Intense Lasers Simulation of
Quantum Cosmology & Gravity

Sang Pyo Kim
Kunsan Nat’l Univ & CoReLS/IBS
IZEST, Embassy of Romania in Paris
September 17-19, 2014
# Overview of Strong Field Physics

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Unified Picture for Pair Production
[SPK, JHEP11('07)]

Vacuum Fluctuations/Unruh Effects

Schwinger Mechanism/Polarization

QED
  - Condensed Matter Analogues
  - Black Holes
  - Black Hole Analogues

Hawking Radiation
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Simulating Quantum Effects in Cosmology via Ion Traps?
Simulating Gibbons-Hawking Effect

[Menicucci, Olson, Milburn, New J. Phys. 12 (‘10)]

- An ion trap experiment is proposed as the analogue of quantum effects in an expanding universe, such as Gibbons-Hawking effect, inflationary structure, etc.

\[ ds^2 = -dt^2 + a^2(t) d\vec{x}^2 \]

- An Unruh-DeWitt detector in a FRW universe

\[ H_{\text{int}} = f(t)m(t)\phi(t, \vec{x}) \]

- A homogeneous, isotropic field \( \Phi \) has the Hamiltonian of time-dependent oscillators

\[ H(t) = \frac{1}{2} \sum_k \frac{\pi_k^2}{a^{3/2}(t)} + a^{3/2}(t) \omega_k^2(t) \phi_k^2 \]
Paul Trap

- Nonrelativistic quantum motion of a charged particle in an rf trap with an electric potential (quantum theory)
  \[ \Phi = \frac{U}{2} \left( \alpha x^2 + \beta y^2 + \gamma z^2 \right) + \frac{\bar{U}}{2} \cos(\omega_{rf} t) \left( \alpha' x^2 + \beta' y^2 + \gamma' z^2 \right) \]

- Maxwell equation at each moment (dynamical trapping)
  \[ \Delta \Phi = 0 \Rightarrow \begin{cases} \alpha + \beta + \gamma = 0 \\ \alpha' + \beta' + \gamma' = 0 \end{cases} \]

Schematic drawing of the electrodes for a cylindrically symmetric 3D rf trap.

\[ r_0 \cong \sqrt{2}z_0 = 100 \mu m \sim 1 \text{cm}, \ \bar{U} = 100 \sim 500 \text{V}, \ U = 0 \sim 50 \text{V}, \ \omega/2\pi = 100 \text{kHz} \sim 100 \text{MHz} \]

[Leibfried et al, Rev. Mod. Phys. 75 (‘03)]
QED in Ultra-Strong, Time-Dependent Magnetic Fields
Sources for Strong EM Fields

Neutron Stars & Magnetars

IZEST
Extreme Light Road Map

Schwinger Limit

[Harding and Lai, Rep. Prog. Phys. 69 ('06)]

[Homma, Habs, Mourou, Ruhl and Tajima, PTP Suppl. 193 ('12)]
Scalar QED in $B(t)$

- A homogeneous, time-dependent, magnetic field $B(t)$ and an induced electric field with the vector potential

$$\vec{A}(t, \vec{r}) = \frac{1}{2} \vec{B}(t) \times \vec{r}$$

- How to find the Landau levels and the QED effective action?
  - The first quantized theory: the Klein-Gordon equation
  - The second quantized theory: the Hamiltonian from field action

- The dynamically coupled Landau levels continuously make transitions among themselves, and the 1st or 2nd quantized theory is similar to a relativistic theory of time-dependent, coupled oscillators coupled to each other.
1st Quantized Formulation

- The two-component, first order wave function of the KG eq expanded by Landau states [SPK, Ann Phys 344 (‘14)]

\[
\begin{pmatrix}
\Psi(t, \bar{x}_\perp) \\
\partial \Psi(t, \bar{x}_\perp) / \partial t
\end{pmatrix} = \begin{pmatrix}
\Phi^T(t, \bar{x}_\perp) & 0 \\
0 & \Phi^T(t, \bar{x}_\perp)
\end{pmatrix}
\]

\[
\times T \exp \left[ \int \begin{pmatrix}
\Omega(t') & I \\
-\omega^2(t') & \Omega(t')
\end{pmatrix} dt' \right] \begin{pmatrix}
\tilde{\psi}(t_0) \\
d\tilde{\psi}(t_0) / dt_0
\end{pmatrix}
\]

- The instantaneous Landau energies

\[
\omega_n^2(t) = qB(t)(2n+1) + m^2 + k_z^2
\]

- The continuous transitions among Landau levels

\[
\langle m,t | \Omega(t) | n,t \rangle = \frac{\dot{B}(t)}{4B(t)} \left( \sqrt{n(n-1)} \delta_{m,n-2} - \sqrt{(n+1)(n+2)} \delta_{m,n+2} \right)
\]
**2nd Quantized Formulation**

- Hamiltonian from the field action [SPK, Ann Phys. 350 in press (‘14)],

\[
H_{\perp} = \int d^2 x_{\perp} \left[ \Pi^2_{\perp} + \Phi_{\perp} \left( \bar{p}^2_{\perp} + \omega^2_L(t)\bar{x}^2_{\perp} + m^2 - 2\omega_L(t)L_z \right) \Phi_{\perp} \right]
\]

- QED in the 2\textsuperscript{nd} quantized formulation
  - becomes the relativistic theory of time-dependent, coupled, oscillators due to the angular momentum in a time-dependent magnetic field with/without an electric field.
  - becomes the relativistic theory of time-dependent, decoupled oscillators in a pure electric field.

- QED in general EM fields is an interesting problem as analog of quantum cosmology or Hawking radiation.
Transitions from Quantum to Classical Cosmology
From QG to SQG to CG

**Quantum Gravity**

\[ \hat{G}_{\mu\nu} = 8\pi G \hat{T}_{\mu\nu} \]

Quantum cosmology (wave function of the universe)

\[ G = \frac{1}{m_p^2} \ll 1 \]

**Semiclassical Quantum Gravity**

\[ G^C_{\mu\nu} + G^Q_{\mu\nu}[G] = 8\pi G \left< \hat{T}_{\mu\nu} \right> \]

QFT in curved spacetime, Hawking radiation, pair production

\[ \hbar \ll 1 \]

**Classical Gravity**

\[ G^C_{\mu\nu} + G^Q_{\mu\nu}[G] = 8\pi G \left( T^C_{\mu\nu} + T^Q_{\mu\nu}[\hat{h}] \right) \]

Inflationary models
What is Quantum Cosmology?
Second Quantized Universe

• The wave function of universe in the superspace for FRW geometry and a minimal scalar

\[ ds^2 = -da^2 + a^2 d\phi^2 \]

• The Hamiltonian constraint and the Wheeler-DeWitt equation

\[
H(a, \phi) = -\left( \pi_a^2 + V_G(a) \right) + \frac{1}{a^2} \left( \pi_\phi^2 + 2a^6 V(\phi) \right) = 0
\]

\[
\left[-\nabla^2 - V_G(a) + 2a^4 V(\phi)\right]\Psi(a, \phi) = 0
\]

\[
\nabla^2 = -\frac{\partial^2}{\partial a^2} + \frac{1}{a^2} \frac{\partial^2}{\partial \phi^2}, \quad V_G(a) = ka^2 - 2\Lambda a^4
\]
Hartle-Hawking Wave Function

Euclidean solutions for a FRW coupled to a massive field scalar [Hawking, NPB 239 (‘84)]
Quantum Universes in Superspace
[SPK, Page, PRD 45 (‘92); SPK, PRD 46 (‘92)]

- The scalar field for single-field inflation model
  \[ V(\phi) = \lambda_{2p} \phi^{2p} / (2p) \]

- The eigenfunctions and the Symanzik scaling law
  \[ H_M(\phi, a)\Phi_n(\phi, a) = E_n(a)\Phi_n(\phi, a) \]
  \[ E_n(a) = \left( \lambda_{2p} a^6 / p \right)^{1/(p+1)} \varepsilon_n \]
  \[ \Phi_n(\phi, a) = \left( \lambda_{2p} a^6 / p \right)^{1/4(p+1)} F_n \left( \left( \lambda_{2p} a^6 / p \right)^{1/(p+1)} \phi \right) \]

- The coupling matrix among the energy eigenfunctions
  \[ \frac{\partial}{\partial a} \Phi(\phi, a) = \Omega(a)\Phi(\phi, a) \]
  \[ \Omega_{mn}(a) = \frac{3}{4(p + 1)a} (\varepsilon_m - \varepsilon_n) \int d\zeta F_m(\zeta)F_n(\zeta)\zeta^2 \]
Quantum Universes in Superspace

- The two-component wave function of the universe
  \[
  \begin{pmatrix}
  \Psi(a, \phi) \\
  \partial \Psi(a, \phi) / \partial a
  \end{pmatrix} =
  \begin{pmatrix}
  \Phi^T (\phi, a) & 0 \\
  0 & \Phi^T (\phi, a)
  \end{pmatrix}
  \]

- The off-diagonal components are the gravitational part equation only with \( V_G(a') - E / a'^2 \).

- The continuous transitions among energy eigenfunctions.
Oscillatory Behavior via Squeezing

• The wave function of the massive scalar field universe near the Big Bang singularity is a squeezed state of harmonic wave functions

\[
\left| \Psi(\alpha, \phi) \right\rangle = \left[ \Phi^{T}(\alpha, \phi) \right] e^{\Omega(\alpha - \alpha_0)} \left[ \left| \psi(\alpha_0) \right\rangle + (\alpha - \alpha_0) \frac{d}{d\alpha_0} \left| \psi(\alpha_0) \right\rangle \right]
\]

\[
\alpha = \ln(a), \quad \Omega = \frac{3}{4} \left( \hat{c}^2 - \hat{c}^+ \hat{c}^2 \right), \quad E = me^{3\alpha} \left( 2\hat{c}^+ \hat{c} + 1 \right) - ke^{4\alpha}
\]

• The oscillatory behavior via squeezing with an almost constant magnitude [SPK, PRD 46 (‘92), NPB Proc. Suppl 246 (‘14)] is similar to the chaotic behavior of a more complex, homogeneous, anisotropic (Mixmaster) universe without matter [Belinski, Khalatnikov, Lifshitz (‘70)].
Wave Packet for FRW with a Minimal Scalar

A closed universe \((k=1), m = 6, \) and \(n = 120\) (harmonic quantum number) [Fig. from Kiefer, PRD 38 (88)]
Third Quantization
# Third Quantization in 3+1 Dimensions

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<td>Field</td>
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<td>Vacuum</td>
<td>Void</td>
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Third Quantization

- The WDW equation from the third quantized Hamiltonian

\[ S = \int d\alpha d\phi \left[ -\left( \frac{\partial \Psi}{\partial \alpha} \right)^2 + \frac{1}{a^2} \left( \frac{\partial \Psi}{\partial \phi} \right)^2 + \left( 2a^4 V(\phi) - V_G(a) \right) \Psi^2 \right] \]

- A massless field is a sum of \( a \)-dependent oscillators [Banks, NPB 309 (‘88); McGuigan, PRD 38 (‘88); Giddings, Strominger, NPB 321 (‘89); Hosoya, Morikawa, PRD 39 (‘89); Abe, PRD 47 (‘93); SPK, Kim, Soh, NPB 406 (‘93); Horiguchi, 48 (‘93)] and is a tachyonic state for the closed universe [SPK, NPB Proc. Suppl. 246 (‘14)].

- The third quantization of a massive field is analogous to the second quantized charged KG in a time-dependent, homogeneous, magnetic field \( \vec{A}(t, \vec{r}) = \vec{B}(t) \times \vec{r} / 2 \).
Third Quantization

- Expand the wave function by the energy eigenfunctions of Hamiltonian, $\Psi(a, \phi) = \Phi^T(\phi, a) \cdot \tilde{\psi}(a)$, for the scalar field to obtain the third quantized Hamiltonian

$$H(a) = \frac{1}{2} \tilde{\pi}^T \cdot \tilde{\pi} - \tilde{\pi}^T \Omega(a) \tilde{\psi} + \frac{1}{2a^2} \tilde{\psi}^T E(a) \tilde{\psi}$$

where

$$\tilde{\pi} = \partial \tilde{\psi}(a) / \partial a + \Omega(a) \tilde{\psi}(a)$$

- The massive scalar quantum cosmology can be solved in the sense that the coupling matrix $\Omega$ and the energy-eigenvalue matrix $E$ are explicitly known.
Intense Lasers Simulation of Quantum Universe
WDW Eq vs KG Eq in B(t)

- Wheeler-DeWitt equation for FRW universe with a massive scalar field

\[
\begin{align*}
- \pi_a^2 & + V_G(a) + \frac{1}{a^2} \left( \pi_\phi^2 + a^6 m^2 \phi^2 \right) \Psi(a, \phi) = 0
\end{align*}
\]

- Transverse motion of a charged scalar in a time-dependent, homogeneous, magnetic field B(t)

\[
\begin{align*}
\frac{\partial^2}{\partial t^2} & + \left( \vec{p}_\perp + \frac{qB(t)}{2} \right)^2 \vec{x}_\perp - qB(t)L_z + m^2 + k_z^2 \Phi_\perp(t, \vec{x}_\perp) = 0
\end{align*}
\]
# Quantum Cosmology vs Scalar QED

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<td>Wave functions of universe</td>
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Conclusion

• **Scalar QED** is an analogue of quantum cosmology with a massive scalar field
  – Homogeneous, time-dependent magnetic field ↔ massive scalar
  – Homogeneous, time-dependent electric field ↔ gravitational potential

• **Quantum universe may be simulated by relativistic quantum motion of charged particles in a homogeneous, time-dependent magnetic and an electric fields.**
Why Massive Scalar Field Quantum Cosmology?
9-Year WMAP: Single-Field Inflation Models
[Astrophys. J. Suppl. 208 (‘13)]
Planck 2013 Results
[arXiv:1303.5082v2]