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Abstract

An open economy version of the Baxter and King's [1993] model is constructed with habit formation to investigate the dynamic and steady-state effects of an expansionary budget policy. In line with empirical evidence, consumption is weakly responsive, investment is crowded out, the drop in savings drives the current account into deficit and government spending multipliers display small values. The sensitivity analysis shows that the effectiveness of the fiscal policy (1) decreases as habit persistence gets stronger, (2) increases with labor supply responsiveness, (3) falls with trade integration. Finally, we find that habit persistence weakens the connection between government spending multipliers and both the elasticity of labor supply and exports-to-GDP ratio.

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1 Introduction

There has been recently a renewed interest in fiscal stimuli among policy makers as an instrument of stabilization. Coincidentally, the exploration of the macroeconomic effects of fiscal shocks has received notable attention in the empirical literature. While there is a dispersion of estimates across countries and time periods, it is possible to draw up a list of major empirical facts. However, as will be shown later, the theoretical explanation of these regularities remains unsatisfactory as the predictions of the open economy version of the Baxter and King's [1993] model are at odds with empirical facts. In this paper, we show that empirical evidence can be easily reconciled with neoclassical theory as long as agents display a habit-forming behavior.

Empirically, four key regularities can be established. A large strand of the empirical literature finds that an expansionary budget policy raises both worked hours and GDP, yet reveal that fiscal multipliers display small values (see e. g. , Perotti [2005] and Mountford and Uhlig [2008]). More precisely, Perotti [2005] documents a decrease in government spending multipliers in the post-1980 period in comparison with the pre-1980 period. One of the most prominent and consistent findings of the empirical literature is that investment is crowded out by public spending in the short-run (see e. g. , Afonso and Sousa [2009], Blanchard and Perotti [2002], Mountford and Uhlig [2008] and Perotti [2005]). In contrast, empirical findings about the impact of fiscal policy on consumption are rather diverse; in our view, the overall conclusion that can be drawn is that consumption is weakly responsive to fiscal policy on impact (see e. g. , Afonso and Sousa [2009], Mountford and Uhlig [2008], Perotti [2005] in the post-1980 period). Finally, for 14 European Union countries over the period 1970-2004, Beetsma, Giuliodori and Klaassen [2008] find that a government spending shock produces a trade balance deficit.

To emphasize the necessity of introducing time non-separable preferences to accommodate these empirical evidence, it is convenient to shed light on the mechanics of transmission of fiscal shocks in the baseline neoclassical RBC framework (see e. g. , Karayalçin [1999]).[1] In the open economy version of Baxter and King's [1993] model, households perceive the rise in public spending as a tax liability. The consecutive fall in the real permanent income induces the representative agent to consume less and work more. Due to the fixity of the rate of time preference, consumption overshoots its steady-state level and thereby releases resources for capital accumulation. Since saving behavior plays a minor role, the resulting investment boom drives the current account into deficit and raises the size of expenditure multipliers.

However, these conclusions enter in sharp contradiction with recent empirical findings. First, consumption reacts very smoothly to a fiscal shock. Second, investment is crowded out by public spending instead of being crowded-in. Third, short-term and long-term spending multipliers display small values since they average less than 0.5 in the long-run (see e. g. , Hemming et al. [2002] for a review). Finally, while estimates suggest that a fiscal shock causes a deterioration in the external asset position, empirical evidence documented by Freund [2005] reveals that short-run current account deficits episodes are more associated with a decline in savings than with an increase in investment.

Our main contribution is to show that the introduction of a habit-forming behavior in a two-good open economy version of the Baxter and King's [1993] can account for the empirical evidence related to the macroeconomic effects of fiscal shocks. Implicitly, in our framework, the consumers' reaction to a fiscal impulse is a mix of Ricardian and Keynesian behaviors. In a Ricardian manner, forward-looking consumers react to the rise in government expenditure by cutting real expenditure. In a Keynesian manner, households reduce their real consumption on impact but by a smaller amount than the decline in their disposable income. As higher government spending withdraws resources to the private sector and households' consumption reacts weakly to the fiscal shock, an excess demand arises in the home goods market. Hence, due to consumption inertia, investment is crowded out by public spending. The fall in investment lowers expenditure multipliers which thereby display smaller sizes than those predicted by standard models assuming time separability in utility. Since households wish to sustain their original standard of living, the resulting drop in private savings worsens the current account. In addition, the economic boom following a fiscal expansion is associated with a decumulation of internationally traded bonds, thus corroborating the counter-cyclicality of the current account found in the data (see e. g. , Backus, Kehoe and Kydland [1994]).

Oddly, so far, the literature has been silent about the connection between households' preference parameters and the effectiveness of fiscal policy. Plotting initial and steady-state expenditure multipliers against the weight of habits in utility, we show that stronger habit persistence lowers the effectiveness of fiscal policy. The reason is that (1) in the short-term, investment is crowded out further by public spending, and (2) in the long-term, consumption and exports fall by a larger amount. A second key policy question is how consumption inertia and labor responsiveness interact in determining the effectiveness of fiscal policy. Like Baxter and King [1993], numerical experiments revealed that government spending multipliers rise as labor supply is more responsive. However, existence of consumption inertia prevents a balanced-budget fiscal policy from displaying an expenditure multiplier greater than unity by moderating the initial stimulus of demand triggered by higher public spending. Finally, estimating the response of multipliers to a greater trade openness, we find that trade integration reduces the effectiveness of fiscal policy. Yet, habit persistence weakens the relationship between multipliers and trade openness.

The remainder of the paper is organized as follows. In section 2, we develop an open economy version of the neoclassical model with habit formation. Section 3 shows how consumption inertia allows to reconcile the predictions of the neoclassical model with recent empirical evidence and discusses the mechanics of transmission of fiscal shocks. In section 4, we conduct a sensitivity analysis by estimating the relationship between the effectiveness of fiscal policy and key preference parameters. Section 5 summarizes our main results and concludes.

2 An Open Economy Model with Habit Formation

We consider a semi-small open economy that is populated by a large number of identical households and firms that have perfect foresight and live forever. The country is assumed to be semi-small in the sense that it is price-taker in international capital markets but is large enough on world good markets to influence the price of its export goods.

2.1 Households

At each instant the representative household consumes domestic goods and foreign goods denoted by c^D and c^F, respectively, which are aggregated by means of a CES function:

$$c = \left[\varphi^{\frac{1}{\phi}} \left(c^D\right)^{\frac{\phi-1}{\phi}} + (1-\varphi)^{\frac{1}{\phi}} \left(c^F\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{1}$$

where φ is the weight of the domestic good in the overall consumption bundle ($0 < \varphi < 1$) and ϕ corresponds to the intratemporal elasticity of substitution between domestic and foreign consumption goods.

The representative agent is endowed with a unit of time, supplies a fraction $n(t)$ as labor, and the remainder $l(t) \equiv 1 - n(t)$ is consumed as leisure. At any instant of time, households derive utility not only from their current real consumption $c(t)$ but also from their current level of habits denoted by $s(t)$. Hence, assuming that the felicity function is additively separable in consumption and labor, the representative household maximizes the following objective function:

$$U = \int_0^{\infty} \left\{ \frac{1}{1-\epsilon} \left[\frac{c(t)}{(s(t))^{\gamma}}\right]^{1-\epsilon} - \gamma_N \frac{n(t)^{1+\frac{1}{\sigma_N}}}{1+\frac{1}{\sigma_N}} \right\} e^{-\beta t} \mathrm{d}t, \tag{2}$$

where β is the consumer's discount rate, $\epsilon > 0$ corresponds to the coefficient of relative risk aversion, and $\gamma > 0$ stands for the weight attached to habits s into utility; $\sigma_N > 0$ is the Frisch elasticity of labor supply (or intertemporal elasticity of substitution for labor supply).

The habitual standard of living is defined as a distributed lag on past real consumption:

$$s(t) = \sigma \int_{-\infty}^{t} c\left(c^D(\tau), c^F(\tau)\right) e^{-\sigma(t-\tau)} \mathrm{d}\tau, \tag{3}$$

where the parameter σ indexes the relative weight of recent consumption in determining the reference stock s. Differentiating equation (3) w. r. t. time gives the law of motion of habit stock:

$$\dot{s}(t) = \sigma \left[c(t) - s(t)\right]. \tag{4}$$

Intuitively, the larger σ, the greater the weight of consumption in the recent past in determining the stock of habits, and the faster the reference stock adjusts to current expenditure.

In line with Carroll, Overland and Weil [2000], we have assumed that the utility derived from current and past consumption takes an iso-elastic form. The felicity function can be rewritten as $u(c,s) = \frac{1}{1-\epsilon}\left[(c)^{1-\gamma}\left(\frac{c}{s}\right)^{\gamma}\right]^{1-\epsilon}$ according to which, agents derive utility from a geometric weighted average of absolute and relative consumption where γ is the weight of relative consumption. If $\gamma = 0$, the comparison of real consumption to the reference stock turns out to be irrelevant and the case of time separability in preferences is obtained. If γ is positive, agents care about their relative consumption in deriving utility. In this case, they are aware that for a given fall in current consumption, the utility loss is now reduced by the consecutive decrease in habits. Because the slope of the indifference curves along a constant consumption path rises, individuals' impatience increases. Therefore, real consumption deviates from the usual perfectly smooth temporal profile and declines over time, whenever the real interest rate is unchanged.

By shifting the time paths for consumption and thereby savings, the introduction of habits affects the long-term response of consumption to a change of private wealth dictated by the long-run intertemporal elasticity of substitution (henceforth, IES). More specifically, by noting that habits coincide with real consumption in the long run, setting $c = s$ into the iso-elastic function above yields a long-run IES denoted by ν equal to $\frac{1}{[\gamma+\epsilon(1-\gamma)]}$ which is higher than the standard IES $1/\epsilon$ as long as $\epsilon > 1$. By contrast, the reference stock is fixed over a short interval of time. Hence, the short-horizon and long-horizon elasticities will not coincide as long as $\gamma > 0$.[2]

Households supply $n(t)$ units of labor services for which they receive the wage rate $w(t)$. They hold the physical capital stock for which they receive the capital rental rate. In addition, they accumulate internationally traded bonds, $b(t)$, that yields net interest rate earnings $r^{\star}b(t)$, expressed in terms of the foreign good. Denoting by T the lump-sum taxes, the flow budget constraint is equal to households' real disposable income less consumption $p_c c$ and investment I expenditure:

$$\dot{b}(t) = \frac{1}{p(t)}\left\{r^{\star}p(t)b(t) + \left(r^K(t)+\delta_K\right)k(t) + w(t)n(t) - p_c\left(p(t)\right)c(t) - I(t) - T(t)\right\}, \quad (5)$$

where p_c is the consumption price index and p the relative price of the foreign good (or the real exchange rate). Investment I leads to capital accumulation:

$$\dot{k}(t) = I(t) - \delta_K k(t), \tag{6}$$

where $0 \leq \delta_K < 1$ is a fixed depreciation rate.

2.2 Firms

A large number of identical and perfectly competitive firms produce a final good which can be consumed domestically, invested or exported. They use physical capital k and labor n, according to a constant returns to scale production function, $Y = F(k, n)$, which is assumed to have the usual neoclassical properties of positive and diminishing marginal products. Factor markets are assumed to be competitive, so that capital and labor are paid their marginal products, i. e. , $F_k = r^K + \delta_K$ for capital and $F_n = w$ for labor.

2.3 Government

The government finances public spending by raising lump-sum taxes T according to the following balanced condition $T = g^D + pg^F$; g^D and pg^F correspond to the government purchases falling on the domestic and the foreign good respectively, measured in domestic units.

2.4 Macroeconomic Dynamics

In an open economy model with a representative agent having perfect foresight, a constant rate of time preference and perfect access to world capital markets, we require that $\beta = r^\star$ in order to generate an interior solution. This standard assumption made in the literature implies that the marginal utility of wealth, λ, will undergo a discrete jump when individuals receive new information and must remain constant over time from thereon.[3]

The adjustment of the semi-small open economy towards the steady-state is described by a fourth order dynamic system which comprises the accumulation equation of habits (i. e. , eq. (4)) and the dynamic equation for the real exchange rate dynamics which equalizes the rates of return on domestic r^K and foreign assets $r^\star + \dot{p}/p$.[4] The third equation is the law of motion of real consumption which relates the optimal profile of c to the difference between the consumption-based real interest rate, $r^c = r^\star + (1-\alpha_c)\frac{\dot{p}}{p}$, and the rate of time preference, denoted by $\rho(t)$:

$$\frac{\dot{c}}{c} = \frac{1}{\epsilon}\left[1+\sigma\xi c^{\epsilon}s^{\gamma(1-\epsilon)}\right]\left[r^\star + (1-\alpha_c)\frac{\dot{p}}{p} - \rho(t)\right], \tag{7}$$

where $0 < \frac{1}{\epsilon}\left[1+\sigma\xi c^{\epsilon}s^{\gamma(1-\epsilon)}\right]$ stands for the short-run IES under time non separable preferences which is smaller than $1/\epsilon$. The key point to note is that the rate of time preference is variable over time. The reason is that habits gradually adjust to their steady-state value which in turn leads to a discrepancy between the current and future marginal utility of consumption.

Denoting by X the exports to the rest of the world, we impose a domestic good market clearing condition to fully describe the macroeconomic equilibrium:[5]

$$\dot{k} = F(k,n) - c^D - X(p) - \delta_k k - g^D, \tag{8}$$

where investment adjusts to equalize supply and aggregate demand along the transitional path. Hence, capital accumulation is affected by habit persistence which impinges on consumption in the domestic good c^D.

Consumption inertia also influences the transitional adjustment of the stock of financial wealth denoted by a and thereby the current account dynamics which is the reflection of savings and investment behavior:

$$ca(t) = p(t)\dot{b}(t) = \dot{a}(t) - \dot{k} - b\dot{p}(t), \tag{9}$$

where we denote by ca the current account measured in terms of the domestic good; the stock of financial wealth $a(t) \equiv p(t)b(t) + k(t)$ evolves according to the following law of motion: $\dot{a}(t) = r^K(t)a(t) + w(t)n(t) - T - p_c(p(t))\,c(t)$ where savings are equal to the real disposable income less total expenditure in consumption goods.

2.5 Steady-State

We now discuss the salient features of the steady-state. The assumptions of constant returns to scale and an exogenously given world interest rate imply that the steady-state capital-labor ratio remains unchanged after a fiscal expansion. Since the steady-state value of the capital-labor ratio remains fixed, i. e. $\frac{\tilde{k}}{\tilde{n}} = \left(\frac{\alpha_K}{r^\star+\delta_K}\right)^{\frac{1}{1-\alpha_K}}$, the marginal product of labor and thereby the wage rate are also unaffected in the long-run. By substituting the wage rate into the labor supply decision evaluated at the steady-state, we get $\tilde{n} = \left[\frac{1}{\gamma_N}\frac{\bar{\lambda}}{\tilde{p}}\left(1-\alpha_K\right)\left(\frac{\tilde{k}}{\tilde{n}}\right)^{\alpha_K}\right]^{\sigma_N}$. Hence, a decrease in private wealth measured in terms of the domestic good (i. e. , a rise in $\bar{\lambda}/\tilde{p}$) stimulates labor supply in the long-run.

Setting $\dot{c} = \dot{p} = 0$ and using the fact that real consumption coincides with the habit stock once economy reaches the long-term equilibrium enables us to derive the steady-state level of real consumption:

$$\tilde{c} = \left[\left(\frac{\beta+\sigma}{\beta+\sigma\left(1-\gamma\right)}\right)\frac{p_c\bar{\lambda}}{\tilde{p}}\right]^{-\nu}. \tag{10}$$

As long as households attribute a positive weight to relative consumption (i. e. , $\gamma > 0$) the rate of change of c is influenced by the long-term IES with time non separable preferences, ν, which exceeds $\nu^{\gamma=0} = 1/\epsilon$. Therefore, confronted to a given reduction of private wealth, habit-forming consumers experience a larger steady-state fall in consumption than households displaying standard preferences.[6]

The market-clearing condition for the home good determines the steady-state value of the real exchange rate. The relative price $\tilde{p}$ adjusts so that the production of domestic goods is exactly outweighed by a demand counterpart:

$$Y\left(\bar{\lambda},\tilde{p}\right) = c^D\left(\bar{\lambda},\tilde{p}\right) + X\left(\tilde{p}\right) + \delta_K k\left(\bar{\lambda},\tilde{p}\right) + g^D, \tag{11}$$

where we have rewritten $\tilde{c}^D = p_c\left(1-\alpha_c\right)\tilde{c}$ as $c^D\left(\bar{\lambda},\tilde{p}\right)$. Equation (11) allows us to express the steady-state value of the real exchange rate as a function of $\bar{\lambda}$ and g^D, i. e. , $\tilde{p} = p\left(\bar{\lambda},g^D\right)$, with $p_{\bar{\lambda}} > 0$ and $p_{g^D} < 0$. Consider a rise in g^D, assuming that $\delta_K = 0$ for simplicity. If preferences are time separable and the economy is closed, a fiscal expansion crowds out consumption in the domestic good c^D by reducing private wealth which results in a multiplier inferior to 1. If preferences are time non separable, c^D declines dramatically in response to the rise in $\bar{\lambda}$ which yields a multiplier much less than 1. Finally, in an open economy with habits, private demand for the domestic good shrinks more in comparison with a closed economy through an additional channel. The long-run real exchange appreciation triggers a larger reduction in the size of the long-run multiplier by causing a decline in exports and by inducing agents to consume less of the domestic good.

In a framework that considers standard preferences, the current account is mainly affected by investment fluctuations, due to the fixity of the time preference rate which implies a flat temporal path for consumption. Instead, the assumption of time non separable preferences implies that both investment decisions and consumption choices affect the external asset position. More precisely, the linearized version of the intertemporal solvency condition allows the steady-state level of net foreign asset position to be determined by the long-run levels of physical capital and habits: $\left(\tilde{b} - b_0\right) = \Phi_1 \left(\tilde{k} - k_0\right) + \Phi_2 \left(\tilde{s} - s_0\right)$ with $\Phi_1 < 0$ and $\Phi_2 > 0$ for all parametrization. The intertemporal budget constraint describes the long-run trade-off between capital and habits on the one hand, and net foreign assets consistent with solvency on the other hand. A steady-state decrease in habits (increase in capital) causes a decumulation of the stock of foreign assets by causing a drop in savings (a rise in investment).

3 Fiscal Expansion and Habit Persistence

In this section, we explore the macroeconomics effects of a rise in government spending on the domestic good (g^D). While the model can be solved analytically, we propose some numerical simulations to illustrate key theoretical results and discusses fiscal policy implications.[7]

3.1 Benchmark Parametrization

We start by describing the calibration that we use as a benchmark. The world interest rate which is equal to the subjective time discount rate β is set to 3.5%. The elasticity of substitution between domestic and foreign goods ϕ is set to 1.5. Exports are assumed to take a power form $X = X(p) = \gamma_X p^{\nu_X}$ with $\gamma_X > 0$ a scaling parameter. The real exchange rate elasticity of exports denoted by ν_X is set to 0.8 in line with estimates by Bayoumi [1999].[8] Government spending as a share of GDP g/Y is 20%. The import content of government expenditure is set to 10%. Finally, the production function takes a Cobb-Douglas form $Y = F(k,n) = k^{\alpha_K} n^{1-\alpha_K}$ with the share of capital in output which is assumed to equal 0.35. Finally, we set δ_K to 0.035 so as to be consistent with a ratio investment-GDP roughly equal to 18%.[9] All the previous parameter values remain unchanged in the analysis below. Next, we turn to the parameters for which we conduct some sensitivity analysis.

Our baseline setting for the weight of the domestic good φ is 0.95 which corresponds to an import content of consumption (α_c) of 15% approximately. Two additional critical parameters are the weight of habits in utility, γ, and the speed σ at which the standard of living catches up with current consumption. From empirical results provided by Sommer [2007], estimations of γ fall in a range between 0.7-0.8. We set γ to 0.8. The relative-risk aversion parameter, ϵ, is set to 2.5, yielding a long-term IES of 0.77. We set σ to 0.65 which means that the time required to close 95% of the discrepancy between $s(t)$ and $c(t)$ following a change in $c(t)$ is roughly four and a half periods. The last critical parameter is the intertemporal elasticity of substitution for labor supply σ_N. In our benchmark parametrization, we set σ_N to 0.4 which is halfway of the values documented by the empirical literature and chosen by RBC models.[10]

Quantitative estimation of macroeconomic effects of a rise in government spending shock by 1 percentage point of initial output are reported in Table 1. We consider seven alternative scenarios: time separable preferences (i. e. , $\gamma = 0$), benchmark parametrization (i. e. , $\gamma = 0.8$, $\sigma = 0.65$, $\sigma_N = 0.4$, $\varphi = 0.95$), a smaller weight of habits in utility (i. e. , $\gamma = 0.3$), a faster speed at which habits catch up with current consumption (i. e. , $\sigma = 0.95$), a weakly responsive labor supply (i. e. , $\sigma_N = 0.2$), a highly responsive labor supply (i. e. , $\sigma_N = 1$), and a larger trade openness (i. e. , $\varphi = 0.825$). Computed transitional paths of key variables are displayed in Figures 1 where we abbreviate time separable preferences in “TS pref.”. In all figures, the responses of consumption, GDP, employment and investment are expressed as deviations from initial steady-state values scaled by initial GDP (in percentage).

3.2 Steady-State Effects of a Permanent Fiscal Expansion

We now discuss the steady-state changes after a fiscal expansion summarized in Table 1A. By increasing lump-sum taxes and thereby lowering the representative agent's real disposable income, a permanent balanced-budget fiscal expansion induces agents to raise labor effort while reducing real consumption. The positive labor supply effect stimulates capital by shifting up its marginal product schedule which boosts output. Whereas households' consumption falls, the stimulus of output is not large enough to compensate the excess of demand in the home good market triggered by higher public spending. This requires a rise in the relative price of the domestic good (i. e. , a real exchange rate appreciation). The fall in p depresses consumption in the domestic good $\tilde{c}^D$ more and causes a decline in exports $\tilde{X}$, regardless of the form of preferences.

Table 1A provides a sensitivity analysis regarding the long-term effects of raising public spending. The first line allows us to compare the long-run adjustment of consumption depending on whether preferences are time separable or time non separable. Whereas consumers cut their real expenditure by around 0.45% of initial GDP if preferences are time separable (i. e. , $\gamma = 0$), $\tilde{c}$ declines by roughly 0.65% for the benchmark parametrization (i. e. , $\gamma = 0.8$). Intuitively, as the weight of habits in utility increases, households are more reluctant to cut their real expenditure in the short-run. Thereby, households decumulate more financial wealth. Since agents must ultimately satisfy their intertemporal solvency condition, a larger steady-state fall in consumption is required. From the fifth line, the necessary decline in the stock of foreign assets rises from 0.3% to about 0.45% of initial GDP as γ is raised from 0 to 0.8. The response of output to a fiscal impulse will be addressed later in section 4.

< Please insert Table 1 about here >

3.3 Habits Reconcile Theory with Empirical Facts

In this subsection, we discuss the short-run reaction of the open economy and show that the introduction of habit persistence in consumption allows to reconcile the predictions of the neoclassical model with recent empirical evidence.

Consumption Reacts Weakly to a Fiscal Expansion

Mountford and Uhlig [2008] found that consumption does not change significantly in response to a positive spending shock in the U.S. This empirical result has been recently confirmed by Afonso and Sousa [2009] for additional OECD countries like the U.K., Germany, and Italy. To address the tenuous response of consumption, it is convenient to simplify the analysis by assuming that the capital stock is fixed, which implies a one-dimensional stable path for consumption. Substituting the stable solution for $c(t)$ evaluated at time $t = 0$ into the households' budget constraint yields the initial optimal reaction of consumption:

$$c(0) = \left(\frac{\sigma + \mu_1}{\sigma}\right)\left[1 + \frac{\tilde{\Pi}}{\tilde{\Lambda}}\frac{r^\star \mu_1}{\sigma}\right] s_0 - \frac{\mu_1}{\sigma}\frac{r^\star - \mu_1}{\tilde{\Lambda}}\frac{\tilde{p}r^\star\left[b_0 + W(0)\right]}{p_c}, \tag{12}$$

where $\tilde{\Pi} \geq 1$, $\tilde{\Lambda} = (r^\star - \mu_1) - \tilde{\Pi}\left(\frac{\sigma+\mu_1}{\sigma}\right) r^\star$; $-\frac{\mu_1}{\sigma}\frac{r^\star-\mu_1}{\tilde{\Lambda}}$ represents the short-run marginal propensity to consume (henceforth, MPC) of the real permanent income $\frac{\tilde{p}r^\star[b_0+W(0)]}{p_c}$. In a small open economy model where the relative price of foreign goods p is exogenous, $\tilde{\Pi} = 1$ and $\tilde{\Lambda} = -\frac{\mu_1}{\sigma}(\sigma + r^\star)$, the short-run MPC simplifies to $\frac{r^\star-\mu_1}{\sigma+r^\star}$. If preferences are time separable (i. e. , $\gamma = 0$), the stable root μ_1 reduces to $-\sigma$ which results in a MPC equal to unity. In contrast, as long as people care about habits in deriving utility (i. e. , $\gamma > 0$), inequality $\sigma > -\mu_1$ holds, which implies that the short-run MPC is unambiguously inferior to one. The explanation relies upon the behavior of the time preference rate which is variable over time. Following a fiscal expansion, agents expect a long-term fall in their usual standard of living. However, when the fiscal expansion is implemented, the stock of habits does not change, so that the marginal utility of current consumption exceeds that of future consumption. This provides a strong incentive to reallocate expenditure in present and yields a smaller than unity short-term MPC. Consequently, real consumption falls on impact but less than the decline in the real disposable income. Considering now the case of an endogenous real exchange rate, the short-term MPC rises. The reason is that the real exchange rate depreciation allows for a smoother real consumption temporal path. Such a consumption behavior is consistent with the intertemporal solvency condition as long as consumption falls by a larger amount in the short-run than if the real exchange rate was constant over time. Yet, the MPC is always smaller than unity, as long as $\tilde{\Pi} < \frac{r^\star}{r^\star - \mu_1}$, which holds for our baseline parametrization.

Figure 1(a) displays the dynamic adjustment of consumption in the benchmark case and compares it to alternative scenarios. In all scenarios, households behave in a Ricardian manner and thus react to the fall in their after-tax lifetime income by reducing their real expenditure. However, in a Keynesian manner, consumption does not adjust immediately and fully, as long as household care about habits in deriving utility. More precisely, the short-run MPC displays a value inferior to one. Hence, habit-consumers prefer to adjust less-than-fully now and more-than-fully later (see the blue line). By contrast, households having time separable preferences exhibit a much larger short-run MPC which results in an over-reaction of consumption on impact (see the red dotted line). Openness is also an important factor in determining the size of the response of consumption. More precisely, a rise in trade integration moderates the drop in consumption by softening the reduction in private wealth measured in domestic units (see the black line). Finally, as expected, the initial response of consumption to a fiscal shock is all the weaker as labor supply is more responsive since the fall in the real permanent income is less pronounced (see the dotted black line).

Public Spending Crowds Out Investment

One of the most prominent and consistent findings of the empirical literature on fiscal policy is the crowding-out of investment by public spending. However, the Keynesian explanation of this empirical fact remains unsatisfactory.[11] Similarly, the prediction of the baseline neoclassical model, as exemplified by Baxter and King [1993], is also at odds with empirical facts. As we shall see now, short-run consumption inertia plays a major role in accommodating this empirical fact. To derive the initial response of investment, we differentiate the market-clearing condition for the home good:

$$\mathrm{d}I(0) = \mathrm{d}Y(0) - \Theta \mathrm{d}p(0) - \left(c^D/c\right) \mathrm{d}c(0) - \mathrm{d}g^D, \tag{13}$$

where $\Theta \equiv \frac{X}{p}\left[\nu_X + \frac{c^D}{X}\phi\alpha_c\right] > 0$ captures the impact of a change in the real exchange rate on exports and consumption. On the one hand, the stimulus of output and the fall in private demand (i. e. , exports and private consumption) releases resources for capital accumulation (see the first and second terms on the RHS of (13)). On the other hand, higher public purchases withdraw resources from the private sector, which implies the possibility that investment falls in the short-term (see the last term on the RHS of (13)). If preferences are time separable (i. e. , $\gamma = 0$), output rises and consumption falls dramatically which results in a short-run stimulus of capital accumulation. In contrast, as long as consumption inertia is strong enough, an excess of demand arises in the home goods market which must be eliminated by a fall in investment.

How do preference parameters influence the size of the crowding-out of investment by public spending? For various values of preference parameters, the resulting dynamic adjustment of investment is illustrated in Figure 1(f). If consumption inertia is strong enough (i e. , $\gamma = 0.8$, $\sigma = 0.95$) and/or labor supply is weakly responsive (i e. , $\sigma_N = 0.2$), investment expenditure is more likely to be crowded out by public spending in line with empirical evidence (see the blue and black lines). The reason is that consumption does not decrease and output does not rise enough to more than offset the rise in public spending. By contrast, an expansionary budget policy stimulates capital accumulation as long as γ approaches 0 and/or labor supply is highly responsive (see the red dotted, black dotted and red lines). Initial investment responses relative to initial GDP are summarized in the fourth line in Table 1B. If habits are irrelevant or weak (i. e. , $\gamma = 0$ or $\gamma = 0.3$), or labor supply reacts strongly (i. e. , $\sigma_N = 1$), investment is crowded in and rises by about 0.10-0.20% of initial GDP. Conversely, in the baseline scenario or assuming a high speed of habits, a rise in government spending by 1% of GDP lowers investment by 0.13-0.15% of GDP and crowds out further investment (by 0.25%) if labor supply is weakly responsive. Trade openness also influences considerably the size of the crowding-out. More precisely, a rise in export-to-GDP ratio from 11% to 21% moderates the drop in investment from 0.13% of GDP to 0.02%. The reason is that a rise in trade openness amplifies the negative impact of the real exchange appreciation on exports.

A Current Account Deficit Attributed to Savings Behavior

An important issue in an open economy framework is the reaction of the external asset position to a fiscal shock. In accordance with estimates by Beetsma, Giuliodori and Klaassen [2008] who find that an increase in public spending yields a fall in trade balance, the open economy experiences a current account deficit in all scenarios, as shown in the sixth line in Table 1B. Yet, the explanation of the current account deficit differs across cases. To see this, recall that the current account balance net of the depreciation of foreign assets, i. e. $ca(t) + b\dot{p}(t)$, is equal to savings $\dot{a}(t)$ less capital accumulation $\dot{k}(t)$. We depicted the current account adjustment for the benchmark scenario and time separable preferences in Figures 1(d) and 1(e) respectively. As portrayed in Figure 1(d), if consumption inertia is strong enough, the initial fall in investment fails to offset the large drop in savings, which results in a current account deficit. This conclusion is supported by empirical evidence documented by Freund [2005] according to which current account deficits are mostly demand-driven in the short-term. As summarized in line 6 of Table 1B, higher values of γ leads to a greater current account deficit triggered by a larger reduction in savings. Conversely, if preferences are time separable, as considered in Figure 1(e), the consumption behavior causes a rise in savings and an investment boom, the latter leading to a current account deficit on impact. Besides habit persistence, trade openness raises the size of the current account deficit as well. The reason is that as the economy is more open, government spending crowds out consumption by a smaller amount which results in a larger reduction in savings while investment falls much less. Our results are in line with those documented by Corsetti and Müller [2006] who find that more open economies than that of the US (Canada and the UK) experience greater current account deficits while the effects of fiscal shocks on investment are moderate.

3.4 Fiscal Policy Transmission with Habits

We now discuss the adjustment of the open economy illustrated in Figures 1. For pedagogical purpose, we begin with the more simple case displayed in the red dotted line where agents do not care about habits (i. e. , $\gamma = 0$). In response to the fall in real disposable income, individuals cut sizeably their real expenditure and work more. As the real exchange rate must overshoot its steady-state value, the fall in the relative price drives down exports and depresses further consumption in the domestic good while stimulating labor supply. The resulting excess of supply in the home good market allows investment to be crowded in which yields a current account deficit. The dynamic response of employment, pictured in Figure 1(c), shows that the real exchange rate depreciation exerts a negative impact on worked hours. Yet, as illustrated in Figure 1(b), capital accumulation is strong enough to push up output which increases monotonically. Over the transition, consumption follows a rising temporal path driven by the higher than steady-state consumption-based real interest rate.

Adding habits change dramatically the mechanics of transmission of fiscal shocks. The benchmark scenario is portrayed in the blue line. On impact, consumption responds weakly so that investment is crowded out. As illustrated in Figure 1(b), the short-run decumulation of physical capital and the gradual decline in employment drives down output over a first phase, which contrasts markedly with the dynamic adjustment without habits. Regarding the consumption-side, the initial rise in the time preference rate ρ is large enough to offset the increase in the consumption-based real interest rate and leads to a decreasing temporal path for households' real expenditure portrayed in Figure 1(a). The gradual decline in the reference stock reduces ρ which falls monotonically. Once consumption has decreased by a sufficient amount, the economy experiences an investment boom, which boosts GDP after five periods and exerts a negative impact on the current account. Finally, after 12 periods, the declining time preference rate equalizes the slippery slope side of the consumption-based real interest rate's hump-shaped transitional path, which induces a rising temporal path for consumption, as illustrated in Figure 1(a).

< Please insert Figures 1(a), 1(b), 1(c), 1(d), 1(e), 1(f) about here >

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4 Effectiveness of the Fiscal Policy and Habit Persistence

Our task in this section is to investigate the connection between the effectiveness of fiscal policy and key households' preference parameters. More specifically, we provide an explanation of the small value of government spending multipliers found in the data and its declining size in the post-1980 period.

4.1 The Long-Term Government Spending Multiplier

We first investigate the size of the long-term balanced-budget expenditure multiplier by differentiating the home good market-clearing condition w. r. t. g^D:[12]

$$\frac{\mathrm{d}\tilde{Y}}{\mathrm{d}g^D} = \frac{\tilde{c}^D}{\tilde{c}}\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^D} + \frac{\tilde{X}}{\tilde{p}}\left(\nu_X + \alpha_c \phi \frac{\tilde{c}^D}{\tilde{p}}\right)\frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^D} + 1. \quad (14)$$

According to the first term on the RHS of (14), the reduction of real consumption which spreads over the two goods, depresses private demand for the domestic good and thereby output. The second term on the RHS of (14) reflects the negative impact of the long-term real exchange rate appreciation on c^D and exports after a rise in g^D. Whereas private demand falls after a fiscal expansion falling on the domestic good, its decline is less than the rise in government spending by 1 percentage point of GDP. Consequently, in either seven scenarios, the expenditure multiplier is positive but inferior to 1. The response of output varies markedly across the seven scenarios. In particular, as habit persistence gets stronger (i. e. , γ gets closer to unity), real consumption and exports fall by a larger amount which result in a smaller spending multiplier.

Lines 2-4 in Table 1A shows the effects on output components. Each component is scaled by initial GDP which enables us to disentangle their contribution to the response of output. The multiplier is equal to nearly 0.7 if preferences are time separable, a value which is much higher than empirical evidence suggests. If $\gamma = 0.8$, output rises by only 0.5%: consumption in the domestic good and exports fall by 0.54% and 0.06% of initial GDP respectively while investment increases by 0.09% of initial GDP. Trade integration also slightly reduces the effectiveness of fiscal policy by reducing exports more, though moderating the drop in $\tilde{c}^D$.

4.2 The Short-Term Government Spending Multiplier

We now investigate the size of the short-term expenditure multiplier by evaluating the home good market-clearing condition at time $t = 0$ and differentiating w. r. t. g^D:

$$\frac{\mathrm{d}Y(0)}{\mathrm{d}g^D} = \frac{c^D}{c}\frac{\mathrm{d}c(0)}{\mathrm{d}g^D} + \Theta\frac{\mathrm{d}p(0)}{\mathrm{d}g^D} + \frac{\mathrm{d}I(0)}{\mathrm{d}g^D} + 1, \tag{15}$$

with $\Theta \equiv \frac{X}{p}\left[\nu_X + \frac{c^D}{X}\phi\alpha_c\right] > 0$. The first term on the RHS of (15) represents the negative influence of the short-term decline in consumption. While at first glance consumption inertia should raise the effectiveness of the fiscal policy by softening the drop in real consumption on impact, the second and the third terms counteract the positive influence of habits. More precisely, as habit persistence gets stronger, the real exchange rate appreciates more on impact which in turn depresses exports more. Additionally, investment is crowded out by a larger amount by public spending. Since these two effects predominate, the fiscal policy is less effective as γ approaches unity and σ takes higher values.

Numerical experiments reported in Table 1C show that across cases, the initial multiplier displays a smaller value than the steady-state multiplier due to the larger drop in private demand. As in the long-run, habits persistence lowers sizeably the effectiveness of fiscal policy. As we can see in lines 2-4 in Table 1B, for the benchmark parametrization, following a rise in government spending by 1 percentage point of GDP, consumption in the domestic good and exports fall by 0.36% and 0.11% respectively, while investment drops by 0.13% of initial GDP. Hence, the fiscal impulse yields a rise in output by only 0.4%, a value close to VAR results summarized by Hemming et al. [2002].[13] Instead, if γ is set to zero, the multiplier exceeds 0.5 as the open economy experiences an investment boom, though consumption declines dramatically.

4.3 Sensitivity Analysis

Our analysis shows that three critical parameters matter in determining the value of government spending multipliers: (1) the weight of relative consumption in utility γ, (2) the elasticity of labor supply σ_N, and (3) trade integration which we measure by the ratio of exports-to-GDP ratio. We have conducted a sensitivity analysis with respect to these three parameters. We allow γ to vary from 0 to 0.94, σ_N from 0.05 to 4, and φ from 0.98 to 0.20 implying an exports-to-GDP ratio which falls between 7%-52%. Figures 2(a)-2(h) plot the long-term and short-term expenditure multipliers against these three key parameters, considering time separable preferences in Figures 2(e) and 2(h).[14]

Figure 2(a) shows that the size of the long-term expenditure multiplier monotonically decreases as individuals pay more attention to relative consumption in deriving utility. The closer to unity γ, the larger the long-term fall in real consumption and thereby the smaller the long-term expenditure multiplier. Interestingly, the short-term expenditure multiplier displays a non monotonic pattern w. r. t. γ. As shown in Figure 2(b), the size of the impact multiplier rises slightly as long as γ exceeds 0.9. As stressed above, stronger habits moderate the short-term fall in consumption but amplify the decline in investment. If consumption inertia is strong enough, the former effect prevails, which results in a positive relationship between habit persistence and spending multipliers.

< Please insert Figures 2(a), 2(b), 2(c), 2(d), 2(e), 2(f), 2(g), 2(h) here >

As illustrated in Figures 2(c)-2(d), like Baxter and King [1993], both long-term and short-term expenditure multipliers rise with the intertemporal elasticity of substitution for labor σ_N. The reason is that following a demand boom triggered by public spending, domestic supply must rise. The more responsive labor supply, the larger the steady-state rise in employment, which boosts further capital accumulation in the long-run. However, as long as γ is set to

0.8, the long-term expenditure multiplier does no longer exceed unity for plausible values of σ_N. More precisely, raising the elasticity of labor supply from 0.1 to 2 yields a rise in the long-term expenditure multiplier from 0.2 to 0.9. Setting γ equal to zero now raises $d\tilde{Y}/dg^D$ from 0.3 to 1.1 as illustrated in Figure 2(e). The explanation is that the steady-state reduction of consumption is more pronounced as long as agents care about past consumption. This consumption adjustment moderates the demand boom for the domestic good and thereby the long-run stimulus of output.

Perotti [2005] finds stronger GDP response to government spending shocks in the pre-1980 period than in the post-1980 period. Since one major development that has occurred since the middle seventies is that all industrialized economies have become more open over time, we now tackle the question: how trade openness influences effectiveness of fiscal policy? Figures 2(f) and 2(g) plot the expenditure multipliers against the exports-to-GDP ratio. Interestingly, both steady-state and impact multipliers fall as trade openness increases. More precisely, a rise in the exports-to-GDP ratio from 10% to 50% lowers the steady-state multiplier from 0.49 to 0.43. The smaller response of GDP to a fiscal impulse in more open economies is supported by recent findings documented by Beetsma et al. [2008]. To understand the negative relationship between government spending multipliers and openness, recall that in a more open economy, a higher import content of consumption (i. e. , an increase in α_c) induces households to substitute further the domestic for the foreign goods after a given real exchange rate appreciation. Additionally, because the exports-to-GDP ratio is higher, a fiscal expansion crowds out exports more. Finally, 2(g) shows that the short-term multiplier reacts more to a rise in trade openness due to the real exchange rate overshooting behavior on impact.

One additional striking result is that habit persistence weakens the relationship between expenditure multipliers and trade openness. More specifically, increasing trade openness from 10% to 37% (by reducing φ from 0.95 to 0.5) lowers the long-run spending multiplier from 0.49 to 0.46 in the case of habits, and from 0.68 to 0.58 if preferences are time separable, as illustrated in Figures 2(f) and 2(h) respectively. Hence, a rise in trade integration should have played an important role in driving down the size of government spending multipliers over time, but it cannot account for the whole decrease in the effectiveness of fiscal policy in the post-1980 period as long as habits are considered.[15] The small reaction of spending multipliers to globalization originates from the strong sensitivity of consumption in the domestic good to trade openness in the case of habits.[16]

5 Conclusion

Open economy versions of Baxter and King's [1993] model considering standard preferences find that consumption reacts strongly to a fiscal shock so that investment is crowded in. The consecutive capital accumulation yields a current account deficit and results in large government spending multipliers (see e. g. , Karayalçin [1999]). However, recent empirical evidence cast doubt over the predictions of the baseline neoclassical model. In our contribution, we have shown that the introduction of a habit index into the utility function helps improving the predictive power of the Baxter and King's [1993] model by modifying the chain of events following a fiscal expansion. More precisely, following a rise in public spending, real consumption exhibits a tenuous response as habits imply a sluggish response of consumption to the decrease in the real disposable income. Since the fall in private consumption is not large enough to offset higher public demand for the domestic good, investment is crowded out by public spending in the short-run, which pushes down the size of the short-term spending multiplier. At the same time, savings drop dramatically, which yields a current account deficit. Because habit-consumers prefer to adjust real consumption less-than-fully now and more-than-fully later, the resulting sizeable steady-state decline in real consumption lowers the size of the long-term spending multiplier.

One additional important issue that our paper addresses is the role of trade openness. We find that larger trade integration drives down spending multipliers by depressing exports more and yields a greater current account deficit. These results are in line with estimates by Beetsma, Giuliodori and Klaassen [2008] which show that a public spending shock has a smaller effect on output of more open economies, and a stronger effect on the trade balance which exhibits a stronger deterioration. Finally, since one major development that has occurred since the middle seventies is that all industrialized economies have become more open over time, the negative connection between trade openness and effectiveness of the fiscal policy could provide an explanation of the weaker GDP response to government spending shocks in the post-1980 period than in the pre-1980 period found in the data.

Notes

[1]Baxter and King [1993], Heijdra [1998] who relaxes the perfect competition assumption on product markets, Galí et al. [2007] who introduce "rule-of-thumb" consumers, Karayalçin [1999] who considers an open-economy version of the Baxter and King's model, commonly assume that preferences are time separable. One notable exception is Karayalçin [2003] who constructs a one good open-economy model in which agents possess habit-forming endogenous time discount rates. Karayalçin's article pursues a different goal and his results differ from ours. Karayalçin aims at providing an explanation of endogenous persistence in response to fiscal shocks; additionally, the savings behavior and the resulting current account adjustment enter in sharp contrast with those derived in our paper. More specifically, Karayalçin finds that consumption over-reacts, private savings rises which together with the fall in investment drives the current account into surplus.

[2]More specifically, if we calculate the IES over a short interval of time, we will find that the short-run IES is smaller than $1/\epsilon$.

[3]We drop the exposition of first-order static conditions for reason of space. Full discussion can be retrieved in a longer manuscript.

[4]Since the number of predetermined variables (s and k) equals the number of negative eigenvalues, and the number of jump variables (c and p) equals the number of positive eigenvalues, there is a unique two-dimensional convergent path towards the steady-state. Equilibrium dynamics and formal solutions can be retrieved in a longer version of the paper.

[5]Exports are positively correlated with the relative price of foreign goods, i. e. $X_p > 0$. An increase in p, makes the domestic good cheaper and thereby stimulates exports.

[6] Additionally, equation (10) shows that $\tilde{c}$ depends on a novel preference parameter σ. A higher σ and thus a faster catching-up of the habit stock softens the change of $\tilde{c}$ by moderating the decumulation or accumulation of financial wealth along the transitional path.

[7]For reason of space, we restricted ourselves to a rise in g^D. Analytical and numerical results after a rise in government spending falling on g^F are available from the author upon request.

[8]For 21 industrialized economies over 1965-1992, Bayoumi [1999] estimates significant export response elasticities ranging from 0.31 contemporaneously to 0.79 after four years. Since the dynamic system is linearized around the steady-state, we choose the long-term value.

[9]For the benchmark parametrization, total consumption expenditure, consumption expenditure, exports, and net exports as a share of initial GDP are 63%, 53%, 11%, and -2% respectively.

[10]While empirical studies based on micro data find small values, say falling in the range 0.1-0.4, the real business cycle literature often sets higher values for σ_N, say broadly equal to or higher than unity. See e. g. , Mankiw and Weinzierl [2006] for a discussion on that subject.

[11]Standard Keynesian theory predicts that public spending crowds out investment through increases in interest rates. However, Mountford and Uhlig [2008] who find that empirically "government spending shocks crowd out both residential and non-residential investment without causing interest rates to rise", cast doubt over the standard Keynesian explanation.

[12]For the sake of analytical simplicity, we abstract from physical capital depreciation in this section and thus set δ_K to zero.

[13]Hemming et al. [2002] review the GDP responses to a fiscal shock across several empirical papers. In Mountford and Uhlig [2008] and Perotti [2005], the impact spending multiplier falls in a range between 0.2-0.4 over 1960-2000, exception for Germany where the spending multiplier exceeds 1. In line with the results of Perotti [2005], Corsetti and Müller [2006] find the reaction of GDP to a fiscal shock is small and only significant for Australia where GDP increases by 0.42.

[14]In a previous version of the paper, we conducted a sensitivity analysis with respect to the speed of adjustment of habits σ. Since government spending multipliers weakly react to a change in σ, we omitted this analysis for reason of space.

[15]See Canzoneri et al. [2007] who suggest several factors like monetary policy explaining the smaller size of government spending multipliers in the post-1980 period.

[16]According to numerical results, a rise in trade integration from 10% to 37% implies that $\tilde{c}^D$ falls by -0.34% of initial GDP instead of -0.54% if $\gamma = 0.8$, and decreases by -0.29% instead of -0.40% if $\gamma = 0$. To see it more formally, differentiate (10) w. r. t. $\tilde{p}$ which yields $\mathrm{d}\tilde{c}/\tilde{c} = \nu\,(1-\alpha_c)\,\mathrm{d}\tilde{p}/\tilde{p}$ with ν the long-run IES and $(1-\alpha_c)$ the domestic content in total consumption expenditure. As γ gets closer to unity, the long-run IES ν takes larger values. Hence, for a given real exchange appreciation, lowering the share of domestic goods $(1-\alpha_c)$ moderates as much the reduction in real consumption as individuals care more about habits in deriving utility. Consequently, habits weaken the sensitivity of spending multipliers to trade integration.

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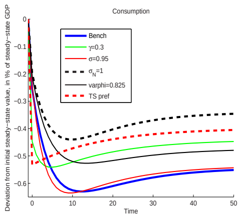
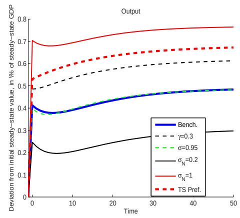
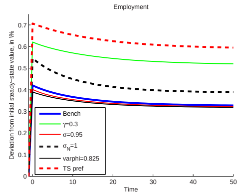


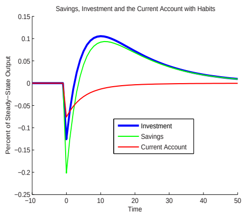
(a) Consumption



(b) Output

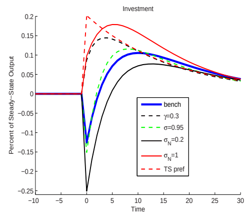


(c) Employment



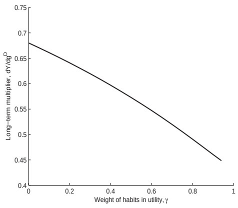
(d) Current Account with Habits

(e) Current Account without Habits

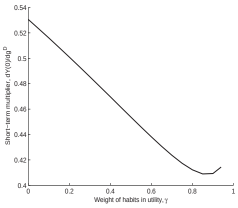


(f) Investment

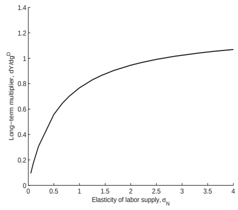
Figure 1: Computed Transitional Paths



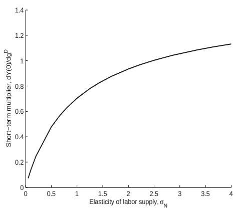
(a) Long-Term Multiplier against γ



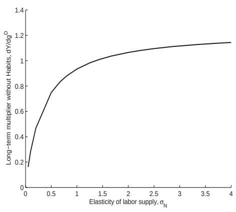
(b) Impact Multiplier against γ



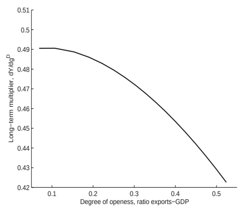
(c) Long-Term Multiplier against σ_N



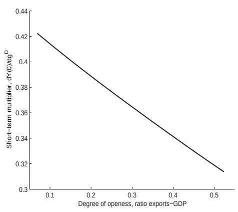
(d) Impact Multiplier against σ_N



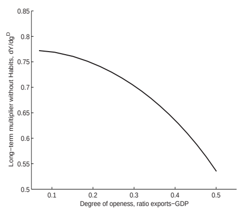
(e) Long-Term Multiplier against σ_N $(\gamma = 0)$



(f) Long-Term Multiplier against Trade Openness



(g) Impact Multiplier against Trade Openness



(h) Long-Term Multiplier against Trade Openness $(\gamma = 0)$

Figure 2: Spending Multipliers: Sensitivity to γ, σ_N and Trade Openness

Table 1: Quantitative Effects of a Fiscal Expansion (in %)

Variables[a]	TS Pref.	Weak Habits	Benchmark	High Speed	Small σ_N	High σ_N	Open
	($\gamma = 0$)	($\gamma = 0.3$)	($\gamma = 0.8$, $\sigma = 0.65$)	($\sigma = 0.95$)	($\sigma_N = 0.2$)	($\sigma_N = 1$)	($\varphi = 0.825$)
A.Long-Term							
Real Consumption $d\tilde{c}$	-0.46	-0.52	-0.66	-0.65	-0.81	-0.43	-0.66
Cons. in home good $d\tilde{c}^D$	-0.40	-0.44	-0.54	-0.54	-0.68	-0.33	-0.48
Exports $d\tilde{X}$	-0.04	-0.05	-0.06	-0.06	-0.07	-0.03	-0.12
Investment, $d\tilde{I}$	0.12	0.11	0.09	0.09	0.05	0.13	0.08
Stock of Foreign Assets, $d\tilde{b}$	-0.32	-0.31	-0.46	-0.36	-0.46	-0.45	-0.75
B.Impact							
Consumption, $dc(0)$	-0.62	-0.52	-0.31	-0.30	-0.35	-0.25	-0.29
Cons. in home good, $dc^D(0)$	-0.58	-0.50	-0.36	-0.35	-0.39	-0.30	-0.39
Exports $dX(0)$	-0.10	-0.10	-0.11	-0.11	-0.11	-0.10	-0.20
Investment, $dI(0)$	0.20	0.09	-0.13	-0.15	-0.25	0.10	-0.02
Savings, $dS(0)$	0.17	0.04	-0.20	-0.23	-0.33	0.02	-0.15
Current Account, $dca(0)$	-0.04	-0.04	-0.08	-0.08	-0.08	-0.07	-0.13
C.Spending Multipliers							
Short-Term Multiplier, $dY(0)/dg^D$	0.53	0.49	0.41	0.40	0.25	0.70	0.39
Long-Term Multiplier, $d\tilde{Y}/dg^D$	0.68	0.62	0.49	0.49	0.31	0.77	0.48

[a] We consider a rise in g^D which raises total government spending by one percentage point of GDP. Impact and steady-state changes are scaled by initial GDP. Government spending multipliers have been obtained by dividing the variation of GDP by the change in public spending. TS. Pref: time separable preferences.

HABIT PERSISTENCE AND EFFECTIVENESS OF FISCAL POLICY IN AN OPEN ECONOMY

TECHNICAL APPENDIX
NOT INTENDED FOR PUBLICATION

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A Preliminary Definitions, Useful Properties and Short-Run Static Solutions

Preliminaries for Consumption-Side

The instantaneous utility is assumed to take an iso-elastic form:

$$u(c,s) = \frac{1}{1-\epsilon}\left[\frac{c}{s^{\gamma}}\right]^{1-\epsilon}. \tag{16}$$

Partial derivatives are given by :

$$\begin{aligned}
u_c &= c^{-\epsilon}s^{-\gamma(1-\epsilon)} > 0, && \text{(17a)}\\
u_s &= -\gamma c^{1-\epsilon}s^{-[\gamma(1-\epsilon)+1]} < 0, && \text{(17b)}\\
u_{cc} &= -\epsilon c^{-(1+\epsilon)}s^{-\gamma(1-\epsilon)} < 0, && \text{(17c)}\\
u_{ss} &= \gamma\left[\gamma(1-\epsilon)+1\right]c^{1-\epsilon}s^{-[\gamma(1-\epsilon)+2]} < 0 \quad \text{i. o. i} \quad \epsilon > \frac{1+\gamma}{\gamma}, && \text{(17d)}\\
u_{cs} &= -\gamma(1-\epsilon)c^{-\epsilon}s^{-[\gamma(1-\epsilon)+1]} > 0 \quad \text{i. o. i} \quad \epsilon > 1. && \text{(17e)}
\end{aligned}$$

Preliminaries for Production-Side

We denote by :

- $\beta_N \equiv -\frac{F_n}{F_{nn}n} > 0$ the absolute value of labor demand elasticity;
- $\alpha_K \equiv \frac{F_k k}{Y} > 0$ the share of output paid out to the capital input and $1-\alpha_K \equiv \frac{F_n n}{Y} > 0$ the share of output paid out to the labor input;
- $\sigma_{KN} \equiv \frac{F_n F_k}{Y F_{kn}} > 0$ the substitution elasticity between capital and labor;
- $\sigma_N = \frac{v_n}{v_{nn}n} = \frac{1}{\epsilon_N} > 0$ the intertemporal elasticity of substitution for labor;
- $\nu_X = \frac{X_p p}{X} > 0$ the elasticity of exports with respect to the real exchange rate.

Useful Properties

Since the production function $Y = F(k,n)$, is assumed to be a linear homogenous production function, it possesses the following useful properties:

$$F_{kn} = -\frac{k}{n}F_{kk} = -\frac{n}{k}F_{nn}, \tag{18a}$$

$$F_{kk}F_{nn} - F_{kn}^2 = 0. \tag{18b}$$

First-Order Conditions

Denoting by λ and ξ the shadow prices of wealth and habits, the macroeconomic equilibrium is described by the following set of equations:

$$u_c(c,s) + \sigma\xi = \frac{p_c(p)\lambda}{p}, \tag{19a}$$

$$v_n(n) = -\frac{\lambda}{p}w, \tag{19b}$$

$$F_k = r^K + \delta_K, \quad F_n = w, \tag{19c}$$

$$\dot{\lambda} = \lambda(\beta - r^{\star}), \tag{19d}$$

$$\dot{\xi} = (\beta+\sigma)\xi - u_s(c,s), \tag{19e}$$

$$\dot{p} = p\left[F_k(k,n) - \delta_K - r^{\star}\right], \tag{19f}$$

$$\dot{k} = F(k,n) - c^D - X(p) - g^D - \delta_K k, \tag{19g}$$

$$\dot{b} = \frac{1}{p}\left[r^{\star}pb + F(k,n) - p_c(p)c - I - \left(g^D + pg^F\right)\right], \tag{19h}$$

together with the accumulation equations of habits (4) and physical capital (6) and the transversality conditions:

$$\lim_{t\to\infty} \xi s \exp\left(-r^{\star}t\right) = \lim_{t\to\infty} \bar{\lambda} b \exp\left(-r^{\star}t\right) = \lim_{t\to\infty} \frac{\bar{\lambda}}{p} k \exp\left(-r^{\star}t\right) = 0. \tag{20}$$

Using the fact that $w = F_n\left(k,n\right)$, the first-order condition (19b) for labor rewrites as $-v_n\left(n\right) = \frac{\lambda}{p}F_n$ which can be solved for employment n:

$$n = n\left(\bar{\lambda}, p, k\right), \tag{21}$$

where the partial derivatives are given by

$$n_{\bar{\lambda}} = \frac{\partial n}{\partial \bar{\lambda}} = \frac{n}{\bar{\lambda}}\chi > 0, \tag{22a}$$

$$n_p = \frac{\partial n}{\partial p} = -\chi\frac{n}{p} < 0, \tag{22b}$$

$$n_k = \frac{\partial n}{\partial k} = \chi\frac{F_{kn}n}{F_n} = \chi\frac{n}{k}\frac{\alpha_K}{\sigma_{KN}} > 0, \tag{22c}$$

where we let

$$\chi = \frac{\sigma_N \beta_N}{\sigma_N + \beta_N} > 0. \tag{23}$$

The market-clearing condition (11) can be solved for investment in physical capital:

$$I = I\left(\bar{\lambda}, c, p, k, g^D\right), \tag{24}$$

where the partial derivatives are given by

$$I_{\bar{\lambda}} = \frac{\partial I}{\partial \bar{\lambda}} = F_n n_{\bar{\lambda}} = \frac{Y}{\bar{\lambda}}\chi\left(1-\alpha_K\right) > 0, \tag{25a}$$

$$I_c = \frac{\partial I}{\partial c} = -\left(1-\alpha_c\right)p_c = -\frac{c^D}{c} \equiv v_c < 0, \tag{25b}$$

$$I_p = \frac{\partial I}{\partial p} = F_n n_p - \left(X_p - pp_c''c\right) = -\frac{Y}{p}\left[\chi\left(1-\alpha_K\right) + \frac{X}{Y}\left(\nu_X + \frac{c^D}{X}\alpha_c\phi\right)\right] < 0, \tag{25c}$$

$$I_k = \frac{\partial I}{\partial k} = F_k + F_n n_k = \frac{\alpha_K Y}{k}\left[1 + \chi\frac{\left(1-\alpha_K\right)}{\sigma_{KN}}\right] > 0, \tag{25d}$$

$$I_{g^D} = \frac{\partial I}{\partial g^D} = -1 < 0. \tag{25e}$$

All parameters are defined above and we have computed expression $\left(X_p - pp_c''c\right)$ as follows:

$$\Theta \equiv X_p - pp_c''c = \frac{X}{p}\left(\nu_X + \frac{c^D}{X}\alpha_c\phi\right) > 0. \tag{26}$$

We used the fact that $-\frac{p_c''p}{p_c'} = \phi\left(1-\alpha_c\right) > 0$, denoting by ϕ the intratemporal elasticity of substitution between the domestic good and the foreign good.

As it will be useful later, we investigate the sign of the following expression by making use of property (18a):

$$F_{kk} + F_{kn}n_k = F_{kk}\left(\frac{\beta_N}{\sigma_N + \beta_N}\right) < 0, \tag{27}$$

and

$$\begin{aligned} & F_{kk}I_p + F_{kn}\left[n_k I_p - \left(I_k - \delta_K\right)n_p\right] \\ = \; & n_p\left[F_{kk}F_n - r^K F_{kn}\right] - \Theta\left(F_{kk} + F_{kn}n_k\right) > 0. \end{aligned} \tag{28}$$

Derivation of the Dynamic Equation for Real Consumption

We establish the law of motion of real expenditure along an optimal path. Our aim is to rewrite the dynamic equation for consumption in a more usual form, making easier the economic interpretation of analytical results.

By differentiating equation (19a) w. r. t. time, substituting (4) and (19e), and making use of (19a) to eliminate ξ, we get:

$$\dot{c} = \frac{1}{u_{cc}} \left[(\beta + \sigma) \left(u_c - \frac{p_c \bar{\lambda}}{p} \right) + \sigma u_s - u_{cs} \sigma (c - s) - \frac{p_c \bar{\lambda}}{p} (1 - \alpha_c) \frac{\dot{p}}{p} \right]. \tag{29}$$

Using the iso-elastic form (16) for utility, we can rewrite (29) as follows:

$$\frac{\dot{c}}{c} = \frac{1}{\epsilon} \left\{ \sigma \gamma \epsilon \left(\frac{c}{s} \right) + \sigma \gamma (1 - \epsilon) + (\beta + \sigma) \left[\frac{p_c \bar{\lambda}}{p} c^{\epsilon} s^{\gamma(1-\epsilon)} - 1 \right] + \frac{p_c \bar{\lambda}}{p} (1 - \alpha_c) \frac{\dot{p}}{p} \right\}. \tag{30}$$

To rewrite the law of motion of real consumption in a more interpretable form, we calculate the time preference rate denoted by ρ, defined as the proportional rate of decrease of marginal utility of real consumption expressed in present value terms (see e. g. Epstein [1987]):

$$\rho \equiv - \left. \frac{\mathrm{d} \ln \left\{ \left[u_c (c, s) + \sigma \xi \right] \exp \left(-\beta t \right) \right\}}{\mathrm{d}t} \right|_{\dot{c}(t)=0}. \tag{31}$$

By substituting the accumulation equation for habits (4) and the dynamic equation for its shadow price (19e), and eliminating ξ by making use of (19a), the rate of time preference writes as follows:

$$\rho(t) = \beta + \frac{p}{p_c \lambda} \left[\sigma u_s - u_{cs} \sigma (c - s) + (\beta + \sigma) \left(u_c - \frac{p_c \lambda}{p} \right) \right]. \tag{32}$$

Using the iso-elastic form (16) for utility we can rewrite (32) as follows:

$$\rho = \beta - \frac{p c^{-\epsilon} s^{-\gamma(1-\epsilon)}}{\bar{\lambda} p_c} \left\{ (\beta + \sigma) \left(\frac{p_c \lambda}{p} c^{\epsilon} s^{\gamma(1-\epsilon)} - 1 \right) + \sigma \gamma \epsilon \frac{c}{s} + \sigma \gamma (1 - \epsilon) \right\} \tag{33}$$

By making use of (33), we wan rewrite the temporal path followed by real consumption (30) as:

$$\dot{c} = \frac{c}{\epsilon} \left(1 + \frac{\sigma \xi}{c^{-\epsilon} s^{-\gamma(1-\epsilon)}} \right) \left[r^{\star} + (1 - \alpha_c) \frac{\dot{p}}{p} - \rho (c, s, p) \right], \tag{34}$$

Consumption rises or falls depending on wether the consumption-based real interest rate (the first term on the RHS in square brackets) is above or below the endogenous time preference rate (the second term on the RHS). The multiplicative term falls between 0 and 1 (since $c^{-\epsilon} s^{-\gamma(1-\epsilon)} = p_c \bar{\lambda} - \sigma \xi > 0$). Hence, the relationship between the rate of change of real consumption and the consumption-based real interest rate is smaller than $1/\epsilon$ which corresponds to the IES with time separable preferences. Consequently, the short-term IES with time non separable is smaller than the inverse of the coefficient of relative risk aversion.

B Equilibrium Dynamics and Formal Solutions

Some Useful Linearized Expressions

Useful expressions evaluated at the steady-state denoted by a tilde write as follows:

$$\tilde{u}_c + \frac{\sigma}{\beta+\sigma}\tilde{u}_s = \tilde{c}^{-[\epsilon+\gamma(1-\epsilon)]}\frac{\beta+\sigma(1-\gamma)}{\beta+\sigma} > 0, \tag{35a}$$

$$\frac{\sigma(\beta+\sigma)}{\tilde{u}_{cc}}\left[\tilde{u}_{cc} + \frac{\beta+2\sigma}{\beta+\sigma}\tilde{\Gamma}\right] = \frac{\sigma}{\epsilon}\left[\gamma+\epsilon(1-\gamma)\right]\left[\beta+\sigma(1-\gamma)\right] > 0, \tag{35b}$$

$$-\frac{\left[\tilde{u}_c + \frac{\sigma}{\beta+\sigma}\tilde{u}_s\right]}{\left[\tilde{u}_{cc} + \frac{\beta+2\sigma}{\beta+\sigma}\tilde{\Gamma}\right]\tilde{c}} = 1/\left[\gamma+\epsilon(1-\gamma)\right], \tag{35c}$$

$$-\frac{p_c\bar{\lambda}}{\tilde{p}\tilde{u}_{cc}\tilde{c}} = -\frac{\tilde{u}_c + \frac{\sigma}{\beta+\sigma}\tilde{u}_s}{\tilde{u}_{cc}\tilde{c}} = \frac{\left[\beta+\sigma(1-\gamma)\right]}{\epsilon(\beta+\sigma)} > 0, \tag{35d}$$

$$-\frac{\sigma(\beta+2\sigma)}{\tilde{u}_{cc}}\tilde{\Gamma} = \frac{\sigma\gamma}{\epsilon}\left\{\sigma\epsilon - (1-\epsilon)\left[\beta+\sigma(1-\gamma)\right]\right\} > 0, \tag{35e}$$

$$\left[\tilde{u}_{cc} + \frac{\beta+2\sigma}{\beta+\sigma}\tilde{\Gamma}\right] = -\frac{p_c\bar{\lambda}}{\tilde{p}}\left[\gamma+\epsilon(1-\gamma)\right]\tilde{c}^{-1} < 0, \tag{35f}$$

where Γ is given by (37).

We first linearize the time preference rate:

$$\rho(t) = \beta + \frac{\tilde{p}u_{cc}}{p_c\bar{\lambda}}(\beta+\sigma)\left(c(t)-\tilde{c}\right) + \frac{\tilde{p}}{p_c\bar{\lambda}}(\beta+2\sigma)\tilde{\Gamma}\left(s(t)-\tilde{s}\right) + \frac{(\beta+\sigma)}{\tilde{p}}\left(p(t)-\tilde{p}\right) \tag{36}$$

with

$$\Gamma = u_{cs} + \frac{\sigma}{\beta+2\sigma}u_{ss} > 0. \tag{37}$$

The sign of Γ depends on the magnitude of the cross partial derivative of the felicity function, u_{cs}. We assume from now and thereafter that the marginal utility of real consumption is sufficiently increasing in the reference stock such that the preferences of the representative agent display adjacent complementarity and Γ is positive (see Ryder and Heal [1973]).

Then, substituting first the dynamic equation for the real exchange rate given by (19f) and the short-run static solution for labor given by (21), we linearize the dynamic equation for real consumption (34):

$$\begin{aligned}\dot{c}(t) &= -\frac{\bar{\lambda}p_c}{\tilde{p}u_{cc}}\Big\{-\rho_c\left(c(t)-\tilde{c}\right) - \rho_s\left(s(t)-\tilde{s}\right) + (1-\alpha_c)\left[F_{kk}+F_{kn}n_k\right]\left(k(t)-\tilde{k}\right)\\ &+ \left[(1-\alpha_c)F_{kn}n_p - \rho_p\right]\left(p(t)-\tilde{p}\right)\Big\}.\end{aligned} \tag{38}$$

Substituting the linearized version of the time preference rate, (38) rewrites as follows:

$$\begin{aligned}\dot{c}(t) &= (\beta+\sigma)\left(c(t)-\tilde{c}\right) + \frac{\beta+2\sigma}{u_{cc}}\tilde{\Gamma}\left(s(t)-\tilde{s}\right) + \frac{\bar{\lambda}p_c}{\tilde{p}^2u_{cc}}(1-\alpha_c)\left[(\beta+\sigma)-\tilde{p}F_{kn}n_p\right]\left(p(t)-\tilde{p}\right)\\ &- \frac{\bar{\lambda}p_c}{\tilde{p}u_{cc}}\left[F_{kk}+F_{kn}n_k\right]\left(k(t)-\tilde{k}\right),\end{aligned} \tag{39}$$

with

$$\frac{\beta+2\sigma}{u_{cc}}\tilde{\Gamma} = -\frac{\gamma}{\epsilon}\left\{\sigma\epsilon-(1-\epsilon)\left[\beta+\sigma(1-\gamma)\right]\right\} < 0$$

$$\frac{\bar{\lambda}p_c}{\tilde{p}u_{cc}} = -\tilde{c}\frac{\left[\beta+\sigma(1-\gamma)\right]}{\epsilon(\beta+\sigma)} < 0.$$

Saddle-Point Stability Conditions

Inserting the short-run static solution for labor (21) into the dynamic equations for real consumption (34) and the real exchange rate (19f), inserting the short-run static solution for investment (24) into the accumulation equation of physical capital (6), linearizing these together with the accumulation equation of habits (4) around the steady-state, and denoting long-term values by a tilde, we obtain in a matrix form:

$$\left(\dot{s},\dot{c},\dot{k},\dot{p}\right)^T = J\left(s(t)-\tilde{s},c(t)-\tilde{c},k(t)-\tilde{k},p(t)-\tilde{p}\right)^T, \tag{40}$$

where J is given by

$$J \equiv \begin{pmatrix} -\sigma & \sigma & 0 & 0 \\ \frac{\beta+2\sigma}{\tilde{u}_{cc}}\Gamma & \beta+\sigma & a_{23} & a_{24} \\ 0 & \tilde{I}_c & \left(\tilde{I}_k-\delta_K\right) & \tilde{I}_p \\ 0 & 0 & \tilde{p}\left[\tilde{F}_{kk}+\tilde{F}_{kn}\tilde{n}_k\right] & \tilde{p}\tilde{F}_{kn}\tilde{n}_p \end{pmatrix}, \tag{41}$$

with

$$a_{23} = -\frac{\bar{\lambda}p_c}{\tilde{p}\tilde{u}_{cc}}\left(1-\alpha_c\right)\left[\tilde{F}_{kk}+\tilde{F}_{kn}\tilde{n}_k\right], \tag{42a}$$

$$a_{24} = \frac{\bar{\lambda}p_c}{\tilde{u}_{cc}\tilde{p}^2}\left(1-\alpha_c\right)\left[(\beta+\sigma)-\tilde{p}\tilde{F}_{kn}\tilde{n}_p\right], \tag{42b}$$

and $\Gamma > 0$ evaluated at the steady-state is given by (37).

Denoting by μ the eigenvalue, the characteristic equation for the matrix J (41) of the linearized system writes as follows:

$$\mu^4 + b_1\mu^3 + b_2\mu^2 + b_3\mu + b_4 = 0, \tag{43}$$

with

$$b_1 = -\text{tr}J = -2r^\star < 0, \tag{44a}$$

$$b_2 = M_2 = (r^\star)^2 - \left\{\frac{\sigma(\beta+\sigma)}{\tilde{u}_{cc}}\left[\tilde{u}_{cc}+\left(\frac{\beta+2\sigma}{\beta+\sigma}\right)\Gamma\right]+\tilde{p}\left(\tilde{F}_{kk}+\tilde{F}_{kn}\tilde{n}_k\right)\left[\tilde{I}_p-\tilde{I}_c\frac{\bar{\lambda}p_c}{\tilde{u}_{cc}\tilde{p}^2}\left(1-\alpha_c\right)\right]\right.$$
$$\left. - \tilde{p}\left(\tilde{I}_k-\delta_K\right)\tilde{F}_{kn}\tilde{n}_p\right\} \gtrless 0 \tag{44b}$$

$$b_3 = -M_3 = r^\star\left\{\frac{\sigma(\beta+\sigma)}{\tilde{u}_{cc}}\left[\tilde{u}_{cc}+\left(\frac{\beta+2\sigma}{\beta+\sigma}\right)\tilde{\Gamma}\right]+\tilde{p}\left(\tilde{F}_{kk}+\tilde{F}_{kn}\tilde{n}_k\right)\left[\tilde{I}_p-\tilde{I}_c\frac{\bar{\lambda}p_c}{\tilde{u}_{cc}\tilde{p}^2}\left(1-\alpha_c\right)\right]\right.$$
$$\left. - \tilde{p}\left(\tilde{I}_k-\delta_K\right)\tilde{F}_{kn}\tilde{n}_p\right\} > 0, \tag{44c}$$

$$b_4 = \text{Det}J = \frac{\sigma(\beta+\sigma)\tilde{p}}{\tilde{u}_{cc}}\left\{\left[\tilde{u}_{cc}+\left(\frac{\beta+2\sigma}{\beta+\sigma}\right)\tilde{\Gamma}\right]\left[\tilde{I}_p\tilde{F}_{kk}+\tilde{F}_{kn}\left(\tilde{I}_p\tilde{n}_k-\tilde{n}_p\left(\tilde{I}_k-\delta_K\right)\right)\right]\right.$$
$$\left. - \tilde{I}_c\frac{\bar{\lambda}p_c}{\tilde{p}^2}\left(1-\alpha_c\right)\left[\tilde{F}_{kk}+\tilde{F}_{kn}\tilde{n}_k\right]\right\} > 0, \tag{44d}$$

where M_2 and M_3 are respectively the sum of all diagonal second and third order minors of J (see Dockner and Feichtinger [1991], p. 45). We used the following property $(I_k - \delta_K) + \tilde{p}F_{kn}n_p = r^\star$ to determine (44c).

For the sake of clarity, we drop from now and thereon the tilde over the partial derivatives evaluated at the steady-state. The positive signs of (44c) and (44d) stems from the following inequality:

$$u_{cc} + \frac{\sigma}{\beta+2\sigma}u_{ss} < -\left(\frac{\beta+\sigma}{\beta+2\sigma}\right)u_{cc}, \tag{45}$$

Imposing (45) ensures that adjacent complementarity is not too strong and that the dynamic system exhibits a saddle point stability (see Becker and Murphy [1988]).

The characteristic polynomial of degree four (43) can be rewritten as a characteristic polynomial of second degree:

$$\theta^2 + \frac{b_3}{r^\star}\theta + b_4 \qquad \text{with} \quad \theta = \mu\left(r^\star - \mu\right). \tag{46}$$

By evaluating first the eigenvalues θ from the second order polynomial and then calculating μ from the definition of θ, the four eigenvalues of the upper-left four by four submatrix in the Jacobian are given by

$$\mu_i \equiv \frac{1}{2}\left\{ r^\star \pm \sqrt{(r^\star)^2 + 2\left(\frac{b_3}{r^\star} \pm \sqrt{\left(\frac{b_3}{r^\star}\right)^2 - 4b_4}\right)}\right\}, \qquad i = 1,2,3,4, \tag{47}$$

with

$$\mu_1 < \mu_2 < 0 < r^\star < \mu_3 < \mu_4. \tag{48}$$

and having the following properties

$$r^\star - \mu_1 = \mu_4, \quad r^\star - \mu_2 = \mu_3. \tag{49}$$

The determinant of the matrix given by (44d) is positive, i. e. $b_4 > 0$. This is consistent with there being either 2 negative and 2 positive roots, 4 positive, or 4 negative roots. Since the trace of the matrix J is equal to $2r^\star = -b_1$ which is positive and the trace is equal to the sum of eigenvalues, only the first two cases must be considered. By Descartes rule of signs, necessary and sufficient conditions for the characteristic equation to have just two positive roots is that either $b_2 < 0$ or $b_3 > 0$. From (44c), $b_3 > 0$ so we can exclude the case of four positive roots. Since the system features two state variables, s and k, and two jump variables, c and p, the equilibrium yields a unique stable saddle-path.

Following Dockner and Feichtinger [1991], the necessary and sufficient conditions for saddle-point stability with real roots are:

$$\frac{b_3}{r^\star} > 0, \tag{50a}$$

$$0 < 4b_4 \leq \left(\frac{b_3}{r^\star}\right)^2. \tag{50b}$$

The first condition (50a) holds if the inequality (44c) is fulfilled. Expression $\left(\frac{b_3}{r^\star}\right)^2 - 4b4$ can be rewritten as:

$$\begin{aligned}\left(\frac{b_3}{r^\star}\right)^2 - 4b_4 &= \left\{\frac{\sigma\left(\beta+\sigma\right)}{u_{cc}}\left[u_{cc} + \left(\frac{\beta+2\sigma}{\beta+\sigma}\right)\tilde{\Gamma}\right]\right. \\ &- \left.\tilde{p}\left[\left(F_{kk} + F_{kn}n_k\right)\left(I_p - I_c\frac{\bar{\lambda}p_c}{u_{cc}\tilde{p}^2}\left(1-\alpha_c\right)\right) - \left(I_k - \delta_K\right)F_{kn}n_p\right]\right\}^2 \\ &- 4\frac{\sigma\left(\beta+2\sigma\right)\tilde{\Gamma}}{u_{cc}}I_c\frac{\bar{\lambda}p_c}{u_{cc}\tilde{p}}\left(1-\alpha_c\right)\left(F_{kk} + F_{kn}n_k\right) \gtreqless 0. \end{aligned} \tag{51}$$

As the second condition may quite plausibly not hold, the system (41) exhibits saddle-point behavior (as we have shown previously) but the stable roots may be either real or complex. In the former case, trajectories may be monotonic or humped. Hence, in the latter case, the dynamics involve cyclical behavior.

Formal Solutions

Setting the constants $A_3 = A_4 = 0$ to insure a converging adjustment for all macroeconomic aggregates, the stable paths are given by:

$$s(t) - \tilde{s} = A_1 e^{\mu_1 t} + A_2 e^{\mu_2 t}, \tag{52a}$$

$$c(t) - \tilde{c} = \omega_2^1 A_1 e^{\mu_1 t} + \omega_2^2 A_2 e^{\mu_2 t}, \tag{52b}$$

$$k(t) - \tilde{k} = \omega_3^1 A_1 e^{\mu_1 t} + \omega_3^2 A_2 e^{\mu_2 t}, \tag{52c}$$

$$p(t) - \tilde{p} = \omega_4^1 A_1 e^{\mu_1 t} + \omega_4^2 A_2 e^{\mu_2 t}, \tag{52d}$$

where the eigenvectors ω_j^i associated with eigenvalue μ_i are given by

$$\omega_2^i = \left(\frac{\sigma+\mu_i}{\sigma}\right), \tag{53a}$$

$$\omega_3^i = \frac{\left(\tilde{p}F_{kn}n_p - \mu_i\right) I_c\left(\sigma+\mu_i\right)}{\sigma\left\{\left(r^\star-\mu_i\right)\mu_i + \tilde{p}\left\{I_pF_{kk}+F_{kn}\left[I_pn_k-\left(I_k-\delta_K\right)n_p\right]\right\}\right\}}, \tag{53b}$$

$$\omega_4^i = -\frac{\tilde{p}\left(F_{kk}+F_{kn}n_k\right)}{\left(\tilde{p}F_{kn}n_p-\mu_i\right)}\omega_3^i, \tag{53c}$$

where we used the fact that $F_nn_k + pF_{kn}n_p = 0$. We normalized ω_1^i to unity. The two constants write as follows:

$$A_1 = \frac{\mathrm{d}\tilde{k} - \omega_3^2\mathrm{d}\tilde{s}}{\omega_3^2-\omega_3^1}, \qquad A_2 = \frac{\omega_3^1\mathrm{d}\tilde{s} - \mathrm{d}\tilde{k}}{\omega_3^2-\omega_3^1}. \tag{54}$$

To determine the signs of eigenvectors, we have to establish the signs of useful expressions. The first one is straightforward:

$$\frac{b_3}{r^\star} > \sqrt{\left(\frac{b_3}{r^\star}\right)^2 - 4b_4} > 0. \tag{55}$$

This inequality holds while the eigenvalues are real.

Making use of (49), we can establish some conditions for the signs of the following expressions:

$$\mu_1\mu_4 + \tilde{p}\left\{I_pF_{kk}+F_{kn}\left[I_pn_k-\left(I_k-\delta_K\right)n_p\right]\right\} \lesseqgtr 0,$$
$$\text{depending on wether} \quad \tilde{p}\left\{I_pF_{kk}+F_{kn}\left[I_pn_k-\left(I_k-\delta_K\right)n_p\right]\right\} \lesseqgtr \sigma\left(\beta+\sigma\right), \tag{56a}$$
$$\mu_2\mu_3 + \tilde{p}\left\{I_pF_{kk}+F_{kn}\left[I_pn_k-\left(I_k-\delta_K\right)n_p\right]\right\} \gtreqless 0,$$
$$\text{depending on wether} \quad \tilde{p}\left\{I_pF_{kk}+F_{kn}\left\{\left[I_pn_k-\left(I_k-\delta_K\right)n_p\right]\right\} \gtreqless \sigma\left(\beta+\sigma\right), \tag{56b}$$

Since for plausible (and a large range of) values of preferences and production-side parameters, inequality $0 < \tilde{p}\left[I_pF_{kk}+F_{kn}\left(I_pn_k-\left(I_k-\delta_K\right)n_p\right)\right] < \sigma\left(\beta+\sigma\right)$ holds, we deduce that $\mu_1\mu_4 + \tilde{p}\left[I_pF_{kk}+F_{kn}\left(I_pn_k-\left(I_k-\delta_K\right)n_p\right)\right] < 0$ and $\mu_2\mu_3 + \tilde{p}\left[I_pF_{kk}+F_{kn}\left(I_pn_k-\left(I_k-\delta_K\right)n_p\right)\right] < 0$ (see inequality (56a)).

In light of the above, we assume that $0 < \tilde{p}\left\{I_pF_{kk}+F_{kn}\left[I_pn_k-\left(I_k-\delta_K\right)n_p\right]\right\} < \sigma\left(\beta+\sigma\right)$ which implies in turn:

$$\begin{aligned} & \left(r^\star-\mu_1\right)\mu_1 + \tilde{p}\left\{I_pF_{kk}+F_{kn}\left[I_pn_k-\left(I_k-\delta_K\right)n_p\right]\right\} \\ = \; & \mu_4\mu_1 + \tilde{p}\left\{I_pF_{kk}+F_{kn}\left[I_pn_k-\left(I_k-\delta_K\right)n_p\right]\right\} < 0, \end{aligned} \tag{57a}$$

$$\begin{aligned} & \left(r^\star-\mu_2\right)\mu_2 + \tilde{p}\left\{I_pF_{kk}+F_{kn}\left[I_pn_k-\left(I_k-\delta_K\right)n_p\right]\right\} \\ = \; & \mu_3\mu_2 + \tilde{p}\left\{I_pF_{kk}+F_{kn}\left[I_pn_k-\left(I_k-\delta_K\right)n_p\right]\right\} < 0. \end{aligned} \tag{57b}$$

In addition, the sign of eigenvector $(\sigma+\mu_1)$ can be established as follows:

$$\left(\sigma+\mu_1\right) \gtreqless 0 \quad \text{depending on wether} \quad \tilde{p}\left\{I_pF_{kk}+F_{kn}\left[I_pn_k-\left(I_k-\delta_K\right)n_p\right]\right\} \lesseqgtr \sigma\left(\beta+\sigma\right). \tag{58}$$

Imposing the inequality has the following implications for eigenvectors ω_2^1 and ω_2^2:

$$\left(\sigma+\mu_1\right) > 0, \quad \left(\sigma+\mu_2\right) > 0 \quad \text{if} \quad \tilde{p}\left\{I_pF_{kk}+F_{kn}\left[I_pn_k-\left(I_k-\delta_K\right)n_p\right]\right\} < \sigma\left(\beta+\sigma\right), \tag{59}$$

where it is straightforward to infer that $(\sigma+\mu_2) > 0$ since $\mu_1 < \mu_2 < 0$.

An another way to show that the eigenvector ω_2^i is positive is to use the second line of $\left(J-\mu_iI_{4\times4}\right)\omega_j^i = 0$ which allows to derive another expression of ω_2^i:

$$\omega_2^i = -\frac{\frac{\beta+2\sigma}{\tilde{u}_{cc}}\tilde{\Gamma} + \frac{\bar{\lambda}p_c}{\tilde{p}^2u_{cc}}\left(1-\alpha_c\right)\left(\beta+\sigma-\mu_i\right)\omega_4^i}{\left(\beta+\sigma-\mu_i\right)} > 0, \quad i=1,2, \tag{60}$$

where we used the fact that $\omega_3^i = -\frac{(\tilde{p}F_{kn}n_p - \mu_i)}{\tilde{p}(F_{kk}+F_{kn}n_k)}$ and we estimated the following expression (see (42a)-(42b)):

$$a_{24} - a_{23}\frac{(\tilde{p}F_{kn}n_p - \mu_i)}{\tilde{p}\left(F_{kk}+F_{kn}n_k\right)} = \frac{\bar{\lambda}p_c}{\tilde{p}^2 u_{cc}}\left(1-\alpha_c\right)\left(\beta+\sigma-\mu_i\right) < 0, \quad i = 1, 2.$$

From expression (60), it is straightforward to show that $\omega_2^i > 0$ for $i = 1, 2$ as long as $\omega_4^i > 0$, by making use of properties (49).

Signs of Eigenvectors

We write out the four eigenvectors ω^i, corresponding with stable eigenvalues μ_i with $i = 1, 2$, to determine their signs

$$\omega^1 = \left(\begin{array}{cc} 1 & (+) \\ \left(\frac{\sigma+\mu_1}{\sigma}\right) & (+) \\ \frac{(\tilde{p}F_{kn}n_p-\mu_1)I_c(\sigma+\mu_1)}{\sigma\{(r^\star-\mu_1)\mu_1+\tilde{p}\{I_pF_{kk}+F_{kn}[I_pn_k-(I_k-\delta_K)n_p]\}\}} & (?) \\ -\frac{\tilde{p}(F_{kk}+F_{kn}n_k)I_c(\sigma+\mu_1)}{\sigma\{(r^\star-\mu_1)\mu_1+\tilde{p}\{I_pF_{kk}+F_{kn}[I_pn_k-(I_k-\delta_K)n_p]\}\}} & (+) \end{array}\right), \tag{61}$$

where the sign of $(\tilde{p}F_{kn}n_p - \mu_1)$ is undetermined, similarly to the case of time separable preferences.

$$\omega^2 = \left(\begin{array}{cc} 1 & (+) \\ \left(\frac{\sigma+\mu_2}{\sigma}\right) & (+) \\ \frac{(\tilde{p}F_{kn}n_p-\mu_2)I_c(\sigma+\mu_2)}{\sigma\{(r^\star-\mu_2)\mu_2+\tilde{p}\{I_pF_{kk}+F_{kn}[I_pn_k-(I_k-\delta_K)n_p]\}\}} & (?) \\ -\frac{\tilde{p}(F_{kk}+F_{kn}n_k)I_c(\sigma+\mu_2)}{\sigma\{(r^\star-\mu_2)\mu_2+\tilde{p}\{I_pF_{kk}+F_{kn}[I_pn_k-(I_k-\delta_K)n_p]\}\}} & (+) \end{array}\right), \tag{62}$$

where the sign of and $(\tilde{p}F_{kn}n_p - \mu_2)$ is undetermined, similarly to the case of time separable preferences.

Formal Solution for Labor

To compute the formal solution for labor, we linearize the short-run static solution (21) for labor around the steady-state:

$$n(t) - \tilde{n} = n_p\left(p(t) - \tilde{p}\right) + n_k\left(k(t) - \tilde{k}\right). \tag{63}$$

Substituting the stable solutions for $p(t)$ and $k(t)$, we get:

$$n(t) - \tilde{n} = L_1 A_1 e^{\mu_1 t} + L_2 A_2 e^{\mu_2 t}, \tag{64}$$

with

$$L_1 = n_k\omega_3^1 + n_p\omega_4^1, \tag{65a}$$
$$L_2 = n_k\omega_3^2 + n_p\omega_4^2. \tag{65b}$$

Formal Solution for the Stock of Foreign Assets

The accumulation equation for traded bonds is:

$$\dot{b}(t) = r^\star b(t) + \frac{X\left(p(t)\right)}{p(t)} - p_c'\left(p(t)\right)c(t) - g^F. \tag{66}$$

We first linearize equation (66) around the steady-state:

$$\dot{b}(t) = r^\star\left(b(t) - \tilde{b}\right) + \tilde{\Omega}\left(p(t) - \tilde{p}\right) - p_c'\left(\tilde{p}\right)\left(c(t) - \tilde{c}\right) \tag{67}$$

with

$$\tilde{\Omega} \equiv \frac{1}{\tilde{p}}\left[\frac{\tilde{X}}{\tilde{p}}\left(\tilde{\nu}_X - 1\right) + \tilde{c}^F\tilde{\nu}_F\right] = \frac{1}{\tilde{p}}\left[\Theta - \frac{\tilde{X}}{\tilde{p}}\right] > 0, \tag{68}$$

where the elasticities of exports and imports w. r. t. the real exchange rate are:

$$\nu_X = \frac{X_p p}{X} > 0, \quad \nu_F = -\frac{p_c'' p}{c^F} = \phi \left(1 - \alpha_c\right) > 0. \tag{69}$$

Expression (68) gives the net exports reaction expressed in terms of the foreign good to a change in the real exchange rate. We assume that the generalized version of the Marshall-Lerner condition, i. e. with an unbalanced trade balance, holds. Hence, a rise in the relative price of import goods leads to an improvement of the trade balance evaluated at the steady-state, so we set $\tilde{\Omega} > 0$.

Inserting stable solutions for $c(t)$ and $p(t)$, given respectively by (52b) and (52d), the solution for the current account writes as follows (we set $A_3 = A_4 = 0$ to eliminate unstable paths):

$$\dot{b}(t) = r^\star \left(b(t) - \tilde{b}\right) + \tilde{\Omega} \sum_{i=1}^{2} A_i \omega_4^i e^{\mu_i t} - p_c' \sum_{i=1}^{2} A_i \omega_2^i e^{\mu_i t} \tag{70}$$

Solving the differential equation yields:

$$\begin{aligned} b(t) - \tilde{b} &= \left[\left(b_0 - \tilde{b}\right) - \frac{N_1 A_1}{\mu_1 - r^\star} - \frac{N_2 A_2}{\mu_2 - r^\star}\right] e^{r^\star t} \\ &+ \frac{N_1 A_1}{\mu_1 - r^\star} e^{\mu_1 t} + \frac{N_2 A_2}{\mu_2 - r^\star} e^{\mu_2 t}, \end{aligned} \tag{71}$$

where

$$N_1 = \tilde{\Omega}\omega_4^1 - p_c'\omega_2^1, \quad N_2 = \tilde{\Omega}\omega_4^2 - p_c'\omega_2^2. \tag{72}$$

Invoking the transversality condition for intertemporal solvency given by (20), we obtain the linearized version of the nation's intertemporal budget constraint:

$$b_0 - \tilde{b} = \frac{N_1 A_1}{\mu_1 - r^\star} + \frac{N_2 A_2}{\mu_2 - r^\star}. \tag{73}$$

For the national intertemporal solvency to hold, the terms in brackets of equation (71) must be null, such that the stable solution for net foreign assets finally reduces to

$$b(t) - \tilde{b} = \frac{N_1 A_1}{\mu_1 - r^\star} e^{\mu_1 t} + \frac{N_2 A_2}{\mu_2 - r^\star} e^{\mu_2 t}. \tag{74}$$

Inserting the values for the constants A_1 and A_2 given by equations (54), we obtain the linearized version of the national intertemporal budget constraint expressed as a function of initial stocks of capital and habits:

$$\tilde{b} - b_0 = \Phi_1 \left(\tilde{k} - k_0\right) + \Phi_2 \left(\tilde{s} - s_0\right), \tag{75}$$

with

$$\Phi_1 = \frac{\left(\mu_1 - r^\star\right) N_2 - \left(\mu_2 - r^\star\right) N_1}{\left(\mu_1 - r^\star\right)\left(\mu_2 - r^\star\right)\left(\omega_3^2 - \omega_3^1\right)}, \tag{76a}$$

$$\Phi_2 = \frac{\left(\mu_2 - r^\star\right) \omega_3^2 N_1 - \left(\mu_1 - r^\star\right) \omega_3^1 N_2}{\left(\mu_1 - r^\star\right)\left(\mu_2 - r^\star\right)\left(\omega_3^2 - \omega_3^1\right)}. \tag{76b}$$

Formal Solution for the Stock of Financial Wealth

Financial wealth measured in terms of the domestic good, $a(t)$, is equal to the sum of the stock of foreign assets, $p(t)b(t)$, measured in terms of the foreign good and the capital stock. The law of motion for financial wealth ($S(t) = \dot{a}(t)$) is given by:

$$\dot{a}(t) = r^K(t) a(t) + w(t) n(t) - p_c\left(p(t)\right) c - T, \tag{77}$$

where lump-sump taxes cover overall government spending, i. e. $T = g^D + pg^F$. Remembering that $w = F_n$ and $r^K = F_k - \delta_K$ and substituting the short-run static solution for labor $n\left(\bar{\lambda}, p, k,\right)$, linearizing (77) in the neighborhood of the steady-state, we get:

$$\begin{aligned} \dot{a}(t) &= r^\star\left(a(t) - \tilde{a}\right) + \left[\left(F_{kk} + F_{kn} n_k\right)\tilde{p}\tilde{b} + F_n n_k\right]\left(k(t) - \tilde{k}\right) - p_c\left(c(t) - \tilde{c}\right) \\ &+ \left[\left(F_{kn}\tilde{p}\tilde{b} + F_n\right) n_p - \left(\tilde{c}^F + g^F\right)\right]\left(p(t) - \tilde{p}\right), \end{aligned} \tag{78}$$

where we used the fact that $\tilde{r}^K \equiv F_k - \delta_K = r^\star$ at the steady-state; we used properties (18) and $a \equiv pb + k$ to rewrite $\left(F_{kk} + F_{kn} n_k\right)\tilde{a} + \left(F_{kn} + F_{nn} n_k\right)\tilde{n}$ and $F_{kn} n_p \tilde{a} + F_{nn} n_p \tilde{n}$ as $\left(F_{kk} + F_{kn} n_k\right)\tilde{p}\tilde{b}$ and $F_{kn} n_p \tilde{p}\tilde{b}$ respectively.

By inserting the stable solutions, using the fact that $\omega_3^i = -\frac{\left(\tilde{p}F_{kn}n_p - \mu_i\right)}{\tilde{p}\left(F_{kk} + F_{kn}n_k\right)}\omega_4^i$, and rearranging terms, the solution for the stock of financial wealth writes as follows:

$$\begin{aligned} \dot{a}(t) &= r^\star\left(a(t) - \tilde{a}\right) - \sum_{i=1}^{2}\left[\left(r^\star - \mu_i\right)\tilde{b} + \frac{\tilde{X}}{\tilde{p}} + \frac{F_n n_k}{\tilde{p}\left(F_{kk} + F_{kn} n_k\right)}\left(\frac{F_{kk}F_n}{F_{kn}} - \mu_i\right)\right]\omega_4^i A_i e^{\mu_i t} \\ &- p_c \sum_{i=1}^{2} A_i \omega_2^i e^{\mu_i t}, \end{aligned} \tag{79}$$

where we used the fact that $\left(\tilde{c}^F + g^F\right) = r^\star\tilde{b} + \frac{\tilde{X}}{\tilde{p}}$.

Solving the differential equation leads to:

$$a(t) - \tilde{a} = \left[\left(a_0 - \tilde{a}\right) - \frac{S_1 A_1}{\mu_1 - r^\star} - \frac{S_2 A_2}{\mu_2 - r^\star}\right] e^{r^\star t} + \frac{S_1 A_1}{\mu_1 - r^\star} e^{\mu_1 t} + \frac{S_2 A_2}{\mu_2 - r^\star} e^{\mu_2 t}, \tag{80}$$

where

$$S_1 = -\left[\mu_4\tilde{b} + \frac{\tilde{X}}{\tilde{p}} + \frac{F_n n_k}{\tilde{p}\left(F_{kk} + F_{kn} n_k\right)}\left(\frac{F_{kk}F_n}{F_{kn}} - \mu_1\right)\right]\omega_4^1 - p_c\omega_2^1 < 0, \tag{81a}$$

$$S_2 = -\left[\mu_3\tilde{b} + \frac{\tilde{X}}{\tilde{p}} + \frac{F_n n_k}{\tilde{p}\left(F_{kk} + F_{kn} n_k\right)}\left(\frac{F_{kk}F_n}{F_{kn}} - \mu_2\right)\right]\omega_4^2 - p_c\omega_2^2 < 0, \tag{81b}$$

with $\left(F_{kk} + F_{kn} n_k\right) < 0$ and $n_k > 0$, and eigenvectors ω_4^1, ω_4^2, ω_2^1, ω_2^2 are positive for all parametrization.

Invoking the transversality condition for intertemporal solvency, we get the linearized version of the households' intertemporal budget constraint:

$$a_0 - \tilde{a} = \frac{S_1 A_1}{\mu_1 - r^\star} + \frac{S_2 A_2}{\mu_2 - r^\star}. \tag{82}$$

For the intertemporal solvency to hold, the terms in brackets of equation (82) must be null, so as the stable solution for the stock of financial wealth finally reduces to:

$$a(t) - \tilde{a} = \frac{S_1 A_1}{\mu_1 - r^\star} e^{\mu_1 t} + \frac{S_2 A_2}{\mu_2 - r^\star} e^{\mu_2 t}. \tag{83}$$

Formal Solution for GDP

Linearize the production function in the neighborhood of the steady-state:

$$Y(t) = \tilde{Y} + F_k\left(k(t) - \tilde{k}\right) + F_n\left(n(t) - \tilde{n}\right).$$

Remembering that $F_k = r^\star + \delta_K$, and using expressions of L_1 and L_2 given by (65), the stable solution for overall output writes as follows:

$$\begin{aligned} Y(t) &= \tilde{Y} + Y_k\left(k(t) - \tilde{k}\right) + Y_p\left(p(t) - \tilde{p}\right), \\ &= \tilde{Y} + Y_k \sum_{i=1}^{2}\omega_3^i A_i e^{\mu_i t} + Y_p \sum_{i=1}^{2}\omega_4^i A_i e^{\mu_i t}, \end{aligned} \tag{84}$$

with

$$Y_k = (r^\star + \delta_K) + F_n n_k = I_k > 0, \quad Y_p = F_n n_p < 0. \tag{85}$$

C Steady-State Changes of Government Spending Shocks

The steady-state of the economy is obtained by setting $\dot{c}, \dot{s}, \dot{k}, \dot{p}, \dot{b} = 0$ and is defined by the following set of equations:

$$u_c(\tilde{c}, \tilde{s}) + \frac{\sigma}{\beta+\sigma} u_s(\tilde{c}, \tilde{s}) = \frac{p_c(\tilde{p})\bar{\lambda}}{\tilde{p}}, \tag{86a}$$

$$\tilde{c} = \tilde{s}, \tag{86b}$$

$$r^\star + \delta_K = F_k\left[\tilde{k}, n\left(\bar{\lambda}, \tilde{p}, \tilde{k}\right)\right], \tag{86c}$$

$$r^\star \tilde{p}\tilde{b} + F\left[\tilde{k}, n\left(\bar{\lambda}, \tilde{p}, \tilde{k}\right)\right] - p_c(\tilde{p})\tilde{c} - \delta_K \tilde{k} - g^D - \tilde{p} g^F = 0, \tag{86d}$$

$$F\left[\tilde{k}, n\left(\bar{\lambda}, \tilde{p}, \tilde{k}\right)\right] = (1-\alpha_c) p_c(\tilde{p})\tilde{c} + \delta_K \tilde{k} + X(\tilde{p}) + g^D, \tag{86e}$$

and the economy's intertemporal budget constraint

$$\left(\tilde{b} - b_0\right) = \Phi_1\left(\tilde{k} - k_0\right) + \Phi_2(\tilde{s} - s_0), \tag{86f}$$

where we used the fact that at steady-state $\tilde{I} = \delta_K \tilde{k}$, and we have substituted the short-run static solution for labor which obviously holds in the long-term.

C.1 Steady-State Changes

Totally differentiating equations (86) yields in matrix form:

$$\begin{pmatrix} \left[u_{cc} + \left(\frac{\beta+2\sigma}{\beta+\sigma}\right)\tilde{\Gamma}\right] & \frac{p_c\bar{\lambda}(1-\alpha_c)}{\tilde{p}^2} & -\frac{p_c}{\tilde{p}} & 0 & 0 \\ 0 & F_{kn}n_p & F_{kn}n_{\bar{\lambda}} & F_{kk} + F_{kn}n_k & 0 \\ -p_c & \left(I_p + \tilde{p}\tilde{\Omega}\right) & I_{\bar{\lambda}} & (I_k - \delta_K) & \tilde{p}r^\star \\ -(1-\alpha_c)p_c & I_p & I_{\bar{\lambda}} & (I_k - \delta_K) & 0 \\ -\Phi_2 & 0 & 0 & -\Phi_1 & 1 \end{pmatrix} \begin{pmatrix} \mathrm{d}\tilde{c} \\ \mathrm{d}\tilde{p} \\ \mathrm{d}\bar{\lambda} \\ \mathrm{d}\tilde{k} \\ \mathrm{d}\tilde{b} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ \mathrm{d}g^D + \tilde{p}\mathrm{d}g^F \\ \mathrm{d}g^D \\ \mathrm{d}b_0 - \Phi_1 \mathrm{d}k_0 - \Phi_2 \mathrm{d}s_0 \end{pmatrix}. \tag{87}$$

We used the fact that $\left(\tilde{c}^F + g^F - r^\star\tilde{b}\right) = \tilde{X}/\tilde{p}$ (see (89)) together with $F_n n_p = I_p + \tilde{\Theta}$ and $\tilde{p}\tilde{\Omega} = \tilde{\Theta} - \tilde{X}/\tilde{p}$ to rewrite $\left(F_n n_p - \frac{\tilde{X}}{\tilde{p}}\right)$ as follows $\left(I_p + \tilde{p}\tilde{\Omega}\right)$.

Domestic Good g^D

The steady-state effects of an unanticipated permanent increase in government expenditure falling on the domestic good are obtained from the total differential of the equilibrium system

(86) w. r. t. g^D:

$$\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^D} = \frac{\mathrm{d}\tilde{s}}{\mathrm{d}g^D} = \frac{p_c}{D}\left(F_{kk} + F_{kn}n_k\right)\tilde{\Omega} - \frac{r^\star p_c \alpha_c}{D}\Phi_1 F_{kn} n_p < 0, \quad (88a)$$

$$\frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^D} = \frac{F_{kn}n_{\bar{\lambda}}}{D}\left[u_{cc} + \frac{\beta+2\sigma}{\beta+\sigma}\Gamma\right] r^\star \tilde{p}\Phi_1 - \frac{\left(F_{kk}+F_{kn}n_k\right)}{D}\frac{p_c}{\tilde{p}}\left(\tilde{p}r^\star\Phi_2 - \alpha_c p_c\right) \lesseqgtr 0, \quad (88b)$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^D} = -\frac{\left(F_{kk}+F_{kn}n_k\right)}{D}\frac{p_c\bar{\lambda}\left(1-\alpha_c\right)}{\tilde{p}^2}\left(\tilde{p}r^\star\Phi_2 - \alpha_c p_c\right)$$
$$-\frac{\left[u_{cc} + \frac{\beta+2\sigma}{\beta+\sigma}\Gamma\right]}{D}\left[F_{kn}n_p r^\star \tilde{p}\Phi_1 - \tilde{p}\tilde{\Omega}\left(F_{kk}+F_{kn}n_k\right)\right] \gtreqless 0, \quad (88c)$$

$$\frac{\mathrm{d}\tilde{k}}{\mathrm{d}g^D} = -\frac{F_{kn}}{D}n_{\bar{\lambda}}\left[u_{cc} + \frac{\beta+2\sigma}{\beta+\sigma}\Gamma\right]\tilde{p}\tilde{\Omega}$$
$$+\frac{F_{kn}}{D'}n_p\frac{\alpha_c p_c}{\tilde{p}}\left(\Phi_2 r^\star \tilde{p} - \alpha_c p_c\right) = \frac{(+)}{(+)} + \frac{(+)}{(+)} > 0, \quad (88d)$$

$$\frac{\mathrm{d}\tilde{b}}{\mathrm{d}g^D} = \Phi_1\frac{\mathrm{d}\tilde{k}}{\mathrm{d}g^D} + \Phi_2\frac{\mathrm{d}\tilde{s}}{\mathrm{d}g^D} < 0, \quad (88e)$$

where $\Phi_1 < 0$, $\Phi_2 > 0$, $\left(\Phi_2 r^\star \tilde{p} - \alpha_c p_c\right) < 0$ and $D > 0$ for all parametrization.

Totally differentiating the short-run static solution for labor (21) and substituting relevant expressions (88), the steady-state change of employment after a permanent rise in g^D is:

$$\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^D} = n_{\bar{\lambda}}\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^D} + n_p\frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^D} + n_k\frac{\mathrm{d}\tilde{k}}{\mathrm{d}g^D},$$
$$= -\frac{F_{kk}}{D}\left\{-\left[u_{cc} + \frac{\beta+2\sigma}{\beta+\sigma}\tilde{\Gamma}\right]n_{\bar{\lambda}}\tilde{p}\tilde{\Omega} + \frac{p_c}{\tilde{p}}\alpha_c n_p\left(\Phi_2 r^\star \tilde{p} - \alpha_c p_c\right)\right\} > 0,$$
$$= -\frac{F_{kk}\chi\tilde{n}}{D}\left(\frac{p_c}{\tilde{p}}\right)^2\left[\gamma + \epsilon\left(1-\gamma\right)\right]\frac{\tilde{X}}{p_c\tilde{c}}\left[\nu_X + \frac{\tilde{p}\tilde{c^F}}{\tilde{X}}\left(\alpha_c\nu + \left(1-\alpha_c\right)\phi\right) - 1\right]$$
$$+\frac{F_{kk}\chi\tilde{n}}{D}\frac{\tilde{c}^F}{\tilde{c}}r^\star\Phi_2 > 0, \quad (89)$$

where the sign of (89) comes from assumption $\left(\Phi_2 r^\star \tilde{p} - \alpha_c p_c\right) < 0$. As long as this inequality holds, and thereby labor rises, as longer as a fall in real consumption by one unit induces an improvement in the balance of trade by a larger size than the fall in interest receipts.

Import Good g^F

The steady-state effects of an unanticipated permanent increase in government expenditure falling on the foreign good are obtained from the total differential of the equilibrium system

(86) w. r. t. to g^F:

$$\frac{d\tilde{c}}{dg^F} = \frac{d\tilde{s}}{dg^F} = -\frac{p_c}{D}\left\{-\tilde{\Theta}\left(F_{kk}+F_{kn}n_k\right)+\alpha_c n_p\left[F_{kk}F_n - F_{kn}r^{\star}\right]\right\} = -\frac{(+)}{(+)} < 0, \quad (90a)$$

$$\frac{d\tilde{p}}{dg^F} = \frac{\tilde{p}}{D}\left[u_{cc}+\frac{\beta+2\sigma}{\beta+\sigma}\Gamma\right]\left[F_{kk}I_{\bar{\lambda}}+F_{kn}\left(n_k I_{\bar{\lambda}} - n_{\bar{\lambda}}\left(I_k-\delta_K\right)\right)\right] - \frac{(p_c)^2}{D}\left(1-\alpha_c\right)\left(F_{kk}+F_{kn}n_k\right) > 0, \quad (90b)$$

$$\frac{d\bar{\lambda}}{dg^F} = -\frac{\tilde{p}\left(F_{kk}+F_{kn}n_k\right)}{D}\left[\frac{\left(1-\alpha_c\right)p_c}{\tilde{p}}\right]^2\bar{\lambda} - \frac{\left[u_{cc}+\frac{\beta+2\sigma}{\beta+\sigma}\Gamma\right]}{D}\tilde{p}\left[\left(F_{kk}+F_{kn}n_k\right)I_p - F_{kn}n_p\left(I_k-\delta_K\right)\right] > 0, \quad (90c)$$

$$\frac{d\tilde{k}}{dg^F} = \frac{F_{kn}}{D}\left\{-\left[u_{cc}+\frac{\beta+2\sigma}{\beta+\sigma}\Gamma\right]n_{\bar{\lambda}}\tilde{p}\tilde{\Theta}+p_c^2\left(1-\alpha_c\right)\alpha_c n_p\right\}, = \frac{F_{kn}\chi\tilde{n}}{D}\frac{p_c}{\tilde{p}}\left[\gamma+\epsilon\left(1-\gamma\right)\right]\frac{\tilde{X}}{\tilde{c}}\left[\nu_X+\alpha_c\frac{\tilde{c}^D}{\tilde{X}}\left(\phi-\nu\right)\right] > 0, \quad (90d)$$

$$\frac{d\tilde{b}}{dg^F} = \Phi_1\frac{d\tilde{k}}{dg^F}+\Phi_2\frac{d\tilde{s}}{dg^F} < 0, \quad (90e)$$

where $\Phi_1 < 0$ and $\Phi_2 > 0$ and $D > 0$ for all parametrization. We used the fact that $\left\{F_{kk}I_{\bar{\lambda}}+F_{kn}\left[n_k I_{\bar{\lambda}} - n_{\bar{\lambda}}\left(I_k-\delta_K\right)\right]\right\} = n_{\bar{\lambda}}\left(F_{kk}F_n - F_{kn}r^{\star}\right) < 0$ to compute the sign of (90b). The sign of (90d) holds, i. e. the capital stock is permanently raised after a rise in government spending in g^F, if $\phi > \nu$, that is if the long-term IES with time non separable preferences is less than the intratemporal elasticity of substitution ϕ. As empirical evidence overwhelmingly suggest, the intertemporal elasticity of substitution for consumption is smaller that unity, i. e. $\nu < 1$; additionally, since domestic and foreign goods are substitutes, we set $\phi > 1$; hence, the inequality $\phi > \nu$ holds.

Totally differentiating the short-run static solution for labor (21) and substituting relevant expressions (90), the steady-state change of employment after a permanent rise in g^F is:

$$\frac{d\tilde{n}}{dg^F} = n_{\bar{\lambda}}\frac{d\bar{\lambda}}{dg^F}+n_p\frac{d\tilde{p}}{dg^F}+n_k\frac{d\tilde{k}}{dg^F}, = -\frac{F_{kk}\chi\tilde{n}}{D}\frac{(p_c)^2}{\tilde{p}}\left[\gamma+\epsilon\left(1-\gamma\right)\right]\frac{\tilde{X}}{p_c\tilde{c}}\left[\nu_X+\alpha_c\frac{\tilde{c}^D}{\tilde{X}}\left(\phi-\nu\right)\right] > 0. \quad (91)$$

The sign of (91) holds, i. e. employment is permanently raised after a rise in g^F as long as the inequality $\phi > \nu$ is fulfilled.

C.2 Wealth Effect and Direct Effect: The Two-Step Solution Procedure

In this section, we calculate the signs of the partial derivatives of the steady-state functions obtained in the first step of the two-step solution procedure by solving system (86a)-(86e) without the intertemporal budget constraint (86f). Totally differentiating the system of equations (86a)-(86e) yields in matrix form:

$$\begin{pmatrix} \left[u_{cc}+\left(\frac{\beta+2\sigma}{\beta+\sigma}\right)\Gamma\right] & \frac{p_c\bar{\lambda}(1-\alpha_c)}{\tilde{p}^2} & 0 & 0 \\ 0 & F_{kn}n_p & F_{kk}+F_{kn}n_k & 0 \\ -p_c & \left(I_p+\tilde{p}\tilde{\Omega}\right) & \left(I_k-\delta_K\right) & \tilde{p}r^{\star} \\ -\left(1-\alpha_c\right)p_c & I_p & \left(I_k-\delta_K\right) & 0 \end{pmatrix}\begin{pmatrix} d\tilde{c} \\ d\tilde{p} \\ d\tilde{k} \\ d\tilde{b} \end{pmatrix} = \begin{pmatrix} \frac{p_c}{\tilde{p}}d\bar{\lambda} \\ -F_{kn}n_{\bar{\lambda}}d\bar{\lambda} \\ dg^D+\tilde{p}dg^F - I_{\bar{\lambda}}d\bar{\lambda} \\ dg^D - I_{\bar{\lambda}}d\bar{\lambda} \end{pmatrix}. \quad (92)$$

The determinant of the system matrix is given by:

$$G \equiv \tilde{p}r^{\star}\left[u_{cc}+\left(\frac{\beta+2\sigma}{\beta+\sigma}\right)\tilde{\Gamma}\right]\left\{F_{kk}I_p+F_{kn}\left[n_kI_p-\left(I_k-\delta_K\right)n_p\right]\right\}$$
$$+\tilde{p}r^{\star}\left[\frac{p_c\left(1-\alpha_c\right)}{\tilde{p}}\right]^2\left(F_{kk}+F_{kn}n_k\right)\bar{\lambda}<0, \tag{93}$$

where the sign of G follows from (26) and (27).

From system (92), we can calculate the partial derivatives of the following steady-state functions:

$$\tilde{k} = k\left(\bar{\lambda},g^D,g^F\right), \tag{94a}$$
$$\tilde{s} = \tilde{c}=m\left(\bar{\lambda},g^D,g^F\right), \tag{94b}$$
$$\tilde{p} = p\left(\bar{\lambda},g^D,g^F\right), \tag{94c}$$
$$\tilde{b} = v\left(\bar{\lambda},g^D,g^F\right), \tag{94d}$$

with

$$m_{\bar{\lambda}} \equiv \frac{\partial\tilde{c}}{\partial\bar{\lambda}}=\frac{p_cr^{\star}}{G}\left\{\left\{F_{kk}I_p+F_{kn}\left[n_kI_p-\left(I_k-\delta_K\right)n_p\right]\right\}\right.$$
$$\left.+\frac{1}{G}\frac{\bar{\lambda}\left(1-\alpha_c\right)}{\tilde{p}}\left\{F_{kk}I_{\bar{\lambda}}+F_{kn}\left[n_kI_{\bar{\lambda}}-n_{\bar{\lambda}}\left(I_k-\delta_K\right)\right]\right\}\right\}<0, \tag{95a}$$

$$m_{g^D} \equiv \frac{\partial\tilde{c}}{\partial g^D}=-\frac{r^{\star}}{G}\frac{p_c\bar{\lambda}\left(1-\alpha_c\right)}{\tilde{p}}\left(F_{kk}+F_{kn}n_k\right)=\frac{(+)}{(-)}<0, \tag{95b}$$

$$m_{g^F} \equiv \frac{\partial\tilde{c}}{\partial g^F}=0, \tag{95c}$$

$$p_{\bar{\lambda}} \equiv \frac{\partial\tilde{p}}{\partial\bar{\lambda}}=-\frac{\left[u_{cc}+\left(\frac{\beta+2\sigma}{\beta+\sigma}\right)\Gamma\right]}{G}r^{\star}\tilde{p}\left[\left(F_{kk}+F_{kn}n_k\right)I_{\bar{\lambda}}-F_{kn}n_{\bar{\lambda}}\left(I_k-\delta_K\right)\right]$$
$$+\frac{r^{\star}}{G}\left(1-\alpha_c\right)\left(p_c\right)^2\left(F_{kk}+F_{kn}n_k\right)=\frac{(-)}{(-)}+\frac{(-)}{(-)}>0, \tag{95d}$$

$$p_{g^D} \equiv \frac{\partial\tilde{p}}{\partial g^D}=\frac{\left[u_{cc}+\left(\frac{\beta+2\sigma}{\beta+\sigma}\right)\Gamma\right]}{G}r^{\star}\tilde{p}\left(F_{kk}+F_{kn}n_k\right)=\frac{(+)}{(-)}<0, \tag{95e}$$

$$p_{g^F} \equiv \frac{\partial\tilde{p}}{\partial g^F}=0, \tag{95f}$$

$$k_{\bar{\lambda}} \equiv \frac{\partial\tilde{k}}{\partial\bar{\lambda}}=\frac{r^{\star}\tilde{p}F_{kn}}{G}\left\{\left[u_{cc}+\left(\frac{\beta+2\sigma}{\beta+\sigma}\right)\Gamma\right]n_{\bar{\lambda}}\tilde{\Theta}-\frac{p_c^2\left(1-\alpha_c\right)}{\tilde{p}}\alpha_cn_p\right\}>0, \tag{95g}$$

$$k_{g^D} \equiv \frac{\partial\tilde{k}}{\partial g^D}=-\frac{r^{\star}\tilde{p}}{G}F_{kn}n_p\left[u_{cc}+\left(\frac{\beta+2\sigma}{\beta+\sigma}\right)\Gamma\right]=\frac{(-)}{(-)}>0, \tag{95h}$$

$$k_{g^F} \equiv \frac{\partial\tilde{k}}{\partial g^F}=0, \tag{95i}$$

where the signs follow from several expressions which have been computed:

$$
\begin{aligned}
& \left\{F_{kk}I_p + F_{kn}\left[n_k I_p - (I_k - \delta_K) n_p\right]\right\} + \frac{\bar{\lambda}(1-\alpha_c)}{\tilde{p}}\left\{F_{kk}I_{\bar{\lambda}} + F_{kn}\left[n_k I_{\bar{\lambda}} - n_{\bar{\lambda}}(I_k - \delta_K)\right]\right\} \\
= \; & -\tilde{\Theta}\left(F_{kk} + F_{kn}n_k\right) + \alpha_c n_p \left[F_{kk}F_n - F_{kn}r^{\star}\right] > 0, \qquad (96a) \\
& n_p I_{\bar{\lambda}} - n_{\bar{\lambda}} I_p = n_{\bar{\lambda}}\tilde{\Theta} > 0, \qquad (96b) \\
& n_p + (1-\alpha_c)\frac{\bar{\lambda}}{\tilde{p}} n_{\bar{\lambda}} = \alpha_c n_p < 0, \qquad (96c) \\
& \left[u_{cc} + \left(\frac{\beta + 2\sigma}{\beta+\sigma}\right)\Gamma\right] n_{\bar{\lambda}}\tilde{\Theta} - \frac{p_c^2(1-\alpha_c)}{\tilde{p}}\alpha_c n_p \\
= \; & \frac{p_c}{\tilde{p}}\frac{\left[\gamma + \epsilon(1-\gamma)\right]}{\tilde{c}} n_p \tilde{X}\left[\nu_X + \alpha_c \frac{\tilde{c}^D}{\tilde{X}}(\phi - \nu)\right] < 0, \qquad (96d) \\
& \left[\left(F_{kk} + F_{kn}n_k\right) I_{\bar{\lambda}} - F_{kn} n_{\bar{\lambda}} (I_k - \delta_K)\right] = n_{\bar{\lambda}}\left[F_{kk}F_n - F_{kn}r^{\star}\right] < 0, \qquad (96e)
\end{aligned}
$$

To determine the sign of (96a), we used the fact that $(I_k - \delta_K) = (F_k - \delta_K) + F_n n_k = r^{\star} + F_n n_k$ at the steady-state. The sign of (96d) holds as long as $\phi > \nu$, i. e. if the long-term IES with time non separable preferences is smaller than the intratemporal elasticity of substitution ϕ.

Substituting the steady-state function for the capital stock (94a) and the real exchange rate (94c) into the static solution for employment (21), the long-term level of labor can be expressed in terms of marginal utility of wealth and exogenous policy parameters:

$$
\tilde{n} = N\left(\bar{\lambda}, g^D, g^F\right), \qquad (97)
$$

where partial derivatives are given by:

$$
\begin{aligned}
N_{\bar{\lambda}} \; & \equiv \; \frac{\partial \tilde{n}}{\partial \bar{\lambda}} = -\frac{\tilde{p}r^{\star}n_{\bar{\lambda}}}{G}\left[u_{cc} + \left(\frac{\beta+2\sigma}{\beta+\sigma}\right)\Gamma\right]\left[F_{kk}\tilde{\Theta} + (1-\alpha_c)n_p\left(F_{kk}F_n - F_{kn}r^{\star}\right)\right] \\
& + \; \frac{n_p F_{kk} r^{\star}}{G} p_c^2 (1-\alpha_c)\alpha_c \gtrless 0, \qquad (98a) \\
N_{g^D} \; & \equiv \; \frac{\partial \tilde{n}}{\partial g^D} = \frac{n_p \tilde{p} r^{\star} F_{kk}}{G}\left[u_{cc} + \left(\frac{\beta+2\sigma}{\beta+\sigma}\right)\Gamma\right] > 0, \qquad (98b) \\
N_{g^F} \; & \equiv \; \frac{\partial \tilde{n}}{\partial g^F} = 0. \qquad (98c)
\end{aligned}
$$

C.3 Short-Term Effects of a Fiscal Expansion: The Case of Inelastic Labor

The derivation of trajectories turn out to be quite complex in the case of elastic labor. By contrast, as long as employment is fixed, we are able to fully characterize transitional dynamics. In this subsection, we characterize the transitional dynamics by assuming that labor is fixed for analytical simplicity. Yet, tt is worthwhile noticing that the results derived in the case of inelastic labor hold with elastic labor.

We start the investigation of transitional dynamics by evaluating the constants after a government spending shock, g^j (with $j = D, F$):

$$
\begin{aligned}
\frac{A_1}{\mathrm{d}g^j} \; & = \; -\frac{\omega_3^2}{\omega_3^2 - \omega_3^1}\frac{\mathrm{d}\tilde{s}}{\mathrm{d}g^j} > 0, \quad j = D, F, \qquad (99a) \\
\frac{A_2}{\mathrm{d}g^j} \; & = \; \frac{\omega_3^1}{\omega_3^2 - \omega_3^1}\frac{\mathrm{d}\tilde{s}}{\mathrm{d}g^j} < 0, \quad j = D, F, \qquad (99b)
\end{aligned}
$$

where $\omega_3^1 > 0$, $\omega_3^2 > 0$, $\mathrm{d}\tilde{s}/\mathrm{d}g^j < 0$.

From (99), we deduce the following property:

$$
\frac{A_2}{\mathrm{d}g^j} = -\frac{\omega_3^1}{\omega_3^2}\frac{A_1}{\mathrm{d}g^j}, \quad j = D, F. \qquad (100)
$$

Setting $t = 0$ into (52d), differentiating with respect to g^D, and noting that $\frac{\mathrm{d}\tilde{k}}{\mathrm{d}g^D} = 0$, the real exchange rate appreciates initially following an unanticipated permanent rise in government spending on the domestic good:

$$\begin{aligned}
\frac{\mathrm{d}p(0)}{\mathrm{d}g^D} &= \frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^D} + \omega_4^1 \frac{A_1}{\mathrm{d}g^D} + \omega_4^2 \frac{A_2}{\mathrm{d}g^D}, \\
&= \frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^D} + \left[\frac{\omega_3^1\omega_4^2 - \omega_3^2\omega_4^1}{\omega_3^2 - \omega_3^1}\right] \frac{\mathrm{d}\tilde{s}}{\mathrm{d}g^D}, \\
&= -\frac{F_{kk}}{D}\frac{p_c}{\tilde{p}} \left\{ (\tilde{p} r^\star \Phi_2 - \alpha_c p_c) + \left[\frac{\omega_3^1\omega_4^2 - \omega_3^2\omega_4^1}{\omega_3^2 - \omega_3^1}\right] \left(I_p + \frac{\tilde{X}}{\tilde{p}}\right) \right\} < 0. \qquad (101)
\end{aligned}$$

Then, we derive the initial reactions of investment and the rate of change of the real exchange rate:

$$\begin{aligned}
\frac{\mathrm{d}\dot{k}(0)}{\mathrm{d}g^D} &= \mu_1 \omega_3^1 \frac{A_1}{\mathrm{d}g^D} + \mu_2 \omega_3^2 \frac{A_2}{\mathrm{d}g^D}, \\
&= \left[\frac{\omega_3^1\omega_3^2}{\omega_3^2 - \omega_3^1}\right] (\mu_2 - \mu_1) \frac{\mathrm{d}\tilde{s}}{\mathrm{d}g^D} < 0, \qquad (102a) \\
\frac{\mathrm{d}\dot{p}(0)}{\mathrm{d}g^D} &= \mu_1 \omega_4^1 \frac{A_1}{\mathrm{d}g^D} + \mu_2 \omega_4^2 \frac{A_2}{\mathrm{d}g^D}, \\
&= -\left[\frac{\mu_1\omega_4^1\omega_3^2 - \mu_2\omega_4^2\omega_3^1}{\omega_3^2 - \omega_3^1}\right] \frac{\mathrm{d}\tilde{s}}{\mathrm{d}g^D} = 0. \qquad (102b)
\end{aligned}$$

We investigate in more details the condition of the non-monotonic adjustment of the investment in physical capital. More specifically, we must determine wether there exists a critical value of time, $t = \hat{t} > 0$, such that investment , i. e. $I\left(\hat{t}\right) = \dot{k}\left(\hat{t}\right) = 0$. Setting the time derivative of the stable solution of physical capital at zero, and solving for $\hat{t}$ yields:

$$\hat{t} = \frac{1}{\mu_1 - \mu_2} \ln \left[-\frac{\mu_2 \omega_3^2 A_2/\mathrm{d}g^D}{\mu_1 \omega_3^1 A_1/\mathrm{d}g^D} \right], \qquad (103)$$

where $\mu_1 - \mu_2 < 0$ and the necessary condition for $\hat{t} > 0$ corresponds to:

$$0 < -\frac{\mu_2 \omega_3^2 A_2/\mathrm{d}g^D}{\mu_1 \omega_3^1 A_1/\mathrm{d}g^D} < 1 \quad \Leftrightarrow \quad \frac{\mathrm{d}\dot{k}(0)}{\mathrm{d}g^D} < 0. \qquad (104)$$

If condition (104) holds, the stock of physical capital initially decreases before reaching a turning point at time $\hat{t}$. Subsequently, investment is crowded-in and the stock of capital goods increases towards its unchanged steady-state level (because we assumed that labor supply is inelastic).

Regarding the transitional path followed by the real exchange rate, it is convenient to rewrite the time derivative of its stable solution by using the fact that $\omega_4^i = \frac{\tilde{p}F_{kk}}{\mu_i}\omega_3^i$:

$$\dot{p}(t) = -\tilde{p} F_{kk} \omega_3^1 \frac{A_1}{\mathrm{d}g^D} \mathrm{d}g^D \left(1 - e^{(\mu_1 - \mu_2)t}\right) e^{\mu_2 t} > 0. \qquad (105)$$

To sum up, after a rise in g^D, the relative price of the foreign good jumps initially downward and then the real exchange rate depreciates along a stable monotonic transitional path.

We have now to derive the initial reaction of the current account. At this end, we differentiate the two-dimensional stable solution for $b(t)$ w. r. t. time, evaluate at $t = 0$ and differentiate w. r. t. g^j (with $j = D, F$):

$$\begin{aligned}
\frac{\mathrm{d}\dot{b}(0)}{\mathrm{d}g^j} &= \mu_1 \frac{N_1}{\mu_1 - r^\star} \frac{A_1}{\mathrm{d}g^j} + \mu_2 \frac{N_2}{\mu_2 - r^\star} \frac{A_2}{\mathrm{d}g^j} \\
&= -\left[\frac{\mu_1 (\mu_2 - r^\star) \omega_3^2 N_1 - \mu_2 (\mu_1 - r^\star) \omega_3^1 N_2}{(\mu_1 - r^\star)(\mu_2 - r^\star)\left(\omega_3^2 - \omega_3^1\right)}\right] \frac{\mathrm{d}\tilde{s}}{\mathrm{d}g^j}, \quad j = D, F \qquad (106) \\
&= -\frac{(-)}{(+)} \times (-) < 0,
\end{aligned}$$

where used the fact that $\left(\omega_3^2 - \omega_3^1\right) > 0$. From (106), the current account unambiguously deteriorates at time $t = 0$.

To investigate in more details the conditions under which the stock of foreign bonds adjusts non monotonically, we have to determine wether there exists a critical value of time, denoted by $t = \tilde{t} > 0$, such that the stock of traded bonds reaches a turning point along the stable trajectory, i. e. $ca\left(\tilde{t}\right) = \dot{b}\left(\tilde{t}\right) = 0$. Setting the time derivative of the stable solution of traded bonds equal to zero and solving for $\tilde{t}$, we get:

$$\tilde{t} = \frac{1}{\mu_1 - \mu_2} \ln\left[-\frac{\mu_2\left(\mu_1 - r^\star\right) N_2 A_2/\mathrm{d}g^j}{\mu_1\left(\mu_2 - r^\star\right) N_1 A_1/\mathrm{d}g^j}\right], \tag{107}$$

where $N_1 < 0$ for all parametrization and $N_2 \gtrless 0$. In the case $N_2 > 0$, the critical value of time along the transitional path does not exist since the term in square brackets is negative. Since N_2 is positive for all parametrization, the current account stays in deficit along entire adjustment.

Setting $t = 0$ into the stable solution of real consumption, differentiating with respect to g^j, and substituting constants given by (99), we obtain the initial response of real consumption following an unanticipated permanent rise in government spending:

$$\begin{aligned}\frac{\mathrm{d}c(0)}{\mathrm{d}g^j} &= \frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^j} + \left(\frac{\sigma + \mu_1}{\sigma}\right)\frac{A_1}{\mathrm{d}g^j} + \left(\frac{\sigma + \mu_2}{\sigma}\right)\frac{A_2}{\mathrm{d}g^j} \\ &= -\left[\frac{\mu_1\omega_3^2 - \mu_2\omega_3^1}{\sigma\left(\omega_3^2 - \omega_3^1\right)}\right]\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^j} = -\frac{(-)}{(+)} \times (-) < 0, \quad j = D, F\end{aligned} \tag{108}$$

where $\omega_3^2 - \omega_3^1 > 0$ and $\mu_1\omega_3^2 - \mu_2\omega_3^1 < 0$ (after tedious computations).

Regarding real consumption's transitional path, there exists a critical value of time, $t = \breve{t} > 0$, such that the real consumption reaches a turning point, i. e. $\dot{c}\left(\breve{t}\right) = 0$. Setting the time derivative of the stable solution of consumption equal to zero and solving for $\breve{t}$, we get:

$$\breve{t} = \frac{1}{\mu_1 - \mu_2} \ln\left[-\frac{\mu_2\omega_2^2 A_2/\mathrm{d}g^j}{\mu_1\omega_2^1 A_1/\mathrm{d}g^j}\right], \tag{109}$$

where $\mu_1 - \mu_2 < 0$ and the necessary condition for $\breve{t} > 0$ corresponds to:

$$0 < -\frac{\mu_2\omega_2^2 A_2/\mathrm{d}g^j}{\mu_1\omega_2^1 A_1/\mathrm{d}g^j} < 1 \quad \Leftrightarrow \quad \frac{\mathrm{d}\dot{c}(0)}{\mathrm{d}g^j} < 0. \tag{110}$$

If condition (110) holds, real consumption initially decreases before reaching a turning point at time $\breve{t}$. Subsequently, the real consumption rises towards its new lower long-run level.

Then, we investigate in more details the condition of the non-monotonic adjustment of the reference stock. Adopting the same procedure than previously, there exists a critical value of time, $t = \acute{t} > 0$, such that the habit stock reaches a turning point, i. e. $\dot{s}\left(\acute{t}\right) = \dot{s}\left(\acute{t}\right) = 0$. Setting the time derivative of the stable solution of the reference stock equal to zero and solving for $\acute{t}$, we get:

$$\acute{t} = \frac{1}{\mu_1 - \mu_2} \ln\left[-\frac{\mu_2 A_2/\mathrm{d}g^j}{\mu_1 A_1/\mathrm{d}g^j}\right], \tag{111}$$

where $\mu_1 - \mu_2 < 0$ and the necessary condition for $\acute{t} > 0$ corresponds to:

$$0 < -\frac{\mu_2 A_2/\mathrm{d}g^j}{\mu_1 A_1/\mathrm{d}g^j} < 1 \quad \Leftrightarrow \quad \frac{\mathrm{d}\dot{s}(0)}{\mathrm{d}g^j} < 0. \tag{112}$$

If condition (112) holds, the stock of habits initially decreases before reaching a turning point at time $\acute{t}$. Subsequently, accumulation of habits turns to be positive and the reference stock increases towards its new lower steady-state level, i. e. s overshoots its long-term value.

C.4 The Short-Term and Long-Term Marginal Propensities to Consume with Habits

In this subsection, we provide the main steps of derivation of the short-run and long-run marginal propensities to consume (MPC). The fact that stable paths are two-dimensional complicate substantially the derivation of MPC. Hence, we found convenient to simplify the model by abstracting from capital accumulation.

Households maximize the following objective function:

$$U = \int_0^{\infty} \left\{ \frac{1}{1-\epsilon} \left[\frac{c(t)}{(s(t))^{\gamma}} \right]^{1-\epsilon} - \gamma_N \frac{n(t)^{1+\epsilon_N}}{1+\epsilon_N} \right\} \exp\left(-\beta t\right) \mathrm{d}t, \tag{113}$$

subject to the flow budget constraint

$$p(t)\dot{b}(t) = r^{\star} p(t) b(t) + w(t) n(t) - p_c c - T, \tag{114}$$

and the accumulation equation of habits (4). The government finances government spending on the domestic good g by levying lump-sum taxes T.

Macroeconomic Equilibrium

The macroeconomic equilibrium is described by the following set of equations:

$$u_c\left(c,s\right) + \sigma\xi = \frac{p_c \lambda}{p}, \tag{115a}$$

$$v_n\left(n\right) = -\frac{\lambda}{p} w, \tag{115b}$$

$$F_n\left(n\right) = w, \tag{115c}$$

$$\dot{\lambda} = \lambda\left(\beta - r^{\star}\right), \tag{115d}$$

$$\dot{\xi} = \left(\beta + \sigma\right)\xi - u_s\left(c,s\right), \tag{115e}$$

$$F\left(n\right) = \left(1-\alpha_c\right) p_c c + X\left(p\right) + g, \tag{115f}$$

together with the accumulation equation of foreign assets (180), the accumulation equation of habits (4) and appropriate transversality conditions; $(1-\alpha_c) p_c c$ stands for the consumption in the domestic good.

Solving (115b) for labor yields $n = n\left(\bar{\lambda}, p\right)$. Substituting the short-run static solution for labor into the market-clearing condition for the home good (115f) and solving for the real exchange rate yields:

$$p = \Pi\left(\bar{\lambda}, c, g^D\right), \tag{116}$$

where

$$\Pi_{\bar{\lambda}} \equiv -\frac{F_n n_{\bar{\lambda}}}{F_n n_p - \Theta} > 0, \tag{117a}$$

$$\Pi_c \equiv \frac{\left(1-\alpha_c\right) p_c}{F_n n_p - \Theta} < 0, \tag{117b}$$

$$\Pi_{g^D} \equiv \frac{1}{F_n n_p - \Theta} < 0, \tag{117c}$$

with $\Theta \equiv \frac{X}{p}\left(\eta_X + \phi\left(1-\alpha_c\right)\frac{p c^F}{X}\right) > 0$.

Differentiating (115a) with respect to time, substituting (4), (115e) and (182), eliminating ξ by making use of (115a), yields the dynamic equation for real consumption:

$$\dot{c} = -\Psi^{-1} \frac{p_c \bar{\lambda}}{p u_{cc}} \left(r^{\star} - \rho\left(c, s, p, \bar{\lambda}\right)\right), \tag{118}$$

where

$$\Psi = 1 + \frac{p_c \bar{\lambda}}{p u_{cc} c} \left(1-\alpha_c\right) \frac{\Pi_c c}{p}. \tag{119}$$

Linearizing (184) together with the accumulation equation of habits (4) around the steady-state, and denoting long-term values by a tilde, we obtain in a matrix form:

$$\left(\dot{s}, \dot{c}\right)^T = J\left(s(t) - \tilde{s}, c(t) - \tilde{c}\right)^T, \tag{120}$$

where J is given by

$$J \equiv \begin{pmatrix} -\sigma & \sigma \\ \tilde{\Psi}^{-1}\frac{\beta+2\sigma}{\tilde{u}_{cc}}\Gamma & (\beta+\sigma) \end{pmatrix}, \tag{121}$$

with

$$\tilde{\Psi} = 1 + \frac{\beta + \sigma(1-\gamma)}{\epsilon(\beta+\sigma)} \frac{\tilde{c}^D(1-\alpha_c)}{\tilde{Y}\left[\chi(1-\alpha_K) + \frac{\tilde{X}}{\tilde{Y}}\nu_X + \frac{\tilde{p}\tilde{c}^F}{\tilde{Y}}\phi(1-\alpha_c)\right]} > 1. \tag{122}$$

Using (35b), the determinant denoted by Det of the linearized 2×2 matrix (188) is unambiguously negative:

$$\text{Det J} = -\tilde{\Psi}^{-1}\frac{\sigma}{\epsilon}\left[\beta + \sigma(1-\gamma)\right]\left\{\left[\gamma + \epsilon(1-\gamma)\right] - (1-\alpha_c)\frac{\Pi_c\tilde{c}}{\tilde{p}}\right\} < 0, \tag{123}$$

where $0 < \tilde{\Psi}^{-1} < 1$. The trace denoted by Tr is given by:

$$\text{Tr J} = \beta = r^\star > 0. \tag{124}$$

The characteristic roots obtained from the 2×2 linearized matrix J write as follows:

$$\mu_i \equiv \frac{1}{2}\left\{\text{Tr}J \pm \sqrt{(\text{Tr}J)^2 - 4\text{Det}J}\right\} \gtrless 0, \quad i = 1, 2. \tag{125}$$

Since the system features one state variable, s, and one jump variable, c, the equilibrium yields a unique one-dimensional stable saddle-path. It is straightforward to show that setting $\gamma = 0$ yields a stable root which simplifies to $\mu_1^{\gamma=0} = -\sigma$.

Stable solutions paths are given by:

$$s(t) - \tilde{s} = B_1 e^{\mu_1 t}, \quad c(t) - \tilde{c} = \omega_2^1 B_1 e^{\mu_1 t}, \tag{126}$$

where we normalized ω_1^i to unity. The eigenvector ω_2^i associated with eigenvalue μ_i is given by

$$\omega_2^i = \left(\frac{\sigma + \mu_i}{\sigma}\right) > 0, \quad i = 1, 2. \tag{127}$$

Eigenvector $\omega_2^1 = \left(\frac{\sigma+\mu_1}{\sigma}\right)$ is positive if and only if:

$$\left[\beta + \sigma(1-\gamma)\right]\left[\gamma + \epsilon(1-\gamma)\right] < \epsilon(\beta+\sigma). \tag{128}$$

It can be shown that condition (128) is satisfied as long as $\epsilon > 1$.

Consumption, Habit Persistence, and Permanent Income

Solving (180) and invoking the transversality condition yields:

$$\int_0^\infty \frac{p_c}{p(\tau)} c(\tau) e^{-r^\star \tau} \mathrm{d}\tau = b(0) + W(0),$$

where $W(0)$ denotes non-financial wealth, defined as the present discounted value of the future flow of real disposable income expressed in terms of the foreign good, i.e.

$$W(0) = \int_0^\infty \left[\frac{w(\tau)n(\tau) - T}{p(\tau)}\right] e^{-r^\star \tau} \mathrm{d}\tau.$$

Linearizing $\frac{p_c(p(t))}{p(t)}c(t)$ around the steady-state, substituting the stable solution for consumption $c(t) = \tilde{c} + \frac{\sigma+\mu_1}{\sigma}\left(s(t) - \tilde{s}\right)$ into the intertemporal budget constraint, this enables us to derive the long-run level of real expenditure, $\tilde{c}$, which satisfies the intertemporal solvency condition given a stable adjustment:

$$\tilde{c} = -\frac{\tilde{\Pi}}{\tilde{\Lambda}} \frac{r^\star (\sigma + \mu_1)}{\sigma} s_0 + \frac{r^\star - \mu_1}{\tilde{\Lambda}} \frac{\tilde{p} r^\star [b_0 + W(0)]}{p_c}, \tag{129}$$

where $\tilde{p}r^\star [b_0 + W(0)] / p_c$ is the permanent income defined as the annuity value of financial and non financial wealth expressed in terms of the foreign good deflated by the consumption price index. Additionally, we set:

$$\tilde{\Pi} = \left[1 - (1-\alpha_c) \frac{\Pi_c \tilde{c}}{\tilde{p}}\right] = 1 + \frac{(1-\alpha_c) \frac{\tilde{c}^D}{\tilde{Y}}}{\left[\chi (1-\alpha_K) + \frac{\tilde{X}}{\tilde{Y}} \nu_X + \frac{\tilde{p}\tilde{c}^F}{\tilde{Y}} \phi (1-\alpha_c)\right]} > 0, \tag{130a}$$

$$\tilde{\Lambda} = (r^\star - \mu_1) - \tilde{\Pi} \left(\frac{\sigma + \mu_1}{\sigma}\right) r^\star, \tag{130b}$$

where $\tilde{\Pi} > 1$.

Before deriving formal expressions of the marginal propensity to consume (MPC), we set the following assumption :

Assumption 1 $\tilde{\Pi} < \frac{r^\star - \mu_1}{r^\star}$.

which in turn insures that $0 < \tilde{\Lambda} < r^\star - \mu_1$. From an economic point of view, inequality $\tilde{\Pi} < \frac{r^\star - \mu_1}{r^\star}$ holds as long as the domestic content of total consumption expenditure $(1-\alpha_c)$ is small, and both the share of exports in GDP $\tilde{X}/\tilde{Y}$ and the import content of total consumption expenditure are large. Yet, as we shall see below, Assumption 1 holds even if the economy displays a small trade openness.

The term in front of the real permanent income $\frac{r^\star - \mu_1}{\tilde{\Lambda}} > 0$ on the RHS of (129) represents the long-run MPC of the real permanent income. Setting $\gamma = 0$ implies the multiplicative term reduces to unity so that the long-run MPC is equal to one. Unlike, relaxing time separability in utility implies that the long-MPC turns out to higher than one, at the condition that $(\sigma + \mu_1) > 0$ which is satisfied.

Evaluating now the stable solution of $c(t)$ at time $t = 0$, using the fact that $\tilde{s} = \tilde{c}$, and substituting (129), this enables us to derive the optimal initial level of real consumption:

$$c(0) = \left(\frac{\sigma + \mu_1}{\sigma}\right) \left[1 + \frac{\tilde{\Pi}}{\tilde{\Lambda}} \frac{r^\star \mu_1}{\sigma}\right] s_0 - \frac{\mu_1}{\sigma} \frac{r^\star - \mu_1}{\tilde{\Lambda}} \frac{\tilde{p} r^\star [b_0 + W(0)]}{p_c}, \tag{131}$$

where $\frac{r^\star - \mu_1}{\tilde{\Lambda}} > 0$ represents the short-run MPC the real permanent income. As long as inequality $0 < \tilde{\Pi} < \frac{r^\star - \mu_1}{r^\star}$ holds (see assumption (1)), optimal consumption at time $t = 0$ is positively correlated with the initial stock of habits and the short-run marginal propensity to consume the real permanent income is smaller than unity. Notice that $0 > \mu_1^{\gamma>0} > \mu_1^{\gamma=0} = -\sigma$. The reason is that without habits, dynamics do not degenerate but the adjustment towards the long-run equilibrium gets faster. Hence, the consumer without habits display a larger short-run marginal propensity to consume.

In a small open economy model where the relative price of domestic goods is exogenous and thereby remains constant over time, we have $\tilde{\Psi} = \tilde{\Pi} = 1$ and $\tilde{\Lambda} = -\frac{\mu_1}{\sigma}(\sigma + r^\star)$. Consequently, the short-run and long-run marginal propensities rewrite as follows:

$$\tilde{c} = \frac{r^\star (\sigma + \mu_1)}{\mu_1 (\sigma + r^\star)} s_0 + \frac{\sigma (r^\star - \mu_1)}{\mu_1 (\sigma + r^\star)} \frac{\tilde{p} r^\star [b_0 + W(0)]}{p_c}, \tag{132a}$$

$$c(0) = \frac{\sigma + \mu_1}{\sigma + r^\star} s_0 + \frac{r^\star - \mu_1}{\sigma + r^\star} \frac{\tilde{p} r^\star [b_0 + W(0)]}{p_c}. \tag{132b}$$

From (132b), the condition for the short-term MPC to be smaller than unity is $\sigma + \mu_1 > 0$ which always holds as long as $\gamma > 0$. In words, the short-run MPC is smaller with an exogenous p than with an endogenous p. The reason is as follows. Along the transitional path, the real exchange depreciates which in turn implies a smoother real consumption adjustment (due to the the rise in the consumption-based real interest rate which compensates the high time preference rate). Such a consumption behavior is consistent with the intertemporal solvency condition as long as the consumption falls by a larger amount in the short-run than that prevailing with a real exchange rate constant over time. Without habits, $\mu_1^{\gamma=0} = -\sigma$ such that the short-run and long-run MPC coincide and are equal to unity.

How is Relevant Assumption 1?

In this paragraph, we provide a numerical analysis which supports Assumption 1 since inequality $\tilde{\Pi} < \frac{r^\star - \mu_1}{r^\star}$ could not be derived analytically. Adopting the benchmark parametrization discussed in section 4.1, we set: the weight of habits into utility γ at 0.8, the relative-risk-aversion coefficient ϵ at 2.5, the speed at which habits catch-up with current consumption σ at 0.65, the weight attached to consumption in the domestic good φ at 0.95, the elasticity of substitution between the foreign and domestic good ϕ at 1.5, the output share of labor income $1-\alpha_K$ at 0.65, the fixed time discount rate $\beta = r^\star$ at 0.035, the elasticity of exports with respect to the real exchange rate ν_X at 0.8, the share of domestic goods in consumption expenditure $(1-\alpha_c)$ at 85%, the ratio of exports to GDP X/Y at 10%, the share of consumption in the domestic good in GDP c^D/Y at 57%, the share of consumption in the foreign good pc^F/Y at 10%, the elasticity of labor supply with respect to the marginal utility of wealth $\chi \equiv \frac{\sigma_N \beta_N}{\sigma_N + \beta_N}$ at 0.35 (setting σ_N and β_N at 0.4 and 2.9). Numerical results for μ_1 given by (125) and $\tilde{\Pi}$ given by (130a) are as follows: $\mu_1 = -0.06$, $\frac{r^\star - \mu_1}{r^\star} = 2.85$, and $\tilde{\Pi} = 2.11$. Hence, assumption 1 is satisfied for our benchmark parametrization. Notice that our calibration uses US data for the period 1980-2008. Since the US economy is less open than most of OECD countries and because $\tilde{\Pi}$ lowers as trade openness rises, we expect that this condition holds for a large range of parametrization.

C.5 Long-Term and Short-Term Effects of a Fiscal Expansion on the Consumption in the Domestic Good

Using the fact that $c^D = (1-\alpha_c)\, p_c c$, the steady-state change of consumption in the domestic good after a permanent rise in g^D is:

$$\frac{\mathrm{d}\tilde{c}^D}{\mathrm{d}g^D} = \phi\,(1-\alpha_c)\, c^F \frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^D} + (1-\alpha_c)\, p_c \frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^D} < 0, \tag{133}$$

where the negative sign follows from (88a) and (88b). Since consumption in the domestic good influences the size of the spending multiplier, we compare the magnitude of the decline in $\tilde{c}^D$ with the drop in total expenditure in consumption goods:

$$\begin{aligned}\frac{\mathrm{d}\tilde{c}^D}{\mathrm{d}g^D} - p_c \frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^D} &= \frac{r^\star F_{kn} n_p \Phi_1 p_c^2 \alpha_c}{D} \left\{\alpha_c + (1-\alpha_c)\,\phi\,[\gamma + \epsilon\,(1-\gamma)]\right\} \\ &\quad - \frac{(F_{kk} + F_{kn} n_k)}{D} \frac{p_c^2 \alpha_c}{\tilde{p}} \left[\frac{\tilde{X}}{\tilde{p}}(\nu_X - 1) + \phi\,(1-\alpha_c)\,\tilde{c} r^\star \Phi_1\right] \lesseqgtr 0,\end{aligned} \tag{134}$$

where we used the fact that $\tilde{p}\tilde{\Omega} = \frac{\tilde{X}}{\tilde{p}}\nu_X + \tilde{c}^F \phi\,(1-\alpha_c) - \frac{\tilde{X}}{\tilde{p}}$ and one useful expression (35f).

Using the fact that $c^F = p_c' c$, the steady-state change of consumption in the foreign good after a permanent rise in g^F is:

$$\frac{\mathrm{d}\tilde{c}^F}{\mathrm{d}g^D} = -\phi\,(1-\alpha_c)\,\frac{\tilde{c}^F}{p}\frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^D} + \frac{\alpha_c p_c}{\tilde{p}}\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^D} \lesseqgtr 0, \tag{135}$$

where the undetermined sign follows from (90a) and (90b).

Linearizing $c^D = (1-\alpha_c)\, p_c c$ around the steady-state, evaluating the resulting expression at time $t = 0$ and differentiating with respect to g^D yields:

$$\frac{\mathrm{d}c^D(0)}{\mathrm{d}g^D} = \phi\,(1-\alpha_c)\,\tilde{c}^F \frac{\mathrm{d}p(0)}{\mathrm{d}g^D} + (1-\alpha_c)\, p_c \frac{\mathrm{d}c(0)}{\mathrm{d}g^D} < 0, \tag{136}$$

where the negative sign follows from (101) and (108).

C.6 Time Preference Rate Dynamics

The specification of a habit-forming behavior implies a variable time preference rate. Following an unanticipated permanent fiscal expansion, the time preference rate rises initially and then decreases over time toward its steady-state value, β. The reaction of ρ reflects the temporary gap between the marginal utility of current consumption and the marginal utility of future consumption. In this section, we determine the stable transitional paths for the time preference rate which have been computed numerically.

The solution of the time preference rate is obtained by linearizing (32) in the neighborhood of the steady-state and using the fact that $\tilde{\rho} = \beta$:

$$\begin{aligned} \rho(t) &= \beta + \frac{\tilde{p} u_{cc}}{p_c \bar{\lambda}} (\beta + \sigma) (c(t) - \tilde{c}) + \frac{\tilde{p}}{p_c \bar{\lambda}} (\beta + 2\sigma) \tilde{\Gamma} (s(t) - \tilde{s}) + \frac{(1 - \alpha_c)}{\tilde{p}} (\beta + \sigma) (p(t) - \tilde{p}), \\ &= \beta + \frac{\tilde{p} u_{cc}}{p_c \bar{\lambda}} (\beta + \sigma) \left[\Xi_1 A_1 e^{\mu_1 t} + \Xi_2 A_2 e^{\mu_2 t} \right], \end{aligned} \tag{137}$$

where

$$\Xi_1 = \omega_2^1 + \frac{(\beta + 2\sigma)}{(\beta + \sigma)} \frac{\tilde{\Gamma}}{u_{cc}} + \frac{p_c \bar{\lambda}}{\tilde{p}^2 u_{cc}} (1 - \alpha_c) \omega_4^1 = -\frac{(\beta + 2\sigma)}{(\beta + \sigma)} \frac{\tilde{\Gamma}}{u_{cc}} \frac{\mu_1}{(\beta + \sigma - \mu_1)} < 0, \tag{138a}$$

$$\Xi_2 = \omega_2^2 + \frac{(\beta + 2\sigma)}{(\beta + \sigma)} \frac{\tilde{\Gamma}}{u_{cc}} + \frac{p_c \bar{\lambda}}{\tilde{p}^2 u_{cc}} (1 - \alpha_c) \omega_4^2 = -\frac{(\beta + 2\sigma)}{(\beta + \sigma)} \frac{\tilde{\Gamma}}{u_{cc}} \frac{\mu_2}{(\beta + \sigma - \mu_2)} < 0, \tag{138b}$$

with Γ given by (37). To determine the signs of Ξ_1 and Ξ_2, we made use of (60), i. e. $\omega_2^i = -\frac{\frac{\beta+2\sigma}{\tilde{u}_{cc}}\tilde{\Gamma} + \frac{\bar{\lambda} p_c}{\tilde{p}^2 u_{cc}}(1-\alpha_c)(\beta+\sigma-\mu_i)\omega_4^i}{(\beta+\sigma-\mu_i)} > 0$.

We estimate the initial reaction of the time preference rate by evaluating (137) at time $t = 0$ and by differentiating w. r. t. g^j $(j = D, F)$:

$$\frac{\mathrm{d}\rho(0)}{\mathrm{d}g^j} = \frac{\tilde{p} u_{cc}}{p_c \bar{\lambda}} (\beta + \sigma) \left[\Xi_1 \frac{A_1}{\mathrm{d}g^i} + \Xi_2 \frac{A_2}{\mathrm{d}g^i} \right]. \tag{139}$$

D Welfare Analysis

In this section, we investigate the welfare effects of an unanticipated permanent rise in government spending, g^j, falling on the domestic good $(j = D)$ or the foreign good $(j = F)$. We denote by ϕ the instantaneous welfare:

$$\phi(t) = u(c(t), s(t)) + v(n(t)), \tag{140}$$

and by U its discounted value over an infinite horizon:

$$U = \int_0^\infty \phi(t) \exp(-\beta t) \, \mathrm{d}t. \tag{141}$$

We repeat by convenience the steady-state value of real consumption (see (10)):

$$\tilde{c} = \left[\left(\frac{\beta + \sigma}{\beta + \sigma (1 - \gamma)} \right) \frac{p_c \bar{\lambda}}{\tilde{p}} \right]^{-\frac{1}{\gamma + \epsilon(1-\gamma)}}. \tag{142}$$

Differentiating the felicity function $u(c)$ w. r. t. . c and s and evaluating at the steady state using (193) yields:

$$\tilde{u}_c = \tilde{c}^{-[\gamma + \epsilon(1-\gamma)]} = \left(\frac{\beta + \sigma}{\beta + \sigma (1 - \gamma)} \right) \frac{p_c \bar{\lambda}}{\tilde{p}} > 0, \tag{143a}$$

$$\tilde{u}_s = -\gamma \tilde{c}^{-[\gamma + \epsilon(1-\gamma)]} = -\gamma \left(\frac{\beta + \sigma}{\beta + \sigma (1 - \gamma)} \right) \frac{p_c \bar{\lambda}}{\tilde{p}} = -\gamma \tilde{u}_c < 0, \tag{143b}$$

where a tilde over partial derivatives indicate that they are evaluated at the steady-state. Furthermore, we have computed several useful expressions:

$$\Delta_1 \equiv \tilde{u}_c\omega_2^1 + \tilde{u}_s = \left(\frac{\beta+\sigma}{\beta+\sigma\,(1-\gamma)}\right)\frac{p_c\bar{\lambda}}{\tilde{p}}\left[\frac{\sigma\,(1-\gamma)+\mu_1}{\sigma}\right] < 0, \tag{144a}$$

$$\Delta_2 \equiv \tilde{u}_c\omega_2^2 + \tilde{u}_s = \left(\frac{\beta+\sigma}{\beta+\sigma\,(1-\gamma)}\right)\frac{p_c\bar{\lambda}}{\tilde{p}}\left[\frac{\sigma\,(1-\gamma)+\mu_2}{\sigma}\right] > 0, \tag{144b}$$

where we imposed the following condition which holds for plausible values of parameters (as shown by numerical experiments):

$$\sigma\,(1-\gamma)+\mu_1 < 0, \quad \sigma\,(1-\gamma)+\mu_2 > 0. \tag{145}$$

D.1 Instantaneous Welfare Effects

We first linearize the instantaneous utility function (190) in the neighborhood of the steady-state:

$$\phi(t) = \tilde{\phi} + u_c\,(\tilde{c},\tilde{s})\,(c(t)-\tilde{c}) + u_s\,(\tilde{c},\tilde{s})\,(s(t)-\tilde{s}) + v_n\,(n(t)-\tilde{n})\,, \tag{146}$$

with $\tilde{\phi}$ given by

$$\tilde{\phi} = u\,(\tilde{c},\tilde{c}) + v\,(\tilde{n})\,, \tag{147}$$

where we used the fact $\tilde{c} = \tilde{s}$ at the steady-state.

By substituting solutions for $s(t)$, $c(t)$ and $n(t)$, we derive the two-dimensional stable solution for instantaneous welfare:

$$\phi(t) = \tilde{\phi} + \left[u_c\omega_2^1 + u_s + v_nL_1\right]A_1e^{\mu_1 t} + \left[u_c\omega_2^2 + u_s + v_nL_2\right]A_2e^{\mu_2 t}, \tag{148}$$

where $L_1 = n_k\omega_3^1 + n_p\omega_4^1$ and $L_2 = n_k\omega_3^2 + n_p\omega_4^2$; we drop the superscript tilde to save notation; we stress that the partial derivatives are evaluated at the steady-state, i. e. $u_c = u_c\,(\tilde{c},\tilde{c})$, $u_s = u_s\,(\tilde{c},\tilde{c})$, and $v_n = v_n\,(\tilde{n})$.

Differentiating (200) w. r. t. g^j yields the steady-state change of instantaneous welfare:

$$\frac{\mathrm{d}\tilde{\phi}}{\mathrm{d}g^j} = (u_c+u_s)\,\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^j} + v_n\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^j} < 0, \quad j = D, F, \tag{149}$$

where $u_c + u_s = (1-\gamma)\,u_c > 0$ and $v_n = -\frac{\bar{\lambda}}{\tilde{p}}F_n < 0$. Since $\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^j} < 0$ and $\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^j} > 0$, households unambiguously experience instantaneous welfare losses in the long-run. As γ gets closer to unity γ and labor supply gets less responsive, i. e. σ_N gets smaller, agents pay less attention to the drop in absolute consumption in deriving utility and supply less labor, which in turn dampen welfare losses.

We estimate the initial reaction of instantaneous welfare by evaluating (201) at time $t = 0$ and differentiating w. r. t. g^j:

$$\frac{\mathrm{d}\phi(0)}{\mathrm{d}g^j} = \frac{\mathrm{d}\tilde{\phi}}{\mathrm{d}g^j} + [\Delta_1 + v_nL_1]\,\frac{A_1}{\mathrm{d}g^i} + [\Delta_1 + v_nL_1]\,\frac{A_2}{\mathrm{d}g^j}, \quad j = D, F, \tag{150}$$

In a semi-small open economy model with fixed labor, ϕ drops on impact and overshoots its new lower steady-state value, whereas the fall in consumption displays a smaller size in the short-term than in the long-term. With elastic labor supply, besides the initial welfare losses induced by the reduction of real consumption on impact, the rise in labor supply in the ST magnifies the fall in utility. Therefore, ϕ falls in the short-term by a larger amount than in the long-term.

D.2 Overall Welfare Effects

So far, we have analyzed the instantaneous welfare implications of an unanticipated permanent fiscal expansion, say at different points of times. To address welfare effects in a convenient way within an intertemporal-maximizing framework, we have to evaluate the discounted value of (190) over the agent's infinite planning horizon. We will see numerically that the sluggish adjustment in welfare that arises from consumption inertia enters in sharp contrast with the adjustment of welfare with time separable preferences.

In order to have a correct and comprehensive measure of welfare, we calculate the discounted value of instantaneous welfare over the entire planning horizon

$$U = \frac{\tilde{u}}{\beta} + \frac{[\Delta_1 + v_n L_1]}{\beta - \mu_1} A_1 + \frac{[\Delta_2 + v_n L_2]}{\beta - \mu_2} A_2. \tag{151}$$

The first term on the right hand-side of (208) represents the capitalized value of instantaneous welfare evaluated at the steady-state. The second and the third term on the RHS of (208) vanish whenever preferences are time separable. The change in overall welfare after a fiscal expansion will be estimated numerically.

E Simulations

E.1 Functional Forms and the Steady-State

Consumption-Side

To conduct the numerical analysis, we assume that the utility function is of the CRRA form:

$$u(c) = \frac{1}{1-\epsilon} \left(\frac{c}{s^{\gamma}} \right)^{1-\epsilon}, \tag{152}$$

where the parameter ϵ corresponds to the coefficient of relative risk aversion and the parameter γ indexes the importance of habit formation in the instantaneous utility function.

We assume that the representative household maximizes a CES function given by:

$$c(.,.) = \left[\varphi^{\frac{1}{\phi}} \left(c^D\right)^{\frac{\phi-1}{\phi}} + (1-\varphi)^{\frac{1}{\phi}} \left(c^F\right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{153}$$

with $\phi > 0$ the intratemporal elasticity of substitution between consumption of domestic and foreign goods, given total expenditure measured in terms of traded goods:

$$E \equiv c^D + pc^F. \tag{154}$$

At the first stage, the household minimizes the cost, $E(t) = c^D(t) + p(t)c^F(t)$, for a given level of subutility, $c(t)$, where $p(t)$ is the relative price of the foreign good or the real exchange rate. For any chosen $c(t)$, the optimal basket $(c^D(t), c^F(t))$ is a solution to:

$$p_c(p(t))\, c(t) = \min_{\{c^D(t), c^F(t)\}} \left\{ c^D(t) + p(t)c^F(t)(t) : c\left(c^D(t), c^F(t)\right) \geq c(t) \right\}. \tag{155}$$

The subutility function $c(.)$ is linear homogeneous implies that total expenditure in consumption goods can be expressed as $E(t) = p_c(p(t))\, c(t)$, with $p_c(p(t))$ is the unit cost function dual (or consumption-based price index) to c. The unit cost dual function, $p_c(.)$, is defined as the minimum total expense in consumption goods, E, such that $c = c\left(c^D(t), c^F(t)\right) = 1$, for a given level of the real exchange rate, p. Its expression is given by

$$p_c = \left[\varphi + (1-\varphi)\, p^{1-\phi} \right]^{\frac{1}{1-\phi}}. \tag{156}$$

The minimized unit cost function depends on the real exchange rate and is expressed in terms of the foreign good. It has the following properties:

$$p'_c = (1-\varphi)\, p^{-\phi} \left[\varphi + (1-\varphi) p^{1-\phi}\right]^{\frac{\phi}{1-\phi}} > 0, \tag{157a}$$

$$\begin{aligned} p''_c &= \phi\,(1-\varphi)\, p^{-(1+\phi)} \left[\varphi + (1-\varphi)\, p^{1-\phi}\right]^{\frac{\phi}{1-\phi}} \left[\frac{(1-\varphi)\, p^{1-\phi}}{\varphi + (1-\varphi)\, p^{1-\phi}} - 1\right] < 0, \\ &= \phi p'_c p^{-1} \left[\frac{(1-\varphi)\, p^{1-\phi}}{\varphi + (1-\varphi)\, p^{1-\phi}} - 1\right]. \end{aligned} \tag{157b}$$

Intra-temporal allocations between non tradable goods and tradable goods follow from Shephard's Lemma (or the envelope theorem) applied to (155):

$$c^F = p'_c c, \quad \text{and} \quad \frac{pc^F}{p_c c} = \alpha_c, \tag{158a}$$

$$c^D = \left[p_c - pp'_c\right] c, \quad \text{and} \quad \frac{c^D}{p_c c} = (1-\alpha_c), \tag{158b}$$

with the shares of the foreign and the domestic goods in consumption expenditure are given respectively by

$$\alpha_c = \frac{(1-\varphi)\, p^{1-\phi}}{\varphi + (1-\varphi)\, p^{1-\phi}},, \tag{159a}$$

$$1-\alpha_c = \frac{\varphi}{\varphi + (1-\varphi)\, p^{1-\phi}}. \tag{159b}$$

Making use of expressions (157), the term $-p''_c p / p'_c$ can be rewritten as follows:

$$-\frac{p''_c p}{p'_c} = \phi\,(1-\alpha_c) > 0, \tag{160}$$

where $\phi\,(1-\alpha_c)$ represents the elasticity of consumption in the foreign good with respect to the real exchange rate.

Production-Side

We assume that the production function takes the Cobb-Douglas form:

$$Y = f(k) = k^{\alpha_K}, \tag{161}$$

where α_K corresponds to the capital share.

Partial and cross-partial derivatives of the production function write as follows:

$$\begin{aligned} F_k &= \alpha_K k^{\alpha_K - 1} = \alpha_K \frac{Y}{k} > 0, \quad F_{kk} = -(1-\alpha_K)\frac{F_k}{k} < 0, \\ F_n &= (1-\alpha_K)\frac{Y}{n} > 0, \quad F_{nn} = -\alpha_K \frac{F_n}{n} < 0, \\ F_{kn} &= \alpha_K \frac{F_n}{k} > 0. \end{aligned}$$

Export Function

Because the economy is semi-small, it is large enough in the production of the domestic good to affect its relative price and thus the nation's real exchange rate p. Following Kollman [2001], we assume that this influence is captured by an export function, $X(p)$ which takes a power form:

$$X = X(p) = \gamma_X p^{\nu_X}, \tag{162}$$

where $\gamma_X > 0$ is a constant and ν_X represents the price elasticity of the home country's exports. Note that in the limiting case, of a perfectly elastic export demand function, i. e. $\nu_X = +\infty$, the real exchange rate is equal to unity.

By making use of the functional form for the export function (162), we can rewrite the reaction of export and consumption in the domestic good w. r. t. the real exchange rate as follows:

$$\Theta = \left(X_p - pp_c''c\right) = \frac{X}{p}\left[\nu_X + \frac{c^D}{X}\phi\alpha_c\right] > 0. \tag{163}$$

Finally, we estimate at the steady-state the net export reaction function with respect to the real exchange rate given by (68) by substituting the functional form for the export function (see eq. (162)):

$$\tilde{\Omega} \equiv -\frac{1}{\tilde{p}}\left[\frac{\tilde{X}}{\tilde{p}} - \tilde{\Theta}\right] = \frac{\tilde{X}}{\tilde{p}^2}\left[\nu_X + \phi\left(1-\alpha_c\right)\frac{\tilde{p}\tilde{c}^F}{\tilde{X}} - 1\right] > 0. \tag{164}$$

where we used the fact that $\frac{c^D}{X}\alpha_c = \frac{pc^F}{X}\phi\left(1-\alpha_c\right)$.

Steady-State

The steady-state (86) can be rewritten as follows:

$$\tilde{c} = \left[\left(\frac{\beta+\sigma}{\beta+\sigma\left(1-\gamma\right)}\right)\frac{p_c\bar{\lambda}}{\tilde{p}}\right]^{-\frac{1}{\gamma+\epsilon(1-\gamma)}}, \tag{165a}$$

$$\tilde{c} = \tilde{s}, \tag{165b}$$

$$\frac{\tilde{k}}{\tilde{n}} = \left(\frac{\alpha_K}{r^\star + \delta_K}\right)^{\frac{1}{1-\alpha_K}}, \tag{165c}$$

$$\tilde{n} = \left[\frac{1}{\gamma_N}\frac{\bar{\lambda}}{\tilde{p}}\left(1-\alpha_K\right)\left(\frac{\tilde{k}}{\tilde{n}}\right)^{\alpha_K}\right]^{\frac{1}{\epsilon_N}}, \tag{165d}$$

$$\tilde{Y} = \tilde{c}^D + \tilde{I} + \gamma_X\left(\tilde{p}\right)^{\nu_X} + g^D, \tag{165e}$$

$$\tilde{b} = -\frac{\left[\tilde{Y} - p_c\left(\tilde{p}\right)\tilde{c} - \tilde{I} - g^D - \tilde{p}g^F\right]}{\tilde{p}r^\star}, \tag{165f}$$

$$\tilde{b} - \Phi_1\tilde{k} - \Phi_2\tilde{s} = b_0 - \Phi_1 k_0 - \Phi_2 s_0. \tag{165g}$$

with

$$\tilde{c}^D = \left(1-\alpha_c\right)p_c\tilde{c}, \quad \tilde{c}^F = \frac{\alpha_c}{\tilde{p}}p_c\tilde{c}, \tag{166a}$$

$$\tilde{Y} = \tilde{k}^{\alpha_K}\tilde{n}^{1-\alpha_K} = \left(\frac{r^\star+\delta_K}{\alpha_K}\right)\tilde{k}, \quad \tilde{I} = \delta_K\tilde{k}. \tag{166b}$$

E.2 Eigenvectors and Real Eigenvalues

The Linearized Matrix

The linearized version of the dynamic system writes as follows

$$\left(\dot{s}, \dot{c}, \dot{k}, \dot{p}\right)^T = J\left(s(t)-\tilde{s}, c(t)-\tilde{c}, k(t)-\tilde{k}, p(t)-\tilde{p}\right)^T, \tag{167}$$

where the linearized elements matrix, denoted by J, is given by

$$J \equiv \begin{pmatrix} -\sigma & \sigma & 0 & 0 \\ a_{21} & (\beta+\sigma) & a_{23} & a_{24} \\ 0 & \tilde{I}_c & \tilde{I}_k - \delta_K & \tilde{I}_p \\ 0 & 0 & a_{43} & a_{44} \end{pmatrix}, \tag{168}$$

with

$$
\begin{aligned}
a_{21} &= -\frac{\gamma}{\epsilon}\left\{\sigma\epsilon-(1-\epsilon)\left[\beta+\sigma(1-\gamma)\right]\right\}, <0,\\
a_{23} &= \frac{\left[\beta+\sigma(1-\gamma)\right]}{\epsilon(\beta+\sigma)}\frac{\tilde{c}^D}{p_c}\left(F_{kk}+F_{kn}n_p\right)<0,\\
a_{24} &= -\frac{\left[\beta+\sigma(1-\gamma)\right]}{\epsilon(\beta+\sigma)}\frac{\tilde{c}^D}{\tilde{p}p_c}\left[(\beta+\sigma)-F_{kn}n_p\right]<0,\\
a_{43} &= \tilde{p}\left(F_{kk}+F_{kn}n_k\right)<0, \quad a_{44}=\tilde{p}F_{kn}n_p,
\end{aligned}
$$

and $\tilde{I}_c = -\tilde{c}^D/\tilde{c} < 0$, $\tilde{I}_k - \delta_K = r^\star + F_n n_k = r^\star + \chi F_{kn}\tilde{n} > 0$, and $\tilde{I}_p = -\frac{\tilde{Y}}{\tilde{p}}\left[\chi(1-\alpha_K)+\frac{\tilde{X}}{\tilde{Y}}\left(\nu_X+\frac{\tilde{c}^D}{\tilde{X}}\alpha_c\phi\right)\right]<0$.

Determinant and Condition for Real Roots

Using functional forms, the determinant J of the matrix of the linearized system denoted by b_4 (see (44d)) writes now as follows:

$$
\begin{aligned}
\text{Det J} = b_4 &= -\frac{\sigma}{\epsilon}\left[\beta+\sigma(1-\gamma)\right]\left[\gamma+\epsilon(1-\gamma)\right]\Big\{\left(F_{kk}+F_{kn}n_k\right)\left[\tilde{p}\tilde{\Theta}+\nu(1-\alpha_c)\tilde{c}^D\right]\\
&+ \tilde{p}n_p\left[F_{kn}r^\star-F_{kk}F_n\right]\Big\}>0, \qquad (169)
\end{aligned}
$$

where $\nu=\frac{1}{[\beta+\sigma(1-\gamma)]}$ is the long-run IES with time non separable preferences.

Adopting a similar procedure, we estimate the term $b_3/r^\star$ (see (44c)):

$$
\begin{aligned}
\frac{b_3}{r^\star} &= \frac{\sigma}{\epsilon}\left[\gamma+\epsilon(1-\gamma)\right]\left[\beta+\sigma(1-\gamma)\right]-\tilde{p}\left(F_{kk}+F_{kn}n_k\right)\left[\tilde{\Theta}+(1-\alpha_c)\frac{\tilde{c}^D}{\tilde{p}}\frac{\left[\beta+\sigma(1-\gamma)\right]}{\epsilon(\beta+\sigma)}\right]\\
&+ \tilde{p}n_p\left[F_{kk}F_n-F_{kn}r^\star\right]>0. \qquad (170)
\end{aligned}
$$

and the condition (51) for real roots:

$$
\begin{aligned}
\left(\frac{b_3}{r^\star}\right)^2-4b_4 &= \Big\{\frac{\sigma}{\epsilon}\left[\gamma+\epsilon(1-\gamma)\right]\left[\beta+\sigma(1-\gamma)\right]-\tilde{p}\left(F_{kk}+F_{kn}n_k\right)\left[\tilde{\Theta}+(1-\alpha_c)\frac{\tilde{c}^D}{\tilde{p}}\frac{\left[\beta+\sigma(1-\gamma)\right]}{\epsilon(\beta+\sigma)}\right]\\
&- \tilde{p}n_p\left[F_{kk}F_n-F_{kn}r^\star\right]\Big\}^2+4\frac{\sigma\gamma}{\epsilon}\left\{\sigma\epsilon-(1-\epsilon)\left[\beta+\sigma(1-\gamma)\right]\right\}\frac{\left[\beta+\sigma(1-\gamma)\right]}{\epsilon(\beta+\sigma)}\\
&\times (1-\alpha_c)\tilde{c}^D\left(F_{kk}+F_{kn}n_k\right)\gtrless 0, \qquad (171)
\end{aligned}
$$

which is positive as long as σ is not too small and γ is not too close from unity.

Eigenvalues

We write out the two stable and two unstable eigenvalues:

$$\mu_1 \equiv \frac{1}{2}\left\{r^\star-\sqrt{(r^\star)^2+2\left(\frac{b_3}{r^\star}+\sqrt{\left(\frac{b_3}{r^\star}\right)^2-4b_4}\right)}\right\}<0, \qquad (172a)$$

$$\mu_2 \equiv \frac{1}{2}\left\{r^\star-\sqrt{(r^\star)^2+2\left(\frac{b_3}{r^\star}-\sqrt{\left(\frac{b_3}{r^\star}\right)^2-4b_4}\right)}\right\}<0, \qquad (172b)$$

$$\mu_3 \equiv \frac{1}{2}\left\{r^\star+\sqrt{(r^\star)^2+2\left(\frac{b_3}{r^\star}-\sqrt{\left(\frac{b_3}{r^\star}\right)^2-4b_4}\right)}\right\}>0, \qquad (172c)$$

$$\mu_4 \equiv \frac{1}{2}\left\{r^\star+\sqrt{(r^\star)^2+2\left(\frac{b_3}{r^\star}+\sqrt{\left(\frac{b_3}{r^\star}\right)^2-4b_4}\right)}\right\}>0, \qquad (172d)$$

where $b_3/r^\star$, $(b_3/r^\star)^2 - 4b_4$ are given by (170) and (171).

If preferences are time separable, the smallest eigenvalue and the highest eigenvalue reduce to:

$$\mu_1^{\gamma=0} = -\sigma < 0, \quad \mu_4^{\gamma=0} = r^\star + \sigma > 0. \tag{173}$$

Eigenvectors

Eigenvector ω_2^i writes as follows:

$$\omega_2^1 = \left(\frac{\sigma + \mu_1}{\sigma}\right) > 0, \tag{174a}$$

$$\omega_2^2 = \left(\frac{\sigma + \mu_2}{\sigma}\right) > 0, \tag{174b}$$

Eigenvector ω_3^i writes as follows:

$$\omega_3^1 = \frac{\left(\tilde{p}F_{kn}n_p - \mu_1\right) I_c \left(\sigma + \mu_1\right)}{\sigma\left\{\mu_1\mu_4 - \tilde{p}\left[\left(F_{kk} + F_{kn}n_k\right)\tilde{\Theta} + n_p\left(F_{kn}r^\star - F_{kn}F_n\right)\right]\right\}} > 0, \tag{175a}$$

$$\omega_3^2 = \frac{\left(\tilde{p}F_{kn}n_p - \mu_2\right) I_c \left(\sigma + \mu_2\right)}{\sigma\left\{\mu_2\mu_3 - \tilde{p}\left[\left(F_{kk} + F_{kn}n_k\right)\tilde{\Theta} + n_p\left(F_{kn}r^\star - F_{kn}F_n\right)\right]\right\}} > 0. \tag{175b}$$

Eigenvector ω_4^i writes as follows:

$$\omega_4^1 = -\frac{\tilde{p}\left(F_{kk} + F_{kn}n_k\right)}{\left(\tilde{p}F_{kn}n_p - \mu_1\right)}\omega_3^1, > 0, \tag{176a}$$

$$\omega_4^2 = -\frac{\tilde{p}\left(F_{kk} + F_{kn}n_k\right)}{\left(\tilde{p}F_{kn}n_p - \mu_2\right)}\omega_3^2, > 0, \tag{176b}$$

where the signs of ω_2^1, ω_2^2, ω_3^2 and ω_4^2 stem from the following condition that we imposed (and holds for a large range of parametrization):

$$0 < -\tilde{p}\left[\left(F_{kk} + F_{kn}n_k\right)\tilde{\Theta} + n_p\left(F_{kn}r^\star - F_{kn}F_n\right)\right] < \sigma\left(\sigma + \beta\right),$$

with $\beta = r^\star$.

Finally, we estimated eigenvectors for the time preference rate:

$$\bar{\Xi}\Xi_1 = \bar{\Xi}\frac{\gamma}{\epsilon\left(\beta + \sigma\right)}\left\{\sigma\epsilon - \left(1 - \epsilon\right)\left[\beta + \sigma\left(1 - \gamma\right)\right]\right\}\frac{\mu_1}{\left(\sigma + \mu_4\right)} > 0, \tag{177a}$$

$$\bar{\Xi}\Xi_2 = \bar{\Xi}\frac{\gamma}{\epsilon\left(\beta + \sigma\right)}\left\{\sigma\epsilon - \left(1 - \epsilon\right)\left[\beta + \sigma\left(1 - \gamma\right)\right]\right\}\frac{\mu_2}{\left(\sigma + \mu_3\right)} > 0, \tag{177b}$$

with

$$\bar{\Xi} = \frac{\tilde{p}u_{cc}}{p_c\bar{\lambda}}\left(\beta + \sigma\right) = -\frac{\left(\beta + \sigma\right)^2}{\left[\beta + \sigma\left(1 - \gamma\right)\right]}\frac{\epsilon}{\tilde{c}} < 0. \tag{178}$$

F The Long-Run Adjustment of the Real Exchange Rate and Habits

In this section, we solve (165a)-(165d) for the steady-state values of consumption, reference stock, labor and capital stock. In a first step, we solve (165c)-(165d) for the steady-state values of physical capital and employment:

$$\tilde{k} = k\left(\bar{\lambda}, \tilde{p}\right), \quad \tilde{n} = n\left(\bar{\lambda}, \tilde{p}\right), \tag{179}$$

with

$$\eta_{k,\bar{\lambda}} \equiv \frac{\partial \tilde{k}}{\partial \bar{\lambda}} \frac{\bar{\lambda}}{\tilde{k}} = -F_{kn} F_{kk} \tilde{k} \frac{v_n}{v_{nn}} = \sigma_N > 0, \tag{180a}$$

$$\eta_{k,p} \equiv \frac{\partial \tilde{k}}{\partial \tilde{p}} \frac{\tilde{p}}{\tilde{k}} = F_{kn} F_{kk} \tilde{k} \frac{v_n}{v_{nn}} = -\sigma_N < 0, \tag{180b}$$

$$\eta_{n,\bar{\lambda}} \equiv \frac{\partial \tilde{n}}{\partial \bar{\lambda}} \frac{\bar{\lambda}}{\tilde{n}} = \frac{v_n}{v_{nn} \tilde{n}} = \sigma_N > 0, \tag{180c}$$

$$\eta_{n,p} \equiv \frac{\partial \tilde{n}}{\partial \tilde{p}} \frac{\tilde{p}}{\tilde{n}} = -\frac{v_n}{v_{nn} \tilde{n}} = -\sigma_N < 0, \tag{180d}$$

where we used the fact that $F_{kn} = -F_{kk} \frac{k}{n}$ (see property (18a)). Substitution of (179) into the production function $Y = F(k, n)$, enables us to solve for output. This yields:

$$\tilde{Y} = Y\left(\bar{\lambda}, \tilde{p}\right), \tag{181}$$

with

$$\eta_{Y,\bar{\lambda}} \equiv \frac{\partial \tilde{Y}}{\partial \bar{\lambda}} \frac{\bar{\lambda}}{\tilde{Y}} = \sigma_N > 0, \quad \eta_{Y,p} \equiv \frac{\partial \tilde{Y}}{\partial \tilde{p}} \frac{\tilde{p}}{\tilde{Y}} = -\sigma_N < 0. \tag{182}$$

In a second step, we solve (165a)-(165b) for the steady-state values of consumption and the reference stock:

$$\tilde{c} = \tilde{s} = c\left(\bar{\lambda}, \tilde{p}\right), \tag{183}$$

with

$$\eta_{c,\bar{\lambda}} \equiv \frac{\partial \tilde{c}}{\partial \bar{\lambda}} \frac{\bar{\lambda}}{\tilde{c}} = -\nu < 0, \quad \eta_{c,p} \equiv \frac{\partial \tilde{c}}{\partial \tilde{p}} \frac{\tilde{p}}{\tilde{c}} = \nu\left(1 - \alpha_c\right) > 0, \tag{184}$$

where $\nu \equiv \frac{1}{\gamma + \epsilon(1-\alpha_c)} > 0$ corresponds to the long-run intertemporal elasticity of substitution for consumption. Substituting (183) into $c^D = (1 - \alpha_c) p_c c$, we can solve for the steady-state value of consumption in the domestic good:

$$\tilde{c}^D = c^D\left(\bar{\lambda}, \tilde{p}\right), \tag{185}$$

with

$$\eta_{c^D,\bar{\lambda}} \equiv \frac{\partial \tilde{c}^D}{\partial \bar{\lambda}} \frac{\bar{\lambda}}{\tilde{c}^D} = -\nu < 0, \quad \eta_{c^D,p} \equiv \frac{\partial \tilde{c}^D}{\partial \tilde{p}} \frac{\tilde{p}}{\tilde{c}^D} = \phi \alpha_c + \nu\left(1 - \alpha_c\right) > 0. \tag{186}$$

Inserting (179), (181) and (185) into the home good market-clearing condition, equation (**??** can be rewritten as follows (equation (11) in the text):

$$Y\left(\bar{\lambda}, \tilde{p}\right) = c^D\left(\bar{\lambda}, \tilde{p}\right) + X\left(\tilde{p}\right) + \delta_K k\left(\bar{\lambda}, \tilde{p}\right) + g^D. \tag{187}$$

In a third step, we insert steady-state functions (186) and (184) into the market-clearing condition for the home good (165e), which may be solved for the long-run value of the real exchange rate:

$$\tilde{p} = p\left(\bar{\lambda}, g^D\right), \tag{188}$$

with

$$\eta_{p,\bar{\lambda}} \equiv \frac{\partial \tilde{p}}{\partial \bar{\lambda}} \frac{\bar{\lambda}}{\tilde{p}} = \frac{\left\{\sigma_N\left(1 - \alpha_K \frac{\delta_K}{r^\star + \delta_K}\right) + \frac{\tilde{c}^D}{\tilde{Y}} \nu\right\}}{\left\{\sigma_N\left(1 - \alpha_K \frac{\delta_K}{r^\star + \delta_K}\right) + \frac{\tilde{c}^D}{\tilde{Y}}\left(\phi \alpha_c + \nu\left(1 - \alpha_c\right)\right) + \frac{\tilde{X}}{\tilde{Y}} \nu_X\right\}} > 0, \tag{189a}$$

$$\eta_{p,g^D} \equiv \frac{\partial \tilde{p}}{\partial g^D} \frac{g^D}{\tilde{Y}} = -\frac{1}{\left\{\sigma_N\left(1 - \alpha_K \frac{\delta_K}{r^\star + \delta_K}\right) + \frac{\tilde{c}^D}{\tilde{Y}}\left(\phi \alpha_c + \nu\left(1 - \alpha_c\right)\right) + \frac{\tilde{X}}{\tilde{Y}} \nu_X\right\}} < 0 \tag{189b}$$

where we have rewritten $\frac{\tilde{k}}{\tilde{Y}}$ as $\frac{\alpha_K}{r^\star + \delta_K}$ by making use of (165c). Note that $\eta_{p,\bar{\lambda}} < 1$ if the long-run IES for consumption is smaller than the intratemporal elasticity of substitution ϕ. This condition holds since c^D and c^F are substitutes (such that $\phi > 1$) and empirical evidence suggest that $\nu < 1$. Additionally, the elasticity of the steady-state value of the real exchange rate w. r. t. $\bar{\lambda}$ rises with ν. Since ν is an increasing function of γ, $\eta_{p,\bar{\lambda}}$ increases as γ approaches unity.

G Effects of a Permanent Rise in Government Spending: The Case of Time Separable Preferences

In this section, we investigate the effects of fiscal shocks by considering the case of time separable preferences (i.e. setting $\gamma = 0$).

G.1 First-Order Conditions

To obtain the macroeconomic equilibrium, we first derive the optimality conditions for households and firms and combine these with the accumulation equations. This leads to the set of equations:

$$u_c(c) = \frac{p_c(p)\lambda}{p}, \tag{190a}$$

$$v_n(n) = -\frac{\lambda}{p}w, \tag{190b}$$

$$F_k = r^K + \delta_K, \quad F_n = w, \tag{190c}$$

$$\dot{\lambda} = \lambda(\beta - r^\star), \tag{190d}$$

$$\dot{p} = p\left[F_k(k,n) - \delta_K - r^\star\right], \tag{190e}$$

$$\dot{k} = F(k,n) - c^D - X(p) - g^D - \delta_K k, \tag{190f}$$

$$\dot{b} = \frac{1}{p}\left[r^\star pb + F(k,n) - p_c(p)c - I - \left(g^D + pg^F\right)\right], \tag{190g}$$

together with the transversality conditions:

$$\lim_{t\to\infty} \bar{\lambda} b \exp(-r^\star t) = \lim_{t\to\infty} \frac{\bar{\lambda}}{p} k \exp(-r^\star t) = 0, \tag{191}$$

where λ is the co-state variable associated with dynamic equation (190g).

We first solve equation (190a) for consumption:

$$c = c\left(\bar{\lambda}, p\right), \tag{192}$$

where the partial derivatives are given by

$$c_{\bar{\lambda}} = \frac{\partial c}{\partial \bar{\lambda}} = \frac{p_c}{pu_{cc}} = -\sigma_c \frac{c}{\bar{\lambda}} < 0, \tag{193a}$$

$$c_p = \frac{\partial c}{\partial p} = \frac{p_c \bar{\lambda}(1-\alpha_c)}{p^2 u_{cc}} = (1-\alpha_c)\sigma_c \frac{c}{p} > 0. \tag{193b}$$

Using the Shephard's Lemma, intra-temporal allocations between domestic goods and foreign goods are: $c^F = p_c' c$ and $c^D = [p_c - pp_c']c$ and substituting (193), we solve for consumption in the domestic and foreign goods:

$$c^D = c^D\left(\bar{\lambda}, p\right), \quad c^F = c^F\left(\bar{\lambda}, p\right), \tag{194}$$

where the partial derivatives are given by

$$c^D_{\bar{\lambda}} = \frac{\partial c^D}{\partial \bar{\lambda}} = -\sigma_c \frac{c^D}{\bar{\lambda}}, \tag{195a}$$

$$c^D_p = \frac{\partial c^D}{\partial p} = \frac{c^D}{p}\left[\phi\alpha_c + (1-\alpha_c)\sigma_c\right] > 0, \tag{195b}$$

$$c^F_{\bar{\lambda}} = \frac{\partial c^F}{\partial \bar{\lambda}} = -\sigma_c \frac{c^F}{\bar{\lambda}} < 0, \tag{195c}$$

$$c^F_p = \frac{\partial c^F}{\partial p} = -\frac{c^F}{p}(1-\alpha_c)\left[\phi - \sigma_c\right] < 0, \tag{195d}$$

where we assumed that $\sigma_c < \phi_c$ as empirical evidence suggest that $\sigma_c < 1$.

Using the fact that $w = F_n(k, n)$, the first-order condition (190b) for labor rewrites as $-v_n(n) = \frac{\lambda}{p} F_n$ which can be solved for employment n:

$$n = n\left(\bar{\lambda}, p, k\right), \tag{196}$$

where the partial derivatives are given by (22).

Using the fact that $I = \dot{k} + \delta_K k$, the market-clearing condition (190f) can be solved for investment in physical capital:

$$I = I\left(\bar{\lambda}, p, k, g^D\right), \tag{197}$$

where the partial derivatives are given by:

$$I_{\bar{\lambda}} = \frac{\partial I}{\partial \bar{\lambda}} = F_n n_{\bar{\lambda}} - c^D_{\bar{\lambda}} = \frac{Y}{\bar{\lambda}} \left[(1 - \alpha_K)\chi + \sigma_c \frac{c^D}{Y} \right] > 0, \tag{198a}$$

$$I_p = \frac{\partial I}{\partial p} = F_n n_p - \left(X_p + c^D_p\right) = -\frac{Y}{p} \left\{ \chi(1 - \alpha_K) + \frac{X}{Y} \left[\nu_X + \frac{c^D}{X} \left(\alpha_c \phi + (1 - \alpha_c)\sigma_c\right) \right] \right\} = \Psi \lessgtr 0, \tag{198b}$$

$$I_k = \frac{\partial I}{\partial k} = F_k + F_n n_k = \frac{\alpha_K Y}{k} \left[1 + \chi \frac{(1 - \alpha_K)}{\sigma_{KN}} \right] > 0, \tag{198c}$$

$$I_{g^D} = \frac{\partial I}{\partial g^D} = -1 < 0. \tag{198d}$$

G.2 Equilibrium Dynamics and Formal Solutions

Saddle-Point Stability and Formal Solutions for $k(t)$ and $p(t)$

Inserting the short-run static solutions for labor (196) into the dynamic equation for the real exchange rate (190e), and inserting the short-run static solution for investment (197) into the accumulation equation of physical capital (6), the dynamic system writes as follows:

$$\dot{p} = p\left\{ F_k\left[k, n\left(\bar{\lambda}, p, k\right)\right] - \delta_K - r^\star \right\}, \tag{199a}$$

$$\dot{k} = F\left[k, n\left(\bar{\lambda}, p, k\right)\right] - c^D\left(\bar{\lambda}, p\right) - X(p) - \delta_K k - g^D. \tag{199b}$$

Linearizing these two equations around the steady-state, and denoting long values by a tilde, we obtain in a matrix form:

$$\left(\dot{k}, \dot{p}\right)^T = J\left(k(t) - \tilde{k}, p(t) - \tilde{p}\right)^T, \tag{200}$$

where J is given by

$$J \equiv \begin{pmatrix} \left(r^\star + \tilde{F}_n \tilde{n}_k\right) & \tilde{\Psi} \\ \tilde{p}\left(\tilde{F}_{kk} + \tilde{F}_{kn}\tilde{n}_k\right) & \tilde{p}\tilde{F}_{kn}\tilde{n}_p \end{pmatrix}, \tag{201}$$

where Ψ is given by (198c).

The determinant denoted by Det of the linearized 2×2 matrix (201) is unambiguously negative:

$$\text{Det J} = \tilde{p} n_p \left(r^\star F_{kn} - F_n F_{kk}\right) + \tilde{p}\left(c^D_p + X_p\right)\left(\tilde{F}_{kk} + \tilde{F}_{kn}\tilde{n}_k\right) < 0, \tag{202}$$

and the trace denoted by Tr given by

$$\text{Tr J} = r^\star + \tilde{F}_n \tilde{n}_k + \tilde{p}\tilde{F}_{kn}\tilde{n}_p = r^\star > 0, \tag{203}$$

where we used the fact that at the long-run equilibrium $\tilde{F}_n \tilde{n}_k + \tilde{p}\tilde{F}_{kn}\tilde{n}_p = 0$.

The characteristic root obtained from J given by (201) writes as follows:

$$\mu_i \equiv \frac{1}{2}\left\{ r^\star \pm \sqrt{\left(r^\star\right)^2 - 4\text{Det J}} \right\} \gtrless 0, \quad i = 1, 2. \tag{204}$$

We denote by $\mu_1 < 0$ and $\mu_3 < 0$ the stable and unstable real eigenvalues, satisfying

$$\mu_1 < 0 < r^\star < \mu_3. \tag{205}$$

Since the system features one state variable, K, and one jump variable, p, the equilibrium yields a unique one-dimensional stable saddle-path.

Stable solutions paths are given by :

$$k(t) - \tilde{k} = B_1 e^{\mu_1 t}, \quad p(t) - \tilde{p} = \omega_2^1 B_1 e^{\nu_1 t}, \tag{206}$$

where we normalized ω_1^i to unity. The eigenvector ω_2^i associated with eigenvalue μ_i is given by

$$\omega_2^i = -\frac{\tilde{p}\left(\tilde{F}_{kk} + \tilde{F}_{kn}\tilde{n}_k\right)}{\left(\tilde{p}\tilde{F}_{kn}\tilde{n}_p - \mu_i\right)} \gtrless 0. \tag{207}$$

Formal Solution for the Stock of Foreign Assets

We first linearize equation (66) around the steady-state:

$$\dot{b}(t) = r^\star \left(b(t) - \tilde{b}\right) + \tilde{\Omega}\left(p(t) - \tilde{p}\right), \tag{208}$$

with

$$\tilde{\Omega} \equiv \frac{1}{\tilde{p}}\left[\frac{\tilde{X}}{\tilde{p}}\left(\tilde{\nu}_X - 1\right) + \tilde{c}^F \tilde{\eta}_F\right] = \frac{1}{\tilde{p}}\left[\tilde{\Theta} - \frac{\tilde{X}}{\tilde{p}}\right] > 0, \tag{209}$$

where the elasticities of exports and imports w. r. t. the real exchange rate are:

$$\nu_X = \frac{X_p p}{X} > 0, \quad \eta_F = -\frac{c_p^F p}{c^F} > 0. \tag{210}$$

The condition under which a real exchange depreciation leads to an improvement of the trade balance evaluated at the steady-state (i. e. $\tilde{\Omega} > 0$) writes as follows:

$$\frac{\tilde{p}}{\tilde{X}}\tilde{\Theta} = \tilde{\nu}_X + \tilde{\eta}^F \frac{\tilde{p}\tilde{c}^F}{\tilde{X}} > 1 \tag{211}$$

Inserting stable solution for $p(t)$, solving the differential equation leads to the expression, and invoking the transversality condition (191), we obtain the linearized version of the nation's intertemporal budget constraint:

$$b_0 - \tilde{b} = \frac{\tilde{\Omega}\omega_2^1}{\mu_1 - r^\star} B_1 = \Phi_1\left(k_0 - \tilde{k}\right). \tag{212}$$

together with the stable solution for the stock of foreign bonds:

$$b(t) - \tilde{b} = \frac{\tilde{\Omega}\omega_2^1 B_1}{\mu_1 - r^\star} e^{\mu_1 t} = \Phi_1\left(k(t) - \tilde{k}\right), \tag{213}$$

where we let $\Phi_1 = \frac{\tilde{\Omega}\omega_2^1}{\mu_1 - r^\star}$. While the sign of eigenvector ω_2^1 remains indeterminate, we will assume thereafter that $\omega_2^1 > 0$ which implies that $\Phi_1 < 0$. Consequently, the current account and capital investment are negatively correlated, in line with empirical evidence.

G.3 Long-Term Effects of a Rise in Government Spending: The Case of Time Separable Preferences

The steady-state of the economy is obtained by setting $\dot{c}, \dot{k}, \dot{p}, \dot{b} = 0$ and is defined by the following set of equations:

$$r^\star + \delta_K = F_k \left[\tilde{k}, n\left(\bar{\lambda}, \tilde{p}, \tilde{k}\right)\right], \tag{214a}$$

$$r^\star \tilde{p}\tilde{b} + F\left[\tilde{k}, n\left(\bar{\lambda}, \tilde{p}, \tilde{k}\right)\right] - p_c\left(\tilde{p}\right) c\left(\bar{\lambda}, \tilde{p}\right) - \delta_K \tilde{k} - g^D - \tilde{p} g^F = 0, \tag{214b}$$

$$F\left[\tilde{k}, n\left(\bar{\lambda}, \tilde{p}, \tilde{k}\right)\right] = c^D\left(\bar{\lambda}, \tilde{p}\right) + \delta_K \tilde{k} + X\left(\tilde{p}\right) + g^D, \tag{214c}$$

and the economy's intertemporal budget constraint

$$\left(\tilde{b} - b_0\right) = \Phi_1\left(\tilde{k} - k_0\right), \tag{214d}$$

where we used the fact that in the long-run $\tilde{I} = \delta_K \tilde{k}$, and we have substituted the short-run static solution for labor and consumption in the domestic good which obviously holds in the long-run.

Totally differentiating equations (137) yields in matrix form:

$$\begin{pmatrix} F_{kn}n_p & F_{kn}n_{\bar{\lambda}} & F_{kk} + F_{kn}n_k & 0 \\ \left(I_p + \tilde{p}\tilde{\Omega}\right) & \left(I_{\bar{\lambda}} - \tilde{p}c^F_{\bar{\lambda}}\right) & \left(I_k - \delta_K\right) & \tilde{p}r^\star \\ I_p & I_{\bar{\lambda}} & \left(I_k - \delta_K\right) & 0 \\ 0 & 0 & -\Phi_1 & 1 \end{pmatrix} \begin{pmatrix} \mathrm{d}\tilde{p} \\ \mathrm{d}\bar{\lambda} \\ \mathrm{d}\tilde{k} \\ \mathrm{d}\tilde{b} \end{pmatrix}$$
$$= \begin{pmatrix} 0 \\ \mathrm{d}g^D + \tilde{p}\mathrm{d}g^F \\ \mathrm{d}g^D \\ \mathrm{d}b_0 - \Phi_1 \mathrm{d}k_0 \end{pmatrix}. \tag{215}$$

We used the fact that $\left(\tilde{c}^F + g^F - r^\star \tilde{b}\right) = \tilde{X}/\tilde{p}$ (see (89)) together with $F_n n_p = I_p + c^D_p + X_p$ and $\tilde{p}\tilde{\Omega} = \tilde{\Theta} - \tilde{X}/\tilde{p}$ and $p_c c_p - c^D_p = \tilde{p}c^F_p$ to rewrite $\left(F_n n_p - p_c c_p - \frac{\tilde{X}}{\tilde{p}}\right)$ as follows $\left(I_p + \tilde{p}\tilde{\Omega}\right)$.

The determinant of matrix of coefficients denoted by D writes as follows:

$$\begin{aligned} D \quad \equiv \quad & -\Phi_1 \tilde{p} r^\star F_{kn} n_{\bar{\lambda}} \tilde{\Theta} - F_{kn}\tilde{p}\left(I_k - \delta_K\right)\left(n_p c^F_{\bar{\lambda}} + n_{\bar{\lambda}}\tilde{\Omega}\right) \\ + \quad & \tilde{p}\left(F_{kk} + F_{kn}n_k\right)\left(I_{\bar{\lambda}}\tilde{\Omega} + I_p c^F_{\bar{\lambda}}\right) \lesseqgtr 0. \end{aligned} \tag{216}$$

We rewrote $\left(I_{\bar{\lambda}} F_{kn} n_p - I_p F_{kn} n_{\bar{\lambda}}\right)$ as follows:

$$I_{\bar{\lambda}} F_{kn} n_p - I_p F_{kn} n_{\bar{\lambda}} = F_{kn} n_{\bar{\lambda}} \tilde{\Theta} > 0, \tag{217}$$

where we let

$$\Theta \equiv X_p + c^D_p - \sigma_c \frac{c^D}{p} = \left[\nu_X + \frac{c^D}{X}\alpha_c\left(\phi - \sigma_c\right)\right] > 0. \tag{218}$$

If $\Phi_1 < 0$ is small enough (in absolute terms), the determinant D is negative. Since for all parametrization, the determinant D is negative, we will set this assumption thereafter.

Domestic Good g^D

The steady-state effects of an unanticipated permanent increase in government expenditure falling on the domestic good are obtained from the total differential of the equilibrium system

(214) w. r. t. g^D:

$$\frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^D} = \frac{\Phi_1 \tilde{p} r^\star F_{kn} n_{\bar{\lambda}}}{D} + \frac{\tilde{p} c^F_{\bar{\lambda}} \left(F_{kk} + F_{kn} n_k\right)}{D} = (+) + (-) \lesseqgtr 0, \tag{219a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^D} = -\frac{\Phi_1 \tilde{p} r^\star F_{kn} n_p}{D} + \frac{\left(F_{kk} + F_{kn} n_k\right) \tilde{p}\tilde{\Omega}}{D} > 0, \tag{219b}$$

$$\frac{\mathrm{d}\tilde{k}}{\mathrm{d}g^D} = -\frac{F_{kn}\tilde{p}}{D} \left[n_p c^F_{\bar{\lambda}} + n_{\bar{\lambda}}\tilde{\Omega}\right] > 0, \tag{219c}$$

$$\frac{\mathrm{d}\tilde{b}}{\mathrm{d}g^D} = \Phi_1 \frac{\mathrm{d}\tilde{k}}{\mathrm{d}g^D} < 0, \tag{219d}$$

where $D < 0$, $\tilde{\Omega} > 0$ and $\Phi_1 < 0$.

Totally differentiating the short-run static solution for consumption and labor given by (192) and (196), respectively, and substituting relevant expressions (219), we obtain the steady-state changes of real consumption and employment after a permanent rise in g^D:

$$\begin{aligned} \frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^D} &= c_{\bar{\lambda}} \frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^D} + c_p \frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^D}, \\ &= -\frac{c_{\bar{\lambda}}}{D} \left\{ \Phi_1 \tilde{p} r^\star F_{kn} n_p \alpha_c - \left(F_{kk} + F_{kn} n_k\right) \frac{\tilde{X}}{\tilde{p}} \left(\tilde{\Omega} + \sigma_c \left(1-\alpha_c\right) \tilde{c}^F\right) \right\} < 0, \end{aligned} \tag{220a}$$

$$\begin{aligned} \frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^D} &= n_{\bar{\lambda}} \frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^D} + n_p \frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^D} + n_k \frac{\mathrm{d}\tilde{k}}{\mathrm{d}g^D}, \\ &= \frac{F_{kk}}{D} \left[n_{\bar{\lambda}} \tilde{p}\tilde{\Omega} + n_p \tilde{p} c^F_{\bar{\lambda}}\right] > 0, \end{aligned} \tag{220b}$$

where we used the fact that

$$-\tilde{p}\tilde{\Omega} + \left(1-\alpha_c\right) c^F_{\bar{\lambda}} \bar{\lambda} = \frac{\tilde{X}}{\tilde{p}} \left[\left(\nu_X + \frac{\tilde{p}\tilde{c}^F}{\tilde{X}} \left(1-\alpha_c\right)\phi\right) - 1\right], \tag{221a}$$

$$\begin{aligned} & n_{\bar{\lambda}} \tilde{p}\tilde{\Omega} + n_p \tilde{p} c^F_{\bar{\lambda}} \\ = \; & n_{\bar{\lambda}} \frac{\tilde{X}}{\tilde{p}} \left\{ \left[\nu_X + \frac{\tilde{p}\tilde{c}^F}{\tilde{X}} \left(\left(1-\alpha_c\right)\phi + \alpha_c \sigma_c\right)\right] - 1 \right\} > 0. \end{aligned} \tag{221b}$$

Making use of (219) and (220b), we can compute the steady-state change of output after a permanent rise in g^D:

$$\frac{\mathrm{d}\tilde{Y}}{\mathrm{d}g^D} = \frac{\tilde{p}}{D} \left[n_{\bar{\lambda}}\tilde{\Omega} + n_p c^F_{\bar{\lambda}}\right] \left[w F_{kk} - \left(r^\star + \delta_K\right) F_{kn}\right] > 0, \tag{222}$$

where $\tilde{p}\left[n_{\bar{\lambda}}\tilde{\Omega} + n_p c^F_{\bar{\lambda}}\right]$ is given by (221b).

Totally differentiating the short-run static solution for consumption and labor given by (192) and (196), respectively, and substituting relevant expressions, we obtain the steady-state changes of real consumption in the domestic good after a permanent rise in g^D:

$$\begin{aligned} \frac{\mathrm{d}\tilde{c}^D}{\mathrm{d}g^D} &= \left(1-\alpha_c\right) \left[\phi \tilde{c}^F \frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^D} + p_c \frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^D}\right], \\ &= \frac{\tilde{c}^D}{\bar{\lambda} D} \left\{ \Phi_1 r^\star F_{kn} n_{\bar{\lambda}} \bar{\lambda} \alpha_c \left(\phi - \sigma_c\right) - \left(F_{kk} + F_{kn} n_k\right) \left[\tilde{p}\tilde{\Omega} + \left(\left(1-\alpha_c\right)\sigma_c + \phi\alpha c\right) \tilde{c}^F\right] \right\} \end{aligned} \tag{223}$$

where

$$\tilde{p}\tilde{\Omega} + \left(\left(1-\alpha_c\right)\sigma_c + \phi\alpha c\right)\tilde{c}^F = \frac{\tilde{X}}{\tilde{p}} \left\{ \left(\nu_X + \frac{\tilde{p}\tilde{c}^F}{\tilde{X}}\phi\right) - 1 \right\} > 0. \tag{224}$$

Foreign Good g^F

The steady-state effects of an unanticipated permanent increase in government expenditure falling on the foreign good are obtained from the total differential of the equilibrium system (214) w. r. t. g^F:

$$\frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^F} = -\frac{\tilde{p}\left[F_{kn}n_{\bar{\lambda}}\left(I_k-\delta_k\right)-I_{\bar{\lambda}}\left(F_{kk}+F_{kn}n_k\right)\right]}{D} > 0, \tag{225a}$$

$$\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^F} = \frac{\tilde{p}\left[F_{kn}n_p\left(I_k-\delta_k\right)-I_p\left(F_{kk}+F_{kn}n_k\right)\right]}{D} > 0, \tag{225b}$$

$$\frac{\mathrm{d}\tilde{k}}{\mathrm{d}g^F} = -\frac{\tilde{p}\left[I_{\bar{\lambda}}F_{kn}n_p-I_pF_{kn}n_{\bar{\lambda}}\right]}{D} > 0, \tag{225c}$$

$$\frac{\mathrm{d}\tilde{b}}{\mathrm{d}g^F} = \Phi_1\frac{\mathrm{d}\tilde{k}}{\mathrm{d}g^F} < 0, \tag{225d}$$

where $D<0$, $\tilde{\Omega}>0$ and $\Phi_1<0$.

G.4 Short-Term Effects of a Rise in Government Spending: The Case of Time Separable Preferences

From (219), a fiscal impulse raises the capital stock in the long-run. Differentiating the stable solution for capital stock (206) yields: $\dot{k}(t)=\mu_1 B_1 e^{\mu_1 t}>0$ with $B_1=-\mathrm{d}\tilde{k}<0$. Consequently, investment is unambiguously crowded-in by public spending on impact and the capital stock rises monotonically towards its new long-run level. From the intertemporal solvency condition (212), the long-run accumulation of physical capital yields a decumulation of traded bonds. Hence, the open economy experiences a current account deficit along the transitional path. Regarding the consumption-side, according to (192), the fall in private wealth together with the real exchange rate appreciation drives down consumption. In the same time, the negative wealth effect induces agents to work more. The real exchange rate appreciation raises further labor supply by amplifying the drop in private wealth measured in terms of the domestic good.

H Effects of a Temporary Rise in Government Spending

Following Schubert and Turnovsky [2002], we define a viable steady-state i starting at time $\mathcal{T}_j$ to be one that is consistent with long-run solvency, given the stocks of capital and habits, $K_{\mathcal{T}_j}$ and $s_{\mathcal{T}_j}$, and foreign bonds, $n_{\mathcal{T}_j}$. We rewrite the system of steady-state equations (137) for an arbitrary period j:

$$u_c\left(\tilde{c}_j,\tilde{s}_j\right)+\frac{\sigma}{\beta+\sigma}u_s\left(\tilde{c}_j,\tilde{s}_j\right)=\frac{p_c\left(\tilde{p}_j\right)\bar{\lambda}_j}{\tilde{p}_j}, \tag{226a}$$

$$\tilde{c}_j=\tilde{s}_j, \tag{226b}$$

$$r^\star+\delta_K=F_k\left[\tilde{k}_j,n_j\left(\bar{\lambda}_j,\tilde{p}_j,\tilde{k}_j\right)\right], \tag{226c}$$

$$r^\star\tilde{p}_j\tilde{b}_j+F\left[\tilde{k}_j,n\left(\bar{\lambda}_j,\tilde{p}_j,\tilde{k}_j\right)\right]-p_c\left(\tilde{p}_j\right)\tilde{c}_j-\delta_K\tilde{k}_j-g_j^D-\tilde{p}_jg^F=0, \tag{226d}$$

$$F\left[\tilde{k}_j,n\left(\bar{\lambda}_j,\tilde{p}_j,\tilde{k}_j\right)\right]=\left(1-\alpha_c\right)p_c\left(\tilde{p}_j\right)\tilde{c}_j+\delta_K\tilde{k}_j+X\left(\tilde{p}_j\right)+g_j^D, \tag{226e}$$

and the economy's intertemporal budget constraint

$$\left(\tilde{b}_j-b_{T_j}\right)=\Phi_1\left(\tilde{k}_j-k_{T_j}\right)+\Phi_2\left(\tilde{s}_j-s_{T_j}\right), \tag{226f}$$

In a **first step**, we solve the system (226a)-(226e) for $\tilde{c}_j$, $\tilde{s}_j$, $\tilde{k}_j$, $\tilde{p}_j$, $\tilde{b}_j$ as functions of the marginal utility of wealth, $\bar{\lambda}_j$, and government spending on the domestic g^D and the foreign good g^F. We obtain the steady-state functions (94). The **second step** consists in determining the equilibrium change of $\bar{\lambda}_j$ by taking the total differential of the intertemporal solvency condition (226f):

$$\left[v_{\bar{\lambda}}-\Phi_1K_\lambda-\Phi_2m_\lambda\right]\mathrm{d}\bar{\lambda}_j=-\left[v_{g^\tau}-\Phi_1K_{g^\tau}-\Phi_2m_{g^\tau}\right]\mathrm{d}g^\tau,\quad \tau=D,F \tag{227}$$

from which may solve for the equilibrium value of $\bar{\lambda}_j$ as a function of government spending g^τ:

$$\bar{\lambda} = \lambda\left(g^D, g^F\right), \quad \lambda_{g^D} > 0, \quad \lambda_{g^F} > 0. \tag{228}$$

We assume that the small open economy is initially in steady-state equilibrium, denoted by the subscript $j = 0$:

$$\begin{aligned} s_0 &= \tilde{s}_0 = m\left(\bar{\lambda}_0, g_0^\tau\right), && (229a)\\ k_0 &= \tilde{k}_0 = k\left(\bar{\lambda}_0, g_0^\tau\right), && (229b)\\ p_0 &= \tilde{p}_0 = p\left(\bar{\lambda}_0, g_0^\tau\right), && (229c)\\ b_0 &= \tilde{b}_0 = v\left(\bar{\lambda}_0, g_0^\tau\right), && (229d)\\ \lambda_0 &= \bar{\lambda}_0 = \lambda\left(g_0^\tau\right), && (229e) \end{aligned}$$

where $\tilde{s}_0 = \tilde{c}_0$, $\tau = D, F$, and $m_{g^F} = p_{g^F} = k_{g^F} = 0$.

We suppose now that government expenditure changes unexpectedly at time $t = 0$ from the original level g_0^τ to level g_1^τ over the period $0 \leq t < T$, and reverts back at time T permanently to its initial level, $g_T^\tau = g_2^\tau = g_0^\tau$.

Period 1 $(0 \leq t < T)$

Whereas the fiscal expansion is implemented, the economy follows unstable transitional paths:

$$\begin{aligned} s(t) &= \tilde{s}_1 + A_1 e^{\mu_1 t} + A_2 e^{\mu_2 t} + A_3 e^{\mu_3 t} + A_4 e^{\mu_4 t}, && (230a)\\ c(t) &= \tilde{c}_1 + \omega_2^1 A_1 e^{\mu_1 t} + \omega_2^2 A_2 e^{\mu_2 t} + \omega_2^3 A_3 e^{\mu_3 t} + \omega_2^4 A_4 e^{\mu_4 t}, && (230b)\\ k(t) &= \tilde{k}_1 + \omega_3^1 A_1 e^{\mu_1 t} + \omega_3^2 A_2 e^{\mu_2 t} + \omega_3^3 A_3 e^{\mu_3 t} + \omega_3^4 A_4 e^{\mu_4 t}, && (230c)\\ p(t) &= \tilde{p}_1 + \omega_4^1 A_1 e^{\mu_1 t} + \omega_4^2 A_2 e^{\mu_2 t} + \omega_4^3 A_3 e^{\mu_3 t} + \omega_4^4 A_4 e^{\mu_4 t}, && (230d)\\ b(t) &= \tilde{b}_1 + \left[\left(b_0 - \tilde{b}_1\right) - \sum_{i=1}^4 \frac{N_\tau A_i}{\mu_i - r^\star}\right] e^{r^\star t} + \sum_{i=1}^4 \frac{N_i A_1}{\mu_i - r^\star} e^{\mu_i t}, && (230e) \end{aligned}$$

with the steady-state values $\tilde{c}_1$, $\tilde{s}_1$, $\tilde{k}_1$, $\tilde{p}_1$, $\tilde{b}_1$ given by the following functions:

$$\begin{aligned} \tilde{s}_1 &= m\left(\bar{\lambda}, g_1^\tau\right), && (231a)\\ \tilde{k}_1 &= k\left(\bar{\lambda}, g_1^\tau\right), && (231b)\\ \tilde{p}_1 &= p\left(\bar{\lambda}, g_1^\tau\right), && (231c)\\ \tilde{b}_1 &= v\left(\bar{\lambda}, g_1^\tau\right), && (231d) \end{aligned}$$

where the marginal utility of wealth remains constant over periods 1 and 2 at level $\bar{\lambda}_1 = \bar{\lambda}_2 = \bar{\lambda}$ after its initial jump at time $t = 0$.

Period 2 $(t \geq T)$

Once government spending reverts back to its initial level, the economy follows stable paths:

$$\begin{aligned} s(t) &= \tilde{s}_2 + A_1 e^{\mu_1 t} + A_2 e^{\mu_2 t}, && (232a)\\ c(t) &= \tilde{c}_2 + \omega_2^1 A_1 e^{\mu_1 t} + \omega_2^2 A_2 e^{\mu_2 t}, && (232b)\\ k(t) &= \tilde{k}_2 + \omega_3^1 A_1 e^{\mu_1 t} + \omega_3^2 A_2 e^{\mu_2 t}, && (232c)\\ p(t) &= \tilde{p}_2 + \omega_4^1 A_1 e^{\mu_1 t} + \omega_4^2 A_2 e^{\mu_2 t}, && (232d)\\ b(t) &= \tilde{b}_2 + \frac{N_1 A_1}{\mu_1 - r^\star} e^{\mu_1 t} + \frac{N_2 A_2}{\mu_2 - r^\star} e^{\mu_2 t}, && (232e) \end{aligned}$$

with the steady-state values $\tilde{c}_2$, $\tilde{s}_2$, $\tilde{k}_2$, $\tilde{p}_2$, $\tilde{b}_2$ given by the following functions:

$$\begin{aligned} \tilde{s}_2 &= m\left(\bar{\lambda}, g_2^\tau\right), && (233a)\\ \tilde{k}_2 &= k\left(\bar{\lambda}, g_2^\tau\right), && (233b)\\ \tilde{p}_2 &= p\left(\bar{\lambda}, g_2^\tau\right), && (233c)\\ \tilde{b}_2 &= v\left(\bar{\lambda}, g_1^\tau\right), && (233d) \end{aligned}$$

where $g_2^\tau = g_0^\tau$.

During the transition period 1, the economy accumulates (decumulates) habits, capital and foreign assets. Since this period is unstable, it would lead the nation to violate its intertemporal budget constraint. By contrast, the adjustment process taking place in period 2 is stable and must satisfy the economy's intertemporal budget constraint. At the same time, the zero-root problem requires the equilibrium value of marginal utility of wealth to adjust once-and-for-all when the shock hits the economy. So λ remains constant over the periods 1 and 2. The aim of the *two-step method* is to calculate the deviation of λ such that the country satisfies one single and overall intertemporal budget constraint, given the new relevant initial conditions, s_T, k_T and b_T, prevailing when the shock ends and accumulated over the unstable period. Therefore, for the country to remain intertemporally solvent, we require:

$$\tilde{b}_2 - b_T = \Phi_1 \left(\tilde{k}_2 - k_T \right) + \Phi_2 \left(\tilde{s}_2 - s_T \right), \tag{234}$$

In order to determine the six constants A_1, A_2, A_3, A_4, B_1, B_2, and the equilibrium value of marginal utility of wealth, we impose three conditions:

1. Initial conditions $s(0) = s_0$, $k(0) = k_0$, $b(0) = b_0$ must be met.
2. The stock of habits and the stock of physical capital s and k remain continuous at time $\mathcal{T}$.
3. The intertemporal solvency constraint (234) must hold implying that the net foreign assets remain continuous at time T.

Set $t = 0$ in solutions (230a),(230c) and (232a),(232c), equate (230a)-(230d) and (232a)-(232d), evaluated at time $t = \mathcal{T}$, one obtains

$$\tilde{s}_1 + A_1 + A_2 + A_3 + A_4 = s_0, \tag{235a}$$
$$\tilde{s}_1 + A_1 e^{\mu_1 T} + A_2 e^{\mu_2 T} + A_3 e^{\mu_3 T} + A_4 e^{\mu_4 T} = \tilde{s}_2 + B_1 e^{\mu_1 T} + B_2 e^{\mu_2 T}, \tag{235b}$$
$$\tilde{c}_1 + \omega_2^1 A_1 e^{\mu_1 T} + \omega_2^2 A_2 e^{\mu_2 T} + \omega_2^3 A_3 e^{\mu_3 T} + \omega_2^4 A_4 e^{\mu_4 T} = \tilde{c}_2 + \omega_2^1 B_1 e^{\mu_1 T} + \omega_2^2 B_2 e^{\mu_2 T}, \tag{235c}$$
$$\tilde{k}_1 + \omega_3^1 A_1 + \omega_3^2 A_2 + \omega_3^3 A_3 + \omega_3^4 A_4 = k_0, \tag{235d}$$
$$\tilde{k}_1 + \omega_3^1 A_1 e^{\mu_1 T} + \omega_3^2 A_2 e^{\mu_2 T} + \omega_3^3 A_3 e^{\mu_3 T} + \omega_3^4 A_4 e^{\mu_4 T} = \tilde{k}_2 + \omega_3^1 B_1 e^{\mu_1 T} + \omega_3^2 B_2 e^{\mu_2 T}, \tag{235e}$$
$$\tilde{p}_1 + \omega_4^1 A_1 e^{\mu_1 T} + \omega_4^2 A_2 e^{\mu_2 T} + \omega_4^3 A_3 e^{\mu_3 T} + \omega_4^4 A_4 e^{\mu_4 T} = \tilde{p}_2 + \omega_4^1 B_1 e^{\mu_1 T} + \omega_4^2 B_2 e^{\mu_2 T}. \tag{235f}$$

Estimating s_T, k_T et b_T from (230a), (232c) et (232e), substituting these into (234), and substituting steady-state functions $\tilde{s}_i$, $\tilde{k}_i$, $\tilde{b}_i$, the intertemporal solvency condition (i.e. $b_T = \tilde{b}_2 + \Phi_1 \left(k_T - \tilde{k}_2 \right) + \Phi_2 \left(s_T - \tilde{s}_2 \right)$) can be rewritten as

$$\begin{aligned} & v\left(\bar{\lambda}, g_1^\tau\right) + \left\{ \left[v\left(\lambda_0, g_0^\tau\right) - v\left(\bar{\lambda}, g_1^\tau\right) \right] - \sum_{i=1}^{4} \frac{N_i A_i}{\mu_i - r^\star} \right\} e^{r^\star T} + \sum_{i=1}^{4} \frac{N_i A_i}{\mu_i - r^\star} e^{\mu_i T} - v\left(\bar{\lambda}, g_2^\tau\right) \\ = \; & \Phi_1 \left[k\left(\bar{\lambda}, g_1^\tau\right) + \sum_{i=1}^{4} \omega_3^i A_i e^{\mu_i T} - k\left(\bar{\lambda}, g_2^\tau\right) \right] + \Phi_2 \left[s\left(\bar{\lambda}, g_1^\tau\right) + \sum_{i=1}^{4} A_i e^{\mu_i T} - s\left(\bar{\lambda}, g_2^\tau\right) \right] \end{aligned} \tag{236}$$

Then, we approximate the steady-state changes of variable $x = c, s, k, p, b$ with the differentials:

$$\tilde{x}_1 - \tilde{x}_0 \equiv z\left(\bar{\lambda}, g_1^\tau\right) - z\left(\lambda_0, g_0^\tau\right) = z_{\bar{\lambda}} \mathrm{d}\bar{\lambda} + z_g \tau \mathrm{d}g^\tau, \quad \tau = D, F, \tag{237a}$$
$$\tilde{x}_2 - \tilde{x}_1 \equiv z\left(\bar{\lambda}, g_2^\tau\right) - z\left(\bar{\lambda}, g_1^\tau\right) = -z_g \mathrm{d}g^\tau, \quad \tau = D, F, \tag{237b}$$

where $\mathrm{d}\bar{\lambda} \equiv \bar{\lambda} - \lambda_0$.

Plugging these expressions into (235), we obtain finally

$$A_1 + A_2 + A_3 + A_4 = s_0 - \tilde{s}_1 = -m_{\bar{\lambda}} \mathrm{d}\bar{\lambda} - m_g \tau \mathrm{d}g^\tau, \tag{238a}$$

$$A_1 e^{\mu_1 T} + A_2 e^{\mu_2 T} + A_3 e^{\mu_3 T} + A_4 e^{\mu_4 T} - B_1 e^{\mu_1 T} - B_2 e^{\mu_2 T} = \tilde{s}_2 - \tilde{s}_1 = -m_g \mathrm{d}g^\tau, \tag{238b}$$

$$\omega_2^1 A_1 e^{\mu_1 T} + \omega_2^2 A_2 e^{\mu_2 T} + \omega_2^3 A_3 e^{\mu_3 T} + \omega_2^4 A_4 e^{\mu_4 t} - \omega_2^1 B_1 e^{\mu_1 T} - \omega_2^2 B_2 e^{\mu_2 T} = \tilde{c}_2 - \tilde{c}_1 \tag{238c}$$

$$\omega_3^1 A_1 + \omega_3^2 A_2 + \omega_3^3 A_3 + \omega_3^4 A_4 = k_0 - \tilde{k}_1 = -k_{\bar{\lambda}} \mathrm{d}\bar{\lambda} - k_g \tau \mathrm{d}g^\tau, \tag{238d}$$

$$\omega_3^1 A_1 e^{\mu_1 T} + \omega_3^2 A_2 e^{\mu_2 T} + \omega_3^3 A_3 e^{\mu_3 T} + \omega_3^4 A_4 e^{\mu_4 T} - \omega_3^1 B_1 e^{\mu_1 T} - \omega_3^2 B_2 e^{\mu_2 T} = \tilde{k}_2 - \tilde{k}_1 = -k_g \mathrm{d}g^\tau, \tag{238e}$$

$$\omega_4^1 A_1 e^{\mu_1 T} + \omega_4^2 A_2 e^{\mu_2 T} + \omega_4^3 A_3 e^{\mu_3 T} + \omega_4^4 A_4 e^{\mu_4 t} - \omega_4^1 B_1 e^{\mu_1 T} - \omega_4^2 B_2 e^{\mu_2 T} = \tilde{p}_2 - \tilde{p}_1 = -z_g \mathrm{d}g^\tau, \tag{238f}$$

together with

$$\begin{aligned} & \sum_{i=1}^{4} \frac{N_i A_i}{\mu_i - r^\star} \left(1 - e^{(\mu_i - r^\star)T}\right) + \Phi_1 \sum_{i=1}^{4} \omega_3^i A_i e^{(\mu_i - r^\star)T} + \Phi_2 \sum_{i=1}^{4} A_i e^{(\mu_i - r^\star)T} + v_{\bar{\lambda}} \mathrm{d}\bar{\lambda} \\ = & -\left\{ v_{g^\tau} - \left[v_{g^\tau} - \Phi_1 K_{g^\tau} - \Phi_2 m_{g^\tau} \right] e^{-r^\star T} \right\} \mathrm{d}g^\tau. \end{aligned} \tag{239}$$

The system composed by seven equations determine the six constants and the change of the equilibrium value of the marginal utility of wealth. Unfortunately, the 7×7 system is too complex to obtain analytical solutions with non-ambiguous signs. Hence, we cannot analyze analytically the impact and dynamic responses of the economy upon temporary government spending policies in a way that could done in a two-good semi-small open setup with time separable preferences. Given the complexity of the system, such an analysis is best done by using numerical simulation methods, which can be applied to the linearized model as presented in this subsection.

H.1 Steady-State Changes

When the model features the zero-root property, transitory expansionary budget policies have permanent effects. To see it more formally, it is convenient to write out first the long-run change of key economic variable $x = c, k, n, p$, following a permanent rise in government spending by differentiating the steady-state function of $x = z\left(\bar{\lambda}, g^\tau\right)$:

$$\left.\frac{\mathrm{d}\tilde{x}}{\mathrm{d}g^\tau}\right|_{perm} = z_{\bar{\lambda}} \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^\tau}\right|_{perm} + z_{g^\tau} = z_{\bar{\lambda}} \lambda_{g^\tau} + z_{g^\tau}. \tag{240a}$$

From (240), after a permanent fiscal expansion, the steady-state change of economic variable x is the result of two influences: a *wealth effect* induced by the change in $\bar{\lambda}$ and a *demand effect* driven by permanently raised government spending. Differently, after a temporary fiscal expansion, the *demand effect* is ineffective since g^τ ($\tau = D, F$) is perfectly expected to revert back to its initial level. The longer-lasting the fiscal expansion, the stronger the *wealth effect* and the greater its impact on $\tilde{x}$.

The once-for-all jump of the marginal utility of wealth after a temporary increase in public spending is a scaled-down version of that following a temporary expansionary budget policy:

$$\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^\tau}\right|_{temp} = \left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^\tau}\right|_{perm} \theta = \lambda_{g^\tau} \theta > 0, \quad \tau = D, F. \tag{241}$$

where $0 < \theta < 1$ decreases as T gets larger. Since $\lambda_{g^\tau} > 0$ denotes the steady-state change of $\bar{\lambda}$ for a permanent variation in g^τ, the change of $\bar{\lambda}$ for a temporary policy is smaller but of the same direction. This is quite intuitive since the *wealth effect* induced by the transitory rise in government spending extends over successively shorter periods as the persistence of the fiscal expansion diminishes.

After a permanent fiscal shock, both the *wealth effect* and the *demand effect* impinge on the steady-state changes of stocks of assets. By contrast, as long as the fiscal impulse is transitory, the *demand effect* turns out to be ineffective since g^τ reverts back to its initial level at time T. Hence, the long-run reactions of $x = c, k, n, p$ are solely driven by the *wealth effect* reflected by a rise in $\bar{\lambda}$. Restricting ourself to a rise in g^D, we get:

$$\left.\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^D}\right|_{perm} = m_{\bar{\lambda}}\lambda_{g^\tau} + m_{g^D} < \left.\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^D}\right|_{temp} = m_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^D}\right|_{temp} < 0, \tag{242a}$$

$$0 < \left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^D}\right|_{temp} = N_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^D}\right|_{temp} < \left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^D}\right|_{perm} = N_{\bar{\lambda}}\lambda_{g^D} + N_{g^\tau}, \tag{242b}$$

$$0 < \left.\frac{\mathrm{d}\tilde{k}}{\mathrm{d}g^\tau}\right|_{temp} = k_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^D}\right|_{temp} < \left.\frac{\mathrm{d}\tilde{k}}{\mathrm{d}g^D}\right|_{perm} = k_{\bar{\lambda}}\lambda_{g^D} + k_{g^\tau}, \tag{242c}$$

$$\left.\frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^\tau}\right|_{perm} = p_{\bar{\lambda}}\lambda_{g^\tau} + p_{g^\tau} < 0 < \left.\frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^\tau}\right|_{temp} = p_{\bar{\lambda}}\left.\frac{\mathrm{d}\bar{\lambda}}{\mathrm{d}g^\tau}\right|_{temp}, \tag{242d}$$

where $m_{\bar{\lambda}} < 0$, $N_{\bar{\lambda}} > 0$ (for all parametrization), $k_{\bar{\lambda}} > 0$, $p_{\bar{\lambda}} > 0$. Since the change of $\bar{\lambda}$ for a temporary policy is smaller that after a permanent fiscal expansion, inspection of equations (242) show that consumption, labor and capital are unambiguously scaled-down versions of the permanent policy effect and work in the same direction.

Like for consumption, labor and capital, we have hysteresis effects for the real exchange rate as well. Yet, a temporary fiscal expansion does not dampen the response of $\tilde{p}$ resulting from a permanent policy, i. e. is scaled down version of the permanent policy effect and work in the same direction. By contrast, the long-term response of $\tilde{p}$ is reversed. More precisely, after a permanent fiscal expansion, the steady-state change of the real exchange rate is the result of two conflictory forces: a *wealth effect* induced by the rise in $\bar{\lambda}$ and a *demand effect* driven by permanently raised government spending. Nevertheless, it can be shown formally that p must fall in the long-term. The explanation stems from the fact that the reduction of real consumption is spread over the two goods. Thus the fall in c^D is not large enough to compensate for the increase in g^D such that there is still an excess of demand which requires a real exchange rate appreciation (i. e. p must fall in the long-run). By contrast, after a temporary fiscal expansion, the *demand effect* which pushes up the relative price is ineffective. Consequently, the **real exchange rate unambiguously depreciates after a temporary rise in government spending on the domestic good**. The explanation is that the *wealth effect* induces agents to raise labor effort while reducing real consumption. The positive labor supply effect stimulates capital by raising its marginal product and boosts output. Since an excess supply arises in the domestic good market, this requires a real exchange rate depreciation which stimulates exports and encourages agents to substitute consumption of the domestic for the imported good.

The steady-state and initial changes of output after a transitory rise in public purchases on the domestic good are unambiguously positive:

$$0 < \left.\frac{\mathrm{d}\tilde{Y}}{\mathrm{d}g^D}\right|_{temp} = (r^\star + \delta_K)\left.\frac{\mathrm{d}\tilde{k}}{\mathrm{d}g^D}\right|_{temp} + \tilde{w}\left.\frac{\mathrm{d}\tilde{n}}{\mathrm{d}g^D}\right|_{temp} < \left.\frac{\mathrm{d}\tilde{Y}}{\mathrm{d}g^D}\right|_{perm}, \tag{243a}$$

$$0 < \left.\frac{\mathrm{d}Y(0)}{\mathrm{d}g^D}\right|_{temp} = \tilde{w}\left.\frac{\mathrm{d}n(0)}{\mathrm{d}g^D}\right|_{temp} < \left.\frac{\mathrm{d}Y(0)}{\mathrm{d}g^D}\right|_{perm}, \tag{243b}$$

The longer-lasting the fiscal expansion is, the stronger the *wealth effect* and the larger the long-run and short-run expenditure multipliers after a temporary rise in g^D. Interestingly, even if the public policy is highly persistent, both the steady-state and impact expenditure multipliers will not equalize to that after a permanent expansionary budget policy. The reason is that the *demand effect* vanishes.

Keeping in mind that government spending is restored to its initial level, differentiating of the home good market-clearing condition at the steady-state w. r. t. g^D yields the long-run expenditure multiplier :

$$\left.\frac{\mathrm{d}\tilde{Y}}{\mathrm{d}g^D}\right|_{temp} = \frac{\tilde{c}^D}{\tilde{c}}\left.\frac{\mathrm{d}\tilde{c}}{\mathrm{d}g^D}\right|_{temp} + \frac{\tilde{X}}{\tilde{p}}\left(\nu_X + \alpha_c\phi\frac{\tilde{c}^D}{\tilde{p}}\right)\left.\frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^D}\right|_{temp}, \tag{244}$$

where we abstract from capital depreciation for simplicity. As long as the fiscal expansion is temporary, the real exchange rate depreciates in the long-run which stimulates exports and consumption in the domestic good (see the second term on the RHS of 244). Moreover, $\tilde{c}^D$ falls in the long-run but by a smaller amount than after a permanent expansionary budget policy. It is interesting to notice that the multiplier given by equation (244) is similar to that after a permanent fiscal expansion falling on the foreign good. More precisely, **the long-run economic boom is driven by exports after a temporary fiscal expansion**.

Finally, we investigate the size of the short-term expenditure multiplier by evaluating the home good market-clearing condition at time $t = 0$ and differentiating this expression w. r. t. g^D:

$$\frac{\mathrm{d}Y(0)}{\mathrm{d}g^D} = \frac{c^D}{c}\frac{\mathrm{d}c(0)}{\mathrm{d}g^D} + \Theta\frac{\mathrm{d}p(0)}{\mathrm{d}g^D} + \frac{\mathrm{d}I(0)}{\mathrm{d}g^D} + 1, \tag{245}$$

with $\Theta \equiv \frac{X}{p}\left[\nu_X + \frac{c^D}{X}\phi\alpha_c\right] > 0$. The first term on the RHS of (245) reflects the negative impact on output triggered by the fall in consumption. The second term on the RHS of (245) represents the influence of the initial change of p on output through its effect on exports and c^D. While the real exchange rate depreciates in the long-run, it appreciates on impact and eventually, by a larger size than that after a permanent fiscal policy if the fiscal impulse is implemented over a short-period. The reason is that the steady-state change of the real exchange rate over period 1 is the result of a *wealth effect* and a *demand effect*. Since the *demand effect* more than offsets the *wealth effect* and since the latter displays a smaller size than after a permanent rise in g^D, the steady-state value of $\tilde{p}$ over period 1 is unambiguously smaller than after a permanent fiscal policy. To see it formally, we approximate its steady-state change by using (229c) and (231c):

$$\tilde{p}_1 - \tilde{p}_0 = p\left(\bar{\lambda}, g_1^D\right) - p\left(\lambda_0, g_0^D\right) = p_{\bar{\lambda}}\mathrm{d}\bar{\lambda}|_{temp} + p_{g_D} < p_{\bar{\lambda}}\mathrm{d}\bar{\lambda}|_{perm} + p_{g_D} < 0, \tag{246}$$

where $p_{\bar{\lambda}} > 0$, $p_{g^D} < 0$ and $0 < p_{\bar{\lambda}}\mathrm{d}\bar{\lambda}|_{temp} < p_{\bar{\lambda}}\mathrm{d}\bar{\lambda}|_{perm}$. However, if the rise in government spending is not short-lived, the real exchange rate appreciates on impact but by a smaller amount than that after a permanent fiscal policy. Consequently, the initial drops in exports and c^D will be moderated compared to those following a permanent fiscal policy. To summarize, if the fiscal shock is not too brief, private demand falls on impact, but by a smaller amount than after a permanent fiscal policy. We have now to determine the direction of the initial reaction of investment. We have shown previously that the short-run expenditure multiplier is positive but displays a smaller size than after a permanent fiscal policy. Since private demand (households' consumption and exports) is crowded-out by a lower amount than after a permanent rise in g^D, the smaller size of the short-run expenditure multiplier after a temporary fiscal shock originates unambiguously from the a fall investment (as long as the shock is not too brief). **If the rise in government spending is not too brief, private demand is crowded-out by a smaller amount than after a permanent fiscal impulse such that investment must be crowded-out on impact by a larger one**. In conclusion, the small size of the initial expenditure multiplier after a temporary fiscal policy relies upon (1) the dramatic drop in exports if the shock is short-lived, and (2) the crowding-out of investment is the fiscal shock is not too short-lived.

I Labor Tax-Financing Fiscal Shocks

In this section, we estimate the long-run effects of a rise in government spending falling on the domestic good g^D associated with a rise in payroll taxes τ^F, which are adjusted accordingly to balance the government budget:

$$g^D + \tilde{p}g^F = g = \tau^F\tilde{w}\tilde{n}. \tag{247}$$

Accounting for payroll taxes in the profit maximization modifies the labor decision as follows $F_n = w\left(1+\tau^F\right)$ which in turn implies that $v_n(n) = -\frac{\lambda}{p}\frac{F_n(k,n)}{(1+\tau^F)}$. Setting

$$B = \left(\frac{\alpha_K}{r^\star + \delta_K}\right)^{\frac{1}{1-\alpha_K}}, \tag{248}$$

and substituting the steady-state level of capital-labor ratio $\frac{\tilde{k}}{\tilde{n}} = B$ into the expression above yields:

$$\tilde{n} = \left[\frac{1}{\gamma_N}\frac{\bar{\lambda}}{\tilde{p}}\left(1-\alpha_K\right)\frac{B^{\alpha_K}}{1+\tau^F}\right]^{\sigma_N} \tag{249}$$

Solving (249) for employment, we get:

$$\tilde{n} = n\left(\bar{\lambda}, \tilde{p}, \tau^F\right), \tag{250}$$

with

$$\epsilon_{\tilde{n},\bar{\lambda}} \equiv \frac{\partial \tilde{n}}{\partial \bar{\lambda}}\frac{\bar{\lambda}}{\tilde{n}} = \sigma_N, \tag{251a}$$

$$\epsilon_{\tilde{n},\tilde{p}} \equiv \frac{\partial \tilde{n}}{\partial \tilde{p}}\frac{\tilde{p}}{\tilde{n}} = -\sigma_N, \tag{251b}$$

$$\epsilon_{\tilde{n},\tau^F} \equiv \frac{\partial \tilde{n}}{\partial \tau^F}\frac{\partial\left(1+\tau^F\right)}{\tilde{n}} = -\sigma_N. \tag{251c}$$

In a second step, using the fact that $\tilde{c} = \tilde{s}$, the following equation $u_c\left(\tilde{c},\tilde{s}\right) + \frac{\sigma}{\beta+\sigma}u_s\left(\tilde{c},\tilde{s}\right) = \frac{p_c(\tilde{p})\bar{\lambda}}{\tilde{p}}$ can be can be solved for steady-state consumption:

$$\tilde{c} = c\left(\bar{\lambda}, \tilde{p}\right), \tag{252}$$

with

$$\epsilon_{\tilde{c},\bar{\lambda}} \equiv \frac{\partial \tilde{c}}{\partial \bar{\lambda}}\frac{\bar{\lambda}}{\tilde{c}} = -\nu < 0, \tag{253a}$$

$$\epsilon_{\tilde{c},\tilde{p}} \equiv \frac{\partial \tilde{c}}{\partial \tilde{p}}\frac{\tilde{p}}{\tilde{c}} = \nu\left(1-\alpha_c\right) > 0. \tag{253b}$$

Using the fact that $c^D = \left(p_c - pp_c'\right)c$, we solve for steady-state consumption in the domestic good:

$$\tilde{c}^D = c^D\left(\bar{\lambda}, \tilde{p}\right), \tag{254}$$

with

$$\epsilon_{\tilde{c}^D,\bar{\lambda}} \equiv \frac{\partial \tilde{c}^D}{\partial \bar{\lambda}}\frac{\bar{\lambda}}{\tilde{c}^D} = -\nu < 0, \tag{255a}$$

$$\epsilon_{\tilde{c}^D,\tilde{p}} \equiv \frac{\partial \tilde{c}^D}{\partial \tilde{p}}\frac{\tilde{p}}{\tilde{c}^D} = \nu\left(1-\alpha_c\right) + \phi\alpha_c > 0, \tag{255b}$$

where we used the fact that $\phi\left(1-\alpha_c\right) \equiv -\frac{p_c'' p}{p_c'}$.

Using the linear homogeneity of the production function, i. e. $Y = F\left(\frac{k}{n}, 1\right) = \left(\frac{k}{n}\right)^{\alpha_K} n$, and substituting the steady-state function derived above, the market-clearing condition can be rewritten as follows:

$$B^{\alpha_K} n\left(\bar{\lambda}, \tilde{p}, \tau^F\right) = c^D\left(\bar{\lambda}, \tilde{p}\right) + X\left(\tilde{p}\right) + \delta_K B n\left(\bar{\lambda}, \tilde{p}, \tau^F\right) + g^D. \tag{256}$$

Denoting by $\omega_C = \frac{p_c c}{Y}$ the share of total consumption expenditure in GDP, $\omega_I = \frac{I}{Y}$ the share of investment in GDP, ω_X the share of exports in GDP, totally differentiating (256) and collecting terms yields:

$$\begin{aligned} & \sigma_N\left(1-\omega_I\right)\left[\hat{\bar{\lambda}} - \hat{\tilde{p}}\right] + \omega_C\left(1-\alpha_c\right)\nu\hat{\bar{\lambda}} - \left\{\omega_C\left(1-\alpha_c\right)\left[\nu\left(1-\alpha_c\right) + \phi\alpha_c\right] + \omega_X\nu_X\right\}\hat{\tilde{p}} \\ = \; & \sigma_N\left(1-\omega_I\right)\hat{\tau^F} + \frac{\mathrm{d}g^D}{Y}, \end{aligned} \tag{257}$$

where $\hat{\tilde{x}} = d\tilde{x}/\tilde{x}$ with $x = c, p, n, k, b, \lambda$ and $\hat{\tau}^F = d\tau^F / \left(1-\tau^F\right)$.

Adopting a similar reasoning, the zero current account at the steady-state can be rewritten as follows:

$$B^{\alpha_K} n\left(\bar{\lambda}, \tilde{p}, \tau^F\right) - p_c\left(\tilde{p}\right) c\left(\bar{\lambda}, \tilde{p}\right) - \delta_K B n\left(\bar{\lambda}, \tilde{p}, \tau^F\right) = g^D + \tilde{p} g^F. \tag{258}$$

Denoting by $\omega_B = \frac{r^\star pb}{Y}$ the share of interest receipts from trade bonds holding in GDP, using the fact that $r^\star \tilde{b} - \tilde{c}^F - g^F = \tilde{X}$, totally differentiating (258) and collecting terms yields:

$$\begin{aligned} & \omega_B \hat{\tilde{b}} + \left[\sigma_N\left(1-\omega_I\right) + \omega_C \nu\right] \hat{\bar{\lambda}} - \left[\sigma_N\left(1-\omega_I\right) + \omega_C\left(1-\alpha_c\right)\nu + \omega_X\right] \hat{\tilde{p}} \\ = \quad & \sigma_N\left(1-\omega_I\right) \hat{\tau^F} + \frac{\mathrm{d}g^D}{Y}. \end{aligned} \tag{259}$$

Finally, substituting steady-state functions into the linearized version of the intertemporal solvency condition and totally differentiating yields:

$$\hat{\tilde{b}} + \left[\Phi_2 \frac{\tilde{c}}{\tilde{b}}\nu - \Phi_1 \frac{\tilde{k}}{\tilde{b}}\sigma_N\right] \hat{\bar{\lambda}} + \left[\Phi_1 \frac{\tilde{k}}{\tilde{b}}\sigma_N - \Phi_2 \frac{\tilde{c}}{\tilde{b}}\nu\left(1-\alpha_c\right)\right] \hat{\tilde{p}} = -\Phi_1 \frac{\tilde{k}}{\tilde{b}}\sigma_N \hat{\tau^F} \tag{260}$$

Steady-State Effects of a Labor Tax-Financing Government Spending Shock

Total differentiation of the steady-state can be written in matrix form:

$$\begin{pmatrix} -\left\{\omega_C\left(1-\alpha_c\right)\left[\nu\left(1-\alpha_c\right)+\phi\alpha_c\right]+\omega_X\nu_X\right\} & \left[\sigma_N\left(1-\omega_I\right)+\omega_C\left(1-\alpha_c\right)\nu\right] & 0 \\ -\left[\sigma_N\left(1-\omega_I\right)+\omega_C\left(1-\alpha_c\right)\nu+\omega_X\right] & \left[\sigma_N\left(1-\omega_I\right)+\omega_C\nu\right] & \omega_B \\ \left[\Phi_1\frac{\tilde{k}}{\tilde{b}}\sigma_N - \Phi_2\frac{\tilde{c}}{\tilde{b}}\nu\left(1-\alpha_c\right)\right] & \left[\Phi_2\frac{\tilde{c}}{\tilde{b}}\nu - \Phi_1\frac{\tilde{k}}{\tilde{b}}\sigma_N\right] & 1 \end{pmatrix} \begin{pmatrix} \hat{\tilde{p}} \\ \hat{\bar{\lambda}} \\ \hat{\tilde{b}} \end{pmatrix}$$

$$= \begin{pmatrix} \sigma_N\left(1-\omega_I\right)\hat{\tau^F} + \frac{\mathrm{d}g^D}{Y} \\ \sigma_N\left(1-\omega_I\right)\hat{\tau^F} + \frac{\mathrm{d}g^D}{Y} \\ -\Phi_1\frac{\tilde{k}}{\tilde{b}}\sigma_N\hat{\tau^F} \end{pmatrix}. \tag{261}$$

where we restricted ourself to the case of a rise in government spending in the domestic good.

Determinant denoted by E writes as follows:

$$\begin{aligned} E \quad \equiv \quad & \left[\sigma_N\left(1-\omega_I\right)+\omega_C\nu\right]\left[\omega_C\left(1-\alpha_c\right)\alpha_c\left(\phi-\nu\right)+\omega_X\left(\nu_X-1\right)\right] \\ & +\omega_B\left[\Phi_1\frac{\tilde{k}}{\tilde{b}}\sigma_N - \Phi_2\frac{\tilde{c}}{\tilde{b}}\nu\right]\left[\omega_C\left(1-\alpha_c\right)\alpha_c\left(\phi-\nu\right)+\omega_X\nu_X\right] \\ & +\nu\alpha_c\left(\omega_C - \omega_B\Phi_2\frac{\tilde{c}}{\tilde{b}}\right)\left[\sigma_N\left(1-\omega_I\right)+\omega_C\nu\left(1-\alpha_c\right)\right]+\omega_c\alpha_c\nu\omega_X > 0, \end{aligned} \tag{262}$$

where the sign of E holds as long as ω_B is small as data suggest; additionally, the term $\left[\omega_C\left(1-\alpha_c\right)\alpha_c\left(\phi-\nu\right)+\omega_X\left(\nu_X-1\right)\right]$ reflects the impact on the trade balance of a change in the real exchange rate which is assumed to be positive and reflects the Marshall-Lerner condition.

The steady-state changes after a rise in government spending are:

$$\frac{\hat{\tilde{p}}}{\frac{\mathrm{d}g^D}{Y}} = -\frac{\alpha_c\omega_C\nu + \omega_B\left[\Phi_2\frac{\tilde{c}}{\tilde{b}}\nu - \Phi_1\frac{\tilde{k}}{\tilde{b}}\sigma_N\right]}{E} < 0, \tag{263a}$$

$$\frac{\hat{\bar{\lambda}}}{\frac{\mathrm{d}g^D}{Y}} = \frac{\omega_C\left(1-\alpha_c\right)\alpha_c\left(\phi-\nu\right)+\omega_X\left(\nu_X-1\right)-\omega_B\left[\Phi_1\frac{\tilde{k}}{\tilde{b}}\sigma_N - \Phi_2\frac{\tilde{c}}{\tilde{b}}\nu\left(1-\alpha_c\right)\right]}{E} > 0 \tag{263b}$$

Combining (263) and (250), the steady-state change of labor after a rise in government spending is:

$$\frac{\hat{\tilde{n}}}{\frac{\mathrm{d}g^D}{Y}} = \frac{\sigma_N}{E}\left\{\alpha_c\omega_C\left[\nu\left(1-\alpha_c\right)+\phi\alpha_c\right]+\omega_X\left(\nu_X-1\right)-\omega_B\Phi_2\frac{\tilde{c}}{\tilde{b}}\nu\alpha_c\right\} > 0. \tag{264}$$

The steady-state changes after a rise in payroll taxes are:

$$\frac{\hat{\tilde{p}}}{\hat{\tau}^F} = -\frac{\omega_C \alpha_c \nu \sigma_N (1-\omega_I)}{E} + \frac{\omega_B \sigma_N \nu \left[\Phi_2 \frac{\tilde{c}}{\tilde{b}} (1-\omega_I) + \Phi_1 \frac{\tilde{k}}{\tilde{b}} \omega_C (1-\alpha_c)\right]}{E} < 0, \tag{265a}$$

$$\frac{\hat{\tilde{\lambda}}}{\hat{\tau}^F} = \frac{\sigma_N (1-\omega_I)}{E} \left[\omega_C (1-\alpha_c) \alpha_c (\phi - \nu) + \omega_X (\nu_X - 1)\right] + \frac{\omega_B}{E} \Phi_2 \frac{\tilde{c}}{\tilde{b}} \nu (1-\alpha_c) \sigma_N (1-\omega_I)$$
$$+ \frac{\omega_B}{E} \Phi_1 \frac{\tilde{k}}{\tilde{b}} \sigma_N \left\{\omega_C (1-\alpha_c) \left[\nu (1-\alpha_c) + \phi \alpha_c\right] + \omega_X \nu_X\right\} > 0. \tag{265b}$$

Combining (265) and (250), the steady-state change of labor after a rise in payroll taxes is:

$$\frac{\hat{\tilde{n}}}{\hat{\tau}^F} = -\frac{\sigma_N}{E} \left\{ \nu \left(\omega_C - \omega_B \Phi_2 \frac{\tilde{c}}{\tilde{b}}\right) \left[\omega_C (1-\alpha_c) \alpha_c \phi + \omega_X \nu_X\right] \right.$$
$$\left. + \omega_X \left[\sigma_N (1-\omega_I) + \omega_C \alpha_c \nu\right] \right\} < 0. \tag{266}$$

Suppose that the policy maker wishes to finance the rise in government spending with an increase in payroll taxes, keeping the budget constraint balanced. For unchanged labor supply, a rise in labor taxes yields a rise in tax revenue commonly labelled the *tax rate effect*. In addition, a change in a distorsionary tax modifies the behavior of households. This induces a *tax base effect* which works in opposite direction of the *tax rate effect* on public revenue. More precisely, a rise in labor taxes raise tax revenue. However, as shown by equation (266), labor supply falls which reduces the labor tax base. In addition, keeping in mind that $\tilde{w} = w\left(\tau^F\right)$ with $\frac{\hat{\tilde{w}}}{\hat{\tau}^F} = -1$, the wage rate must fall by the same proportion than the rise in payroll taxes which lowers further the labor tax base. As long as σ_N takes a plausible value, the *tax rate effect* more than offsets the *tax base effect* which implies that a rise in labor taxes raises fiscal revenues. This assumption reflects the fact that the economy moves along the positively slopped side of the Laffer curve.

Keeping in mind that $\hat{\tilde{n}}\big|^{g,F} = \frac{\hat{\tilde{n}}}{\hat{\tau}^F} \hat{\tau}^F\big|^{g,F} + \frac{\hat{\tilde{n}}}{\frac{\mathrm{d}g^D}{Y}} \frac{\mathrm{d}g^D}{Y}$ and by rearranging terms, we can determine the size of the rise in payroll taxes $\tau^F\big|^{g,F}$ after a rise in government spending $\frac{\mathrm{d}g^D}{Y}$ such that the government budget constraint (247) remains balanced:

$$\hat{\tau}^F\big|^{g,F} = \frac{1 - \omega_G \frac{\hat{\tilde{n}}}{\frac{\mathrm{d}g^D}{Y}}}{(1-\alpha_K) - \omega_G + \omega_G \frac{\hat{\tilde{n}}}{\hat{\tau}^F}} \frac{\mathrm{d}g^D}{Y} > 0. \tag{267}$$

According to (267) payroll taxes must rise after an increase in government spending.

Substituting (267) into $\hat{\tilde{n}}\big|^{g,F} = \frac{\hat{\tilde{n}}}{\hat{\tau}^F} \hat{\tau}^F\big|^{g,F} + \frac{\hat{\tilde{n}}}{\frac{\mathrm{d}g^D}{Y}} \frac{\mathrm{d}g^D}{Y}$, we get:

$$\left.\frac{\tilde{n}}{\frac{\mathrm{d}g^D}{Y}}\right|^{g,F} = \frac{\frac{\hat{\tilde{n}}}{\frac{\mathrm{d}g^D}{Y}} \left[(1-\alpha_K) - \omega_G\right] + \frac{\hat{\tilde{n}}}{\hat{\tau}^F}}{(1-\alpha_K) - \omega_G + \omega_G \frac{\hat{\tilde{n}}}{\hat{\tau}^F}} \lesseqgtr 0, \tag{268}$$

where the sign will be determined thanks to numerical experiments.

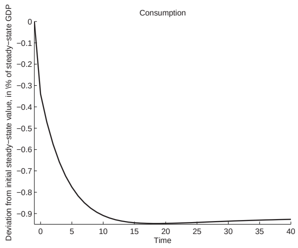
Numerical results for a labor tax-financed government spending policy are reported in Table 2. A rise in government spending by 1 percentage point of GDP requires an increase in the labor tax τ^F by 0.032 (τ^F rises from 0.444 to 0.476). While a rise in lump-sum taxes produces only a *wealth effect*, a rise in labor taxes induces additional effects, say distorsionary effects, which lower labor supply by offsetting the positive effect triggered by the reduction in private wealth. We find that in the long-run, the negative influence of the rise in payroll taxes necessary to finance the increase in public spending is large enough to more than offset the positive influence driven by the *wealth effect*. This results in a steady-state fall in employment

Table 2: Quantitative Effects of an Unexpected Fiscal Expansion Financed by a Rise in Payroll Taxes (in %)

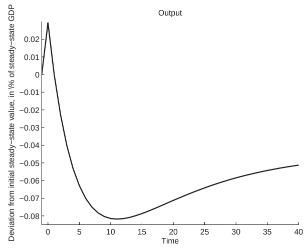
Variables[a]	($\gamma = 0.8$, $\sigma = 0.65$)
A.Long-Term	
Cons. in home good $d\tilde{c}^D$	-0.93
Exports d$\tilde{X}$	-0.11
Investment, $d\tilde{I}$	-0.01
RER, $d\tilde{p}$	-0.32
Labor d$\tilde{n}$	-0.04
Stock of Foreign Assets, $d\tilde{b}$	-0.47
B.Impact	
Consumption, $dc(0)$	-0.42
Labor d$n(0)$	0.05
RER, $dp(0)$	-0.38
Cons. in home good, $dc^D(0)$	-0.46
Exports d$X(0)$	-0.13
Investment, $dI(0)$	-0.38
Savings, $dS(0)$	-0.47
Current Account, $dca(0)$	-0.09
C.Multipliers	
Short-Term Multiplier, d$Y(0)$/dg^D	0.03
Long-Term Multiplier, d$\tilde{Y}$/dg^D	-0.04

[a]We consider a rise in g^D which raises total government spending by one percentage point of GDP and is financed by a rise in payroll taxes by an amount dictated by (267). Parameters are those in the benchmark case. Impact and steady-state changes are scaled by initial GDP. The fiscal multiplier has been obtained by dividing the variation of GDP by the change in public spending.

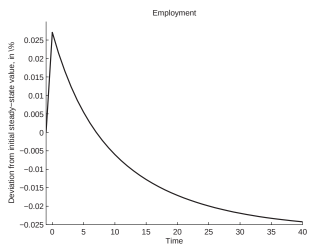
and physical capital, and thereby yields a negative steady-state balanced-budget multiplier (-0.04). In contrast, in the short-run, as the real exchange rate overshoots its steady-state level and pushes down further private wealth measured in terms of the domestic good, the effect on employment of the sizeable real exchange rate appreciation counteracts the distorsionary effect. This allows for an initial rise in employment and thereby a positive impact balanced-budget multiplier, yet displaying a much smaller size (0.03) than that after a lump-sum tax-financed public spending policy.



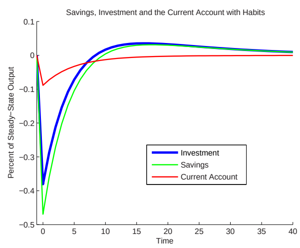
(a) Consumption



(b) Output



(c) Employment



(d) Current Account with Habits

Figure 3: Computed Transitional Paths after a Labor Tax-Financed Government Spending Shock

J Habits in Leisure and Fiscal Shocks

In this subsection, we provide a discussion about the effects of fiscal shocks by assuming that households display a habit-forming behavior in leisure. We abstract from capital accumulation in order to simplify the study of saddle-path stability conditions.

J.1 The Framework

The representative agent is endowed with a unit of time, supplies a fraction $n(t)$ as labor, and the remainder $l(t) \equiv 1 - n(t)$ is consumed as leisure. At any instant of time, households derive utility from consumption $c(t)$, current leisure $l(t)$ and are confronted to disutility from their level of habits in leisure denoted by $s(t)$. Hence, assuming that the felicity function is additively separable in consumption and leisure, the representative, household maximizes the following objective function:

$$U = \int_0^{\infty} \{u(c) + v(l, s)\} \exp(-\beta t) \, \mathrm{d}t, \tag{269}$$

where β is the consumer's subjective time discount rate, $\sigma_c > 0$ corresponds to the intertemporal elasticity of substitution for consumption, and $\gamma > 0$ stands for the weight attached to habits in leisure s into utility.

The stock of habits in leisure is defined as a distributed lag on past leisure' levels:

$$s(t) = \sigma \int_{-\infty}^{t} l(\tau) \exp(-\sigma (t - \tau)) \, \mathrm{d}\tau, \tag{270}$$

where the parameter σ indexes the relative weight of recent leisure in determining the reference stock s. Differentiating equation (270) w. r. t. time gives the law of motion of habit stock:

$$\dot{s}(t) = \sigma [l(t) - s(t)]. \tag{271}$$

The flow budget constraint writes in the usual form:

$$p(t)\dot{b}(t) = r^{\star} p(t) b(t) + w(t)(1 - l(t)) - p_c(p(t))c(t) - T, \tag{272}$$

where $1 - l = n$. The government finances government spending on the domestic good g by levying lump-sum taxes T, i. e. $g = g^D = T$.

The macroeconomic equilibrium is described by the following set of equations:

$$u_c(c) = \frac{p_c \lambda}{p}, \tag{273a}$$

$$v_l(l, s) + \sigma \xi = -\frac{\lambda}{p} w, \tag{273b}$$

$$F_n(n) = w, \tag{273c}$$

$$\dot{\lambda} = \lambda (\beta - r^{\star}), \tag{273d}$$

$$\dot{\xi} = (\beta + \sigma) \xi - v_s(l, s), \tag{273e}$$

$$F(n) = (1 - \alpha_c) p_c c + X(p) + g, \tag{273f}$$

together with the accumulation equation of foreign assets (272), the accumulation equation of habits (271) and appropriate transversality conditions.

We first solve equation (273a) for consumption:

$$c = c(\bar{\lambda}, p), \tag{274}$$

where partial derivatives are given by (193). Using the Shephard's Lemma, intra-temporal allocations between domestic goods and foreign goods are: $c^F = p_c' c$ and $c^D = [p_c - p p_c'] c$ and substituting (193), we solve for consumption in the domestic and foreign goods:

$$c^D = c^D(\bar{\lambda}, p), \quad c^F = c^F(\bar{\lambda}, p), \tag{275}$$

where partial derivatives are given by (195).

Substituting first the short-run static solution for consumption in the domestic good (275), and solving the home good market-clearing condition (273f) for the real exchange rate yields:

$$p = p\left(\bar{\lambda}, l, g\right), \tag{276}$$

where

$$\epsilon_{p,\lambda} \equiv \frac{\partial p}{\partial \lambda}\frac{\lambda}{p} = -\frac{\left(1-\alpha_c\right)\omega_c\sigma_c}{F} > 0, \tag{277a}$$

$$\epsilon_{p,l} \equiv \frac{\partial p}{\partial l}\frac{l}{p} = -\frac{l}{n}\frac{1}{F} < 0, \tag{277b}$$

$$\epsilon_{p,g} \equiv \frac{\partial p}{\partial g}\frac{Y}{p} = -\frac{1}{F} < 0, \tag{277c}$$

with $F \equiv \left(1-\alpha_c\right)\left[\phi\alpha_c + \left(1-\alpha_c\right)\sigma_c\right] + \omega_X\nu_X > 0$; we specified the production function as follows $Y = n^{alpha_N}$, with $0 < \alpha_N < 1$.

To write the law of motion of real consumption in a more interpretable form, we calculate the time preference rate denoted by ρ^L, defined as the proportional rate of decrease of marginal utility of leisure expressed in present value terms (see e. g. Epstein [1987]):

$$\rho^L \equiv -\left.\frac{\mathrm{d}\ln\left\{\left[v_c\left(l,s\right)+\sigma\xi\right]\exp\left(-\beta t\right)\right\}}{\mathrm{d}t}\right|_{\dot{l}(t)=0}. \tag{278}$$

By substituting the accumulation equation for habits (271) and the dynamic equation for its shadow price (273e), and eliminating ξ by making use of (273b), the rate of time preference writes as follows:

$$\rho^L(t) = \beta + \frac{p}{w\lambda}\left[\sigma v_s - v_{ls}\sigma\left(l-s\right) + \left(\beta+\sigma\right)\left(v_l - \frac{w\lambda}{p}\right)\right], \tag{279}$$

where $\rho^L_l = \frac{p}{w\lambda}\left(\beta+\sigma\right)v_{ll} < 0$, $\rho^L_s = \frac{p}{w\lambda}\left(\beta+2\sigma\right)\Gamma^L > 0$, $\rho^L_p = \frac{1}{p}\left[\left(\rho^L-\beta\right)+\left(\beta+\sigma\right)\right]$, $\rho^L_\lambda = -\frac{1}{p}\left[\left(\rho^L-\beta\right)+\left(\beta+\sigma\right)\right]$ and we set

$$\Gamma^L = v_{ls} + \frac{\sigma}{\beta+2\sigma}v_{ss} > 0. \tag{280}$$

Differentiating (273b) with respect to time, substituting (271) and (273b), eliminating ξ by making use of (273b), yields the dynamic equation for leisure:

$$\dot{l} = -\left(\Psi^L\right)^{-1}\frac{\frac{\lambda}{p}w}{v_{ll}l}\left(r^\star - \rho\left(l,s,p,\lambda\right)\right), \tag{281}$$

where

$$\Psi^L = 1 + \frac{\frac{\lambda}{p}F_{nn}}{v_{ll}} + \frac{\frac{\lambda}{p}w}{v_{ll}l}\frac{p_l l}{p} > 0, \tag{282}$$

with $F_{nn} < 0$, $v_{ll} < 0$ and $p_l < 0$.

We assume that the instantaneous utility function takes the following form:

$$u\left(c\right) + v\left(l,s\right) \equiv \frac{1}{1+\frac{1}{\sigma_c}}c^{1+\frac{1}{\sigma_c}} + \frac{1}{1+\epsilon}\left[\frac{l}{\left(s\right)^{\gamma}}\right]^{1+\epsilon}, \tag{283}$$

where σ_c stands for the intertemporal elasticity of substitution for consumption.

Useful expressions evaluated at the steady-state denoted by a tilde write as follows:

$$\tilde{v}_l + \frac{\sigma}{\beta+\sigma}\tilde{v}_s = \tilde{l}^{-[\epsilon+\gamma(1-\epsilon)]}\frac{\beta+\sigma(1-\gamma)}{\beta+\sigma} > 0, \tag{284a}$$

$$-\frac{\tilde{\lambda}\tilde{w}}{\tilde{p}\tilde{v}_{ll}\tilde{l}} = -\frac{\tilde{v}_l + \frac{\sigma}{\beta+\sigma}\tilde{v}_s}{\tilde{v}_{ll}\tilde{l}} = \frac{[\beta+\sigma(1-\gamma)]}{\epsilon(\beta+\sigma)} > 0, \tag{284b}$$

$$-\frac{(\beta+2\sigma)}{\tilde{v}_{ll}}\tilde{\Gamma}^L = \frac{\gamma}{\epsilon}\left\{\sigma\epsilon - (1-\epsilon)[\beta+\sigma(1-\gamma)]\right\} > 0. \tag{284c}$$

$$-\frac{\tilde{v}_l + \frac{\sigma}{\beta+\sigma}\tilde{v}_s}{\tilde{v}_{ll}\tilde{l}} = \frac{\beta+\sigma(1-\gamma)}{\epsilon(\beta+\sigma)} > 0. \tag{284d}$$

Linearizing (281) together with the accumulation equation of habits (271) around the steady-state, and denoting long-term values by a tilde, we obtain in a matrix form:

$$\left(\dot{s}, \dot{l}\right)^T = J\left(s(t)-\tilde{s}, l(t)-\tilde{l}\right)^T, \tag{285}$$

where J is given by

$$J \equiv \begin{pmatrix} -\sigma & \sigma \\ \left(\tilde{\Psi}^L\right)^{-1}\frac{\beta+2\sigma}{\tilde{v}_{ll}}\tilde{\Gamma}^L & \left(\tilde{\Psi}^L\right)^{-1}\left[(\beta+\sigma) - \frac{[\beta+\sigma(1-\gamma)]}{\epsilon}\frac{p_l\tilde{l}}{\tilde{p}}\right] \end{pmatrix}, \tag{286}$$

where

$$\tilde{\Psi}^L = 1 + \frac{\beta+\sigma(1-\gamma)}{\epsilon(\beta+\sigma)}\frac{\tilde{l}}{\tilde{n}}\left\{\frac{1}{\beta_N} + \frac{1}{\omega_X\nu_X + \omega_C(1-\alpha_c)[\phi\alpha_c + (1-\alpha_c)\sigma_c]}\right\} > 1, \tag{287}$$

and

$$\left(\tilde{\Psi}^L\right)^{-1}\frac{\beta+2\sigma}{\tilde{v}_{ll}}\tilde{\Gamma}^L = -\left(\tilde{\Psi}^L\right)^{-1}\frac{\gamma}{\epsilon}\left\{\sigma\epsilon - (1-\epsilon)[\beta+\sigma(1-\gamma)]\right\} \gtrless 0, \tag{288a}$$

$$\left(\tilde{\Psi}^L\right)^{-1}\left[(\beta+\sigma) - \frac{[\beta+\sigma(1-\gamma)]}{\epsilon}\frac{p_l\tilde{l}}{\tilde{p}}\right] = \left(\tilde{\Psi}^L\right)^{-1}(\beta+\sigma)\tilde{\Psi}^L = \beta+\sigma. \tag{288b}$$

The determinant denoted by Det of the linearized 2×2 matrix (286) is unambiguously negative:

$$\text{Det J} = -\left(\tilde{\Psi}^L\right)^{-1}\frac{\sigma}{\epsilon}[\beta+\sigma(1-\gamma)]\left\{[\gamma+\epsilon(1-\gamma)] - \epsilon_{\tilde{p},\tilde{l}}\right\} < 0, \tag{289}$$

where $0 < \tilde{\Psi}^{-1} < 1$ and $\epsilon_{\tilde{p},\tilde{l}} < 0$. The trace denoted by Tr is given by:

$$\text{Tr J} = \beta = r^\star > 0. \tag{290}$$

The characteristic roots obtained from the 2×2 linearized matrix J write as follows:

$$\mu_i \equiv \frac{1}{2}\left\{\text{Tr}J \pm \sqrt{(\text{Tr}J)^2 - 4\text{Det}J}\right\} \gtrless 0, \quad i=1,2. \tag{291}$$

We denote by $\mu_1 < 0$ and $\mu_2 > 0$ the stable and unstable real eigenvalues, satisfying

$$\mu_1 < 0 < r^\star < \mu_2. \tag{292}$$

Since the system features one state variable, s, and one jump variable, l, the equilibrium yields a unique one-dimensional stable saddle-path. It is straightforward to show that setting $\gamma = 0$ yields a stable root which simplifies to $\mu_1^{\gamma=0} = -\sigma$.

Stable solutions paths are given by:

$$s(t)-\tilde{s} = B_1 e^{\mu_1 t}, \quad l(t)-\tilde{l} = \omega_2^1 B_1 e^{\mu_1 t}, \tag{293}$$

where we normalized ω_1^i to unity. The eigenvector ω_2^i associated with eigenvalue μ_i is given by

$$\omega_2^i = \left(\frac{\sigma + \mu_i}{\sigma}\right) > 0, \quad i = 1, 2. \tag{294}$$

Formal Solution for the Stock of Foreign Assets

Substituting the domestic good market clearing condition (273f) into the accumulation equation of foreign assets (272), and substituting the short-run static solution for consumption in the foreign good (275) yields:

$$\dot{b} = r^\star b + \frac{X(p)}{p} - c^F\left(\bar{\lambda}, p\right). \tag{295}$$

Substituting the short-run static solution for the relative price p (276), and linearizing equation (295) around the steady-state leads to:

$$\dot{b}(t) = r^\star\left(b(t) - \tilde{b}\right) + \tilde{\Omega} p_l \left(l(t) - \tilde{l}\right), \tag{296}$$

where

$$\tilde{\Omega} \equiv \frac{\tilde{Y}}{\tilde{p}^2}\left[\omega_X\left(\nu_X - 1\right) + \omega_C \alpha_c \left(1 - \alpha_c\right)\left(\phi - \sigma_c\right)\right] > 0. \tag{297}$$

Inserting stable solution for $l(t)$, solving the differential equation, and invoking the transversality condition, yields the linearized version of the nation's intertemporal budget constraint:

$$b_0 - \tilde{b} = \frac{\tilde{\Omega} p_l \omega_2^1}{\mu_1 - r^\star} B_1 = \Phi_1\left(s_0 - \tilde{s}\right). \tag{298}$$

together with the stable solution for the stock of foreign bonds:

$$b(t) - \tilde{b} = \frac{\tilde{\Omega} p_l \omega_2^1 B_1}{\mu_1 - r^\star} e^{\mu_1 t} = \Phi_1\left(s(t) - \tilde{s}\right), \tag{299}$$

where we let $\Phi_1 = \frac{\tilde{\Omega} p_l \omega_2^1}{\mu_1 - r^\star} > 0$.

J.2 Habits in Leisure and the Long-Term Effects of Fiscal Shocks

Substituting first the short-run static solution for consumption, the steady-state of the economy is obtained by setting $\dot{l}, \dot{s}, \dot{b} = 0$ and is defined by the following set of equations:

$$\tilde{l} = \left[\left(\frac{\beta + \sigma}{\beta + \sigma(1 - \gamma)}\right)\frac{\bar{\lambda} F_n\left(1 - \tilde{l}\right)}{\tilde{p}}\right]^{-\nu}, \tag{300a}$$

$$F\left(1 - \tilde{l}\right) = c^D\left(\bar{\lambda}, \tilde{p}\right) + +X\left(\tilde{p}\right) + g, \tag{300b}$$

$$r^\star \tilde{p}\tilde{b} + F\left(1 - \tilde{l}\right) - p_c\left(\tilde{p}\right) c\left(\bar{\lambda}, \tilde{p}\right) - g = 0, \tag{300c}$$

and the economy's intertemporal budget constraint

$$\left(\tilde{b} - b_0\right) = \Phi_1\left(\tilde{s} - s_0\right), \tag{300d}$$

where $\alpha_N = \frac{F_n n}{Y}$.

Total differentiation of the steady-state (300) can be written in matrix form:

$$\begin{pmatrix} 1+\frac{\nu}{\beta_N}\frac{\tilde{l}}{\tilde{n}} & -\nu & \nu & 0 \\ \alpha_N\frac{\tilde{l}}{\tilde{n}} & \left\{\omega_C\left(1-\alpha_c\right)\left[\phi\alpha_c+\left(1-\alpha_c\right)\sigma_c\right]+\omega_X\nu_X\right\} & -\omega_C\left(1-\alpha_c\right)\sigma_c & 0 \\ -\alpha_N\frac{\tilde{l}}{\tilde{n}} & -\omega_C\left[\alpha_c+\left(1-\alpha_c\right)\sigma_c\right] & \omega_C\sigma_c & \omega_B \\ -\Phi_1\frac{\tilde{l}}{\tilde{b}} & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \hat{\tilde{l}} \\ \hat{\tilde{p}} \\ \hat{\tilde{\lambda}} \\ \hat{\tilde{b}} \end{pmatrix}$$
$$= \begin{pmatrix} 0 \\ \frac{\mathrm{d}g}{Y} \\ -\frac{\mathrm{d}g}{Y} \\ 0 \end{pmatrix}, \tag{301}$$

where we restricted ourself to the case of a rise in government spending in the domestic good.

Determinant denoted by H writes as follows:

$$\begin{aligned} H \equiv\; & \Phi_1\frac{\tilde{l}}{\tilde{b}}\nu\omega_B\left\{\omega_C\left(1-\alpha_c\right)\alpha_c\left(\phi-\sigma_c\right)+\omega_X\nu_X\right\}+\nu\alpha_N\frac{\tilde{l}}{\tilde{n}}\left\{\omega_C\alpha_c\alpha_c\left[\left(1-\alpha_c\right)\phi+\alpha_c\sigma_c-1\right]+\omega_X\nu_X\right\} \\ & +\left(1+\frac{\nu}{\beta_N}\frac{\tilde{l}}{\tilde{n}}\right)\omega_c\sigma_c\left\{\omega_C\left(1-\alpha_c\right)\alpha_c\left(\phi-1\right)+\omega_X\nu_X\right\}>0, \end{aligned} \tag{302}$$

where $\Phi_1 > 0$.

The steady-state changes after a rise in government spending are:

$$\frac{\hat{\tilde{l}}}{\frac{\mathrm{d}g^D}{Y}} = -\frac{\nu}{H}\left\{\alpha_c\omega_C\left[\left(1-\alpha_c\right)\left(\phi-\sigma_c\right)+\sigma_c-1\right]+\omega_X\nu_X\right\}<0, \tag{303a}$$

$$\frac{\hat{\tilde{p}}}{\frac{\mathrm{d}g^D}{Y}} = -\frac{1}{H}\left\{\left(\frac{\nu}{\beta_N}\frac{\tilde{l}}{\tilde{n}}\right)\alpha_c\omega_c\sigma_c-\nu\omega_B\Phi\frac{\tilde{l}}{\tilde{b}}\right\}<0, \tag{303b}$$

$$\frac{\hat{\tilde{\lambda}}}{\frac{\mathrm{d}g^D}{Y}} = \frac{1}{H}\left\{\left(\frac{\nu}{\beta_N}\frac{\tilde{l}}{\tilde{n}}\right)\omega_C\left[\alpha_c\left[\left(1-\alpha_c\right)\left(\phi-\sigma_c\right)-1\right]+\omega_X\nu_X\right]+\nu\omega_B\Phi_1\frac{\tilde{l}}{\tilde{b}}\right\}>0. \tag{303c}$$

According to (303), a fiscal expansion lowers private wealth which induces agents to reduce their consumption in leisure and work more. While the wealth effect impinges negatively on consumption, its drop is not large enough for the market-clearing condition to hold. Hence, the real exchange rate appreciates which depresses exports and pushes down further consumption in the domestic good. Differentiating the intertemporal solvency condition (300d) with respect to government spending, we find that the fall in the stock of habits in leisure drives down the stock of foreign assets. The explanation is that while consumption falls markedly on impact, its drop is not large enough to compensate the decrease in the real disposable income as labor supply exhibits a tenuous response in the short-run. In conclusion, we derive similar results to those in the case of habits in consumption. Leisure and consumption fall while labor rises in the long-run. The open economy experiences a current account deficit over the transition towards the steady-state.

Yet, it is worthwhile noticing that leisure initially decreases but by a smaller amount than in the long-run:

$$\frac{\mathrm{d}l(0)}{\frac{\mathrm{d}g^D}{Y}} = -\frac{\mu_1}{\sigma}\frac{\mathrm{d}\tilde{l}}{\frac{\mathrm{d}g^D}{Y}}<0, \tag{304}$$

where $0 < -\frac{\mu_1}{\sigma} < 1$ and we used the fact that $B_1 = s_0 - \tilde{s}$. Consequently, instead of over-reacting in the short-run, output rises gradually towards its new long-run equilibrium. Additionally, the wage rate decreases but exhibits a tenuous response. Linearizing the short-run static solution for the relative price in the neighborhood of the steady-state, evaluating at time

$t = 0$ and differentiating w. r. t. $g^D = g$, yields:

$$\frac{\mathrm{d}p(0)}{\mathrm{d}\frac{g^D}{Y}} = \frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^D} - p_l \omega_2^1 \frac{\mathrm{d}\tilde{s}}{\mathrm{d}g^D} < \frac{\mathrm{d}\tilde{p}}{\mathrm{d}g^D} < 0, \tag{305}$$

where $p_l < 0$ and $\omega_2^1 > 0$. From (305), the real exchange rate overshoots its steady-state level. The reason is that the tenuous reaction of leisure and thereby the weak reaction of labor yields a small rise in output. Though consumption falls strongly on impact, the rise in government spending triggers an excess of demand in the home good market because higher domestic supply fails to offset greater demand for the domestic good. For the market-clearing condition to hold, a real exchange appreciation that lowers further c^D and depresses exports is required.

Finally, habits in leisure lower the size of short-run government spending multipliers by moderating the initial reaction of labor in comparison with the case without habits. However, likewise consumption habits which imply that habit-forming consumers are confronted to a larger steady-state fall in consumption than consumers having time separable preferences, the presence of habits in leisure yields a larger steady-state fall in leisure and thereby a greater long-run increase in labor. Hence, habits in leisure raises the size of the long-run government spending multipliers in comparison with the case without habits.

If we allow for capital accumulation in a model with habits in leisure, the two-dimensional stable solution would imply that labor displays a hump-shaped adjustment, i. e. overshoots its steady-state level. Hence output peaks after a certain delay. Therefore, the introduction of habits in leisure allows for giving rise to an output response in line with recent VAR responses. Finally, we may expect that the weak response of output on impact yields a crowding-out of investment expenditure by public spending in the short-run, stemming from the gradual and sluggish adjustment of leisure.

K Debt-Financing Fiscal Shocks

The baseline neoclassical RBC model features infinitely-lived Ricardian households, whose consumption choices are based on an intertemporal budget constraint. According to Ricardian equivalence, the particular method used to finance public spending does not matter. Hence, wether government expenditure are financed by means of public debt or taxation, a fiscal expansion reduces the present value of after-tax income, thus producing a negative wealth effect that induces agents to cut consumption expenditure by the same size. While we consider that households display a habit-forming behavior, individuals behave like Ricardian-consumers because we assume that the economy is populated by a large number of identical households and firms that have perfect foresight and live forever. Hence, our results generalize in the presence of non-balanced government fiscal expansion. More precisely, wether the government budget is balanced or not does not affect qualitatively and quantitatively our results.

So far, we have assumed the government budget constraint is balanced. We now relax this assumption and consider that the government may issue traded bonds measured in terms of the domestic good, $D(t)$, to finance its expenditure net of lump-sum taxes,

$$\dot{D}(t) = r^\star D(t) + g^D + p(t) g^F - T. \tag{306}$$

Government expenditure consists in three components: purchases of domestic goods, g^D, and import goods, g^F, and net interest payments on outstanding public debt, $r^\star p(t) D(t)$.

To rule out the possibility that the government ends with a positive debt or credit, we impose the following condition:

$$\lim_{t \to \infty} D(t) \exp\left(-r^\star t\right) = 0. \tag{307}$$

We first linearize (306) around the steady-state:

$$\dot{D}(t) = r^\star \left(D(t) - \tilde{D}\right) + g^F \left(p(t) - \tilde{p}\right).$$

Inserting the stable solution for $(p(t)-\tilde{p})$ given by (52d), the solution for the stock of public (traded) bonds writes as follows:

$$\dot{D}(t) = r^\star \left(D(t) - \tilde{D} \right) + g^F \sum_{i=1}^{2} \omega_4^i A_i e^{\mu_i t}.$$

Solving the differential equation leads to:

$$\begin{aligned} D(t) - \tilde{D} &= \left[\left(D_0 - \tilde{D} \right) - \frac{V_1 A_1}{\mu_1 - r^\star} - \frac{V_2 A_2}{\mu_2 - r^\star} \right] e^{r^\star t} \\ &+ \frac{V_1 A_1}{\mu_1 - r^\star} e^{\mu_1 t} + \frac{V_2 A_2}{\mu_2 - r^\star} e^{\mu_2 t}, \end{aligned} \tag{308}$$

with

$$V_1 = g^F \omega_4^1 > 0, \quad V_2 = g^F \omega_4^2 > 0, \tag{309}$$

where the signs follow from $\omega_4^1 > 0$ and $\omega_4^2 > 0$.

Invoking the condition for intertemporal solvency (307), we obtain the linearized version of the government's intertemporal budget constraint:

$$D_0 - \tilde{D} = \frac{V_1 A_1}{\mu_1 - r^\star} + \frac{V_2 A_2}{\mu_2 - r^\star}. \tag{310}$$

The stable solution for net stock of public traded bonds finally reduces to:

$$D(t) - \tilde{D} = \frac{V_1 A_1}{\mu_1 - r^\star} e^{\mu_1 t} + \frac{V_2 A_2}{\mu_2 - r^\star} e^{\mu_2 t}. \tag{311}$$

Inserting constants A_1 and A_2 given by (54), we obtain the linearized version of the intertemporal budget constraint of government expressed as a function of initial stocks of capital and habits

$$\tilde{D} - D_0 = \Psi_1 \left(\tilde{k} - k_0 \right) + \Psi_2 \left(\tilde{s} - s_0 \right), \tag{312}$$

with

$$\Psi_1 = \frac{\left(\mu_1 - r^\star\right) V_2 - \left(\mu_2 - r^\star\right) V_1}{\left(\mu_1 - r^\star\right)\left(\mu_2 - r^\star\right)\left(\omega_3^2 - \omega_3^1\right)}, \tag{313a}$$

$$\Psi_2 = \frac{\left(\mu_2 - r^\star\right) \omega_3^2 V_1 - \left(\mu_1 - r^\star\right) \omega_3^1 V_2}{\left(\mu_1 - r^\star\right)\left(\mu_2 - r^\star\right)\left(\omega_3^2 - \omega_3^1\right)}, \tag{313b}$$

where $\left(\omega_3^2 - \omega_3^1\right) > 0$, $V_1 > 0$, $V_2 > 0$.

Differentiating the government budget constraint (312) w. r. t. g^D yields:

$$\frac{\mathrm{d}\tilde{D}}{\mathrm{d}g^D} = \Psi_1 \frac{\mathrm{d}\tilde{k}}{\mathrm{d}g^D} + \Psi_2 \frac{\mathrm{d}\tilde{s}}{\mathrm{d}g^D}, \tag{314}$$

where the long-run change in net government debt will be estimated numerically. Differentiating (311) w. r. t. time, we obtain the transitional path for public debt:

$$\dot{D}(t) = \mu_1 \Psi_1 e^{\mu_1 t} + \mu_2 \Psi_2 e^{\mu_2 t}. \tag{315}$$

Differentiating the steady-state government budget constraint w. r. t. g^D, i. e. $r^\star \tilde{p}\tilde{a} + g^D + \tilde{p} g^F - \tilde{T}$, enables us to derive the long-run change of the required lump-sum tax:

$$\frac{\mathrm{d}\frac{\tilde{T}}{\tilde{Y}}}{\frac{\mathrm{d}g^D}{\tilde{Y}}} = 1 + \omega_D \frac{\mathrm{d}\tilde{D}}{\frac{\mathrm{d}g^D}{\tilde{Y}}} \frac{1}{\tilde{D}} + \omega_G^F \frac{\mathrm{d}\tilde{p}}{\frac{\mathrm{d}g^D}{\tilde{Y}}} \frac{1}{\tilde{p}}, \tag{316}$$

where $\omega_D = \frac{r^\star \tilde{D}}{\tilde{Y}}$ is the share of interest payments on outstanding debt in GDP and ω_G^F is the share of government spending on the foreign good in GDP. The details of derivation show that long-run changes of public debt $\tilde{D}$ and lump-sum taxes $\tilde{T}$ do not affect consumption and investment decisions. If the government decides to finance a rise in government spending through issue of public debt, taxes must adjust so that the government's budget restriction is met. However, the sequence of taxes does not affect consumption choices as we shall show now more clearly.

Solving (306) and invoking (307), we obtain the government's budget restriction:

$$D_0 = \int_0^\infty (T - g)^{-r^\star t} \, \mathrm{d}t, \tag{317}$$

where we denoted by $g = g^D + pg^F$ total government expenditure. According to (317), intertemporal solvency requires that the initial debt D_0 is equal to the present value of future primary surpluses.

Combining the accumulation equation of financial wealth (77) and invoking the transversality condition yields:

$$\int_0^\infty p_c\left(p(t)\right) c\left(t\right) e^{-R^K(t)} \mathrm{d}t = a(0) + W(0), \tag{318}$$

where $W\left(0\right)$ denotes human wealth, defined as the present discounted value of the future flow of real disposable income measured in terms of the domestic good, i. e.

$$W(0) = \int_0^\infty \left[w(t)n(t) - T\right] e^{-R^K(t)} \mathrm{d}t. \tag{319}$$

with R^K the discounting factor:

$$R^K(t) = \int_0^t r^K(\tau) \mathrm{d}\tau.$$

By substituting the government's budget restriction (317) into (319), the expression of human wealth can be rewritten as:

$$W(0) = \int_0^\infty \left[w(t)n(t) - g\right] e^{-R^K(t)} \mathrm{d}t - D_0 - \int_0^\infty D(t) \left(r^K - r^\star\right) e^{-R^K(t)} \mathrm{d}t. \tag{320}$$

It it straightforward to see that the path of lump-sum taxes completely vanishes from the expression of human wealth (320). Because the particular path for taxes does not alter households' disposable income, real consumption choices are not affected either.

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