = p'_1 q > 0 \tag{48}$$

But the situation for N is quite different. Indeed one has, according to the fact that q is smaller or larger than $q_{\dot{1}}$:

- in case of perfect mobility, from (28) and (9) :

$$\Delta W^M(q) \equiv W^M(q) - W(0) = \begin{cases} \rho p_1 q(w_2 - w_1)/\Delta > 0 \\ \ell_1(w_2 - w_1) > 0 \end{cases} \tag{49}$$

- in case of perfect rigidity, from (33) and (9) :

$$\Delta W^R(q) \equiv W^R(q) - W(0) = \begin{cases} -\rho p_1 q/(1-\mu) < 0 \\ -\ell_1 w_1 < 0 \end{cases} \tag{50}$$

($W(0) = \ell_1 w_1 + \ell_2 w_2$ is also the income of N in the autarkic situation), which means that N can be winning or loosing to trade. Looking at (49) and (50) however leads us to introduce the two following additional definitions :

[3] The choice of V can result either from an autonomous decision of N (or S) or from some international negociation such as the Multi Fiber Agreement.

Definition 8 *We call* <u>*q-equilibrium with constant income for N*</u> *a pair* $(\lambda_1^N, \lambda_2^N)$ *belonging to* Q *and satisfying in addition :*

$$\lambda_1^N w_1 + \lambda_2^N w_2 \equiv W^N(q) = W(0) \tag{51}$$

Definition 9 *We call* <u>*q-equilibrium with constant global income*</u> *a pair* $(\lambda_1^G, \lambda_2^G)$ *belonging to* Q *and satisfying in addition :*

$$\lambda_1^G w_1 + \lambda_2^G w_2 \equiv W^G(q) = W(0) - p_1' q \tag{52}$$

Given $W(0)$ and (48) the two definitions are self-explanatory. They allow us to prove the following proposition.

Proposition 2 *There exist two limit sensitivities of the mobility of labour* s^N *and* s^G*, with* $s^N > s^G$*, such that :*

- *for* $s \geq s^N$ *(respectively* $s < s^N$*), at each stage of the corresponding trajectory of opening, the income of* N *is equal to or greater than (respectively smaller than) its autarky level ;*
- *for* $s \geq s^G$ *(respectively* $s < s^G$*), at each stage of the corresponding trajectory of opening the global income (North plus South) is equal to or greater than (respectively smaller than) its autarky level*[4]

<u>Proof</u> Using equation (20) defining q-equilibria and together (51) or (52), one gets easily :

- for q between 0 and the value $q_1^\cdot$ which makes $\lambda_1^\cdot$ equal to zero :

$$\begin{aligned} \lambda_1^N &= \ell_1 - \rho p_1 q / w_1 \\ \lambda_2^N &= \ell_2 + \rho p_1 q / w_2 \end{aligned} \tag{53}$$

or

$$\begin{aligned} \lambda_1^G &= \ell_1 - [\rho p_1 + \mu p_1'] q / w_1 \\ \lambda_2^G &= \ell_2 + [\rho p_1 - (1-\mu) p_1'] q / w_2 \end{aligned} \tag{54}$$

- for $q \geq q_1^\cdot$ and until the value $q_2^\cdot$ corresponding to free trade :

$$\lambda_1^N = 0 \qquad \lambda_2^N = \ell_2 + \ell_1 w_1 / w_2 \tag{55}$$

or

$$\lambda_1^G = 0 \qquad \lambda_2^G = \ell_2 + (\ell_1 w_1 - p_1' q) / w_2 \tag{56}$$

with :

$$q_1^N = \ell_1 w_1 / \rho p_1 \qquad q_1^G = \ell_1 w_1 / (\rho p_1 + \mu p_1') \tag{57}$$

and, using (11):

$$q_2^N = \ell_1 w_1 / p_1' \qquad q_2^G = \ell_1 w_1 / (1+\mu) p_1' \tag{58}$$

Straightforward calculations then give

$$\begin{aligned} q_1^R < q_1^G < q_1^N < q_1^M \\ q_2^R < q_2^G < q_2^N < q_2^M \end{aligned} \tag{59}$$

[4]We shall use the notation $s \geq s'$ for $s(q) \geq s'(q) \forall q$ and $s > s'$ for $s \geq s'$ and $s(q) > s'(q)$ on some non zero interval $[0, \cdots]$.

Using Definition 6 , one then sees easily that the two sensitivities of the mobility of labour associated with the trajectories of opening defined above are :

$$s^N = \begin{cases} \rho p_1 / w_2 \ell_1 & \text{for } 0 \leq q < q_1^N \\ 0 & \text{for } q_1^N < q \leq q_2^N \end{cases} \tag{60}$$

$$s^G = \begin{cases} [\rho p_1 - (1-\mu) p_1'] / w_2 \ell_1 & \text{for } 0 \leq q < q_1^G \\ -p_1' / w_2 \ell_1 & \text{for } q_1^G < q \leq q_2^G \end{cases} \tag{61}$$

and one can check, using (59), that

$$s^M > s^N > s^G \tag{62}$$

Conversely, given s^N or s^G, one obtains from Proposition 1 the trajectory of opening defined by λ_j^N or λ_j^G where clearly the income of N or the global income (N plus S) are preserved. Last, let us consider a trajectory of opening defined by a sensitivity s with:

$$s^M \geq s \geq s^N, \quad (\text{respectively } s < s^N)$$

It is clear from (41) and (42) in the proof of Proposition 1 that the λ_j it defines satisfy

$$\lambda_j \geq \lambda_j^N \quad (\text{respectively } \lambda_j < \lambda_j^N) \forall j, q$$

so that we have obviously from (9) and (51)

$$W(q) \geq W^N(q) = W(0) \quad (\text{respectively } <) \quad \forall q$$

Similarly, with $s^M \geq s \geq s^G$ (respectively $s < s^G$) one gets from (9) and (52):

$$W(q) \geq W^G(q) = W(0) - p_1' q \quad (\text{respectively } <) \quad \forall q$$

so that, from (48), the global income $W + W'$ of North plus South remains equal to or greater than (respectively smaller than) the autarky level. ∎

Then we can state first :

Theorem 1 *Given a dynamics of opening of the economy of N of the form (44) with a speed V, let v_L be the speed of the mobility of labour generated along a trajectory of opening. Then there exist two limit speeds of the mobility of labour, v_L^N and v_L^G with $v_L^N > v_L^G$ such that :*

- *the level of income of N will be saved or increased at each stage between autarky and free trade iff $v_L \geq v_L^N$;*
- *the level of the global income of N and S will be saved or increased at each stage between autarky and free trade iff $v_L \geq v_L^G$.*

Proof Let us choose

$$v_L^N = V s^N \quad v_L^G = V s^G$$

Clearly from (59) and (62) $v_L^N > v_L^G$. Theorem 1 is then a direct consequence of (46) and of Proposition 2. ∎

In other words, while opening N to the textile of S, the speed of conversion from unskilled to skilled manpower, is the key variable to control the evolution of income of N or N plus S : $v_L \geq v_L^N$ allows to get at each stage a Pareto superior situation for each zone ; so does v_L with $v_L^N \geq v_L \geq v_L^G$ in case South accepts cooperatively to give back to N a part of its surplus sufficient to maintain the income of N not below its autarky level.

Now considering in addition Assumption 2, one can also state the following partial converse.

Theorem 2 *Given an endogeneous maximal speed of the mobility of labour v_L^0 in N, considering dynamics of opening of the economy of the form (44) with a speed V, then exist two limit speeds of opening of N, V^N and V^G with $V^N < V^G$ such that :*

- *the level of income of N will not be preserved between autarky and free trade if $V > V^N$, it can be kept or preserved at each stage if $V \leq V^N$;*
- *the level of the global income of N and S will not be preserved between autarky and disappearance of active unskilled manpower in N if $V > V^G$, it can be kept or preserved at each stage between autarky and free trade if $V \leq V^G$.*

Proof Let us define

$$V^N = v_L^0/s^N \quad V^G = v_L^0/s^G \tag{63}$$

with the convention that $V^{\cdot} = +\infty$ in $]q_{\dot 1}, q_{\dot 2}[$ where $s^{\cdot}(q) \leq 0$. Then clearly, from (59) and (62), $V^N < V^G$. Let us then supppose that $V > V^{\cdot}$. For any trajectory of opening associated with V, this means that in $[0, q_{\dot 1}[$:

$$V = v_L/s > v_L^0/s^{\cdot}$$

When $v_L > 0$, then $s > 0$, $s < (v_L/v_L^0)s^{\cdot} \leq s^{\cdot}$ from (47) ; when $v_L \leq 0$, then directly $s \leq 0 < s^{\cdot}$. Thanks to Proposition 2 this proves the first part of the two indents, noting in addition that, in $[q_{\dot 1}, q_{\dot 2}]$, $V \geq V^N = +\infty$ can be read $s \leq 0 = s^N$ (which is not true with s^G which is negative then). Conversely, if $V \leq V^{\cdot}$, let us choose the trajectory of opening defined by $s = v_L^0/V$ in $[0, q_{\dot 1}[$ and 0 after. Then clearly $s \geq s^{\cdot}$ which, from Proposition 2 again, finishes the proof. ∎

In other words, opening N to the textile of S at a sufficiently low speed allows to control the evolution of income of N or N plus S. More precisely, if possibilities of mobility of labour are fully used, $V \leq V^N$ allows to get at each stage a Pareto superior situation for each zone ; so does V with $V^G \geq V \geq V^N$ in case South again accepts to share suitably its surplus.

5. Pareto optimality for agents

We shall now use as a criterium for welfare the level of utility of each agent. The situation is then much more complex and subtle.

First the situation is more complex because of unemployment. Except in case of perfect mobility, all q-equilibria with $q > 0$ are such that some unskilled workers are unemployed : see Figure 1. Thus, from (6), they have a zero level of utility, lower than in case of autarky. That means that except for opening processes ending in point C of Figure 1, there is no direct hope to have any free trade situation Pareto superior to autarky. This leads to introduce the additional assumption that there exists in N possible redistribution policies. More precisely we now consider the following four types of agents $j' = 1, 2, 3, 4$ where:

- $j' = 1$ corresponds to the active unskilled agents (they are λ_1),
- $j' = 2$ corresponds to the initially active skilled agent (they are ℓ_2),
- $j' = 3$ corresponds to the unemployed unskilled agents (they are $\ell - \lambda_1 - \lambda_2$),
- $j' = 4$ corresponds to active skilled but initially unskilled agents (they are $\lambda_2 - \ell_2$) .

We then set:

Assumption 3 *Given any q-equilibrium and the associated income $W(q)$ of N, the authorities of N can proceed to redistributions of $W(q)$ giving to an agent of type j' the income $w^r_{j'}(q)$ where the $w^r_{j'}, (q)$ satisfy :*

$$\lambda_1 w^r_1(q) + \ell_2 w^r_2(q) + (\ell - \lambda_1 - \lambda_2) w^r_3(q) + (\lambda_2 - \ell_2) w^r_4(q) = W(q) \tag{64}$$

Note that the situation considered in Fuchs [1997b] of an unemployment benefit αw_1 for non active unskilled workers ($0 < \alpha < 1$), financed by a solidarity tax proportional to activity revenues, is just a particular case of the situation above (with $w^r_3(q) = \alpha w_1, w^r_2(q) = w^r_4(q) = w_2 - (\ell - \lambda_1 - \lambda_2)\alpha w_1 \times w_2/W(q), w^r_1(q) = w_1 - (\ell - \lambda_1 - \lambda_2)\alpha w_1 \times w_1/W(q)$).

Then we can state the following Lemma.

Lemma 1 *Redistribution does not affect total demands. Thus levels of unemployment and employment are unchanged and the new situation obtained is a q-equilibrium corresponding to the new income distribution.*

Proof Keeping the rationing mechanism introduced in Section I, the individual demand functions associated with the income distribution $(w^r_{j'}(q))$ have the same form as has been already calculated, with $w^r_{j'}(q)$ instead of w_j and $c^{j'}_q = w^r_{j'}(q)q/W(q)$ instead of c^j_q. From (12), (14) and (18) we then have using (7) (which derives from (6)) the property that :

$$c_i(p_i, p'_1, q, w^r_{j'}) = w^r_{j'} c_i(p_i, p'_1, q, 1) \tag{65}$$

i.e. the demand functions are homogenous of degree 1 in income[5]. The lemma is then just a consequence of this property which, associated with (64), implies that :

$$\lambda_1 c_i(p_i, p'_1, q, w_1) + \lambda_2 c_i(p_i, p'_1, q, w_2) =$$
$$\lambda_1 c_i(p_i, p'_1, q, w^r_1(q)) + \ell_2 c_i(p_i, p'_1, q, w^r_2(q))$$
$$+(\ell - \lambda_1 - \ell_2) c_i(p_i, p'_1, q, w^r_3(q)) + (\lambda_2 - \ell_2) c_i(p_i, p'_1, q, w^r_4(q))$$

[5]Note that this property can be obtained from utility functions of a much more general form than (6).

which achieves the proof. ∎

But next the situation is also more subtle because the utility level of an agent depends not only on income effects (related to possible unemployment and redistribution) but also on price effects (linked to the access to q at price p'_1). To discuss the two effects together, let us calculate the utility level of an agent of type j' in the framework of a trajectory of opening (λ_j) satisfying Definition 4 and, for each value of q, using a redistribution process satisfying Assumption 3.

For q between 0 and q_1, using (6), (7) and (14), one gets :

$$u(c_1, c_2) \equiv u(q, w^r_{j'}(q)) = A w^r_{j'}(q)[1 + (p_1 - p'_1)q/W(q)] \tag{66}$$

(where $A = \mu^\mu(1-\mu)^{1-\mu}/p_1^\mu p_2^{1-\mu}$). For this utility level to be equal to the autarky level, the trajectory of opening and the redistribution process have to be such that :

$$w^r_{j'}(q)[1 + (p_1 - p'_1)q/W(q)] = w_j \tag{67}$$

with $j = 1$ for $j' = 1, 3$ or 4 and $j = 2$ for $j' = 2$ If this is true, multiplying each equality (67) by the number of corresponding agents and adding them, leads, thanks to (64), to the necessary condition :

$$W(q) + (p_1 - p'_1)q = W(0) \tag{68}$$

Let us then consider the pair (λ_j^P) satisfying together equation (20) and, in addition

$$\lambda_1^P w_1 + \lambda_2^P w_2 \equiv W^P(q) = W(0) - (p_1 - p'_1)q \tag{69}$$

One finds easily the solutions

$$\begin{aligned} \lambda_1^P &= \ell_1 - p_1 q/w_1 \\ \lambda_2^P &= \ell_2 + p'_1 q/w_2 \end{aligned} \tag{70}$$

which are consistent with all the conditions above and define the precise value of q_1:

$$q_1^P = \ell_1 w_1/p_1 \tag{71}$$

which gives the upper bound of the domain in which we have been working. The position of q_1^P with respect to the ranking already obtained in (59) is subtle too. Indeed, one can obtain :

$$q_1^R < q_1^P < q_1^G < q_1^N < q_1^M \tag{72}$$

but the second inequality is valid only if $\pi < \dfrac{1}{2}$.

Similarly, for q between q_1^P and some q_2, using again (6), (7) and this time (18), one gets

$$u(c_1, c_2) \equiv u(q, w^r_{j'}(q)) = B w^r_{j'}(q)[q/W(q)]^\mu[1 - p'_1 q/W(q)]^{1-\mu} \tag{73}$$

(where $B = (1/p_2)^{1-\mu}$). For this utility level to be equal to the level obtained in q_1^P, the trajectory of opening and the redistribution process have to be such that :

$$w^r_{j'}(q)q^\mu[W(q)-p'_1q]^{1-\mu}/W(q) = w^r_{j'}(q_1^P)(q_1^P)^\mu[W^P(q_1^P)-p'_1q_1^P]^{1-\mu}/W^P(q_1^P) \tag{74}$$

Again, multiplying each equality (74) by the number of corresponding agents and adding them leads to

$$q^\mu[W(q)-p'_1q]^{1-\mu} = (q_1^P)^\mu[W^P(q_1^P)-p'_1q_1^P]^{1-\mu}$$

which,using (70 and (71), gives the necessary condition:

$$W(q) = p'_1q + \ell_2 w_2(q_1^P/q)^{\mu/1-\mu}$$

Let us then consider the pair (λ_j^P) defined by :

$$\lambda_1^P = 0 \qquad \lambda_2^P \equiv W^P(q)/w_2 = [p'_1q + \ell_2 w_2(q_1^P/q)^{\mu/1-\mu}]/w_2 \tag{75}$$

(by construction the values $\lambda_j^P(q_1^P)$ and $W^P(q_1^P)$ in (69), (70) and (75) are identical). It is consistent with the conditions above and, through (11), gives the precise value of q_2 :

$$q_2^P = \ell_1 w_1/(p'_1)^{1-\mu}p_1^\mu \tag{76}$$

The position of q_2^P with respect to the ranking of (59) is this time frankly complex. It is worth being calculated though, because it will turn out to be quite useful for our final discussion. The result can be sumarized through the following sequence of equalities:

$$q_2^P = \pi^\mu q_2^N = \pi^\mu(1+\mu)q_2^G = \pi^\mu q_2^R/(1-\mu) \tag{77}$$

In other words we always have $q_2^P < q_2^N$ but q_2^P can be below or above q_2^G and q_2^R. For instance :

$$q_2^P < q_2^R \qquad \text{iff} \qquad \pi^\mu - (1-\mu) < 0 \tag{78}$$

We are now in position to introduce :

Definition 10 *We call q-equilibrium with constant satisfaction for each agent a pair* $(\lambda_1^P, \lambda_2^P)$ *belonging to* Q *and satisfying in addition :*

$$\begin{array}{ll} \lambda_1^P w_1 + \lambda_2^P w_2 = W(0) - (p_1 - p'_1)q & \text{for } 0 \le q \le q_1^P \\ \lambda_1^P = 0, \qquad \lambda_2^P w_2 = p'_1 q + \ell_2 w_2 (q_1^P/q)^{\mu/1-\mu} & \text{for } q_1^P \le q \le q_2^P \end{array} \tag{79}$$

Keeping in mind the construction above, the definition is self explanatory. It allows us to prove the following proposition:

Proposition 3 *There exists a limit sensitivity of the mobility of labour* s^P *such that for* $s \ge s^P$ *(respectively* $s < s^P$*) at each stage of the corresponding trajectory of opening, a suitable redistribution of income can be found which guarantees that the utility level of each agent is equal to or greater than its autarky level (respectively, whatever redistribution of income is used the utility level of some agents will be lower than the autarky level).*

Proof Using (70) or (75) which characterize the trajectory of opening associated with Definition 10 and calling s^P the associated sensitivity of the mobility of labour defined through (39) one gets :

$$s^P = \begin{cases} p'_1/w_2\ell_1 & \text{for} \quad 0 \le q < q_1^P \\ p'_1/w_2\ell_1 - w_1(q_1^P/q)^{\mu/1-\mu}/w_2 q & \text{for} \quad q_1^P < q \le q_2^P \end{cases} \tag{80}$$

One checks easily that $s^P < s^M$ and that conversely, given s^P, one obtains from Proposition 1 the trajectory of opening (λ_j^P) which satisfies Proposition 3. If we now consider a trajectory of opening defined by a sensitivy s with:

$$s^M \geq s \geq s^P \qquad \text{(respectively } s < s^P\text{)}$$

it is again clear from the proof of Proposition 1 that the λ_j it defines satisfy:

$$\lambda_j \geq \lambda_j^P \qquad \text{(respectively } \lambda_j < \lambda_j^P\text{)} \quad \forall j, q > 0$$

so that we have obviously from (8) and (79)

$$W(q) \geq W^P(q) \qquad \text{(respectively } <\text{)} \quad \forall q$$

This in turn means that it is possible (impossible) to find $w_{j'}^r(q)$ satisfying (64) and higher than or equal to the values defined by (67) or (74) with W^P instead of W, which achieves the proof. ∎

Then we can state to begin with :

Theorem 3 *Given a dynamics of opening of the economy of N defined by (44), let v_L be the speed of the mobility of labour generated along a trajectory of opening. Then there exists a limit speed v_L^P of the mobility of labour such that at each stage between autarky and free trade a suitable redistribution of income can be found which guarantees that the utility level of each agent is kept or increased iff $v_L \geq v_L^P$.*

Proof Let us choose $v_L^P = Vs^P$. Theorem 3 is a direct consequence of (46) and Proposition 3. ∎

The comment now is parallel to the one after Theorem 1. And we also have, considering Assumption 2, the partial converse :

Theorem 4 *Given an endogenous maximal speed of the mobility of labour v_L^0 in N, considering dynamics of opening of the economy of the form (44) with the speed V, there exists a limit speed V^P of opening of N such that no redistribution of income exists which avoids that the utility level of some agents will be decreased between autarky and disappearance of active unskilled manpower if $V > V^P$; there exists redistributions such that the utility level of each agent is kept or increased at each stage if $V \leq V^P$.*

Proof Let us define $V^P = v_L^0/s^P$ where, as in the proof of Theorem 2, we adopt the convention that $V^P = +\infty$ in $]q_1^P, q_2^P[$ where $s^P \leq 0$. Then if $V > V^P$, for any scenario of opening associated with V, this means that in $[0, q_1^P[$:

$$V = v_L/s > v_L^0/s^P$$

When $v_L > 0$, then $s > 0$ and $s < (v_L/v_L^0)s^P \leq s^P$ from (47) ; when $v_L \leq 0$, then directly $s \leq 0 < s^P$. Thanks to Proposition 3, this proves the first part of Theorem 4. Conversely, if $V \leq V^P$, let us choose the trajectory of opening defined by $s = v_L^0/V$ in $[0, q_1^P[$ and 0 after.

Thus, clearly, $s \geq s^P$ and again Proposition 3 applies. ∎

Thus,opening N to the textile of S at a speed below V^P guarantees that, if possibilities of mobility of labour are fully used and choosing suitable redistributions of income, Pareto superior situations can be obtained at each stage, including the final free trade situation where quotas are no more binding.

Last, it is worth to be noted that from (61),(62),(80) and (72), one has if $\pi < \frac{1}{2}$:

$$V^P > V^G > V^N \tag{81}$$

6. Comments and conclusion

Beyond our Theorems, which already bring a clear information by themselves, two series of comments deserve to be presented to appreciate completely the relation between the existence of unemployment and the possibility to keep some Paretian features to international trade. The first series concerns the final levels of unemployment which are compatible with the Theorems. The final level corresponding to a definite trajectory of opening is given by:

$$U(q_2) = \ell - \lambda_2(q_2) \tag{82}$$

where q_2 is the corresponding solution of (11). Using (23) we can write (82) as:

$$U(q_2) = \ell - q_2 p_1'/\mu w_2 \tag{83}$$

From this analytical expression, or Figure 1, one can see that the more the final point C' of a given scenario is "on the left", thus with a lower q_2, the greater is U. In the light of our results and of (83), we can then comment relations (59) and (77) as follows :

- if unemployment generated by free trade is smaller than $U(q_2^N)$ (point C^N of Figure 2), trajectories of opening can be found along which Pareto optimality for zones is saved or increased ;
- if unemployment generated by free trade is between $U(q_2^N)$ and $U(q_2^G)$ (points C^N and C^G) Pareto optimality for zones can be saved or increased along some trajectories of opening if South accepts to give North a suitable part of its surplus ;
- if unemployment generated by free trade is smaller than $U(q_2^P)$ (point C^P) trajectories of opening can be found along which, with suitable redistributions of income, Pareto optimality for agents is saved or increased.

Conversely, none of the three possibilities above is available in case the final level of unemployement belongs to $[U(q_2^P), \ell]$ and $q_2^P < q_2^G < q_2^N$.

Now let us keep the same approach but this time in terms of dynamics of opening such as defined by (44). We are then led to our second series of comments, which concern the old discussion between the advocates of bing-bang procedures to go from autarky to free trade and the supporters of progresive opening. In our model, clearly, a big-bang attitude means the choice of an infinite speed of opening in (44) which, from (46) and supposing that Assumption 2 holds, corresponds to a trajectory of opening with a zero sensitivity of the mobility of labour, i.e. to the scenario of perfect rigidity of labour described by the path AB^RC^R of Figures 1 or 2 (it is quite logical that, since training takes time, big-bang opening implies constancy of the volume of skilled labour force). Revisiting our comments above then allows us to say, keeping in mind Definition 3, (59) and (77):

- a big bang dynamics creates a level ℓ_1 of unemployment ;
- a big bang dynamics can in no way allow to preserve the income of North, even if South accepts to share its surplus ;
- a big bang dynamics may lead to a situation where no redistribution of income exists allowing free trade to be Pareto superior (from the point of view of all agents) to autarky.

The last two statement just reflect the fact that, on Figure 2, C^R is to the left of C^N and C^G and may be to the left of C^P too. More precisely, this last situation will occur any time inequality (78) is violated, i.e. in the hatched zone of Figure 3, where the separating line is the curve of equation $\pi^\mu = 1 - \mu$.

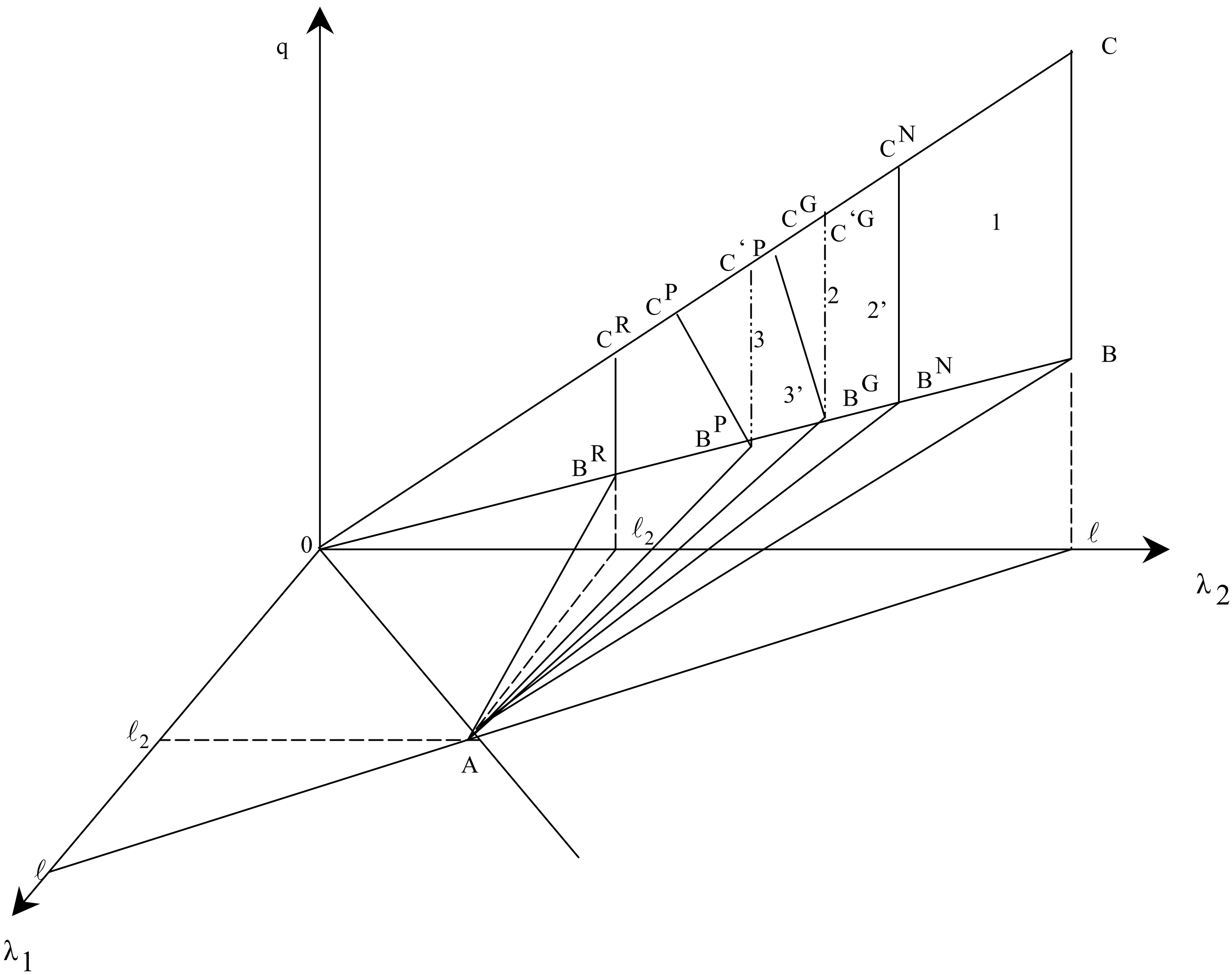

Figure 2

Figures concerning the actual values of π can be found in U.S. Department of Labor [2002]. Roughly speaking, in terms of cost of labour, there appears to be two categories of countries in the South : Newly Industrialized Countries (such as Taiwan, South Korea, Singapour...) where the cost of labour is around 30 to 40 % of what it is in the North, and countries just beginning to enter international trade (such as Bulgaria, China, Roumania or India) for which the cost of labour is below 10 % of what it is in the North. Just for the image then, in Figure 3, $\mu = 0$ gives $\pi = 1/e = 0,37$, $\mu = 1/2$ gives $\pi = 0,25$ (remember that, from (6), μ measures in some sense the interest of the consumers for textile).

Now, coming back to our discussion, to save or increase both incomes of N and S, or at least to increase the global income of N plus S, to be sure that income redistributions exist which make free trade preferable to autarky for all agents, and, in any case, to limit the number of unemployed people, in a situation of non activity and assistance, a dynamics of progressive opening of North appears to be the best way.

Final rewording of our theorems can then be:

- given a definite dynamics of opening with speed V, there is a minimum speed of the mobility of labour

$$v_L^M = V/s^M \tag{84}$$

  (where s^M is given by (40), $1/s^M = +\infty$ where $s^M = 0$) which allows to preserve full employment and thus income of all agents in N ; for smaller values of v_L, if trajectories are in zone 1 of Figure 2, the income of N is preserved ; in zone 2, the income of N plus S ; in zone 3, the utility level of all agents (using suitable redistributions of income) ; to the left of AB^PC^P, nothing can be preserved ; all along unemployment has been increasing.
- given a definite maximal speed of the mobility of labour v_L^0, there is a maximal speed of opening

$$V^M = v_L^0/s_M \tag{85}$$

  (with $V^M = +\infty$ where $s_M = 0$) which preserves full employment and all incomes ; for higher values of V, with $s = v_L^0/V$, trajectories will be respectively in zone 1 ($V \geq V^N$), 2' (with $C^{'G}$ instead of C^G : $V^G \geq V \geq V^N$) or 3' (with $C^{'P}$ instead of C^P : $V^P \geq V \geq V^G$). The properties of all these zones can then easily be compared to those of the big bang trajectory AB^RC^R.

As an empirical conclusion, we shall thus say that the optimal opening policy which can be deduced from our model and could be the goal for the authorities of N is a policy which controls the speed of opening V in such a way that, at each moment, all initially unskilled workers are either training or trained and employed. Such a policy allows to follow the trajectory ABC which, for sure, represent here "the best of worlds". Then, the higher

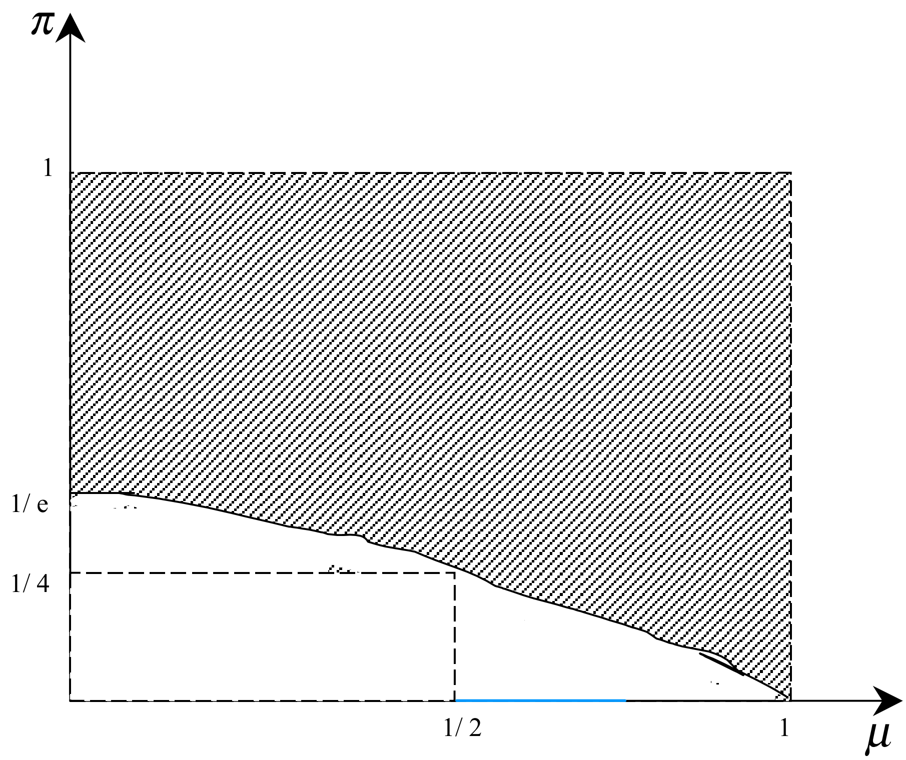

Figure 3

is V above this limit, the greater is the task of the authorities of N: a simple redistribution improving initial nominal incomes first ; an additional negociation with the authorities of S next ; a redistribution improving real but not necessarily nominal incomes, thus more difficult to explain to the different agents, last.

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