Entrepreneurship, technological change and endogenous returns to ability

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Résumé Cet article propose un modèle de choix entrepreneurial mettant en évidence une relation non monotone entre le progrès technique et la sélection des entrepreneurs sur leurs capacités individuelles. Les choix de création d'entreprises sont examinés dans un modèle à deux périodes avec incertitude dans lequel les entrepreneurs décident de continuer ou abandonner leur projet en fonction de l'environnement technologique et de leurs compétences. On analyse comment le progrès technique modifie l'avantage comparatif des entrepreneurs et la dynamique de création d'entreprises dans l'économie. Au-delà d'un seuil de progrès technique, un taux de progrès technologique rapide accroît l'efficacité des projets créés et réduit le nombre d'entrepreneurs qui choisissent de poursuivre, dans l'étape de développement, les projets les moins efficaces. Un progrès technique endogène, fondé sur un mécanisme d'apprentissage par la pratique, renforce ce résultat. Le progrès technique a tendance à exercer un effet "apurant" sur l'activité entrepreneuriale et modifie la perception du marché vis-à-vis de la création d'entreprises.

Abstract This paper proposes a model of entrepreneurial activity highlighting a non-monotone relationship between technological change and ability-based sorting into entrepreneurial types. Entrepreneurial decisions are examined in a two-stage model under uncertainty in which entrepreneurs decide to abandon a project and start a new venture depending on technological change and on ability. This paper investigates how technological change affects the comparative advantage of entrepreneurs thereby shaping entrepreneurial dynamism. Under exogenous technological change, we show that above a threshold level, rapid technological progress increases the number of entrepreneurs undertaking the most efficient projects in the research stage and decreases the number of entrepreneurs of low-adaptative firms who choose to continue their initial business in the development stage. Technological change, endogenized in a learning-by-doing mechanism reinforces these results. Overall, a higher rate of technological change tends to induce a cleansing effect on entrepreneurial activity and alters the market perception of business creation.

Mots clés Progrès technique, activité entrepreneuriale, stigmatisation

Keywords Technological change, Entrepreneurship, Stigma

Classification JEL O33, M13

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This paper contributes to the debate on the determinants of entrepreneurial activity by focusing on the complementarity between credit market conditions, technological environment and individual ability to run firms. For Lazear (2002) “entrepreneurs are jacks-of-all-trades who may not excel in any one skill, but are competent at many”, suggesting that individuals with experience of many different roles are more likely to become entrepreneurs. The obvious implication is that a less specialist and more versatile education may help to spur the level of entrepreneurial activity\(^1\). Yet, one may wonder why much more technology-intensive businesses are undertaken in the US compared to Europe, even though these economies have comparable levels of skills and human capital\(^2\).

Many arguments have been developed in the literature to explain differences in entrepreneurship across countries: individual characteristics, institutional constraints (credit market frictions, administrative costs and barriers to entry), social environment (market’s perception of failure), competition, technology and growth, business cycles, information asymmetry, corporate governance, etc. In this paper, we focus on the interaction between three main determinants of differences in entrepreneurial activities: individual characteristics (ability), technical change and credit market conditions. Considering that entrepreneurs are agents of change, their ability to respond to new opportunities is one of the principal determinants of how well modern economies perform. In our analysis, an increase in the rate of technological growth raises the relative return to entrepreneurial ability, and this affects the selection of entrepreneurs by generating a concentration of high ability entrepreneurs in the most efficient sectors. Consequently, the endogenous sorting of entrepreneurs results in much poorer access to capital and at poorer terms for failed entrepreneurs. From an entrepreneurship perspective, rapid technological growth can account for an increase in inequality in start-up finance and different levels of entrepreneurial activity across countries. By focusing on the interaction between individual ability, technical change and credit market conditions to explain the creation of business enterprises, our approach leans upon two lines of arguments developed in the literature on the determinants of entrepreneurial activity.

The first line of argument focuses on individual characteristics and ability and mobilizes the literature on the links between entrepreneurship and human capital. The attractiveness of entrepreneurial occupations to talent would depend on returns to ability. According to Rosen’s (1981) “superstars” effect, the ablest individuals choose occupations where having marginally greater talent leads to significantly

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\(^1\) The rate of entrepreneurial activity in 2001 indeed varies from 7.2% in Europe (4.5% in Belgium, 7% in France, 7.5% in the UK, 12% in Ireland) to 12.2% in the US. Similarly, between 1995 and 2000, the annual average rate of new enterprise formation ranges from 6.5% in Denmark to 11.7% in France and 15.7% in Germany (European Commission, 2002).

\(^2\) The labor force participation rates of individuals aged 25-64 with a tertiary level of education in 1998 was 86.3% in OECD countries, 87.7% in the US and 87.3% in European economies (OECD Employment Outlook, 1999). Similarly, the average annual employment growth of high-skilled workers over the 1995-2001 period equals 2.79% both in the US and in Europe (OECD Labor force statistics, July 2004)
higher payoff and an economy stagnates if human capital is allocated improperly for growth. For Murphy, Schleifer and Vishny (1991), when productive entrepreneurship offers higher returns to ability than rent-seeking, which may stem from complementary investments in human and physical capital, then the allocation of talent will foster growth. The differential allocation of talent in an economy, which would be due to differences in returns to ability across sectors, may then explain differences in entrepreneurship and growth between apparently similar countries.

But if the allocation of talent to entrepreneurship matters for growth, the reverse relationship, i.e. the fact that growth affects the decision to become an entrepreneur, is also subject to a large attention in the literature. As highlighted by Hassler and Rodriguez-Mora (1999), the allocation of individuals over social positions (becoming an entrepreneur or a worker) depends on the level of entrepreneurial difficulty driven by the rate of technological growth. When the rate of technical change or growth is high, the allocation of individuals over occupations depends more on innate ability and less on social background and this fosters future growth. The latter effect relies on a feedback mechanism whereby the entrepreneurs are the most able and talented individuals in society and when they choose to become entrepreneurs they improve the technology in the line of their business, which increases productivity, innovation and growth. By increasing returns to productive entrepreneurship, ability-biased technical change may thus explain differences in the creation of business enterprises and growth. Following the literature on skill- or ability-biased technical change (e.g. Galor and Tsiddon (1997), Galor and Moav (2000), Rubinstein and Tsiddon (1999)), our model relies on this idea that technological change affects entrepreneurial activity. However, our ambition remains different since we focus on the sorting of individuals to different (“nascent”) entrepreneurial types rather than on occupational choices on the labor market. Moreover, we analyze the determinants of the refinancing decision to determine whether growth induces selection and/or stigmatization regarding entrepreneurial decisions. More precisely, we propose a model in which the endogenous sorting of individuals to entrepreneurial types depends both on the complementarity between ability and growth and on credit market conditions captured through banks’ interest rates.

The second line of argument explains the different levels of entrepreneurial activity according to differences in credit and labor market conditions (Rajan and Zin-
gales (1998), and Bhattacharya and Chakraborty (2004)) or social norms (Landier (2001), Gromb and Scharfstein (2001)). Acemoglu (2001) studies this effect on the labor market and shows that a change in the pattern of comparative advantage affects employment differently in economies with and without credit market frictions. In his model, two economies, Europe and the US, which are identical except for credit market frictions, will generate different responses to a common shock. The US economy, which benefits from fewer credit market frictions than Europe, responds to the arrival of new technologies without an increase in unemployment. At the opposite, technological change can have a persistent effect on European unemployment because, in the absence of efficient credit markets, agents who need funds for startups cannot borrow the necessary amount. Finally, social interactions also matter in the decision to become an entrepreneur because they create social norms and affect reputation and tolerance toward failure. For example, career concerns can induce inefficient continuation of investments as entrepreneurs may be reluctant to abandon their initial project when this is perceived as recognizing an error was made thereby generating an adverse signal for ability (Boot (1992), Holmstrom (1999)). Similarly, entrepreneur’s failure may be highly stigmatized (implying a high cost of capital after failure) or considered as part of the learning process, leading to different types of entrepreneurial regimes and possibly too much or too little entrepreneurship in equilibrium (Landier (2001), Gromb and Scharfstein (2001)). In our model, the market’s perception of business failure (capital or labor market) matters but we focus on a quite different aspect of entrepreneurship. While Gromb and Scharfstein model entrepreneurship as a choice of an organizational form (entrepreneurship vs. intrapreneurship), Landier focuses on capital markets for start-up finance (endogenous -financial- stigmatization of failure). Here, we focus on the returns to ability in entrepreneurship and endogenize the interactions between ability and technical change. Entrepreneurial decisions are determined by the cost of capital but also by the rate of technological progress. In particular, we characterize entrepreneurs by their ability to implement a business strategy, which is defined as a two-stage decision (research and development) under uncertainty: a research stage in which the business idea is implemented, and a development stage in which the project is fully developed and production takes place. The entrepreneurial decision is thus determined by the impact of technological progress on returns to ability.

More precisely, we propose a model in which entrepreneurs differ in their ability to implement business enterprises and the rate of technological progress complements ability in the returns to entrepreneurial decisions. Two types of entrepreneurial

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6In a different perspective, a growing literature considers the financing of new ventures through venture capital and focuses on the mechanisms behind financing arrangements such as the allocation of control rights and the staging of investments over time (Berglof (1994), Gompers (1995), Hellman (1998)). Our model abstracts from the details of venture capital financing and uses a simple contracting model to capture the interaction between ability, technical change and credit conditions in the creation of business enterprises.
projects may be chosen by individuals: a “high adaptative” (type H) project, suited for entrepreneurs able to take correct decisions in rapidly changing environments, and a “low adaptative” (type L) project, suited for lower levels of ability to run efficiently projects in changing environments. Projects last for two stages, a research stage and a development stage. The success of the research venture is determined both by the entrepreneur’s ability and by the technological environment. After initiating the project, the entrepreneur privately observes the probability of success of the initial research venture. If the research stage is successfully completed a product can be developed and sold on the market. Otherwise, if the probability of success of the initial project is too low, the entrepreneur may decide to liquidate it and ask for a refinancing to start a new research venture.

We show that growth affects the comparative advantage of entrepreneurs and shapes the type of business creation. First, entrepreneurs of “high adaptative” firms are better suited to an innovative environment: as the rate of technical change accelerates the number of the most efficient firms increases as well. Second, growth lowers the probability of success for entrepreneurs of “low adaptative” firms. The interest rate charged to failed entrepreneurs willing to restart their initial business is greater than the prevailing interest rates charged to first-timers. Finally, faster growth reduces the number of entrepreneurs of “low adaptative” entrepreneurs who choose to continue their initial business. Overall, rapid technological progress affects sorting and induces a cleansing effect on business creation, thereby affecting the stigmatization toward failure. This in turn may account for an increase in inequality in start-up financing of small and innovative firms and different levels of entrepreneurial activity across countries.

The remainder of the paper is organized as follows. Section 1 presents the basic set up. Section 2 describes the research and development stages. Section 3 examines the equilibrium of the model and section 4 develops the main results with both exogenous and endogenous technical change. Section 5 concludes the paper.

1 Basic set up

1.1 Overview

The model has three dates, t=0,1,2. All agents are risk neutral and the risk-free interest rate is normalized to zero. There is no discounting. The economy is composed of a continuum of entrepreneurs and investors. Projects last for two periods: the first period is a research stage in which entrepreneurs implement their business idea, the second period is a development stage in which production takes place. At date 0, entrepreneurs are endowed with one research project each and lack any source of finance. Bankers are endowed with plenty of funds but are short of research ideas. At date 1, after observing privately the business’s probability of success, entrepre-
neurs decide to either continue the existing business or liquidate it and ask for a refinancing to start a new one. Banks decide whether to grant a new loan to second timers entrepreneurs and set interest rates. At the end of the second period, cash flows and repayments are realized.

The sequence of events may be summarized as follows:

- $t = 0$:
  - Entrepreneurs ask for loans
  - Entrepreneurs choose between two types of business creation
  - Banks set interest rates
  - Business’s probability of success is private information to entrepreneurs

- $t = 1$:
  - Entrepreneurs choose whether to continue their initial business or liquidate it and start a new one
  - Banks decide to refinance the entrepreneur or not

- $t = 2$: Cash flows and repayments are realized

1.2 Contractual variables

In the research stage, the decision to become an entrepreneur implies to choose among two types of firms or projects: a “high-adaptative” (type H) or a “low-adaptative” (type L) firm. In the type H firm, entrepreneurs can spread their ability advantage in the sense that adaptativity to technological environment (and therefore the firm’s returns) increases with individual ability. In type L firms, entrepreneurs have to spend time learning and adapting to complexity. This learning process increases the firm’s returns, but reduces available time for running it and therefore decreases the probability of success of the business which is discovered at the end of the research stage.

In the development stage, a final good is produced using two different types of intermediate goods: goods produced by continued firms (labelled $j = c$) and goods produced by refinanced firms (labelled $j = r$). Hence, we denote the inputs in the production of intermediate goods as “refinanced firms’ goods” and “continued firms’ goods”.

The financing contract between the entrepreneur and the investor, which is signed at date 0, specifies an initial investment for the research venture of $1, generates a cash flow $V_j$ and final repayment $R_j$ at date 2 ($j = c, r$).

Each entrepreneur can run only one business at a time. At date 1, after observing

\footnote{The financing contract can be interpreted as debt or equity. Under risk neutrality, a null transfer in case of termination is not restrictive. Since the abandoned project has a zero reservation value, whether the investor can seize it or not is irrelevant.}
privately the business’s probability of success, entrepreneurs choose whether to continue the initial business \((j = c)\) or liquidate it and ask for a refinancing to start a new business \((j = r)\). In this case, the initial business is terminated and its liquidation value is normalized to zero. The new business again requires an initial investment of $1 and yields a cash flow of \(V_j\) at date 2. Hence, the new business becomes a one-period project. Banks decide whether to grant a new loan to second timers entrepreneurs at an interest rate of \(R_r\).

If the business is abandoned or generates no cash flow at date 2, the repayment to the bank is 0. We thus assume that the entrepreneur is liable for payments to the lender only to the extent of current revenues. Therefore, the firm is restricted to a nonnegative cash flow. Hence, we only need to characterize the repayment for the first-timers (those who carry on the initial project until date 2) and second-timers (entrepreneurs who abandon the first business and start again at date 1), \(R_c\) and \(R_r\). We assume that at date 0, the average project has a positive net present value.

The following figure reproduces the timing and the variables of the model. In the research stage, if we denote by \(\theta\) the firm’s type, \(\theta = H, L\), the expected value of a firm, entrepreneurial payoffs and probabilities of success are denoted respectively as \(V^{\theta}\), \(\lambda^{\theta}\) and \(\pi^{\theta}\), \(\theta = H, L\). In the development stage, entrepreneurs choose to continue the initial business or abandon it and ask for a refinancing to start a new one. The firm’s expected values, entrepreneurial payoffs and probabilities of success are denoted as \(V^c_{\theta}\), \(\lambda^c_{\theta}\) and \(\pi^c_{\theta}\), \(\theta = H, L\), for continued businesses and as \(V_r\), \(\lambda_r\) and \(\pi_r\) for refinanced businesses.

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8At date 1, banks can distinguish between new entrants and failed entrepreneurs who are willing to start a new business. We thus omit news entrepreneurs applying for a loan at the beginning of the second period.

9Formally, see assumption 3 below .
Business probability of success is private information to entrepreneurs.

Cash flows and repayments:

Entrepreneurs choose to continue initial business or liquidate and start a new one.

Refinance:

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<th>Refinance:</th>
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<tr>
<td></td>
<td>$V_r, \lambda_r, \pi_r, R_r$</td>
<td>$V_c^L, \lambda_c^L, \pi_c^L, R_c$</td>
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<tr>
<td></td>
<td>$V_c^H, \lambda_c^H, \pi_c^H, R_c$</td>
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Entrepreneurs ask for loans.
Entrepreneurs choose firm’s type.

Type H: $V^H, \lambda^H, \pi^H$

Type L: $V^L, \lambda^L, \pi^L$

Banks set interest rates.

Banks decide to refinance the entrepreneur or not.

Business probability of success is private information to entrepreneurs.
2 The research and development stages

2.1 The research stage

At date 0, there is a continuum of mass one of wealthless entrepreneurs who may create “high-adaptative” (type H) or “low-adaptative” (type L) firms. The complementarity between ability and technology in the individuals’ decision to become entrepreneurs of type H or type L firms relies on the idea that basic skills become rapidly obsolete in a rapidly changing environment. The most able individuals indeed have a comparative advantage in choosing to become an entrepreneur of type H firms from too perspectives. On the one hand, having marginal greater ability is rewarded in a type H firm, even in a stationary environment (where the rate of technological progress is constant) whereas learning and adapting to new environments is rewarded in type L firms only when growth improves and provided that time is spent to “absorb” new technologies. On the other hand, technological progress exerts an erosion effect on the probability of success of a type L firm whereas it does not affect that of a type H firm.

In other words, we propose a model of “nascent” entrepreneurship (i.e. the entrepreneurs who are currently taking explicit steps to start a new business) in which individuals make the choice of creating a new venture and, as in Lazear’s view of jack-of-all-trades, those who have sufficient knowledge in a variety of areas to put together the many ingredients needed for survival and success in a business will choose a type H firm, while those who have not these skills will choose to create a type L firm in which they will not be able to spread their ability advantage

The firm’s expected value $V^\theta$ depends on the ex ante continuation value $V_c$ (in case of refinancing) and on the entrepreneur’s adaptative capacity $\rho^\theta$:

\[
V^\theta = V_c \cdot \rho^\theta, \quad \theta = H, L
\]

and the entrepreneur’s expected payoff is defined as:

\[
\pi^\theta = \lambda^\theta \cdot V^\theta, \quad \theta = H, L
\]

where $\lambda^\theta$ denotes the probability of success of the current business, which is discovered privately by the entrepreneur after setting up the firm but before date 1.

Type H firms are run by entrepreneurs able to make difficult decisions in complex environments. They adapt instantaneously to new environments and the time needed to learn new technologies is null. Their adaptative capacity depends

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10Our definition of nascent entrepreneurship focuses on the ability to adapt to a changing environment but a more complete theory would encompass more individual characteristics like for example risk-aversion, previous employment status, regional characteristics or gender. Since we focus on the interplay between ability, technical change and credit condition, we rely on a more reduced-form to characterize entrepreneurship.
only on their individual ability: \( \rho^H = h(a_i) \) with \( 0 < h(a_i) < 1 \), \( h'(a_i) > 0 \), and where the entrepreneur’s ability \( a_i \) is distributed uniformly over the unit interval\(^{11}\). A type H firm’s expected value is then defined as:

\[
V^H = V_e \cdot h(a_i)
\]

In type H firms, the most able individuals have the highest adaptative capacity whatever the rate of technological progress, the time needed to adapt to new technologies is null and the entrepreneur’s available time is entirely devoted to run the firm. The probability of success of a type H firm is constant and defined by \( \lambda^H = \lambda \), where \( 0 < \lambda < 1 \). This assumption captures the idea that in type H firm, having marginally greater ability is rewarding, and that other factors such as luck or the technological environment do not affect the probability of success.

In type L firms, entrepreneurs do not have the same competence to take correct decision in rapidly changing environments and learning is necessary to adapt to the technological environment. We assume that the learning process involves an opportunity cost such that the higher the rate of technological progress, the higher the time spent learning the new environment, therefore the higher the adaptative capacity of type L entrepreneurs, but the lower the time remaining for running efficiently the project. We normalize entrepreneurs’ time to 1, so that in L type firms, a fraction \( \delta \cdot \gamma \) of this time is devoted to adapt to new technologies and the remaining fraction \( 1 - \delta \gamma \) is devoted to run the project. The parameter \( \delta \) is thus such that

**Assumption 1.** \( 0 < \delta \gamma < 1 \)

The adaptative capacity of entrepreneurs running type L firms is proportional to the rate of technological progress as follows: \( \rho^L = \delta \cdot \gamma \). The firm’s expected value then is defined as:

\[
V^L = V_e \cdot \delta \gamma
\]

In type L firms, entrepreneurs must spend time to learn and adapt to changing environments and the marginal return to ability is null. The higher the time spent learning, the higher the adaptativity, but the lower the time available for running the firm \((1 - \delta \gamma)\). In turn, the probability of success of type L firm is given by \( \lambda^L = \lambda \cdot (1 - \delta \gamma) \). Hence, there is an erosion effect due to technological progress for type L firms which affects the business’s probability of success.

We assume that the average project has a positive net present value:

**Assumption 2. Project’s positive net present value:** \( \lambda \cdot (2 - \delta \gamma) \cdot V^\theta > 1 \)

In sum, expected entrepreneurial payoffs are given by:

\(^{11}\)Qualitative results are not affected by a more general distribution function for ability as long as it is continuous.
\[ \pi^L(a_i) = \lambda \cdot (1 - \delta \gamma) \cdot \delta \gamma \cdot V_c \]
\[ \pi^H(a_i) = \lambda \cdot h(a_i) \cdot V_c \]  

(1)  
(2)  

At date 0, given the above expected payoffs, entrepreneurs choose to engage in a type H firm rather than in a type L firm if and only if, given their ability and the rate of technological progress, the expected payoff from running a type H firm is higher than the expected payoff from running a type L firm, that is:

\[ \pi^H(a_i) \geq \pi^L(a_i) \iff \lambda \cdot h(a_i) \cdot V_c \geq \lambda \cdot (1 - \delta \gamma) \cdot \delta \gamma \cdot V_c \]

Given that 0 < \lambda < 1 and 0 < \delta \cdot \gamma < 1, this inequality implies that there is a unique threshold level of ability, \( a^* \), such that 0 < \( a^* < 1 \). All individuals with ability above the threshold, \( a^* \) choose to run type H firm, while all individuals with ability below \( a^* \) run type L firms: \( \pi^H(a^*) \geq \pi^L(a^*) \), where:

\[ a^* = h^{-1}(\delta \gamma(1 - \delta \gamma)) \]  

(3)

Since the population mass is normalized to one and ability uniformly distributed over the unit interval, the number of type L entrepreneurs is \( a^* \) and the number of type H entrepreneurs is \( 1 - a^* \). Since function \( h(.) \) is continuous and strictly increasing, the inverse function \( h^{-1} \) is also continuous and strictly increasing.

2.2 The development stage

At date 1, entrepreneurs choose whether to continue their initial business (\( j = c \)) or liquidate it and start a new one (\( j = r \)). Banks decide to refinance the entrepreneur or not.

We consider that the probability of success for refinanced firms is the same independently of firms’ initial types. In other words, refinanced firms are closed down and a new business starts. Hence, the entrepreneur generates the same level of output whatever his ability. The probability that the business succeeds determines the distribution of payoffs that are generated at date 2. At this date, conditional on success, each entrepreneur running a firm of type \( \theta \) will receive a net cash flow of \( \pi^\theta_j = \lambda^\theta_j \cdot (V_j - R_j) \) where \( R_j \) and \( V_j \) represent the repayment to the bank and the value of each type of business. If the business is abandoned or fails at date 2, no repayments are made to the bank. The nonnegativity constraint on entrepreneur’s cash flows requires the following assumption:

Assumption 3. Non-negative cash flows: \( V_j \geq R_j, \ j = c, r \)
The probabilities of success of each type of business enterprises are labelled $\lambda_r$ for a refinanced firm, $\lambda_c^H$ for a continuing firm of type H and $\lambda_c^L$ for a continuing firm of type L. We assume that $\lambda_r = \lambda_c^L \cdot \kappa$, with $\kappa \leq 1$. Hence we have:

$$\lambda_c^H = \lambda > \lambda_c^L \geq \lambda_r$$  \hfill (4)

The expected payoff for each type of entrepreneur then writes:

$$\pi_c^H = \lambda_c^H \cdot (V_c - R_c), \quad \pi_c^L = \lambda_c^L \cdot (V_c - R_c), \quad \pi_r = \lambda_r \cdot (V_r - R_r)$$  \hfill (5)

with $R_j$ and $V_j$ the interest rates and the firms’ values. We assume that a continued project has a positive net present value:

**Assumption 4. Continued project’s positive net present value:**

$$(\lambda_c^H + \lambda_c^L)V_c > 1$$

After substituting for $\pi_c$ and $\pi_r$ from equations (19) and (20) in appendix (6.1), entrepreneurial payoffs from running each type of firm are given by:

$$\pi_c^H = \lambda_c^H \frac{1 - \alpha}{\alpha} \cdot x_c - \lambda_c^H \cdot R_c$$  \hfill (6)

$$\pi_c^L = \lambda_c^L \frac{1 - \alpha}{\alpha} \cdot x_c - \lambda_c^L \cdot R_c$$  \hfill (7)

$$\pi_r = \lambda_r \frac{1 - \alpha}{\alpha} \cdot x_r - \lambda_r \cdot R_r$$  \hfill (8)

### 3 Equilibrium

#### 3.1 Resources constraints

Firms differ in their type (H or L) and at date 1 and we can thus distinguish three categories of firms: those that need a refinancing, firms of type L that are continued and firms of type H that are continued. The population mass is normalized to one, there is a proportion $1 - a^*$ of type H entrepreneurs. We denote by $H_c$ (respectively $L_c$) the fraction of entrepreneurs with type H (respectively type L) who continue their business. Since the number of entrepreneurs is normalized to 1, the resources constraints write:

$$n_c^H + n_c^L + n_r \equiv 1 \quad \text{with} \quad \begin{cases} n_c^H = (1 - a^*) \cdot H_c \\ n_c^L = a^* \cdot L_c \\ n_r = (1 - a^*) \cdot (1 - H_c) + a^* \cdot (1 - L_c) \end{cases}$$  \hfill (9)

where $n_c^H$ (respectively $n_c^L$) is the number of type H (respectively type L) firms that are continued and $n_r$ is the number of firms that are refinanced.
3.2 Refinancing decisions

We consider a first-time entrepreneur’s decision to continue or abandon his initial business at date 1. An equilibrium is determined by the strategy (continuation or abandon) of a first-time entrepreneur who observes the probability of success of his current business and by the cost of capital, \( R_c \) and \( R_r \) for first-timers and second-timers. Entrepreneurs of type L choose to continue their business as long as the expected income, \( \pi^L_e \), is higher than that of a refinanced firm, \( \pi_r \). In equilibrium, this condition is binding, implying that the number of entrepreneurs of type L who choose to continue their business satisfies the following indifference condition: \( \pi^L_e = \pi_r \). Regarding entrepreneurs of type H, the assumption that \( \lambda^H_e > \lambda^L_e \) implies that \( \pi^H_e > \pi^L_e \).

Refinancing decisions in turn satisfy the following rule:

\[
\pi^H_e > \pi^L_e = \pi_r
\]  
(10)

This rule implies that, all firms of type H choose to continue and firms of type L are indifferent between continuing or refinancing, that is:

\[
H_e = 1
\]

\[
\pi^L_e = \pi_r \iff \lambda^L_e \frac{1 - \alpha}{\alpha} \cdot x_c - \lambda^L_e \cdot R_c = \lambda_r \frac{\alpha}{1 - \alpha} \cdot x_r - \lambda_r \cdot R_r
\]  
(12)

Taking into account the resources constraints (9), the market clearing condition writes:

\[
x_c = \lambda^H_e n^H_e + \lambda^L_e n^L_e, \quad x_r = \lambda_r n_r
\]  
(13)

Substituting for (13) into (12) finally allows determining the number \( L_e \) of firms of type L that choose to continue:

\[
\iff L_e = \frac{a\frac{\alpha}{1 - \alpha}k^2 + a^*\frac{1 - \alpha}{\alpha} \frac{1 - \alpha}{\alpha} + \frac{R_c - R_r}{\lambda(1 - \delta)} a^*}{\frac{\alpha}{1 - \alpha}k^2 + \frac{1 - \alpha}{\alpha}}
\]  
(14)

3.3 Banks’ decision

Banks maximize their profits and offer interest rates to first-timer entrepreneurs (those who carry on the initial project until date 2) and to second-timers (the ones who ask for a refinancing at date 1). The loan contract is parameterized by \( R_c \) for the entrepreneurs who continue their initial project and by \( R_r \) for those who want to start again an investment project. At the beginning of the first period, if the loan contract specifies a promised repayment such that lenders get an expected return at least equal to the amount of the initial loan, $1, the loans are granted. If banks receive repayment lower than the initial loan, they refuse to finance the entrepreneur’s investment project. Banks set an identical single rate for both types.
since they can not distinguish between the two types of entrepreneurs at a time when the loan is granted.

The loan contract involves liquidation for all payment lower than the face value of the debt, $R_c$ and $R_r$. Since liquidation destroys all profits from a project, including the entrepreneur’s repayment, both lenders and borrowers receive zero payments whenever there is liquidation. This implies that entrepreneurs will honor their obligation of repayment whenever their projects deliver sufficient return and in equilibrium, there is liquidation only when the value of the project’s cash flow is zero. Entrepreneurs behave as price takers, that is, they can not affect the interest rates charged by the bank. They always borrow from the competing bank offering the lowest interest rate.

We have shown that all firms of type H continue rather than refinance. In addition, the number of firms of type L that choose to continue ($L_c$) is given by equation (14). In that case, the number of type L firms which ask for a refinancing is given by: $a^*(1 - L_c)$. In comparison, the number of entrepreneurs who continue is given by $(1 - a^*) \cdot H_c + a^* \cdot L_c$. We now have to determine equilibrium interest rates charged by the bank in period 2.

For a second-timer, in a competitive financial market the bank’s break-even rate satisfies:

$$R_r = \frac{1}{\lambda_r}$$  \hspace{2cm} (15)

where $\lambda_r$ is the probability of success for a type L second timer, so that the entrepreneur can meet his obligation of repayment toward the bank.

At the first period, banks charge an interest rate $R_c$ prevailing for first-timers. The bank’s break-even rate for a loan made to a randomly selected entrepreneur at the first period insures that the unconditional probability of success $\lambda_c^H + \lambda_c^L$ equals the initial investment. Solving for $R_c$ yields:

$$R_c = \frac{1}{\lambda_c^H + \lambda_c^L}$$  \hspace{2cm} (16)

Note that $R_c < V_c$ from assumption 4. From equation (4) we get $\lambda_c^H > \lambda_c^L \geq \lambda_r$. In turn we have $R_c < R_r$, which implies that the interest rates charged for a failed entrepreneur who restarts a project is greater than the prevailing interest rates for a first timer entrepreneur.

The incentive compatibility constraint ensuring that type L firms with a low probability of success choose to continue is:

$$a^* L_c \lambda_c^L (V_c - R_c) > a^* (1 - L_c) \lambda_r (V_r - R_r)$$
In equilibrium, second-time entrepreneurs can refinance their business only if, \( R_r < V_r \). Otherwise, no feasible payment allows the creditor to break even. If, \( R_r > V_r \), entrepreneurs willing to start again their business are unable to refinance. Consequently, the incentive compatible constraint can be rewritten as: \( a^* L_c \lambda^L_c (V_c - R_c) > 0 \), which holds from assumption 4. Hence, the market does not refinance failed businesses. This situation arises when the expected value of the initial business for failed entrepreneurs is negative, i.e., \( \lambda^L_c \cdot V_c < 1 \).

Without altering the qualitative result of the model but to simplify exposition we will consider for the rest of the paper that \( h(a_i) = a_i \), that is:

\[
a^* = h^{-1}(\delta\gamma(1 - \delta\gamma)) = \delta\gamma(1 - \delta\gamma)
\]  

(17)

**Definition 1. Equilibrium**

An equilibrium in this economy is characterized by equations (3), (9), (11), (14), (15) and (16) defining the values of \( a^* \), \( n^H_c \), \( n^L_c \), \( n_r \), \( H_c \), \( L_c \), \( R_c \) and \( R_r \), that is:

1. At date 0, all individuals with ability below \( a^* \) run type L firms, with:

\[
a^* = \delta\gamma(1 - \delta\gamma)
\]

2. At date 1, the number of firms of type L that choose to continue is given by:

\[
L_c = \frac{\alpha \kappa^2 (1 - a^*) a^{\alpha} - \frac{1 - a^*}{1 - \delta\gamma} \frac{1 - a^*}{\alpha} - \frac{R_c - \kappa R_r}{\lambda(1 - \delta\gamma)a^*}}{\frac{1}{1 - \alpha} \kappa^2 + \frac{1 - a^*}{\alpha}}
\]

3. The interest rate charged to continued businesses is given by:

\[
R_c = \frac{1}{(\lambda^H_c + \lambda^L_c)} = \frac{1}{\lambda(2 - \delta\gamma)}
\]

4. The interest rate charged to refinanced businesses is given by:

\[
R_r = \frac{1}{\lambda_r} = \frac{1}{\lambda \kappa(1 - \delta\gamma)}
\]

**Lemma 1:** A continuation equilibrium exists if:

\[
a^* L_c \lambda^L_c (V_c - R_c) > a^* (1 - L_c) \lambda_r (V_r - R_r)
\]

If \( R_r > V_r \), entrepreneurs willing to start a new business at date 1, are not refinanced.
4 The cleansing effect of growth and the selection of business enterprises

4.1 Exogenous technological change

We derive in this section the analytical results of our model with exogenous technological change, which are illustrated with numerical simulations reproduced in appendix 6.2.

Proposition 1. Growth and firms’ types.

Faster growth, in the sense of a higher rate of technical change increases (respectively decreases) the number of entrepreneurs that choose to run type H firms when \( \gamma > \tilde{\gamma} = 1/2\delta \) (respectively \( \gamma < \tilde{\gamma} = 1/2\delta \)).

Proof: see appendix 6.2.

Proposition 1 states that above a threshold level, as the rate of technological progress \( \gamma \) increases, the number of entrepreneurs running a type H firm \( 1 - a^* \) increases as well. This phenomenon can thus be interpreted as follows. A faster technical change induces a selection effect on the number of type H projects that are undertaken in the first period. Type H firms are considered as high-adaptative because they are run by entrepreneurs able to take the correct decision in difficult situations. As the rate of technological change accelerates, this confers a comparative advantage to entrepreneurs running a type H firm compared to those running a type L firm.

This proposition captures the stylized fact that a change in technology affects which agents have a comparative advantage in entrepreneurship and this in turn affects business creation. Assuming that innovative activities are more likely to be undertaken by type H entrepreneurs, the view that technological progress favor skilled agents is supported by a considerable literature on skill-biased technical change (see e.g. the survey by Acemoglu (2001)). In Acemoglu (1998), an exogenous increase in the supply of skills makes skill-biased techniques more profitable and firms have greater incentives to develop and adopt such techniques. In our model, rapid technological progress increases the comparative advantage of the most able entrepreneurs to run firms and endogenously raises the number of type H firms.

Proposition 2. Growth, interest rates and composition effects

The interest rates charged to failed entrepreneurs who restart their initial business, \( R_r \), is greater than the prevailing interest rates for first timer entrepreneurs, \( R_c \). Both interest rates are increasing in the rate of technical change.

Proof: \( R_r > R_c \) is obtained using equations (4) and (9). The derivative of \( R_r \) and \( R_c \) with respect to \( \gamma \) is immediate (see appendix 6.2 for details).

Note first that both interest rates, \( R_r \) and \( R_c \), are increasing with the rate of technical change. This highlights the complementary role between entrepreneurs’
ability and the rate of technological progress. As suggested above by equations (1) and (2), faster technological change increases the relative return to ability for type H firms compared to type L firms. This in turn translates into a lower probability of success for type L entrepreneurs (due to the erosion effect that affects the business’s probability of success), and for refinanced firms which exerts an upward pressure on corresponding interest rates. Hence, faster growth exerts an erosion effect on entrepreneurs’ ability to run correctly type L or refinanced firms, and interest rates adjust upward\(^{12}\).

Proposition 2 also states that the interest rate charged to first timer entrepreneurs is lower than the interest rate charged to second timer entrepreneurs. This suggests that the cost of capital rises when the credit history of an entrepreneur includes a failure to honor its debt obligation toward the creditor. Because of the uncertainty about the borrower’s type, lenders will downgrade their beliefs about the borrower’s quality when default occurs and will upgrade their beliefs when payment was made the previous periods. High-quality borrowers therefore expect to receive lower cost of capital than low-quality borrowers. The latter in contrast are more likely to have their credit rationed or to be excluded from the credit market (Stiglitz and Weiss (1981)). For many inexperienced entrepreneurs, building up a good credit history through hard work and conservative investing is crucial as borrowers with unfavorable credit histories (e.g. past bankruptcies and delinquent payments) typically have poorer access to credit and at poorer terms. Bankruptcy in particular has harsh consequences since it normally involves a temporary ban from borrowing and a relatively higher cost of capital afterwards (Vercammen (1995)).

The above observations also reflect the fact that firms which have only recently started their business have short credit history and thus have lower credit ratings than firms which have been trading for a long time. As a result, the latter benefit from lower cost of capital compared to the former. Furthermore, in the event of an aggregate shock, evidence suggest that an established firm with a good track record is less likely to have its credit rating downgraded than a recent start-up, without an established credit record (Japelli and Pagano (2000)). This also suggests that reputation effects play a crucial role in providing entrepreneurs incentives to perform. Diamond (1989) illustrates that reputation effects in credit markets strengthen over time. In his model, high-risk borrowers are gradually excluded from the credit market as their types become revealed to lenders. Hence, the increasing concentration of low-risk borrowers results in a decline in the average cost of capital, and this induces some borrowers to switch from choosing a high-risk to a low-risk project\(^{13}\).

\(^{12}\)In Galor and Moav (2000), ability-biased technical change modifies the comparative advantage of high-skilled workers and raises inequality both between and within groups. Here, we highlight a different channel through which ability-biased technical change may affect business creation, namely the cost of capital.

\(^{13}\)Yet, Holmstrom (1982) examines reputation effects in labour market and argues that reputation-related incentives staledly decline as information about the manager’s type is revealed over time.
Proposition 3. Selection and the cleansing effect of growth

Faster growth, in the sense of a higher rate of technological change, reduces the number of entrepreneurs of type \( L \) that choose to continue their initial business when \( \gamma > \tilde{\gamma} = 1/2\delta \).

Proof: see appendix 6.2. □

Proposition 3 suggests that growth has an ambiguous impact on the number of entrepreneurs of type \( L \) that choose to continue their initial business. This impact depends on the opportunity cost of running a type \( H \) firm rather than a type \( L \) firm in the initial phase of the investment decision. This opportunity cost is related to the marginal effects of both learning and erosion of productivity due to technological progress and affects the business climate toward failure. More specifically, let \( \tilde{\gamma} = 1/2\delta \) be the threshold value above which the number of type \( H \) firm increases. Similarly, below this threshold the number of type \( H \) firm decreases with the growth rate.

When the rate of technical change is greater than the threshold, level \( \gamma > \tilde{\gamma} \), a higher rate of technical change \( \gamma \) results in lower probabilities of success for type \( L \) entrepreneurs. Hence, the number of entrepreneurs running a type \( L \) firm \( a^* \) decreases with growth. This translates into a lower number of low-adaptative firms that are continued and a high rate of firm destruction.\(^{14}\) Also, there is a high concentration of type \( H \) entrepreneurs who continue their initial business (from (11) we have \( H_c = 1 \)). Overall, in this economy the average value of businesses and the pool of entrepreneurs are of high quality. In other words, growth induces a cleansing effect on entrepreneurial activity which translates into a climate of tolerance toward failure.\(^{15}\) When business failure is weakly stigmatized, terminating a project does not damage entrepreneurs’ reputation such that they abandon more easily a project with poor future prospects.

A large body of evidence suggests that failure is highly stigmatized in Europe and in some Asian countries, whereas the American’s entrepreneurial regime considers failure as a valuable entrepreneur’s learning experience (Landier (2001), and Saxenian (1994)). In particular, the share of nascent entrepreneurs is equal to 8.3% in the US, 3.4% in Germany and the UK and 0.9% in France, and only 23% of the failed entrepreneurs are nascent entrepreneurs today in Germany (Wagner, 2004). The fear of failure would actually prevent more than 35% of Europeans from starting a new business (Global Entrepreneurship Monitor, 2002). In the US entrepreneurial regime, new firms have an innovative advantage and therefore undertake risky projects which leads to a high mortality rate. In contrast, in the European regime, innovation is undertaken by large established firms and new firms have a higher

\(^{14}\)In the simulations (6.3), for all negative values the optimal decision is simply \( L_c = 0 \). This illustrates the fact that no low-adaptative firms are continued when entrepreneurs receive bad news about the firm’s future prospects.

\(^{15}\)Note that for \( \gamma < 1/2\delta \), we do not have a simple analytical result and this case is only illustrated graphically with numerical simulations, which are reproduced in appendix 6.3.
survival rate. Winter (1984), and Audretsch (1995) provide empirical evidence on the rate of creation and destruction of firms in Europe and US. They show that even though much fewer firms are created in Europe, the proportion of small firms among existing firms is much higher in Europe than in the US. The OECD Small and Medium Enterprise outlook reports that in the late 1990s, 31.7 % (respectively 29.5 %) of French employees worked in an enterprise of less than 20 employees (respectively 500), versus 19.5 % (respectively 47.5 %) for the American employees.

In summary, growth has a non-monotone effect on the creation and destruction of businesses depending on the threshold \( \hat{\gamma} = 1/2\delta \). When \( \gamma > \hat{\gamma} \) faster growth by modifying the comparative advantage of entrepreneurs’ types induces entrepreneurs to take the correct decision regarding the continuation or liquidation of their initial business. Rapid technological growth affects sorting and thus participates in filtering out the least efficient types of entrepreneurs. This effect translates into a lower number of entrepreneurs of low-adaptative firms who choose to continue their initial business.

### 4.2 Endogenous technological change

In this section we endogenize the rate of technological change to analyze the feedback mechanism from the allocation of entrepreneurial talents to innovation and growth. As before, there are two periods (or stages): entrepreneurs develop their business idea in the research stage and production occurs in the development stage. The final good sector and the intermediate goods sector are described in appendix 6.1. We now assume that the rate of technological progress is a positive linear function of the number of type H entrepreneurs who pursue their business idea over both stages: \( \gamma = \varepsilon \cdot \lambda^H \cdot n^H \) with \( \varepsilon > 0 \). This assumption captures the idea of learning-by-doing: as entrepreneurs of type H are the most able to adapt and spread their ability advantage, they contribute to increasing knowledge and growth in the economy.

Given the equilibrium values obtained in section 3, we can substitute for the values of \( \lambda^H \) and \( n^H \) to get the following endogenous growth rate:

\[
\gamma = \varepsilon \lambda (1 - \delta \gamma + \delta^2 \gamma^2)
\]

The existence of a positive equilibrium rate of technological progress requires a further restriction on the value of the parameter \( \varepsilon \):

**Assumption 5.**

\[
\varepsilon < \tilde{\varepsilon} \equiv \frac{1}{\lambda (1 - \delta + \delta^2)}
\]

**Proposition 4.** Endogenous technological progress
Under Assumption 1 and 4, there exists a unique equilibrium rate of technological progress $\hat{\gamma} \in (0, 1)$. The equilibrium rate of technological change increases with the probability of success of the business project ($\lambda$) and the spillover rate of learning-by-doing ($\varepsilon$) and decreases with the adaptative capacity of entrepreneurs of type L ($\delta$).

**Proof:** see appendix 6.4 □

We consider an endogenous rate of technological change which takes the form of a learning-by-doing mechanism proportional to the fraction of type H entrepreneurs. We find that a higher spillover rate of learning-by-doing (i.e. an increase in its efficiency) improves the rate of technological progress. Similarly, a rise in the projects’ probability of success increases the rate of technical change directly by improving the learning curve (i.e. the learning externality). Interestingly, the rise in $\lambda$ also affects the composition of the pool of entrepreneurs in the first period by reducing the threshold of ability $a^*$ above which individuals choose to run type H projects. This mechanism reinforces the feedback loop by which technical change tends to generate discipline: a rise in $\lambda$, though affecting apparently similarly all types of projects, tends to reduce the number of entrepreneurs of type L that choose to continue.

Moreover, we find that the rate of technical change decreases when the spillover rate of learning-by-doing $\varepsilon$ goes down, which occurs for instance due to knowledge obsolescence. Through its negative impact on the rate of technological progress, knowledge obsolescence hence negatively affects the comparative advantage of type H entrepreneurs. This leads to a higher stigma of failure and thereby a higher number of entrepreneurs of type L who choose to continue their initial business.

Finally, the higher the marginal adaptativity cost of type L entrepreneurs $\delta$, the lower the rate of technical progress. A higher value of $\delta$ increases both the adaptative capacity of type L firms and the erosion effect on the probability of success. Overall, the erosion effect dominates and an increase in $\delta$ affects business creation by improving the comparative advantage of type H entrepreneurs and reducing the number of entrepreneurs of type L firms who choose to continue their business.

## 5 Conclusion

This paper analyzes the determinants of entrepreneurial activity by focusing on the complementarity between credit market conditions, technological environment and individual ability to run firms. We develop a two-stage project development set up. We show that rapid technological change affects sorting. First, an acceleration in the growth rate modifies the comparative advantage of individuals in running high versus low-adaptative firms. Above (below) a threshold level, the number of entrepreneurs who choose to run the most efficient projects increases (decreases)
with the growth rate. Second, a higher rate of technical change results in lower probabilities of success for type L entrepreneurs, and this in turn justifies a high cost of capital for undertaking a new venture. Lastly, above the threshold level, faster growth reduces the number of entrepreneurs of low-adaptative firms who choose to continue their initial business. This cleansing effect on entrepreneurial activity highlights that higher levels of growth may reduce stigmatization of failure. When failure is considered as part of the learning process, entrepreneurs abandon more easily an inefficient project. Moreover, an endogenous rate of technological change reinforces the mechanism through which the allocation of entrepreneurial talents affects entrepreneurial dynamism.

The fact that rapid technological progress can account for an increase in inequality across countries is supported by numerous studies. The conventional view is that innovation stimulates job creation but induces simultaneously the destruction of other (older) jobs. Too little entrepreneurial activity can then result from a lack of innovation and hence of job creation (Aghion and Howitt (1992), and Mortensen and Pissarides (1998)). The way the growth rate affects entrepreneurs’ ability thereby influencing entrepreneurial dynamism is the central theme of this paper. We show that when the rate of technological progress is high, the return to entrepreneurs’ ability increases. This in turn affects entrepreneurial sorting in a way that either boosts or hinders entrepreneurship. As an example, the entrepreneurial dynamism of the US economy is in sharp contrast to the relatively low level of firm creation in Europe. A survey on entrepreneurship conducted in 2002 reports that while an estimated 10% of a representative sample of the US stated that they were currently engaged in the process of creating a nascent business, this figure is below 2% for Japan and France, and below 4% for most European countries. We thus mention that the complementarity between technical change and ability in the decision to create a business venture may influence the nature of nascent entrepreneurship in an economy. Our model focuses on a specific characteristics of nascent entrepreneurship based on adaptability to a changing environment. Several extensions could be considered to enlarge this analysis. In particular, introducing other factors such as competition between firms could yield insightful results and constitutes an area for our future research.
6 Appendix

6.1 Description of the development stage

In the development stage, we consider an economy composed of two sectors: a final good sector and an intermediate goods sector. Intermediate goods are used as factors of production in the final good sector. The final good is produced using two different types of intermediate goods: The goods produced by continued firms (labelled c) and the goods produced by refinanced firms (labelled r).

The production function is a Cobb-Douglas:

$$y = x_c^\alpha x_r^{1-\alpha}$$

where $y$ is the final good production in a competitive environment using both continued firms’ goods, $x_c$, and refinanced firms’ goods, $x_r$, and where $0 < \alpha < 1$.

The profit maximization problem by a representative firm in this sector leads to the following inverse demand for inputs:

$$p_c = \frac{\partial y}{\partial x_c} = \alpha x_c^{\alpha-1} x_r^{1-\alpha}$$

$$p_r = \frac{\partial y}{\partial x_r} = (1 - \alpha) x_c^\alpha x_r^{-\alpha}$$

where $p_c$ denotes the price of continued firms’ goods and $p_r$ the price of refinanced firms’ goods. Consequently, the equilibrium price of each intermediate good, $x_c$ and $x_r$, is given by its marginal product.

Intermediate goods are used to produce the final good according to a one-for-one technology. In particular, it is assumed that $x$ units of final good requires $x$ units of intermediate goods. Given the inverse demand for intermediate goods in the final good sector (19) and (20), the optimization program for continued firms, $c$ and for refinanced firms, $r$ is given by:

$$\max_{x_c} p_c x_c - x_c = \alpha x_c^\alpha x_r^{1-\alpha} - x_c.$$  

$$\max_{x_r} p_r x_r - x_r = (1 - \alpha) x_c^\alpha x_r^{1-\alpha} - x_r.$$  

from where we obtain the profit-maximizing prices and the flow of profits for
each type of business:

\[
p_c = \frac{1}{\alpha}, \quad p_r = \frac{1}{1 - \alpha} \\
V_c = \frac{1 - \alpha}{\alpha} \cdot x_c, \quad V_r = \frac{\alpha}{1 - \alpha} \cdot x_r
\]  

(21)

6.2 Proofs

6.2.1 Proof of Proposition 1

The proof is made for a general set of functions \( h(.) \), in particular when \( h(x) = x \) as in (3).

Note that function \( h(.) \) is continuous and strictly increasing. Then, the inverse function, \( h^{-1} \) is also continuous and strictly increasing. From equation (3) we get

\[
\frac{\partial a^*}{\partial \gamma} = h^{-1}'(\delta \gamma (1 - \delta \gamma)) \cdot \delta (1 - 2 \delta \gamma)
\]

where \( h^{-1}'(.) > 0 \).

Given that the number of entrepreneurs that choose to run type H firms is equal to \( 1 - a^* \), we have

\[
\frac{\partial H}{\partial \gamma} > 0 \iff (1 - 2 \delta \gamma) < 0
\]

6.2.2 Proof of Proposition 2

(i) After simple manipulation, the interest rates defined by equations (15) and (16) write:

\[
R_r = \frac{1}{\lambda \kappa (1 - \delta \gamma)} \\
R_c = \frac{1}{\lambda (2 - \delta \gamma)}
\]

After some simple algebra we have

\[
R_r - R_c = \frac{1 - \delta \gamma + 1 - \kappa (1 - \delta \gamma))}{\lambda \kappa (1 - \delta \gamma)(2 - \delta \gamma)}
\]
We thus get $R_c < R_r$, which implies that the interest rates charged to a failed entrepreneur who restarts his project is greater than the prevailing interest rates for a first timer entrepreneur.

(ii) The derivative of $R_r$ and $R_c$ with respect to $\gamma$ is immediate from equations (15) and (16): both interest rates are increasing in the rate of technical change.

6.2.3 Proof of Proposition 3

Substituting for (15) and (16) into (14) and rewriting yields:

$$L_c = \Lambda \cdot \{\Phi - \Psi(a^*(\gamma), \gamma) - \Omega(a^*(\gamma), \gamma)\}$$

where given (17)

$$a^*(\gamma) = \delta \gamma (1 - \delta \gamma)$$

$$\Lambda = \frac{1}{\frac{a}{1-a} \kappa^2 + \frac{1-a}{\alpha}}$$

$$\Phi = \frac{\alpha}{1 - \alpha^2}$$

$$\Psi(a^*(\gamma), \gamma) = \frac{1 - a^*(\gamma)}{a^*(\gamma)} \frac{1}{1 - \delta \gamma} \frac{1 - \alpha}{\alpha}$$

$$\Omega(a^*(\gamma), \gamma) = -\frac{R_c - \kappa R_r}{\lambda(1 - \delta \gamma) a^*(\gamma)} = \frac{1}{\lambda^2(1 - \delta \gamma)^2(2 - \delta \gamma) a^*(\gamma)}$$

Deriving with respect to $\gamma$ then gives:

$$(L_c)' = \frac{\partial L_c}{\partial \gamma} = \Lambda\{-\Phi' - \Omega'\}$$

$$(\Psi)' = \frac{\partial \Psi(a^*(\gamma), \gamma)}{\partial \gamma} = \frac{1 - \alpha - (a^*)'}{\alpha} \frac{1}{(a^*)^2} \frac{1}{1 - \delta \gamma} + \frac{1 - \alpha}{\alpha} \frac{1 - a^*}{a^* (1 - \delta \gamma)^2} \delta$$

$$(\Omega)' = \frac{\partial \Omega(a^*(\gamma), \gamma)}{\partial \gamma} = -\frac{(a^*)'(1 - \delta \gamma)(2 - \delta \gamma) + a^* \delta (1 - \delta \gamma) + 2a^* \delta (2 - \delta \gamma)}{\lambda^2(a^*)^2(1 - \delta \gamma)^3(2 - \delta \gamma)^2}$$

$$(a^*)' = \frac{\partial a^*(\gamma)}{\partial \gamma} = \delta (1 - 2 \delta \gamma)$$

From these equations, we can state that whenever $(a^*)' < 0$, then $(L_c)' < 0$. In other words, when $\gamma > 1/2\delta$, $L_c$ decreases with the growth rate $\gamma$. The opposite case where $\gamma < 1/2\delta$ does not yield a simple analytical result and is only illustrated graphically with numerical simulations, reproduced in appendix 6.3.
6.3 Numerical simulations with exogenous technical change

To run the numerical simulations, the parameters values are then given as follows: \( \alpha = 0.97, \delta = 5, \lambda = 0.9, \kappa = 1 \). The following figures illustrate the links we established between the different variables explored in this paper. They also serve as an illustration of the propositions stated above.

We then get the following curves corresponding to the different effects highlighted in propositions 1 to 3. Figure A and B draw \( a^{ast} \) and \( H \) as functions of \( \gamma \). We observe that \( a^{ast} \) is a inverted-U shaped curve and \( H \) is a U-shaped curve. Regarding figure C, plotting \( L_c \): when \( \gamma > \tilde{\gamma} = 1/2\delta = 0.10 \), \( L_c \) decreases with the growth rate. On the other hand, when \( \gamma < \tilde{\gamma} = 1/2\delta = 0.10 \), \( L_c \) first increases (stigmatization is high) and as \( \gamma \) gets closer to the threshold value \( \tilde{\gamma} \), it starts decreasing (stigmatization starts decreasing).
The rate of technological progress is governed by the equation \( \gamma = \varepsilon \lambda (1 - \delta \gamma + \delta^2 \gamma^2) \).

Let analyze the function \( \phi(\gamma) = \gamma - \varepsilon \lambda (1 - \delta \gamma + \delta^2 \gamma^2) \).

We have:

- \( \phi(0) = -\varepsilon \lambda < 0 \), \( \phi(1) = 1 - \varepsilon \lambda (1 - \delta + \delta^2) \)
- \( \phi'(\gamma) = 1 + \varepsilon \lambda \delta - 2\delta^2 \varepsilon \lambda \gamma \), \( \phi'(0) = 1 + \varepsilon \lambda \delta > 0 \)
- \( \phi''(\gamma) = -2\varepsilon \lambda \delta^2 < 0 \)

Under assumption 4, \( \varepsilon < \frac{1}{\lambda(1-\delta+\delta^2)} \), we get: \( \phi(1) = 1 - \varepsilon \lambda (1 - \delta + \delta^2) > 0 \).

Furthermore,

- \( \varepsilon \lambda < \frac{1}{1 - \delta + \delta^2} = \frac{1}{(1 - \delta)^2 + \delta} < \frac{1}{\delta} \)

and

- \( \varepsilon \lambda < \frac{1}{\delta} \Rightarrow 1 + \frac{1}{\varepsilon \lambda \delta} > 2 \)

Finally, under assumption 1, \( 0 < \delta \gamma < 1 \Rightarrow 2\delta \gamma < 2 \), we therefore get

- \( 2\delta \gamma < 2 < 1 + \frac{1}{\varepsilon \lambda \delta} \Rightarrow 1 + \varepsilon \lambda \delta - 2\delta^2 \varepsilon \lambda \delta > 0 \Leftrightarrow \phi'(\gamma) > 0 \)

In sum, we have shown that under assumption 1 and 4, the equation governing the rate of technological progress \( \phi(\gamma) = \gamma - \varepsilon \lambda (1 - \delta \gamma + \delta^2 \gamma^2) \) is such that:

- \( \phi(0) < 0 \), \( \phi(1) > 0 \), \( \phi'(\gamma) > 0 \), \( \phi'(0) > 0 \), \( \phi''(\gamma) < 0 \)

Hence, the rate of technical change \( \hat{\gamma} \) that solves \( \phi(\gamma) = 0 \) exists, is unique and such that \( \hat{\gamma} \in (0, 1) \).

To illustrate the static comparative of \( \hat{\gamma} \) with respect to the parameters of the model, some numerical simulations are reported below. The parameters values are then given as follows: \( \alpha = 0.97 \), \( \delta = 5 \), \( \kappa = 1 \). The variable \( \tilde{\varepsilon} \) corresponds to the threshold level defined in assumption 5: \( \varepsilon < \tilde{\varepsilon} \equiv \frac{1}{\lambda(1-\delta+\delta^2)} \).

Table I: Rates of technological change for different values of \( \lambda \), \( \delta \) and \( \varepsilon \)

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>( \lambda )</th>
<th>( \tilde{\varepsilon} )</th>
<th>( \varepsilon )</th>
<th>( \gamma )</th>
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<td>0.37</td>
<td>0.3</td>
<td>0.204</td>
</tr>
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<td>0.024</td>
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<td>0.003</td>
<td>0.029</td>
<td>0.0025</td>
</tr>
<tr>
<td>5</td>
<td>0.9</td>
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<td>0.038</td>
</tr>
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References


