Controling externalities with asymmetric information: Ferrous Scrap Recycling and the Gold Rush Problem

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Décembre 2005

Cahier n° 2005-030
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Résumé: Nous proposons un modèle de l'organisation monopsonistique d'une filière de recyclage des métaux ferreux. Ce type d'activité se distingue par des externalités négatives propres à la collecte et de fortes asymétries d'information sur la qualité des matières collectées. Après avoir mis en lumière un effet de "ruée vers l'or" - la relation entre le niveau de prix et les externalités négatives de collecte - nous expliquons comment un recycleur monopsoniseur régule l'activité de collecte en contrôlant le degré d'asymétrie d'information. En particulier, plus la valeur d'une ferraille est élevée, plus les asymétries d'information doivent être importantes. En termes de bien-être, ceci peut être efficace mais induit un dilemme équité-efficacité, lequel est d'autant plus marqué que l'on intègre la dimension environnementale du problème.

Abstract: We develop a model of the monopsonistic organization of a ferrous scrap recycling branch. Negative externalities in the collection activity and information asymmetries on scrap quality are the distinctive features of the branch. After shedding light on the gold rush problem - the interplay between the collection externalities and the price of the good - we explain how a monopsonistic recycling firm regulates the market for scrap collection. The strategic use of expertise transfer to its suppliers is the recycler's control lever to overcome the potential gold rush externalities. The social consequences of this informational solution are inquired stressing a strong equity vs efficiency dilemma, even more pervasive when accounting for the environmental dimension.

Mots clés : Information asymétrique, externalités négatives, recyclage, monopsone

Key Words : Asymmetric information, negative externalities, recycling, monopsony

Classification JEL: L15, D62, Q53, Q33

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Great wastes arise from the suddenness and unexpectedness of mineral discoveries, leading to wild rushes, immensely wasteful socially, to get hold of valuable property.  
(Harold Hotelling, 1931)

You can know the name of a bird in all the languages of the world, but when you’re finished, you’ll know absolutely nothing whatever about the bird... So let’s look at the bird and see what it’s doing – that’s what counts. I learned very early the difference between knowing the name of something and knowing something.  
(Richard Feynman, 1988)

1 Introduction

Mines, fisheries and pastures are canonical examples illustrating the adverse consequences of unrestricted access to resource exploitation. Far from the early concerns of Hotelling (1931), Gordon (1954) and Hardin (1968), ferrous scrap collection is one modern ”commons problem”. The economic and environmental stakes of metal recycling and some theoretically puzzling features of its organization call for a positive analysis relying on the tools of information economics.

The organization of reusable matter collection by ragpickers or by immigrants settling down in industrial basins, does not, at first glance, share common features with the modern organization of waste collection and recycling. Historical insights suggest that the organization of reusable matter collection aimed at overcoming potential conflicts among collectors by allocating and restricting access to the coveted resources. Nowadays¹, independent ferrous scrap collectors performs the collection effort that initiate the recycling of ferrous scrap into a secondary raw material. They compete in a common bounded geographic area to access scrap sources, the so-called ”surface mines”. No comparison holds with the forerunners of refuse collection but on one point: the collection is organized to steer clear of the consequences of unrestricted competition. Limited information diffusion - a trait frequently associated with the profession - is the lever used to regulate the access to ferrous scrap ”lodes”. The reason is that waste collection is subject to a scavenger hunt - the gold rush problem - whose consequence may be a socially inefficient outcome.

¹We rely on an in depth field studies of scrap iron recycling conducted in Belgium and France (Adant and Gaspart, 2002; Adant and Godard, 2004).
A gold rush is characterized by four elements: high value of one resource, free competition for resource extraction, congestion externality and factor transfer towards the rushed in activity. Congestion arises because one can not control directly the extraction effort. It originates in limited geographic availability, inevitably leading to territory intersections. Factor transfer adversely affecting alternative activities in the economy is a second externality of the rush. Whether congestion inefficiency and factor transfers are important depends on the value of the matter to the competitors. If it is low, congestion inefficiency is limited and there is no interest in changing the allocation of factors from one activity to another one. On the contrary, if the matter is highly valuable, social inefficiencies are potentially severe. The first one is a strong overcrowding inefficiency. The second adds to the first one: easily redeployable production factors are shifted towards the highly valued activity. Historical examples are the Californian gold rush and, more recently, the Amazonian gold rush and the Great Lakes "black gold rush" in central Africa\textsuperscript{2}. Those rushes exhibit extracting level beyond the socially optimal threshold.

Under such circumstances, it could be socially beneficial to reduce the efficiency of some participants. Hence an informational solution to the gold rush problem. To our knowledge, it has not yet been emphasized. Economic agents that do not know where the gold is, or can not recognize gold from mica (tantalum from common stone) will have no incentives to bear the costs necessary to search for the highly valued resource. In other words, non expert people will not enter into a gold rush as expert people may. From the historical examples mentioned above, it is clear that the quality of information available is a key determinant in the extraction activity. It is outstanding in the case of ferrous scrap collection.

A simple model of the transactions between a recycling plant and independent ferrous scrap collectors offers a striking example of such outcome and its solution. We study the role of a principal (the recycling firm) relying on information transfer to the agents (the collectors), in addition to price setting, to prevent a gold rush in scrap collection.

The paper is organized as follows. Section two introduces the reader to ferrous scrap recycling and the expertise dimension of scrap collection. Section three focuses on the collection game between the collectors. Section four derives the recycler’s optimal strategy in regulating the game between collectors, given price and expertise transfer as control variables. In section five, we develop a normative analysis to inquire the social efficiency of this organization and considers the environmental dimension. The results are discussed in the last section. We discuss the relevant literature as we go along.

\textsuperscript{2}“Black Gold” refers to Colombite-Tantalite, a mineral from which one extracts two precious metals, Tantalum and Columbium. Both are used in the contruction of electronic components such as capacitors for mobile phones and computers.
2 A glimpse into ferrous scrap recycling

Metal scrap is a reusable resource. Ferrous waste collection and recycling won progressively acclaim with the increasing pressure on the inputs of integrated steel production (iron ore, coal and coke) and the development of mini-mills. In order for recycling to be a sustainable solution to natural resource exhaustion and environmental pollution generated by their extraction, the operators of the recycling branches face a series of challenges. One of them, and not the least, being the good functioning of their input market, the upstream market of the recycling branch.

On this market, independent collectors are supplying a recycling firm ("the recycler" in the sequel) with ferrous scrap batches. Collection consists in searching for ferrous scrap sources, in sorting the materials and in delivering it to the recycler. The sources accessed by the collectors are heterogeneous in terms of available quality, quantity, seasonal variability and access cost (whether the collector has to pay for the scrap or not). The collector’s revenue is simply the weight of the batch (net of observed abnormal waste such as wooden battledore, paving stone, etc.) times the unit price of the quality.

The quality of the batch is evaluated by the buyer. The first characteristic entering the definition of the quality of the scrap is the percentage of iron, that is the matter to be separated and recovered by the recycling technology. The higher its proportion, the higher the value of the material. Given the recycling processes used (shredding or balling and shearing), other characteristics are taken into account to define a unique nomenclature of grades: the length of the pieces, their thickness, their cleanliness and the presence of abnormal waste. Supplied with high quality materials, the recycling firm will be able to produce a high quality secondary raw material. Conversely, low quality materials containing abnormal components reduce the quality of the re-usable output below the level required by a producer of new goods and increase the industrial risks borne by the operators of the branch.

The most important traits of the market are its geographical dimension and the informational conditions prevailing during the transactions between the collectors and the recycler.

The geographical characteristics of the recycling activity are twofold. Firstly, the input market of a recycling firm is defined by its geographical perimeter. The agents search for sources and collect scrap in this common area. There is no legal access restriction to the different sources of scrap, hence the collectors freely access the deposits and compete for them: the pieces of metal a collector has picked up, another can not. Secondly, the collectors are bound to a given area because of the high transportation costs compared to the value of scrap. This confers on the recycler a monopsonistic position in the collection zone.\(^3\)

\(^3\)The localization with respect to the upstream and downstream markets and, particularly, privileged access to the least costly transportation modes (water- and railways) are factors consolidating the monopsonistic
The distinctive informational conditions that prevail on a market for ferrous scrap can be summarized as follows. A menu of grades and their respective unit price is publicly set by the recycler. The recycling firm has a huge discretion to decide upon this menu, thanks to his geographical monopsonistic position. During each spot transaction, the buyer evaluates silently and visually the collector’s batch, then announces the denomination of the quality and the corresponding ”take-it or leave-it” price according to the public menu. There does exist any technological means to assess objectively the quality of scrap batches. Furthermore, the characteristics defining the grades used by the buyer to categorize the pieces of metal into the nomenclature are not public information. The buyer never justifies the categorization of the batch into the nomenclature of qualities. Each collector has to infer what are the characteristics of the pieces taken into account by the buyer from the observation of the denomination announced. Frequent transactions is thus a key determinant of the collector’s expertise, that is his ability to predict from the observation of the characteristics of the matter the categorization chosen by the buyer. Hence the importance of learning and the resulting difference in expertise between collectors.

There is the rub. Because he knows better than the collector the relevant characteristics to evaluate the batch, the buyer can cheat the collector by downgrading the batch and paying the corresponding (lower) price. But a non expert collector will have no incentive to exert the collection effort required to obtain higher qualities, and will supply only low quality. In the eyes of the buyer, such possible market failure calls for expertise transfers leveling up the abilities of the collectors. Hence two conflicting issues in expertise transfer clearly identified by the buyers of recycling plants. More expertise on the collectors’s side enhances effort provision, through the overcoming of the commitment problem. But on the other hand, expertise transfer paves the way to a gold rush by increasing the potential competition for access to scrap sources. The next section is dedicated to model the collection game between the collectors, in which the gold rush problem arises.

3 The collection game

The core of the problem of the branch we consider is the collection technology. It is rather unsophisticated: the collectors own trucks, they spend the day collecting pieces of metal in a urban and industrial areas. Once the truck is full, they go to the recycler’s scarpyard to sell their ”harvest”, or they store it until the transaction with the recycling plant takes place. So position of the recycling firm. Given the importance of transportation costs relative to the value of the material, this preserves the recycling firm from any competition of others recyclers.
the source of their costs is mainly transportation cost, e.g. the time and the fuel they consume. They can pay to have access to some areas, too, like plants to be demolished. Depending on the distance they decide to cover and/or the access fees, they might find batches with higher ferrous fraction, which are more valuable in the recycler’s eyes. The amount of high quality accessible is assumed to be fixed, setting an upper bound to the collected volume.

Formally, we consider two collectors $C_i$, $i \in \{1, 2\}$, who exert an effort $e_i \in [0, 1]$, to increase the probability of getting high quality, at a convex cost $c(e_i)$. This cost encompasses transportation costs and potential access fees. But they may instead simply bring low quality scrap, at a cost normalized to 0. The convexity of the costs stands for the time constraint that the collectors face every day. We often use the notation $e = (e_1, e_2)$. We use quadratic costs, $c(e_i) = \frac{b}{2}e_i^2$, and restrict to binary quality, high or low, which will allow for explicit solutions. In this section, the price is exogenous: it is either 0 for the low quality or $p > 0$ for the high quality. Now, we shall include in the model the particular kind of competition between the collectors.

**Geographical Externalities**

The first ingredient is the limited amount of scrap available. This amounts to say that every day the households and industry generate a given flow of scrap, in a delimited populated area. The second ingredient is the absence of territory allocation. Hence collection takes place on a common area, and what one agent collects is no more available to the other. Each collector decides individually which fraction of the total area to cover, without coordinating with the other. Thus the collectors are subject to negative geographical externalities, a form of congestion.\(^4\)

We model this through the probability for each agent to find a high quality batch. This depends on both efforts; for collector 1, we have $q_1(e_1, e_2)$ with $\frac{\partial q_1}{\partial e_1} \geq 0$, and $\frac{\partial q_1}{\partial e_2} \leq 0$. Moreover, one must reasonably have $\frac{\partial q_1}{\partial e_2}(q_1 + q_2) \geq 0$ for $i = 1, 2$, so that any effort is always productive.

We use a simple geometric representation to capture this geographical interaction. Assume that the collection area is a square of area 1. A first collector starts from the West and chooses how far East he goes, gathering everything in the area between the south side and the north side. The other starts from the South with the same gathering technology toward the North. Efforts are thus linked mainly to covered distances in this interpretation. This is pictured in figure 1.

In the light grey zones, one collector gathers alone, while they both cover the heavy grey zone. There is no collection in the blank area. We assume that the probability of getting

\(^4\)The reader is referred to the early works by Haveman (1974) and Newbery (1975) for a detailed classification.
a high quality batch is proportional to the area covered. The return is split equally in the common area. This amounts to consider a uniform distribution of high quality spread over the unit square. Overall, the probability of getting a good batch for collector $i$ is thus $q_i = e_i(1 - e_{-i}) + \frac{e_1 e_2}{2}$. This yields:

$$
q_1 = e_1(1 - \frac{e_2}{2}) \\
q_2 = e_2(1 - \frac{e_1}{2})
$$

(1)

Negative externalities arise from the specificity of the collection effort. Expression (1) implies that the effort of an agent lowers the marginal productivity of the other’s. There is an equivalent feature in contest situations (e.g. Dixit, 1987), where two agents exert efforts to increase the probability of winning a unique prize (or the share of a fixed-size pie). However, in such settings one has naturally $q_1 + q_2 = 1$, which is not the case here: all high quality is not necessarily collected. The equality is here relaxed to $q_1 + q_2 \leq 1$. In addition, the resources serve here a production purpose.

An additional comment is in order about this technology. It is not a generic formulation but contains the key elements, and besides, it has the very nice property to allow for explicit solutions. It can indeed be really problematic to have tractable results in contest-like situations\(^5\). The problem under study being far more complex than a standard contest, it is not worth using a more generic formulation. All results would qualitatively go through by considering any kind of geometric intersections. The important point is that complete collection necessarily entails a significant overlapping of collection areas.

\(^5\)See Hirshleifer (1989) and Skaperdas (1996) for a discussion of this purely technical point.


**Heterogeneous abilities**

Finally, the collectors (may) differ through an additional efficiency parameter\(^6\). Formally, collector \(i\) is characterized by an expertise level, \(\theta_i \in [0, 1]\). In the case of fisheries, this dimension is emphasized by Durrenberger and Palsson (1987, p.510), quoting Johnson (1979):

"the producers (...) occasionally can control certain production-related information which governs access to this resources. Thus production-related knowledge, such as the specific location of fish and the most effective tactics for catching them, becomes a scarce capital good."

We will mainly focus on the interpretation of \(\theta_i\) as an informational expertise parameter\(^7\), but in this section, the only relevant point is that it is some productivity index. A bigger truck, a better knowledge of the locations to scour or a better skill at discerning scrap along the road would all correspond to a higher ability \(\theta_i\).

We have all the elements to state the following utility functions for the agents in the game for given prices (recall that the price of low quality is normalized to 0 and the price for high quality is \(p\)):

\[
U_1(p, \theta, e) = \theta_1 q_1(e_1, e_2).p - c(e_1)
\]

\[
U_2(p, \theta, e) = \theta_2 q_2(e_1, e_2).p - c(e_2)
\]  

(2)

Let us denote by \(G(p, \theta)\) the corresponding game for each expertise configuration \(\theta\) and price \(p\).

**The collection equilibrium**

We are interested in the Nash equilibria of \(G(p, \theta)\). Since the utility functions are concave with respect to \(e_i\), the best-reply of collector \(i\) when the other exerts effort \(e_{-i}\) is given by the following first-order condition \(\frac{\partial U_i}{\partial e_i}(e_i, e_{-i}, \theta_i) = 0\). The (unique) Nash equilibrium \((e_1^N, e_2^N)\) is thus given by the simultaneous first-order system:

\[
\begin{align*}
(1 - \frac{\theta_1}{2})\theta_1 p &= \mu e_1^N \\
(1 - \frac{\theta_2}{2})\theta_2 p &= \mu e_2^N
\end{align*}
\]

Which yields the unique solution (when interior):

\[
e_i^N(p, \theta) = \frac{2\theta_i p(2\mu - \theta_{-i} p)}{4\mu^2 - \theta_i \theta_{-i} p^2}
\]  

(3)

---

\(^6\) Few papers deal with such asymmetry; see Baik (1994) and Peña-Torres (1999).

\(^7\) In the next section, we specify an corresponding information structure adding further content to this parameter.
When \((1 - \frac{\epsilon^N}{\tilde{T}})\eta_i, p \geq \mu\), we have \(e^N_i = 1\), and the other effort is given by the best-reply: 
\[\mu e^N_i = \frac{1}{2}\theta_i, p\], which can also be 1 when \(\frac{1}{2}\theta_i, p \geq \mu\).

One can check that the equilibrium efforts are increasing in own expertise and decreasing in the other’s. But the effect of \(p\) is not monotone. It depends on both types in a non-trivial way.

To analyze the effect of the price, we study the surplus of collection. We define the Hicks quantity as:

\[H(p, \theta, e) = U_1(p, \theta, e) + U_2(p, \theta, e)\]

and indicate the equilibrium values by a \(N\) superscript. In this case \(e\) is replaced by the unique Nash equilibrium, \(e^N(p, \theta)\), and is suppressed from the arguments. The next proposition gives the main result of this section. It characterizes in particular the social effect of high price leading to fierce competition.

**Proposition 1 (Gold Rush Problem)**

The expertise profile \(\theta^*(p)\) yielding the highest total collection profit is decreasing in the price. Let \(p_{GR} = 2(\sqrt{2} - 1)\mu\). The equilibrium collection surplus \(H^N(p, \theta)\) is maximized for:

\[\theta^* = (1, 1) \quad \text{if } p < p_{GR}\]
\[\theta^* = (1, 0) \text{ or } (0, 1) \quad \text{if } p \geq p_{GR}\]

**Proof.** It is shown in the appendix that the only candidate optimal points are \(\theta = (1, 1)\) and \(\theta = (1, 0)\). Then, the comparison of equilibrium payoffs writes:

\[H^N(p, 1, 1) \geq H^N(p, 1, 0) \iff \frac{4\mu^2}{(p + 2\mu)^2} - \frac{p^2}{2\mu} \geq 0\]

\[\iff 8\mu^2 - (p + 2\mu)^2 \geq 0\]

Let \(p_{GR} = 2(\sqrt{2} - 1)\mu\) be the unique positive root of this polynom. It is clear that the polynom is positive when \(p\) is between 0 and \(p_{GR}\) and negative when \(p\) is above \(p_{GR}\). \(\square\)

It must be stressed that both the limited access and the high value are necessary conditions for a socially sub-optimal rush to occur. This is why one can see this effect as a "Gold" rush. The result is better understood in considering the efforts. In the optimal setting, they are:

\[e^*_1 = e^*_2 = \frac{2p}{2\mu + p} < 2 - \sqrt{2} \quad \text{if } p < p_{GR}\]
\[e^*_1 = 1, \ e^*_2 = 0 \quad \text{if } p \geq p_{GR}\]

If the price is low, the efforts are smaller than 1 (the area is not fully covered), and despite the negative externalities, there are economies of scale with two collectors, because of the
Figure 2: Iso-Hicks quantity in equilibrium.

convexity of the costs. This makes competition desirable. Conversely, if the price is high, the incentives to collect are strong and the effort levels are high. This creates a strong externality, which is sub-optimal. It is clear in the limit case where both collectors exert full effort: then the costs are purely duplicated, and one is thus a pure social loss. In such case, Nash equilibrium (competition) is less efficient than a monopolistic collection. This introduces a discontinuity in the collected volume, too:

\[
q_1^* + q_2^* = \frac{s_{pm}}{(2\mu+p)^2} < 2(\sqrt{2} - 1) \quad \text{if } p < p_{GR} \\
q_1^* = 1, \ q_2^* = 0 \quad \text{if } p \geq p_{GR}
\]

It is somewhat surprising that the optimal trade-off between economies of scale and externalities is so discontinuous in terms of the variable \( \theta \). Given the discrete optimal values, one way of formulating the paradox is: the optimal number of workers is decreasing in the value of the good. Figure 2 illustrates graphically the problem. The iso-Hicks quantity in equilibrium are represented in the space \((\theta_1, \theta_2)\), for two different prices. The dots A,B,C indicate the Hicks-optimal points. At some point, the curvature of the social indifference curves make the optimal point suddenly jump from one corner to another (from A to B or C).

One remarkable point, that we discuss at the end, is that the optimal configuration takes the form of (endogenous) licences. Indeed, setting \( \theta_i = 1 \) amounts to give a licence to \( C_i \), while setting \( \theta_i = 0 \) amounts to completely exclude \( C_i \) from the collection of high quality.

\[\text{It has to be stressed however that the agents never internalize the congestion, and that the Nash equilibrium is never "first-best" (a form of standard commons problem).}\]
Scrap collection and gold rushes

To establish the relevance of individual incentives and their variability\(^9\) in ferrous scrap collection, let us quote Pounds (1959, p. 251):

"Scrap tends to flow unevenly, and its price (...) tends to fluctuate very widely. (...) At lowest, the price of scrap is the cost of collecting it, and at the opposite end, the marginal cost of sorted and graded scrap is higher than pig iron, because it represents pig iron that has been further refined. These extreme fluctuations in the price of scrap greatly influence the supply of obsolete scrap; it is not worth while scouring the farms and quarries for abandoned equipment when prices are low."

This dimension of individual incentives is also the driving force in gold rushes. Moreover as Meade (1897, p. 4) explained in the case of gold, "the necessary implements are few and simple, and they are operated mainly by manual labor and require little skill in their use", which clearly establishes the technological similarity between gold extraction and scrap collection.

The surprising result of our comparative statics is a plausible rationale for the intuitive view that "rushes" may be socially harmful\(^10\), despite the huge fortunes they generated. The inefficiency can be seen in the case where value to the collectors is high: both collectors cover the field entirely, no matter what the other does. Gold rushes have this feature insofar as gold feverishly attracted people, whatever the number of diggers already active. In both cases, all the elements are present to generate the "wild rushes, immensely wasteful socially" of Hotelling (1929, p. 144).

In the vein of natural resource economics, three arguments are worth considering to analyze this effect. First, Cornes, Mason & Sandler (1986) derive a relation between the optimal number of competing firms on the commons and the elasticity of demand. But at the time of gold rushes (1847 to 1855 in California, 1896 to 1899 in Klondike), gold was the numeraire value. Given these short rush periods, one can reasonably consider that the value of extracted gold was inelastic\(^11\), and this discards this approach. Second, gold and ferrous scrap are inert exhaustible resources (e.g. Dasgupta & Heal 1979, chapter 6), thus postponing extraction has no reason to be optimal, except for technological purpose. Third, the potential inefficiency is not one of overconsumption with respect to future opportunities (e.g. Stiglitz, 1976). No one seems thus relevant here.

\(^9\)Chinese demand for scrap dramatically increased between may 2002 and may 2004, and drove world ferrous scrap prices from 808 to 2808.

\(^{10}\)A vast strand of literature exists in general equilibrium theory on the effect of booms. In particular, on the relevancy of the institutional setting see Robinson, Torvik and Verdier (2002) and the references therein.

\(^{11}\)Mitchell (1896) and Meade (1897, p.2-3 and 6) confirmed this assumption.
Closest to our argument is the recent one given by Clay and Wright (2003) in a study of the Californian gold rush. It is related to free entry as a source of inefficiency. The absence of property rights over deposits led to violent conflicts between incumbents and newcomers. This kind of territory intersections induced externalities comparable to that in collection activity. The study insists on the open-access nature of the resource, and the progressive setting up of ownership from initial informal claims. However, there is no point in considering property rights over land in the case of scrap collection taking place in public streets. No authority exists that has either physical or legal enforcement power for territory restrictions. Still, as we will see next, alternative means exist to regulate indirectly the access to the resource.

Considering at a micro scale the transmission of gold fever, it is clear that information played a crucial role in this historical example. As soon as the newspapers released news about gold findings near Sutter’s Fort, people rushed to the location to dig for gold. In 1848, on June 10th, the newspaper *California Star*, commented the rush as follows: ”It is quite unnecessary to remind our readers of the ’prospects of California’ at this time, as the effects of this gold washing enthusiasm, upon the country, through every branch of business are unmistakably apparent to every one [...]. Every seaport as far south as San Diego, and every interior town, and nearly every rancho [...] has become suddenly drained of human beings”.

The very ones that warned their readers finally gave in to the lure of gold, as Bieber (1948, p. 10) indicates:

"the *Californian* issued its last sheet on May 29 and the *California Star* on June 14; editors, typesetters, printer’s devil, and most of the reading public had decamped for the land of gold.”

The next section is dedicated to the positive analysis of the organization of collection, where the recycling firms plays a central role as an information regulator.

4 The recycler’s regulation

A monopsonistic market

The recycling firm, $R$, faces the two scrap collectors $C_1$ and $C_2$. The scrap batches they supply all have the same size, normalized to 1. A batch is either of low or high value to the recycler: $v \in \{v, \bar{v}\}$. This valuation $v$ encompasses the price the recycler gets in the downstream market (the price paid by a metallurgist) and the cost of processing raw materials. The recycler, acting in a monopsonistic way, offers a price menu $(\bar{p}, \bar{p})$ to the agents.

We assume that the reservation utility of the collectors is zero. The value of low quality is also normalized to $\bar{v} = 0$ (remember collecting low quality costs nothing). It is thus clear
that the recycler will always set $p = 0$. Finally, we use the notations $v = \overline{v}, p = \overline{p}$ in the following.

**Informational setting**

While the recycler perfectly knows the value of a batch, many collectors cannot rely on an expert appraisal of the quality of the material found. They do not discriminate between all grades making up the public nomenclature of quality. To be precise, they all know the relationship between the denominations of quality and the corresponding prices, but some lack the knowledge necessary to categorize each batch under its denomination. Sometimes, they are able to prove that the batch corresponds to a high quality denomination by stressing the right specifications.

Such a situation common to many markets. For example, it arises when an antique dealer meets someone who wants to sell the content of an attic. If the seller can exhibit a proof of authenticity for an old painting or furniture, he will get a high price. But it is also likely the case if he is able to perfectly explain why this chair is recognizable as Louis XIV style, by putting forward the relevant characteristics and using keywords of the profession. It is this second kind of transaction we want to capture.

We assume in the following that the recycler is able to selectively educate some collectors, by explaining in each transaction the criteria used to evaluate the batch. Then, in Feynman’s words, they know the bird and not only its name. Of course, the recycler is aware that these educated collectors will be less easily ”cheated”.

Formally, the collectors differ through their expertise levels $\theta_i \in [0, 1]$, as in section 3. The expertise configuration $\theta = (\theta_1, \theta_2)$ is known to all players. Now, the expertise level $\theta_i$ is the probability that the collector distinguishes the value of a batch (e.g. gets an informative signal). Moreover, when he distinguishes the true value, we assume that he is also able to demonstrate it. Conversely, when he is not able to identify the batch, he can not prove anything, and receives a low price.

This is represented by the following hard information structure:

Let $s_i \in \{\emptyset, \overline{v}, \overline{\overline{v}}\}$ denote the signal produced by agent $i$ about his batch. The (conditional) distribution of $s_i$ is given by:

\[
\begin{align*}
\text{Prob}(s_i = \overline{v} | \overline{\overline{v}}) &= \theta_i \\
\text{Prob}(s_i = \emptyset | \overline{v}) &= \theta_i \\
\text{Prob}(s_i = \emptyset | \overline{\overline{v}}) &= (1 - \theta_i) \\
\text{Prob}(\sigma_i = \emptyset | \overline{v}) &= (1 - \theta_i)
\end{align*}
\]

At first glance, the recycler’s superior expertise gives him the opportunity of getting high quality for low price. But this would in turn discourage the collectors’ ex-ante incentives to
exert effort. This corresponds to a form of holdup inefficiency\textsuperscript{12}: indeed if $\theta_i = 0$, agent $i$ will have no incentive to exert effort at all. Put differently, from the agent's point of view, $\theta_i$ is the probability of obtaining the "right" price.

The utility functions of the collectors in the (sub-)game for given prices are then formally given by (1), as in section 3. However, the interpretation is now a bit narrower: with probability $q_i \theta_i$, the batch is high quality and the collector proves it, hence he gets $\underline{p} = p$. In all other cases, he gets $\underline{p} = 0$. Note finally that this setting is consistent with any interpretation where $\theta_i$ is an abstract bargaining power of $C_i$, once a public price $p$ is given.

The recycler's program

The previous section showed that depending on the price of high quality, the rush towards high quality may lead to an inefficient equilibrium. Now, the natural question is: to what extent does the recycler internalize this problem when he chooses prices and controls expertise? From his point of view, the price has an ambiguous effect: he obviously does not want to give too high prices, but for incentives reasons, he has to set a price gap between the different qualities.

Formally, the principal will select a price/expertise scheme such that the Nash equilibrium played by the agents in $\mathcal{G}(p, \theta)$ maximizes the following expected utility:

$$V(p, \theta, e) = (v - p\theta_1)q_1(e_1, e_2) + (v - p\theta_2)q_2(e_1, e_2)$$

(4)

The payoff of the recycler is simply the volume of supply times the net value for him. Since all batches have the same unit size, the expected value coming from collector $i$ is equal to $q_i v$, while the expected cost is $p\theta_i q_i$, because the high price is paid only with probability $\theta_i$. The corresponding program is:

$$\begin{align*}
\text{Max}_{\theta, p} & \quad V(p, \theta, e) \\
\text{s.t.} & \quad p > 0 \quad (IR) \\
& \quad e = e^N(p, \theta) \quad (IC)
\end{align*}$$

(PR)

where the first constraint is the participation of the agents, and the second can be interpreted as an incentive constraint. It requires that the efforts pair is the unique Nash equilibrium of the subgame $\mathcal{G}(p, \theta)$, denoted by $e^N(p, \theta)$. We can substitute the incentive constraint in the objective of the recycler (uniqueness of the equilibrium allows to use the so-called "first-order approach"). He maximizes the following quantity:

$$V^N(p, \theta) = V(p, \theta, e^N(p, \theta))$$

\textsuperscript{12}Lau (2003) develops a model of intermediate holdup in a principal-agent relationship, which shares some common features with our informational setting.
Because both the recycler and the collectors are assumed risk-neutral, the risk associated with the collection technology is not an issue. However, for realism and to get a unique solution to \( (P_R) \), we state:

**Assumption 1** When indifferent between two solutions \( (p, \theta) \) and \( (p', \theta') \), the recycler selects the one which minimizes the associated risk.

In practice, this criterion will amount to select among two solutions having the same expectation the one with the greatest level of expertise and smallest price. It is possible to endogenize this assumption by accounting for risk-aversion of *any one* of the players. This assumption is thus not restrictive, and essentially allows easier proofs.

The main issue in solving \( (P_R) \) is that the recycler accounts for the congestion through the cross incentives effect. Indeed, a more expert collector exerts more effort, but this in turn lowers the incentives of the other. In parallel, raising price increases individual incentives only insofar as the other collector is not too much incited to increase effort, too. On top of that, the hold-up dimension makes an increase in price or expertise desirable for incentives, but also increases the costs. It is thus somewhat remarkable that we reach an extreme solution in terms of \( \theta \) as in the preceding section. The complete solution to the recycler’s problem \( (P_R) \) is given in the next proposition.

**Proposition 2** Under Assumption 1, the recycler chooses optimally the following price and expertise levels:

\[
\begin{align*}
\theta^{**} &= (1, 1) \\
p^{**} &= \frac{2pv}{v+4\mu} & \text{If } v < 2\mu
\end{align*}
\]

\[
\begin{align*}
\theta^{**} &= (1, 0) \text{ or } (0, 1) \\
p^{**} &= \mu & \text{If } v \geq 2\mu
\end{align*}
\]

The full proof is relegated to the appendix, but the sketch is as follows. First, it can be noticed that the price and the expertise levels are substitutes in the principal’s objective. The recycler has in fact two variables to choose, \( r_1 = p\theta_1 \) and \( r_2 = p\theta_2 \). Indeed, it follows from (3) and (4) that his profit when the collectors play the Nash equilibrium of \( G(p, \theta) \) can be rewritten as a function of \( r_1, r_2 \) only. This function is a rational fraction the concavity of which depends on \( v \) and \( \mu \). Then we find interior solutions in terms of \( r_1, r_2 \) for some ranges of \( v \) and \( \mu \). This gives the first part of the proposition. When \( r_i \) is on the frontier, this means that one \( \theta_i \) is equal to zero, and we are able to find the second part. There are two kinds of solution, depending on the value \( v \) relative to the cost parameter \( \mu \).
When the value of scrap is high, intuition suggests that the recycler would like to get a lot. At the extreme, he may want everything to be gathered. This can be done with a single expert collector at a price exactly equal to the marginal cost when \( e_i = 1 \), which is \( \mu \). This is indeed the strategy preferred by the recycler when \( v \geq 2\mu \). However it is not clear before a careful analysis. For example, he could have preferred to have one perfectly expert collector, say \( C_1 \), covering the whole zone, and \( C_2 \) less expert, covering a small area. In this small area, the expected cost for the recycler is \( p \frac{1 + \theta^2}{2} \), which is less than \( p\theta_1 = p \) in the other area. But this gain happens to be offset overall: to gather everything, the recycler has to compensate the externality exerted by \( C_2 \) on \( C_1 \) by setting a higher price.

When \( v \) is low, at the other extreme, the incentive cost is too high to make complete collection desirable. Then, given the convexity of the collection efforts, it is the case that small level of efforts are optimal, despite the externalities. The right trade-off between the two extreme situations happens overall to be very discontinuous, despite the smooth nature of the model (as was the case in the first proposition). In terms of payoffs, \( V^* \) is of course continuous, but the equilibrium payoffs of the collectors do jump at \( v = 2\mu \) (one falls, the other rises). As is always the case with moral hazard, the collectors make a strictly positive profit as soon as they are incited to exert some effort.

The efforts in the optimal scheme are:

\[
\begin{align*}
    e_1^{*} &= e_2^{*} = \frac{v}{2\mu + v} < \frac{1}{2} \quad \text{if } v < 2\mu \\
    e_1^{*} &= 1, \quad e_2^{*} = 0 \quad \text{if } v \geq 2\mu
\end{align*}
\]

The total quantity of scrap gathered naturally increases with \( v \). The maximal quantity collected when \( v < 2\mu \) is \( \frac{3}{4} \), and it jumps to 1 when \( v \geq 2\mu \).

We can now answer the question about a potential gold rush problem in the equilibrium selected by the recycler. Indeed, intuition suggests that he may be tempted to exploit competition between the collectors to obtain the high quality scrap at a low price. It turns out however that he never induces a gold rush through the price schedule and expertise configuration he prefers.

**Corollary 1** *The recycler never triggers a gold rush.*

**Proof.** A sufficient condition is that the highest price set by the recycler when he chooses \( \theta^{**} = (1, 1) \) is too low to induce a Gold Rush. This price is offered when \( v \rightarrow 2\mu \); it is

\[
p^{**} = \frac{2\mu v}{v + 4\mu} = \frac{2}{3} \mu < 2(\sqrt{2} - 1) \mu = p_{GR}.
\]

This comes from the difference between our setting and a tournament (or an auction). Had the recycler to buy only the best harvest, he would have induced a gold rush. But he
internalizes (partly) the externalities between the collectors. To be precise, by valuing the
sum of output he cares about the link in the production function, but not directly about the
optimal repartition of costs. This induces a shift from the social optimum as will be studied
in the next section.

Two additional remarks about the robustness of the conclusions reached here are in order.
First, The collectors have no interest in sharing expertise, as was seen in the first part. Thus
collusive strategies have no bite on the solution to the recycler’s program \( P_R \).

Second, the absence of a technology to measure quality deserves attention. Would such a
tool exist, all the collectors would be able to get the public price. Thus the recycler would
not have any discrimination mean. Given the assumption that expert collectors produce hard
information signal, and that the recycler chooses at no cost whom to make an expert, it is
better for him to keep this flexibility, rather than developing a measuring technology.

**Corollary 2** The recycler is better off without a technology to measure quality.

We have thus given an explanation to the puzzling observation that the recycler prefers
to leave some room for a hold-up inefficiency. In doing so, he is able to better control the
collection externalities. Incentive inefficiency with some collectors is only one side of the coin;
the other is greater efficiency of the others when value is high. In the next part, we inquire
whether this original regulation through indirect price discrimination is socially efficient or
not. The discussion will lead us to additional considerations, such as environmental concern,
that the recycler does not take into account, perhaps not so paradoxically.

## 5 Welfare and Asymmetric Information as a Licensing

**Device**

To conduct a welfare analysis, we define first a simple utilitarian surplus of the industry:
\( S = H + V \). This surplus is the pure economic value created by collection ferrous scrap and
transformation into secondary raw material. For the same reasons that the recycler can not
impose the efforts to the collectors, a regulator can not reasonably use more than the *second best*
tools of prices and expertise. We inquire the social solution in the same way as for the
recycler, except of course for the objective function. The planner’s program is then:

\[
\begin{align*}
\max_{\theta, p} & \quad H^N(p, \theta) + V^N(p, \theta) \\
\text{s.t.} & \quad p > 0
\end{align*}
\]

\((P_S)\)

This program is different in that a social planner directly cares about the real cost of
collection. That implies, for example, that if one collector covers the whole area in equilibrium,
we know for sure that the other should not exert any effort. It is thus clear that when it is worth collecting all scrap, only one collector should do the job, the other having no expertise, or ”licence to collect”. Under the parallel of assumption 1 applied to the social planner (which is justified exactly the same way), the solution is given in the next proposition.

**Proposition 3** When maximizing the economic surplus \( S \), the social planner optimally sets:

\[
\begin{align*}
\theta^* &= (1, 1) \\
\frac{p^*}{2\mu} &= \frac{v+2\mu}{v} \\ 
\theta^* &= (1, 0) \text{ or } (0, 1) \\
p^* &\geq \mu \tag{\text{If } v \geq \mu}
\end{align*}
\]

Moreover, under this scheme, both the collectors and the recycler make a positive profit.

The proof is in appendix. Albeit formally similar to the program of the recycler, this program is solved very differently. Indeed, we show in the appendix that it can be solved in terms of desired levels of effort, as if they were directly controllable. Then it is sufficient to find the right configuration \((p^*, \theta^*)\) that implements those efforts. This is so because the price is neutral in terms of surplus, and only serves as a rush disciplining device. We reach here also the conclusion that the use of licences is a sufficient tool to achieve efficiency, given the irradicable externalities.

What may seem counter-intuitive, is that the recycler is sometimes willing to make his suppliers harder to cheat than socially optimal, namely when \( \mu < v < 2\mu \). However, it is well understood if one accounts for the fact that this also induces more competition between collectors, which the recycler values.

A more detailed comparison of the socially optimal situation with that implemented by the recycler is in order. We state it as the following:

**Corollary 3** The recycler always sets a socially sub-optimal price: \( p^{**} < p^* \) (except when \( v = \mu \)).

Still, the recycler induces too much competition: \( \theta^{**} \geq \theta^* \) (component-wise).

Thus we do not obtain an efficiency result in the case of monopsonistic control, contrary to Schworm (1983) (see also Clark and Munro, 1980). This comes from the difference in cost structure. Schworm, following Dasgupta and Heal (1979, chap. 3), uses linear extraction costs, while we use convex collection costs. The dimension of economies of scale (or cost splitting between collectors) alters the conclusion.
Put together, the comparisons in corollary 3 indicate that even if a social planner would be able to fix a price, it would not be sufficient to restore the first-best levels of effort, if the recycler still keeps the control on expertise transmission. The control of both tools of price and expertise transmission (or licences) is required if one seeks to impose the social optimum.

The full comparison is done in the next table:

<table>
<thead>
<tr>
<th>$v$</th>
<th>$H + V$</th>
<th>$V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\theta^* = (1, 1)$</td>
<td>$\theta^{**} = (1, 1)$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$p^*(v)$</td>
<td>$p^{**}(v) &lt; p^*(v)$</td>
</tr>
<tr>
<td>$2\mu$</td>
<td>$\theta^* = (1, 0)$</td>
<td>$\theta^{**} = (1, 0)$</td>
</tr>
<tr>
<td></td>
<td>$p^* \geq \mu$</td>
<td>$p^{**} = \mu$</td>
</tr>
</tbody>
</table>

In terms of high quality collected, the recycler does not perform as well as would like a social planner. The recycler uses his monopsony power to extract collection rents by setting too low prices. (A comparison of all setting is pictured in figure 3 at the end of the section).

**Other relevant externalities**

The first element that we do not incorporate yet in the model is the environmental value of recycling. Indeed we only focused on market value of scrap ($v$), which is the main concern of the recycler as a standard economic agent. However, at least three components of the social value deserve attention.

First, the lower price of recycled scrap compared to that of pig iron produced from ores. At the steel industry level, using recycled material corresponds to a (unitary) efficiency gain of $(v_0 - v)$ where $v_0$ denotes the cost of primary iron.

Second, given that the natural deposits are exhaustible, a (unitary) opportunity cost of $s$ is recovered by recycling.

Third, when disposed, ferrous scrap imposes direct environmental damages, at a unitary rate $d$.

Let us denote by $q$ the total quantity collected, out of 1, given the total amount generated in the area. Overall, the total loss caused by the ferrous scrap not collected is:

$$E(1 - q) = d(1 - q) + (v_0 - v)(1 - q) + s(1 - q)$$

A global concern of a social planner would be to maximize the following:

$$W = H + V - E$$
This modification of the planner’s objective shifts the social value of high quality from \( v \) to \( v + \delta \). The optimization problem remains the same with this translated variable, and we can thus adapt the result of proposition 3 to:

**Proposition 4** When maximizing \( W \), the social planner would like to set:

\[
\begin{align*}
\theta^* &= (1, 1) \\
p^* &= \frac{2\mu(v + \delta)}{v + \delta + 2\mu} \quad \text{If } v < \mu - \delta \\
\theta^* &= (1, 0) \text{ or } (0, 1) \\
p^* &\geq \mu \quad \text{If } v \geq \mu - \delta
\end{align*}
\]

However, this scheme does not allow the recycler to break even when \( v < \frac{2}{7} (\sqrt{\delta^2 + 8\mu\delta} - \delta) \).

Accounting for the environmental value (in all its forms) of ferrous scrap, the switch between the two expertise configurations \((1, 1)\) and \((1, 0)\) occurs for lower economic value \( v \).

When \( v < \mu - \delta \), the optimal price desired by the planner may not allow the recycler to break even. The budget constraints of the recycler writes:

\[
\frac{2\mu(v + \delta)}{v + \delta + 2\mu} \leq v
\]

It is only satisfied for \( v \) sufficiently high, which simply emphasizes that when the matter has low economic value relative to the environmental value, it is necessary to use external funding to make the branch function optimally. Hence the second part of proposition 4.

A first solution is to balance the budget of the branch by respecting the preceding constraint, namely make it bind when it would be violated. This is the solution graphed in grey in figure 3. A second one is to subsidize the branch to restore collection incentives, as represented by the dotted grey line.

## 6 Discussion

That the ferrous scrap market is spoilt with quality uncertainty is well-known. These informational conditions speak in favour of a market failure. In fact, the asymmetry is the reverse of that in Akerlof (1970), and this leads to a holdup problem on the collection effort. However this (endogenous) informational problem is well understood in considering the collection externalities: information may have a negative social value, an assertion that made is way since Hirshleifer (1971).
In our multiagent setting, the competition for resource exploitation exhibits a trade-off between economies of scale and crowding externalities. High value of scrap can provoke a "gold rush", that the recycler controls by specializing the agents, mimicking a licence to operate. This captures one observed feature: the presence of non expert collectors, called "gleaners", locked in a situation where they collect low quality sources and exit the market when their collection is not profitable.

A solution relying on exclusion would also be implemented by a benevolent planner. In both cases, the surprising result is that the bigger the pie, the less people should have access to it. The strength of the equity vs efficiency dilemma contrasts with an angelic conception of traditional economic solutions for sustainable development. In common pool resource problems, this has been remarked since a long time that inequality can be efficient\footnote{This line of research is still extremely active (e.g. Aggarwal and Narayan, 2004)) but results can go in both direction as illustrated in Baland and Platteau(1997), and Bardhan and Ghatak (1999) among others.} (Olson, 1965).

\textbf{Are price and expertise really substitutable tools?}

In our static model, the recycler as well as a social planner use indifferently expertise and price to control the level of incentives for collection. In some cases, many solutions are then equivalent and we used assumption 1 to select a unique one, the less risky one, which would endogenously be selected if some risk-aversion of any player was present. But in a dynamic setting, this assumption may not be appropriate because the substitution between \( p \) and \( \theta \) would no more be perfect. While the price can change in both directions to adapt to \( v \), the
expertise is an irreversible parameter because it can only be increasing. This would probably bias the results towards even less knowledge transfer. A dynamic model incorporating endogenous learning is part of our ongoing research.


definition of the collection areas, thus it would not eliminate the inefficiency. At the other extreme, a joint incentive scheme, depending only on the sum of collected quantities, would induce cooperation. But it would also leave some room for free-riding strategies, creating another kind of inefficiency. These aspects require more insights from team theory, but it seems that negative externalities are not a good driving force for integration, contrary to positive ones.

There remains collective agreement on territorial delimitations as potential solution. We argue that it is not feasible by the recycler, because he is not credible when committing to buy a limited amount from a supplier and/or he is unable to verify the geographical origin of a batch. The limits of territory thus cannot be enforced ex-post, and monitoring of a collector is obviously too costly. Hence the recycler is unable to control this aspect.

As is expected in the case a competitive fringe faces a monopsony, syndication of the collectors should enhance their position. Indeed, fieldwork stress the presence of an informal institution: a network of (expert) scrap merchants, reminiscent of historical examples (Greif, 1993; Greif, Milgrom and Weingast, 1994; Dessi and Ogilvie 2004). This group of tied merchants can easily monitor the access to stable scrap sources. Hence, territory allocation and enforcement are possible among them. Fields studies and ongoing research give a promising explanation of the network of scrap merchants: with respect to expertise transfer - or licence granting, they complement the role of the principal in the present model. Both parties have a common interest in regulating entry on the market for ferrous scrap collection to prevent gold rushes.
7 Appendix

Proof of Proposition 1:

Proof. We separate the analysis into three parts.

- First, we consider the case where at least one equilibrium effort, say \( e_i^N \), is 1. In this case, all is collected, so any effort of the other collector would be purely wasted. Thus the optimal configuration in that case is clearly \( \theta = (1, 0) \).

- Second, we consider the case where both effort are interior: \( e_i^N < 1 \) for \( i = 1, 2 \).
From the first-order conditions, we have \( p\theta_i q_i^N = \mu(e_i^N)^2 \). Thus the equilibrium Hicks-quantity is:

\[
H^N(p, \theta) = p\theta_1 q_1^N + p\theta_2 q_2^N - \frac{\mu}{2}((e_1^N)^2 + (e_2^N)^2) = \frac{4\mu p^2}{(4\mu^2 - \theta_1 \theta_2 p^2)^2} ((\theta_1 \theta_2 p - \mu (\theta_1 + \theta_2))^2 + \mu^2(\theta_1 - \theta_2)^2)
\]

We can compute the partial derivative of \( H^N(p, \theta) \) with respect to \( \theta_1 \):

\[
\frac{\partial H^N}{\partial \theta_1} = \frac{8\mu^2 p^2 (2\mu - \theta_2 p)}{(4\mu^2 - \theta_1 \theta_2 p^2)^3} ((2\mu - \theta_2 p)^2 + 2\mu \theta_2 p) \theta_1 - 2\mu \theta_2 p)
\]

Because the efforts are assumed interior, we have necessarily \( p\theta_i < 2\mu \), for \( i = 1, 2 \). The first factor of \( \frac{\partial H^N}{\partial \theta_1} \) is then positive. Overall \( \frac{\partial H^N}{\partial \theta_1} \) is positive if and only if \( \theta_1 \geq \tilde{\theta} \equiv \frac{2\mu \theta_2 p}{(2\mu - \theta_2 p)^2 + 2\mu \theta_2 p} \).
Given that \( \theta_2 \leq 1, 2\mu \theta_2 p \leq 2\mu \theta_2 p \), so we have \( 0 \leq \tilde{\theta} \leq 1 \) for all \( p \) and \( \theta_2 \). We conclude that \( H^N \) is first decreasing, then increasing when \( \theta_1 \) moves from 0 to 1. This implies that whatever the values of the other variables, \( H^* \) will be maximized either when \( \theta_1 = 0 \), or when \( \theta_1 = 1 \). The same reasoning also applies to \( \theta_2 \) by symmetry.

- Overall there remains to compare \( H^N(p, 0, 0) \), \( H^N(p, 1, 0) \) and \( H^N(p, 1, 1) \) (of course \( H^N(p, 1, 0) = H^N(p, 0, 1) \) by symmetry).

One sees immediately that \( H^N(p, 0, 0) = 0 \), and is thus uninteresting.

We have \( H^N(p, 1, 1) = \frac{4\mu p^2}{(4\mu^2 - p^4)^3} (p^2 - 4\mu p + 4\mu^2) = \frac{4\mu^2 p^2}{(4\mu^2 - p^4)^2} \) and \( H^N(p, 1, 0) = \frac{4\mu^2 2\mu p^2}{(4\mu^2 - p^4)^3} = \frac{p^2}{2\mu} \).

The comparison ends the proof. \( \blacksquare \)
Proof of Proposition 2:

**Proof.** We first use the variable change \( r_i = p \theta_i \). Considering the equilibrium of the collection game (3), we have

\[
e^*_i(p, \theta_1, \theta_2) = \frac{2 \theta_i p (2 \mu - \theta_{-i} p)}{4 \mu^2 - \theta_1 \theta_2 p^2}
= \frac{2 r_i (2 \mu - r_{-i})}{4 \mu^2 - r_1 r_2} = \tilde{e}_i(r_1, r_2)
\]

Replacing in the recycler’s profit:

\[
V^N(p, \theta_1, \theta_2) = (v - p \theta_1) e_1^N (1 - \frac{e_2^N}{2}) + (v - p \theta_2) e_2^N (1 - \frac{e_1^N}{2})
= (v - r_1) \tilde{e}_1 (1 - \frac{\tilde{e}_2}{2}) + (v - r_2) \tilde{e}_2 (1 - \frac{\tilde{e}_1}{2})
\equiv \tilde{V}(r_1, r_2)
\]

Thus the optimization problem can equivalently be solved with only two variables, \( r_1 \) and \( r_2 \), under the constraints \( r_i \geq 0 \). We split the problem into two parts, as in the proof of Proposition 1.

- Suppose first that (at least) one effort is maximal, assume wlog \( \tilde{e}_1 = 1 \). From the best reply of collector 1, this happens when:

\[
r_1 (1 - \frac{\tilde{e}_2}{2}) \geq \mu
\]

In this case, all is collected anyway, what matters is the collection cost for the recycler:

\[
\tilde{C}(r_1, r_2) = r_1 (1 - \tilde{e}_2) + \frac{1}{2} (r_1 + r_2) \tilde{e}_2
\]

The first term corresponds to the area covered by a single collector, the second corresponds to the area where both are collecting. In the former case, the average price for the recycler is \( r_1 \), while it is \( \frac{1}{2} (r_1 + r_2) \) in the latter.

If \( \tilde{e}_2 = 1 \), then necessarily \( r_1, r_2 \geq 2 \mu \), and the total cost is then at least \( 2 \mu \). But it is possible to collect everything for a cost \( \mu \), when \( \tilde{e}_2 = 0 \). Thus it can not be optimal that \( \tilde{e}_2 = 1 \). We can restrict ourselves to:

\[
\begin{align*}
\tilde{e}_1 &= 1 \\
\tilde{e}_2 &= \frac{r_2}{2 \mu} < 1
\end{align*}
\]

The recycler then minimizes:

\[
\tilde{C}(r_1, r_2) = \frac{1}{4 \mu} (r_1 (4 \mu - r_2) + r_2^2)
\]

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under the constraint \( r_1(1 - \frac{\bar{c}}{\bar{e}_2}) \geq \mu \), or \( r_1 \geq \frac{4\mu^2}{4\mu - r_2} \). It is clear that this constraint will bind because the coefficient of \( r_1 \) is positive.

Replacing yields:

\[
\tilde{C}(r_1, r_2) = \mu + \frac{r_2}{4\mu}
\]

which is trivially minimized when \( r_2 = 0 \).

- We consider now the case where both efforts are strictly smaller than 1. Then the efforts are given by (3), and we have:

\[
\tilde{V}(r_1, r_2) = \frac{4\mu [(v - r_1)r_1(2\mu - r_2)^2 + (v - r_2)r_2(2\mu - r_1)^2]}{4\mu^2 - r_1 r_2^2}
\]

The system of first-order conditions

\[
\begin{align*}
\frac{\partial \tilde{V}}{\partial r_1}(r_1, r_2) &= 0 \\
\frac{\partial \tilde{V}}{\partial r_2}(r_1, r_2) &= 0
\end{align*}
\]

has solutions:

\[
s = \begin{cases} 
 r_i = 2\mu \\
 r_{-i} = \frac{2\mu}{v} \end{cases} \quad \text{or} \quad t = \begin{cases} 
 r_1 = \frac{2\mu}{v+4\mu} \\
 r_2 = \frac{2\mu}{v+4\mu}
\end{cases}
\]

In the candidate solutions \( s \), one effort is equal to 1, because when \( r_i = 2\mu \), the best-reply of collector \( i \) is \( e_i = 1 \) whatever the other’s strategy, contradicting the assumption of strictly interior efforts. We can rule out the points \( s \).

In the candidate solution \( t \), a sufficient condition for the efforts to be strictly smaller than 1 is \( r_i < \mu \), which is equivalent to \( v < 4\mu \).

The second partial derivative of \( \tilde{V} \) at point \( t \) is:

\[
\frac{\partial^2 \tilde{V}}{\partial r_i^2}(t) = \frac{1}{128} \frac{(v^2 - 8\mu^2)(v + 4\mu)^4}{\mu^4(v + 2\mu)^3}
\]

which is negative when \( v < 2\sqrt{2}\mu \).

The determinant of the Hessian matrix at point \( t \) is:

\[
Det(H_{\tilde{V}})(t) = \frac{1}{1024} \frac{(2\mu - v)(v + 4\mu)^8}{\mu^6(v + 2\mu)^5}
\]

So when \( v < 2\mu \), both first and second order conditions for a local maximum are met. Given \( t \) is the only local maximum, it is the global one.

In turn, if \( v \geq 2\mu \), the maximum is necessarily on the frontier, meaning at a point with \( r_i = 0 \) for some \( i \). The recycler then maximizes:

\[
\tilde{V}(r, 0) = \tilde{V}(0, r) = \frac{(v - r)r}{\mu}
\]
since \( v \geq 2\mu \), the best strategy of the recycler is to choose \( r = \mu \). Then the equilibrium efforts of the collection game are \((1, 0)\) or \((0, 1)\).

- Reverting the variable change and selecting the less risky schemes, we have overall:
  If \( v < 2\mu \), the recycler sets \( p^{**} = \frac{2\mu v}{v+4\mu} \) and \( \theta^{**} = (1, 1) \).
  If \( v \geq 2\mu \), the recycler sets \( p^{**} = \mu \) and \( \theta^{**} = (1, 0) \) or \((0, 1)\). ■

**Proof of Proposition 3:**

**Proof.** Consider the social value of collection as a function of \( e \):

\[
\widehat{S}(e) = v(e_1 + e_2 - e_1 e_2) - \frac{1}{2}\mu(e_1^2 + e_2^2)
\]

Would a social planner be able to impose efforts, he would maximize \( \widehat{S} \) with respect to \( e \). The Hessian for \( \widehat{S} \) is:

\[
H_{\widehat{S}} = \begin{bmatrix}
-\mu & -v \\
-v & -\mu
\end{bmatrix}
\]

Thus \( \widehat{S} \) is strictly concave if and only if \( \text{Det}(H_{\widehat{S}}) = \mu^2 - v^2 > 0 \), i.e. for \( v < \mu \).

- The case \( v < \mu \)

The first-order system is:

\[
\begin{cases}
\mu e_1 = v(1 - e_2) \\
\mu e_2 = v(1 - e_1)
\end{cases}
\]

Which unique solution is:

\[
\begin{cases}
\widehat{e}_1 = \frac{v}{v+\mu} \\
\widehat{e}_2 = \frac{v}{v+\mu}
\end{cases}
\]

The question is whether it is possible to induce these levels of effort, given that they must form a Nash equilibrium of the collection game. The answer is yes. Simply consider the (unique) Nash equilibrium of \( G(p, \theta) \), when the efforts are interior. First note that because \( \widehat{e}_1 = \widehat{e}_2 \) and the price should be the same for both collectors, one must have \( \theta_1 = \theta_2 \). Let us define \( r = \theta_1p = \theta_2p \). It must be the case that \( r < 2\mu \), since the efforts have to be interior. Using equilibrium efforts given by (3), there remains to solve:

\[
\frac{v}{v+\mu} = \frac{2r(2\mu - r)}{4\mu^2 - r^2} \text{ with } r < 2\mu
\]

\[\iff r = \frac{2\mu v}{v+2\mu}\]

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which is less than $\mu$ when $v < \mu$. In other words, any configuration with $p\theta_1 = p\theta_2 = \frac{2v\mu}{v+2\mu}$
implements the target interior efforts. Among those, the one exhibiting the smallest risk,
both for the collectors and for the recycler, is that with $\theta^* = (1, 1)$ and $p^* = \frac{2v\mu}{v+2\mu}$. Note that
$p^* < v$, so the recycler breaks even.

- The case $v \geq \mu$

We have seen that in this case no interior level of effort can be optimal.

Assume that one effort is equal to 1. Then it would be obviously wasteful that the other
collector exerts any effort, because all is already collected. Thus an optimal configuration will
be such that one collector has $\theta_i = 0$, and the other collects everything, requiring $p\theta_{-i} \geq \mu$.
Among those, the less risky one is that with $\theta_{-i} = 1$ and $p \geq \mu$.

Assume now that one effort is 0, say $e_2 = 0$. Then $\hat{S}(e_1,0) = ve_1 - \frac{1}{2}\mu e_1^2$. Given that
$v > \mu$, it is obviously optimal that $e_1 = 1$, and we the solution is the same as above.

Overall, any corner solution has the form $p^* \geq \mu$ and $\theta^* = (1,0)$ or $(0,1)$. (Note that,
reasonably, one could impose in addition $p^* \leq v$, so that a budget constraint holds for the
recycler.)

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References


