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Providing Incentives For Informative Voting

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Résumé: Suivant le théorème du jury de Condorcet, le vote informatif sous la règle majoritaire agrège efficacement l'information: si la taille du jury tend vers l'infini la probabilité d'une décision erronée tend vers zéro. Cependant, comme il est maintenant bien connu, des électeurs rationnels et possédant une information privée vont conditionner leurs votes au fait d'être décisif, ce qui peut détruire leur incitation à voter de manière conformément à leur information privée. Ici nous retrouvons le résultat asymptotique de Condorcet en modifiant la règle d'agrégation de telle manière que (a) les votants sont incités à voter de manière informative et (b) la décision collective est asymptotiquement efficace. Le mécanisme est une randomisation ex post entre la règle majoritaire appliquée sur tout les votes et la règle majoritaire appliquée à un échantillon aléatoire des votes.

Abstract: According to Condorcet's jury theorem, informative voting under majority rule leads to asymptotically efficient information aggregation: As the jury size tends to infinity, the probability of a wrong decision goes to zero. However, as is well-known by now, rational and privately informed voters will condition their votes on being pivotal, and this may destroy their incentive to vote informatively (according to their private information). We here restore Condorcet's asymptotic efficiency result by way of modifying the aggregation rule in such a way that (a) voters have an incentive to vote informatively and (b) the collective decision is asymptotically efficient. The mechanism is an ex-post randomization between majority rule applied to all votes and majority rule applied to a randomly sampled subset of votes.

Mots clés : vote stratégique, Condorcet, jury, agrégation de l'information

Key Words : strategic voting, Condorcet, jury, information aggregation.

Classification JEL: D71, D72

PROVIDING INCENTIVES FOR INFORMATIVE VOTING

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Abstract. According to Condorcet's jury theorem, informative voting under majority rule leads to asymptotically efficient information aggregation: As the jury size tends to infinity, the probability of a wrong decision goes to zero. However, as is well-known by now, rational and privately informed voters will condition their votes on being pivotal, and this may destroy their incentive to vote informatively (according to their private information). We here restore Condorcet's asymptotic efficiency result by way of modifying the aggregation rule in such a way that (a) voters have an incentive to vote informatively and (b) the collective decision is asymptotically efficient. The mechanism is an *ex-post* randomization between majority rule applied to all votes and majority rule applied to a randomly sampled subset of votes.

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1. Introduction

The "Condorcet Jury" is a situation where a binary decision has to be taken collectively and individuals have the same goal: to "find the truth," although they each have some private information about this truth. The classical XVIII-th century theorem states that, under conditional independence of individuals' information, aggregation by way of majority rule is asymptotically efficient in the sense that the probability for a mistaken collective decision goes to zero as the number of voters tends to infinity (Condorcet, 1785). The modern strategic analysis of jury voting, initiated by Austen-Smith and Banks (1996), has pointed out a major weakness of the classical result. An unstated hypothesis underlying Condorcet's analysis is that jury members vote informatively, that is, base their votes on their own private information without regard to other jury members' potential information and votes. This hypothesis may seems innocuous, since all jury members share the same goal. However, a careful analysis shows that this is not always the case.

More exactly, Austen-Smith and Banks noticed that, if the number of jury members is large, informative voting is in general not a Nash equilibrium of the Bayesian game that corresponds to Condorcet's setting. This negative result emanates from the following observation: An individual vote makes a difference under majority rule only if the other votes are split even. Hence, as a voter I should reason as if I knew that the others are in a tie. But if these other voters are many, the information that all they are tied is more informative than my own signal. It is thus rational for me to neglect my own signal. Suppose, for instance, that it is easier to recognize a guilty defendant than an innocent defendant — the former may have evidence speaking against him while there may be little evidence for the innocence of an innocent defendant. Then I should vote "innocent" even if my impression is that the defendant is guilty, because the information that half the other jury members have the opposite impression is hardly compatible with the defendant's guilt, but is quite compatible with his innocence. If the jury is large enough, this argument in favor of "not guilty" becomes overwhelming. Consequently, informative voting is not a Nash equilibrium, and Condorcet's information-aggregation argument fails.

We here propose a remedy to this form of "the swing voter's curse".[1] The idea is simple: let everybody cast their votes simultaneously, then toss a calibrated coin and let the collective decision be determined by majority rule among all votes if "head" shows up, otherwise let the decision be determined by majority rule applied to a randomly sampled subset of votes (for instance 1 randomly drawn vote, a "random dictator"). We show that careful calibration of the coin makes informative voting strategically rational and yet information aggregation will be asymptotically efficient.

More precisely, suppose that each voter receives a binary signal, for example ("for" or "against" the defendant being guilty). Suppose that these signals are identically distributed and conditionally independent, given the actual "state of the world" (such as the defendant's guilt or innocence). Suppose also that each voter's signal is informative in the sense that the conditionally expected utility, given the voter's signal, from voting according to the signal is higher than that from abstaining or voting against the signal. The collective decision can lead to two types of error, and these may be equally or unequally costly. For instance, if acquitting a guilty defendant gives the voter disutility α and condemning an innocent defendant gives the voter disutility β (arguably a larger number), then the conditionally expected utility in question is calculated by way of multiplication of each of these two disutilities with the respective conditional probability.

Under these assumptions, it would be optimal for a jury member to vote informatively if that member were asked to make the collective decision single-handedly.

[1] Another "swing voter's curse" concerns the incentive to abstain from voting, see Feddersen and Pesendorfer (1996).

Hence, if instead of majority rule, all voters were asked to cast their votes simultaneously, and the collective decision would be determined by a randomly drawn single vote (with positive probability for each vote), then each voter would vote informatively. Indeed, the incentive for informative voting may remain even if instead the collective decision were determined by majority rule applied to a randomly drawn sample of size k, granted k is not too large. However, a drawback with such schemes is that the information available to the $n-k$ non-selected voters is lost. Consequently, such a mechanism does not, in general, result in efficient information aggregation. However, asymptotic efficiency can be obtained as follows. Let D be a mechanism as described above, that is, majority rule in a randomly selected subset of votes. Let M be majority rule among all n voters in the electorate (assume n odd). Let $p(n)$ be the probability of a tie among $n-1$ voters. Then $p(n)$ tends to zero as n goes to infinity. For each n, let $r(n)$ be some number that also tends to zero but that exceeds $p(n)$. Now consider the mechanism $R = r(n)D + [1-r(n)]M$, the mechanism where all individuals cast their votes simultaneously, thereafter mechanism D is applied with probability $r(n)$ and otherwise mechanism M is applied. It is easily verified that informative voting is a Nash equilibrium in R and that R is asymptotically efficient. This is what we propose in this paper.

The most relevant related literature is that on voting under private information. The two seminal papers are Austen-Smith and Banks (1996), mentioned above, and Feddersen and Pesendorfer (1996).[2] In the latter paper, the so-called swing voter's curse refers to the following phenomenon. If jury members, among whom there are partisans and non-partisans, are allowed to abstain from voting, poorly informed non-partisans may use the following mixed strategy. They probabilistically balance their votes in such a way that they collectively compensate for the presence of partisan voters (who support a given candidate in any case) and leave room for the better informed non-partisan voters. This mixed strategy of poorly informed non-partisan voters involves abstention with positive probability. Subsequent theoretical research on jury behavior mainly concern the relative merits of different voting rules (Feddersen and Pesendorfer, 1998) and on the role of debates before voting (see, for instance, Austen-Smith and Feddersen, 2005).

2. The basic model

Suppose that there is an odd number $n = 2m+1$ of voters, for some non-negative integer m. Each voter i receives a private signal $s_i \in \{0,1\}$ which is correlated with

[2] See also Bade (2006).

the true state of nature $\omega \in \{0,1\}$ according to:

$$\begin{aligned} \Pr[s_i = 0 \mid \omega = 0] &= q_0 \geq 1/2 \\ \Pr[s_i = 1 \mid \omega = 1] &= q_1 \geq 1/2 \end{aligned}$$

Write q for the vector (q_0, q_1).[3] Suppose that that the signals received by different voters are conditionally independent, given the state of nature. We denote by μ_0 the prior probability of state $\omega = 0$, with $\mu_1 = 1 - \mu_0$ and $0 < \mu_0 < 1$. Write μ for the vector (μ_0, μ_1).

The voters are going to take a collective decision $x \in \{0,1\}$. Suppose that each individual is only interested in the decision being correct and that both mistakes are equally costly: the utility to individual i is $u_i = 1$ if $x = \omega$ and $u_i = 0$ if $x \neq \omega$.[4]

2.1. Informativeness of the signal. Suppose first that a single individual is to make the decision — the case of "dictatorship" ($n = 1$ and hence $m = 0$). Under what conditions should he or she decide according his or her signal? Clearly, if the signal is very noisy and the prior is biased, the right decision is to sometimes not follow the signal.

More precisely, by Bayes' rule:

$$\begin{aligned} \Pr[\omega = 0 \mid s_i = 0] &= \frac{\mu_0 \Pr[s_i = 0 \mid \omega = 0]}{\Pr[s_i = 0]} \\ &= \frac{\mu_0 q_0}{\mu_0 q_0 + \mu_1 (1 - q_1)}. \end{aligned}$$

Hence,

$$\begin{aligned} \Pr[\omega = 0 \mid s_i = 0] - \Pr[\omega &= 1 \mid s_i = 0] \\ &= 2\Pr[\omega = 0 \mid s_i = 0] - 1 \\ &= \frac{\mu_0 q_0 - \mu_1 (1 - q_1)}{\mu_0 q_0 + \mu_1 (1 - q_1)}, \end{aligned}$$

and likewise for the case when the signal $s_i = 1$. Consequently, the decision rule $x \equiv s_i$ is optimal if and only if

$$\mu_0 q_0 \geq \mu_1 (1 - q_1) \text{ and } \mu_1 q_1 \geq \mu_0 (1 - q_0),$$

[3] Hence, all voters have the same "competence" in the sense that the conditional probability of receiving a "correct" signal is the same for everyone.

[4] Hence, in the notation of the introduction, we here focus on the special case when $\alpha = \beta$.

or, equivalently, if and only if

$$\frac{1-q_1}{q_0} \leq \frac{\mu_0}{\mu_1} \leq \frac{q_1}{1-q_0}. \tag{1}$$

We will henceforth assume condition (1) to be met and will refer to this as the *informativeness* condition.

We note that the left-most quantity in (1) is a number between zero and one and that the right-most quantity is a number between one and plus infinity. In particular, both inequalities are met when the prior is uniform, that is, when $\mu_0 = \mu_1 = 1/2$. Hence, in this case, it is optimal to vote according to the signal. By contrast, if the prior is biased in favor of one of the states, then it may be optimal to neglect one of the signals. For example, if $\mu_0 < 1/2$, then it is optimal to neglect the signal $s_i = 0$ if both signals are relatively uninformative (that is, if q_0 and/or q_1 are close to $1/2$).

We will say that the signal 0 is *more informative* than signal 1 if

$$\Pr\left[\omega = 0 \mid s_i = 0\right] > \Pr\left[\omega = 1 \mid s_i = 1\right]$$

or, equivalently, if

$$\frac{\mu_0 q_0}{\mu_0 q_0 + \mu_1 (1-q_1)} > \frac{\mu_1 q_1}{\mu_1 q_1 + \mu_0 (1-q_0)}$$
$$\mu_0^2 q_0 (1-q_0) > \mu_1^2 q_1 (1-q_1)$$

In particular, under the uniform prior, $\mu_0 = \mu_1$, signal 0 is more informative than signal 1 if and only if $q_0 < q_1$.

3. Majority rule

Assume informative signals, that is, that condition (1) is met, and suppose that the collective decision is taken according to the majority rule among the $n = 2m+1$ voters, for an arbitrary non-negative integer m.

Given her signal, each voter i then casts a vote $v_i \in \{0,1\}$, and $x = 0$ results if a majority of voters cast the vote 0, while $x = 1$ results in the opposite case. Notice that ties are ruled out if no voter abstains, since there is an odd number of voters.[5] We denote by $M = \Gamma(\mu, q, n)$ the n-player game so defined.

By *informative voting* for voter i we mean to vote $v_i = s_i$ for each $s_i \in \{0,1\}$. Suppose that all voters but i vote informatively. Denote by $\mathcal{T}$ the event of a tie among the others, or, more precisely, that exactly m of the other voters receive the signal 0

[5] We close the model by assuming that the collective decision would be determined by the toss of a fair coin if there would be a tie due to abstention.

and equally many receive the signal 1. Suppose that i received the signal $s_i = 0$. The probability for the joint event that $s_i = 0$ and that there is a tie among the others, conditional on the state $\omega = 0$, is

$$\Pr\left[\mathcal{T} \ \wedge \ s_i = 0 \mid \omega = 0\right] = \binom{2m}{m} q_0^{m+1} \left(1 - q_0\right)^m .$$

Likewise, conditional on the state $\omega = 1$, we have

$$\Pr\left[\mathcal{T} \ \wedge \ s_i = 0 \mid \omega = 1\right] = \binom{2m}{m} q_1^{m} \left(1 - q_1\right)^{m+1} .$$

Therefore, the probability for the joint event that i receives the signal 0 and that there is a tie among the others is

$$\Pr\left[\mathcal{T} \ \wedge \ s_i = 0\right] = \binom{2m}{m} \left[\mu_0 q_0^{m+1} \left(1 - q_0\right)^m + \mu_1 q_1^m \left(1 - q_1\right)^{m+1}\right] .$$

Since the probability of receiving the signal 0 is

$$\Pr\left[s_i = 0\right] = \mu_0 q_0 + \mu_1 \left(1 - q_1\right) ,$$

the probability that i's vote is pivotal, conditional upon $s_i = 0$, is

$$p_0(m) = \Pr\left[\mathcal{T} \mid s_i = 0\right] = \binom{2m}{m} \frac{\mu_0 q_0^{m+1} \left(1 - q_0\right)^m + \mu_1 q_1^m \left(1 - q_1\right)^{m+1}}{\mu_0 q_0 + \mu_1 \left(1 - q_1\right)} .$$

We are now in position to compute the difference in expected utility for voter i between casting the informative vote $v_i = 0$ instead of the vote $v_i = 1$, conditional upon the signal $s_i = 0$:

$$\Delta u_0 = \mathbb{E}\left[u_i \mid s_i = v_i = 0\right] - \mathbb{E}\left[u_i \mid s_i = 0 \ \wedge \ v_i = 1\right] .$$

Because i's vote affects the collective decision x only in the event $\mathcal{T}$, we have

$$\Delta u_0 = p_0(m) \cdot \left(\mathbb{E}\left[u_i \mid \mathcal{T} \ \wedge \ s_i = v_i = 0\right] - \mathbb{E}\left[u_i \mid \mathcal{T} \ \wedge \ s_i = 0 \ \wedge \ v_i = 1\right]\right) .$$

Conditional upon the event $\mathcal{T}$ and the signal $s_i = 0$, the expected utility to individual i associated with each vote $v_i \in \{0, 1\}$ equals the probability for the correct decision:

$$\begin{aligned} \mathbb{E}\left[u_i \mid \mathcal{T} \ \wedge \ s_i = v_i = 0\right] &= \Pr\left[\omega = 0 \mid \mathcal{T} \ \wedge \ s_i = 0\right] \\ \mathbb{E}\left[u_i \mid \mathcal{T} \ \wedge \ s_i = 0 \ \wedge \ v_i = 1\right] &= \Pr\left[\omega = 1 \mid \mathcal{T} \ \wedge \ s_i = 0\right] \end{aligned}$$

These probabilities are easily found by Bayes' rule (factorials cancel):

$$\begin{aligned}\Pr\left[\omega=0 \mid \mathcal{T} \ \wedge \ s_i=0\right] &= \frac{\mu_0 \Pr\left[\mathcal{T} \ \wedge \ s_i=0 \mid \omega=0\right]}{\Pr\left[\mathcal{T} \ \wedge \ s_i=0\right]} \\ &= \frac{\mu_0 q_0^{m+1}\left(1-q_0\right)^m}{\mu_0 q_0^{m+1}\left(1-q_0\right)^m+\mu_1 q_1^m\left(1-q_1\right)^{m+1}}.\end{aligned}$$

Computing the difference

$$\begin{aligned}\Pr\left[\omega=0 \mid \mathcal{T} \ \wedge \ s_i=0\right]-\Pr[\omega &= 1 \mid \mathcal{T} \ \wedge \ s_i=0] \\ &= 2\Pr\left[\omega=0 \mid \mathcal{T} \ \wedge \ s_i=0\right]-1 \\ &= \frac{\mu_0 q_0^{m+1}\left(1-q_0\right)^m-\mu_1 q_1^m\left(1-q_1\right)^{m+1}}{\mu_0 q_0^{m+1}\left(1-q_0\right)^m+\mu_1 q_1^m\left(1-q_1\right)^{m+1}}\end{aligned}$$

one obtains

$$\Delta u_0=\binom{2m}{m}\frac{\mu_0 q_0^{m+1}\left(1-q_0\right)^m-\mu_1 q_1^m\left(1-q_1\right)^{m+1}}{\mu_0 q_0+\mu_1\left(1-q_1\right)}. \tag{2}$$

The condition for Δu_0 to be positive, that is, for i to want to truthfully report the signal $s_i=0$, is thus

$$\mu_0\, q_0^{m+1}\left(1-q_0\right)^m \geq \mu_1 q_1^m\left(1-q_1\right)^{m+1},$$

which can be written

$$\left[\frac{q_0\left(1-q_0\right)}{q_1\left(1-q_1\right)}\right]^m \geq \frac{\mu_1}{\mu_0}\frac{1-q_1}{q_0}. \tag{3}$$

Hence, informative voting on 0 (that is, to chose $v_i=0$ when $s_i=0$) is optimal if and only if condition (3) is met. In particular, if $q_1>q_0$ then $q_1\left(1-q_1\right)\leq q_0\left(1-q_0\right)$ and hence (3) is met for all m large enough, whereas (3) is violated for all m large enough in the opposite case, $q_0>q_1$.

Likewise, informative voting on 1 is optimal if and only if

$$\left(\frac{q_1\left(1-q_1\right)}{q_0\left(1-q_0\right)}\right)^m \geq \frac{\mu_0}{\mu_1}\frac{1-q_0}{q_1}. \tag{4}$$

By symmetry, one of conditions (3) and (4) must fail for m large enough, except in the knife-edge case when $q_0=q_1$. We thus arrive at the following result, due to Austen-Smith and Banks (1996):

Theorem 1 [Austen-Smith and Banks]**.** *If $q_0\neq q_1$, there exists $m_0\in\mathbb{N}$ such that, for all $m>m_0$, informative voting is not a Nash equilibrium of $\Gamma(\mu,q,n)$.*

4. Delegation to a random sample

If informative voting is not an equilibrium, then an alternative to majority rule is to first ask everybody to cast a vote and then randomly select a subset consisting of k votes, where $k < n$ is an odd number, and apply majority rule to the so selected subset. If each vote has a positive probability of being so selected, and k is sufficiently small, then informative voting is again a Nash equilibrium. More precisely, let $q_0, q_1 < 1$. Informative voting on alternative 0 is optimal if

$$\left[\frac{q_0(1-q_0)}{q_1(1-q_1)}\right]^k \geq \left(\frac{\mu_1}{\mu_0}\right)^2 \frac{1-q_0}{q_0} \cdot \frac{1-q_1}{q_1} \tag{5}$$

Likewise, informative voting on alternative 1 is optimal if

$$\left[\frac{q_1(1-q_1)}{q_0(1-q_0)}\right]^k \geq \left(\frac{\mu_0}{\mu_1}\right)^2 \frac{1-q_0}{q_0} \cdot \frac{1-q_1}{q_1} \tag{6}$$

When both inequalities are met, informative voting constitutes a Nash equilibrium. We note, in particular, that under the maintained informativess hypothesis (1), both inequalities are met for $k = 1$. Indeed, both inequalities are then met for all odd integers k that are sufficiently small. Let k^* be the maximal odd $k \in \mathbb{N}$ for which both (5) and (6) are met (and note that k^* is independent of n).

For any odd number $k \leq \min\{k^*, n\}$, let $D = \Gamma(\mu, q, n, k)$ be the game in which all n players vote simultaneously, k votes are drawn at random from among all n votes with equal probability for each vote, this random selection is statistically independent of the state of nature and of the signals and votes, and the collective decision x is determined by majority rule applied to the selected k votes. We have established

Proposition 1. *Informative voting is a Nash equilibrium of* $\Gamma(\mu, q, n, k)$ *for all odd numbers* n *and* $k \leq \min\{k^*, n\}$.

A drawback with this procedure is evidently that it is not asymptotically efficient. The maximal "committee size" k^* is independent of n. Hence, while the collective information tends asymptotically to the truth as the number n of voters tends to infinity, just as claimed by Condorcet, the committee's decision will remain bounded away from being fully informed.

5. Probabilistic delegation to a random sample

Instead of letting a randomly selected subset of size $k \leq \min\{k^*, n\}$ make the decision for sure, suppose that, after everybody in the electorate has cast his or her vote, a randomization device is employed to determine whether to let x be determined by the majority of a randomly selected subset of k votes, as described in section 4, or to let x be determined by the majority of all n votes, as described in section 3. It turns out that under such random delegation, Condorcet's claim can be restored: the collective decision will almost surely be fully informed in the limit as $n \to \infty$.

For the sake of definiteness and ease of notation, let $k = 1$. In other words, with probability $r > 0$ the decision is taken according to the vote of a randomly chosen individual voter, and with probability $1 - r$ according to majority rule among all n voters (and this draw is statistically independent of all other events). Denote by $R = \Gamma(\mu, q, n, k, r)$, for $k = 1$, this new game.

We first investigate the conditions for informative voting to be a Nash equilibrium in this game. Suppose that voter i has received the signal $s_i = 0$. We denote by Δu_0^R the difference in expected utility, for that voter, to cast the informative vote $v_i = 0$ rather than the vote $v_i = 1$, in game R. Voter i is thus the "dictator" with probability r/n, and majority rule is used with probability $1-r$. If another individual is selected as dictator, then i's vote does not matter. It follows that

$$\begin{aligned}
\Delta u_0^R &= \frac{r}{n}\left(\Pr\left[\omega = 0 \mid s_i = 0\right] - \Pr\left[\omega = 1 \mid s_i = 0\right]\right) + (1-r)\Delta u_0 \\
&= \frac{r}{n}\frac{\mu_0 q_0 - \mu_1\left(1 - q_1\right)}{\mu_0 q_0 + \mu_1\left(1 - q_1\right)} - \binom{2m}{m}\frac{\mu_0 q_0^{m+1}\left(1 - q_0\right)^m - \mu_1 q_1^m\left(1 - q_1\right)^{m+1}}{\mu_0 q_0 + \mu_1\left(1 - q_1\right)},
\end{aligned}$$

where $n = 2m + 1$ for some $m \in \mathbb{N}$. The first term is non-negative if and only if

$$\mu_0 q_0 \geq \mu_1\left(1 - q_1\right),$$

The corresponding condition for signal 1 is:

$$\mu_1 q_1 \geq \mu_0\left(1 - q_0\right).$$

Both conditions are satisfied if and only if the informativeness condition (1) is met. In this case, Δu_0^R is positive if r is large enough. Because the second term in Δu_0^R tends quickly to 0 when m tends to infinity, r can actually be small if m is large. This is the content of the following result, where we note that

$$a = 4\max\{q_0(1 - q_0), q_1(1 - q_1)\}$$

is smaller than 1 for all $q_0, q_1 > 1/2$.

Theorem 2 [Informative voting]**.** *Suppose that* $q_0, q_1 > 1/2$ *and that the informativeness condition (1) is met. Let* $b \in (a, 1]$. *There exists an* $m^0 \in \mathbb{N}$ *such that, for all* $m > m^0$ *and all* $r \geq b^m$, *informative voting is a Nash equilibrium of* $\Gamma(\mu, q, 2m+1, 1, r)$.

Proof. The difference Δu_0^R is positive if and only if:

$$r \geq (2m+1)\binom{2m}{m}\frac{\mu_0 q_0^{m+1}(1-q_0)^m - \mu_1 q_1^m (1-q_1)^{m+1}}{\mu_0 q_0 - \mu_1 (1-q_1)}. \tag{7}$$

By Stirling's formula, $m! \simeq \sqrt{2\pi m}(m/e)^m$:

$$\binom{2m}{m} = \frac{(2m)!}{(m!)^2} \simeq \frac{4^m}{\sqrt{\pi m}}(1+o(m))$$

so the right-hand side of (7) is less than

$$\begin{aligned} & (2m+1)\frac{4^m}{\sqrt{\pi m}}\frac{\mu_0 q_0^{m+1}(1-q_0)^m}{\mu_0 q_0 - \mu_1 (1-q_1)}(1+o(m)) \\ = & \frac{2}{\sqrt{\pi}}\frac{\mu_0 q_0 (1+o(m))}{\mu_0 q_0 - \mu_1 (1-q_1)}[4q_0(1-q_0)]^m\sqrt{m} \\ \leq & A(1+o(m))a^m\sqrt{m}, \end{aligned}$$

where

$$A = \frac{2}{\sqrt{\pi}}\frac{\mu_0 q_0}{\mu_0 q_0 - \mu_1 (1-q_1)}.$$

For any $b > a$,

$$\left(\frac{b}{a}\right)^m \frac{1}{\sqrt{m}} \to +\infty$$

as $m \to \infty$. Hence, for all m large enough,

$$b^m > 2Aa^m\sqrt{m} > A(1+o(m))a^m\sqrt{m}.$$

The same reasoning applies to the difference in expected utility upon receiving the signal $s_i = 1$. **QED**

Because the probability r of choosing a dictator can be very made very small when n and hence m is large, the system can mimic the "classical" Condorcet jury and asymptotically reach fully informed decisions.

Theorem 3 [Asymptotic efficiency]. *Suppose that* $q_0, q_1 > 1/2$ *and that the informativeness condition (1) is met. Let* $b \in (a, 1]$, $r_m \in [b^m, 1)$ *for all* $m \in N$ *with* $\lim_{m\to\infty} r_m = 0$*. Let* X_m *be the collective decision in the informative voting equilibrium of the game* $\Gamma(\mu, q, 2m+1, 1, r_m)$*. Then*

$$\lim_{m\to\infty} \Pr[X_m \neq \omega] = 0.$$

Proof. Suppose first that $\omega = 0$ and consider the game $\Gamma(\mu, q, 2m+1, 1, r_m)$ for $m \in N$ fixed. The probability that voter i votes $s_i = 1$ is, by definition $1 - q_0$. If the collective decision is taken by majority rule, the probability of a wrong decision, $X_m = 1$, is some number $Q_0^{(m)}$. So the probability of a wrong decision, $X_m \neq \omega$, given $\omega = 0$, is

$$r_m(1 - q_0) + (1 - r_m)Q_0^{(m)}$$

By the Condorcet Jury Theorem, $Q_0^{(m)} \to 0$ almost surely as $m \to \infty$. Since also $r_m \to 0$, the probability of a wrong decision in the state $\omega = 0$ tends to 0. The same argument is valid for $\omega = 1$. **QED**

6. Conclusion

The above analysis is restricted to a special case. However, we believe that the qualitative conclusions hold more generally. In particular, while we here assume the costs of the two types of mistake, denoted α and β in the introduction, to be identical, we believe that the results are similar when $\alpha \neq \beta$. We here also assumed all voters to be equally "competent" in the sense that their signals were equally informative. Suppose, by contrast, that voters have different competence. If these differences are common knowledge, then weighted majority rule, where more competent voters are given higher weights, may be used to obtain more efficient information aggregation (see Ben-Yashar and Milchtaich (2006)). We believe that our qualitative results carry over also to such cases. If differences in competence are not known, then again we believe that our qualitative conclusions hold. For if voters' individual vectors $q^i = (q_0^i, q_1^i)$ are identically and independently drawn from some fixed probability distribution, and this distribution is not too dispersed, then an application of the law of large numbers will presumably lead to qualitatively the same asymptotic result.

Another direction for generalization, which would be most valuable but is less evident, concerns the binary nature of both signals and choices. Can the present analysis be generalized to choice among finitely many alternatives, with a richer set of signals?

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