# Intense Lasers Simulation of Quantum Cosmology & Gravity

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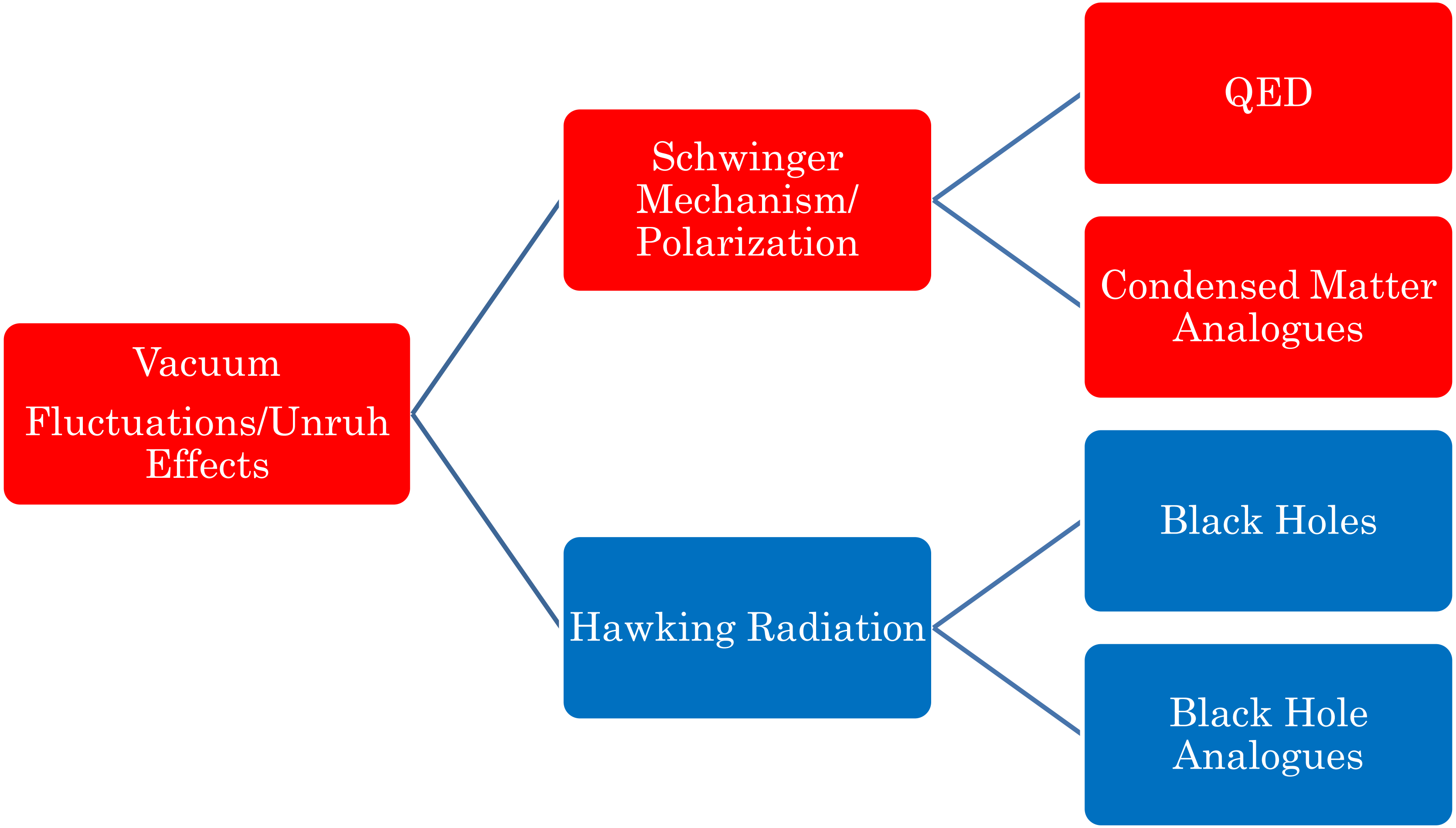
# Overview of Strong Field Physics

| QED | QFT in Curved Spacetime | Quantum Cosmology |
|---|---|---|
| (gauge) 4-vector potential | Curved spacetimes | Superspace of geometry and matters |
| Charged fields in Minkowski spacetimes | Neutral /charged fields in curved spacetimes | Wave functions of the universe in superspace |
| Dirac/Klein-Gordon equation | Klein-Gordon/Dirac/ graviton equation | Wheeler-DeWitt equation |
| Vacuum polarization<br>Schwinger mechanism | Hawking radiation<br>Cosmic radiation | Creation of universes<br>Structure of spacetime |

# Unified Picture for Pair Production

[SPK, JHEP11('07)]



# Weak QED vs Strong QED

| Weak QED | Strong QED |
| --- | --- |
| Pauli Hamiltonian | Second quantized field Hamitonian |
| Schrödinger equation | Functional Schrödinger equation |
| Paul traps in weak EM fields | Charged fields in strong (relativistic) EM fields |

# Simulating Quantum Effects in Cosmology via Ion Traps?

# Simulating Gibbons-Hawking Effect

[Menicucci, Olson, Milburn , New J. Phys. 12 ('10)]

- An ion trap experiment is proposed as the analogue of quantum effects in an expanding universe, such as Gibbons-Hawking effect, inflationary structure, etc.

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

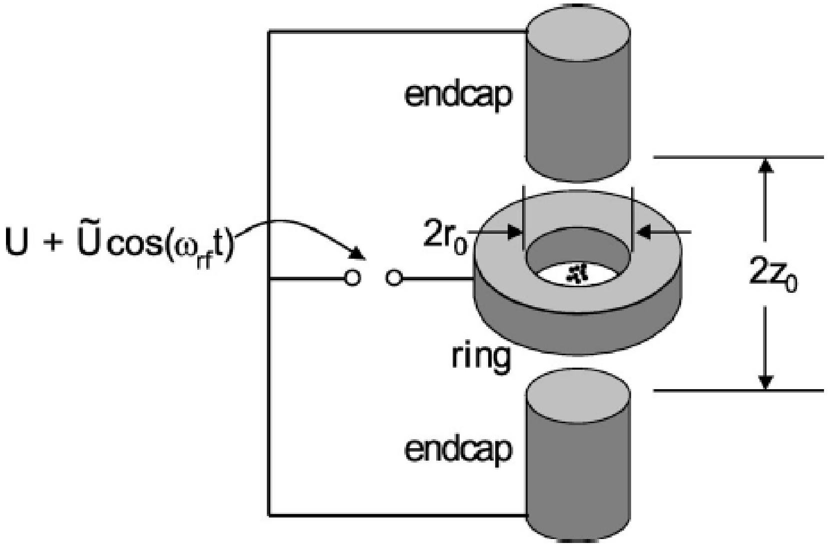
- An Unruh-DeWitt detector in a FRW universe

$$H_{\text{int}} = f(t)m(t)\phi(t,\vec{x})$$

- A homogeneous, isotropic field $\Phi$ has the Hamiltonian of time-dependent oscillators

$$H(t) = \frac{1}{2}\sum_k \frac{\pi_k^2}{a^{3/2}(t)} + a^{3/2}(t)\omega_k^2(t)\phi_k^2$$

# Paul Trap




Schematic drawing of the electrodes for a cylindrically symmetric 3D rf trap

$r_0 \cong \sqrt{2}z_0$ =100μm~1cm, Û=100~500V, U=0−50V, ω/2π = 100kHz ~100MHz [Leibfried et al, Rev. Mod. Phys. 75 ('03)]

- Nonrelativistic quantum motion of a charged particle in an rf trap with an electric potential (quantum theory)

$$\Phi = \frac{U}{2}\left(\alpha x^2 + \beta y^2 + \gamma z^2\right) + \frac{\tilde{U}}{2}\cos(\omega_{rf}t)\left(\alpha' x^2 + \beta' y^2 + \gamma' z^2\right)$$

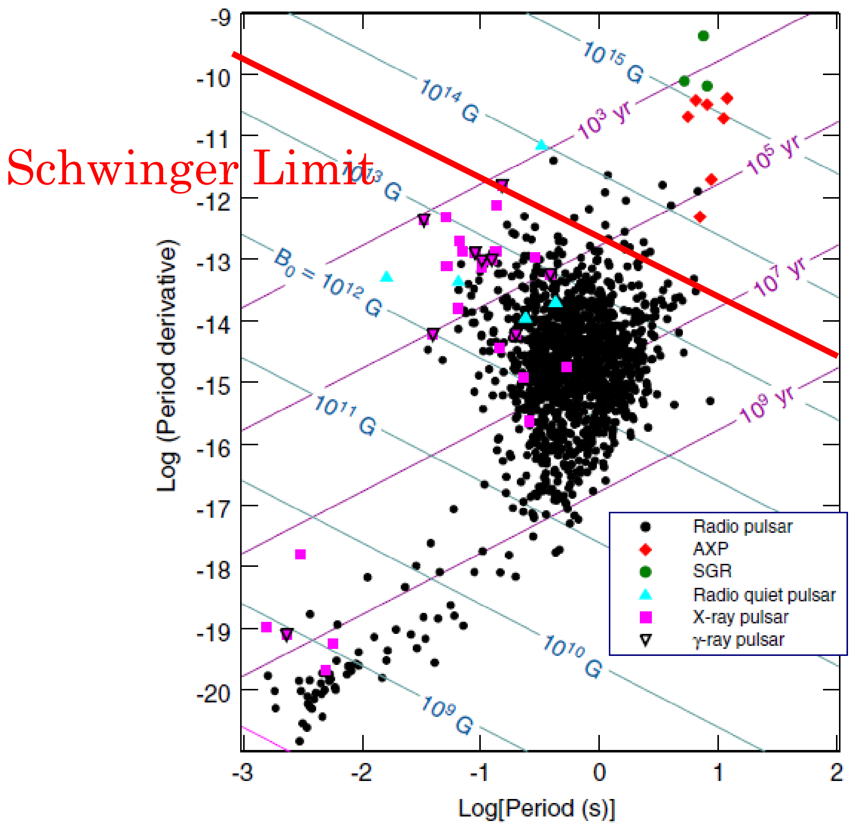
- Maxwell equation at each moment (dynamical trapping)

$$\Delta\Phi = 0 \Rightarrow \begin{cases} \alpha + \beta + \gamma = 0 \\ \alpha' + \beta' + \gamma' = 0 \end{cases}$$

# QED in Ultra-Strong, Time-Dependent Magnetic Fields

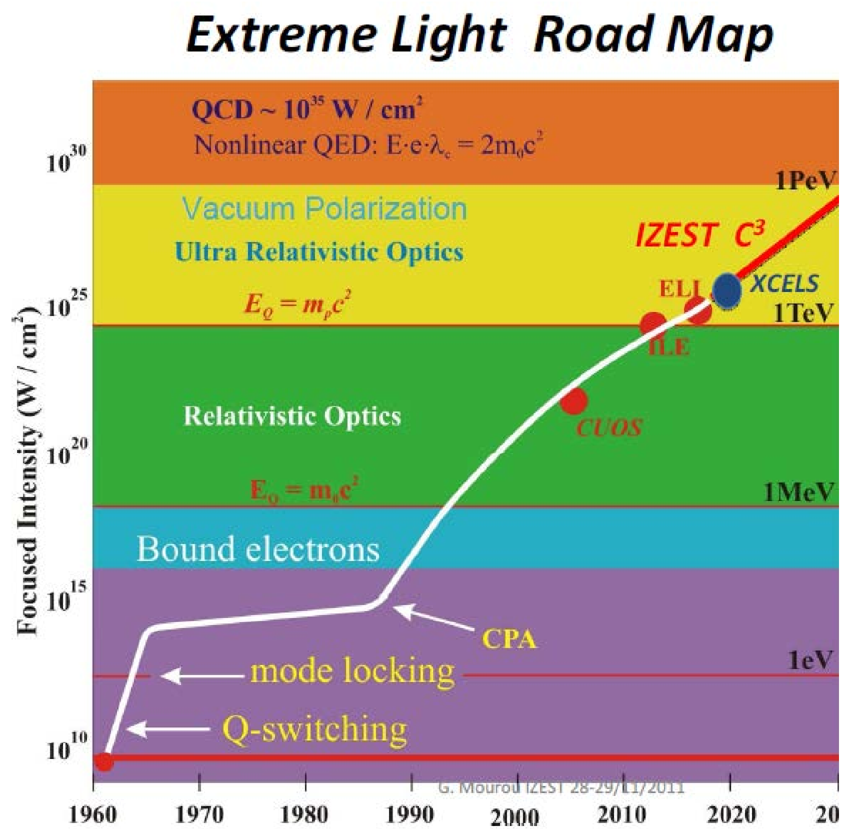
# Sources for Strong EM Fields

## Neutron Stars&Magnetars




[Harding and Lai, Rep. Prog. Phys. 69 ('06)]

## IZEST




[Homma, Habs, Mourou, Ruhl and Tajima, PTP Suppl. 193 ('12)]

# Scalar QED in B(t)

- A homogeneous, time-dependent, magnetic field B(t) and an induced electric field with the vector potential

$$\vec{A}(t, \vec{r}) = \frac{1}{2} \vec{B}(t) \times \vec{r}$$

- How to find the Landau levels and the QED effective action?
  - The first quantized theory: the Klein-Gordon equation
  - The second quantized theory: the Hamiltonian from field action
- The dynamically coupled Landau levels continuously make transitions among themselves, and the $1^{st}$ or $2^{nd}$ quantized theory is similar to a relativistic theory of time-dependent, coupled oscillators coupled to each other.

# $1^{st}$ Quantized Formulation

- The two-component, first order wave function of the KG eq expanded by Landau states [SPK, Ann Phys 344 ('14)]

$$\begin{pmatrix} \Psi(t,\vec{x}_\perp) \\ \partial\Psi(t,\vec{x}_\perp)/\partial t \end{pmatrix} = \begin{pmatrix} \vec{\Phi}^T(t,\vec{x}_\perp) & 0 \\ 0 & \vec{\Phi}^T(t,\vec{x}_\perp) \end{pmatrix} \times T\exp\left[\int \begin{pmatrix} \Omega(t') & I \\ -\omega^2(t') & \Omega(t') \end{pmatrix} dt'\right] \begin{pmatrix} \vec{\psi}(t_0) \\ d\vec{\psi}(t_0)/dt_0 \end{pmatrix}$$

- The instantaneous Landau energies

$$\omega_n^2(t) = qB(t)(2n+1) + m^2 + k_z^2$$

- The continuous transitions among Landau levels

$$\langle m,t|\Omega(t)|n,t\rangle = \frac{\dot{B}(t)}{4B(t)}\left(\sqrt{n(n-1)}\,\delta_{m,n-2} - \sqrt{(n+1)(n+2)}\,\delta_{m,n+2}\right)$$

# 2nd Quantized Formulation

- Hamiltonian from the field action [SPK, Ann Phys. 350 in press ('14)],

$$H_{\perp} = \int d^2 x_{\perp} \left[ \Pi_{\perp}^2 + \Phi_{\perp} \left( \vec{p}_{\perp}^{\,2} + \omega_L^2(t) \vec{x}_{\perp}^{\,2} + m^2 - 2\omega_L(t) L_z \right) \Phi_{\perp} \right]$$

- QED in the 2nd quantized formulation
  - becomes the relativistic theory of time-dependent, coupled, oscillators due to the angular momentum in a time-dependent magnetic field with/without an electric field.
  - becomes the relativistic theory of time-dependent, decoupled oscillators in a pure electric field.
- QED in general EM fields is an interesting problem as analog of quantum cosmology or Hawking radiation.

# Transitions from Quantum to Classical Cosmology

# From QG to SQG to CG

## Quantum Gravity

| | |
|---|---|
| $\hat{G}_{\mu\nu} = 8\pi G \hat{T}_{\mu\nu}$ | Quantum cosmology (wave function of the universe) |

$$G = 1/m_{\mathrm{P}}^2 << 1$$

## Semiclassical Quantum Gravity

| | |
|---|---|
| $G^{\mathrm{C}}_{\mu\nu} + G^{\mathrm{Q}}_{\mu\nu}[G] = 8\pi G \left\langle \hat{T}_{\mu\nu} \right\rangle$ | QFT in curved spacetime, Hawking radiation, pair production |

$$\hbar << 1$$

## Classical Gravity

| | |
|---|---|
| $G^{\mathrm{C}}_{\mu\nu} + G^{\mathrm{Q}}_{\mu\nu}[G] = 8\pi G \left( T^{\mathrm{C}}_{\mu\nu} + T^{\mathrm{Q}}_{\mu\nu}[\hbar] \right)$ | Inflationary models |

# What is Quantum Cosmology?

# Second Quantized Universe

- The wave function of universe in the superspace for FRW geometry and a minimal scalar

$$ds^2 = -da^2 + a^2 d\phi^2$$

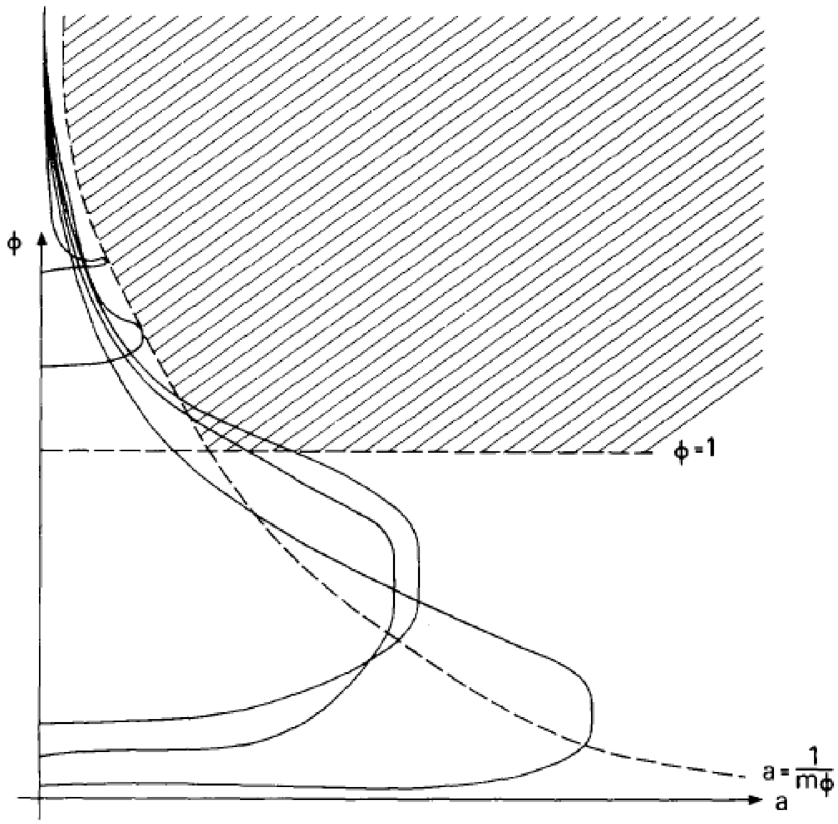
- The Hamiltonian constraint and the Wheeler-DeWitt equation

$$H(a,\phi) = \underbrace{-\left(\pi_a^2 + V_G(a)\right)}_{\text{gravity part } H_G} + \frac{1}{a^2}\underbrace{\left(\pi_\phi^2 + 2a^6 V(\phi)\right)}_{\text{scalar field part } H_M} = 0$$

$$\left[-\nabla^2 - V_G(a) + 2a^4 V(\phi)\right]\Psi(a,\phi) = 0$$

$$\nabla^2 = -\frac{\partial^2}{\partial a^2} + \frac{1}{a^2}\frac{\partial^2}{\partial \phi^2}, \quad V_G(a) = ka^2 - 2\Lambda a^4$$

# Hartle-Hawking Wave Function




Euclidean solutions for a FRW coupled to a massive field scalar
[Hawking, NPB 239 ('84)]

# Quantum Universes in Superspace

[SPK, Page, PRD 45 ('92); SPK, PRD 46 ('92)]

- The scalar field for single-field inflation model

$$V(\phi) = \lambda_{2p}\phi^{2p}/(2p)$$

- The eigenfunctions and the Symanzik scaling law

$$H_M(\phi,a)\Phi_n(\phi,a) = E_n(a)\Phi_n(\phi,a)$$

$$E_n(a) = \left(\lambda_{2p}a^6/p\right)^{1/(p+1)}\varepsilon_n$$

$$\Phi_n(\phi,a) = \left(\lambda_{2p}a^6/p\right)^{1/4(p+1)}F_n\left(\left(\lambda_{2p}a^6/p\right)^{1/(p+1)}\phi\right)$$

- The coupling matrix among the energy eigenfunctions

$$\frac{\partial}{\partial a}\vec{\Phi}(\phi,a) = \Omega(a)\vec{\Phi}(\phi,a)$$

$$\Omega_{mn}(a) = \left(3/4(p+1)a\right)\left(\varepsilon_m - \varepsilon_n\right)\int d\varsigma F_m(\varsigma)F_n(\varsigma)\varsigma^2$$

# Quantum Universes in Superspace

- The two-component wave function of the universe

$$\begin{pmatrix} \Psi(a,\phi) \\ \partial\Psi(a,\phi)/\partial a \end{pmatrix} = \begin{pmatrix} \vec{\Phi}^T(\phi,a) & 0 \\ 0 & \vec{\Phi}^T(\phi,a) \end{pmatrix}$$

$$\times T\exp\left[\int \begin{pmatrix} \Omega(a') & I \\ V_G(a') - E/a'^2 & \Omega(a') \end{pmatrix} da'\right] \begin{pmatrix} \vec{\psi}(a_0) \\ d\vec{\psi}(a_0)/da_0 \end{pmatrix}$$

- The off-diagonal components are the gravitational part equation only with $V_G(a) - E/a^2$.
- The continuous transitions among energy eigenfunctions.

# Oscillatory Behavior via Squeezing

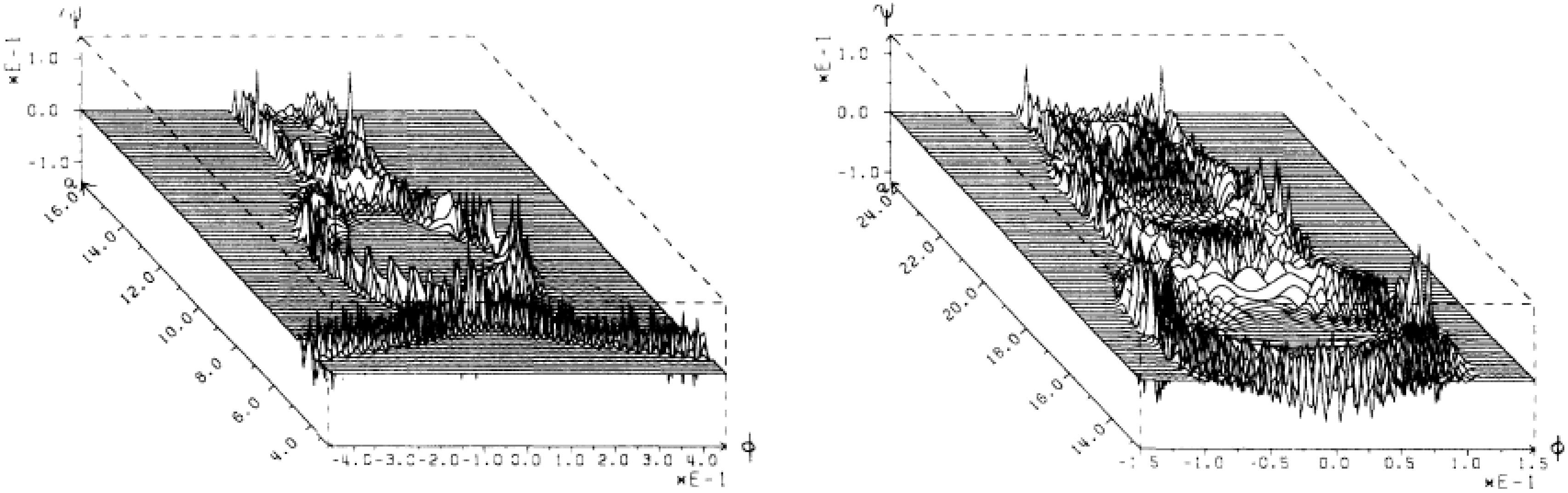
- The wave function of the massive scalar field universe near the Big Bang singularity is a squeezed state of harmonic wave functions

$$\left|\Psi(\alpha,\phi)\right\rangle = \underbrace{\left|\Phi^T(\alpha,\phi)\right\rangle}_{\text{harmonic functions}} \underbrace{e^{\Omega(\alpha-\alpha_0)}}_{\text{squeezing operator}} \underbrace{\left[\left|\vec{\psi}(\alpha_0)\right\rangle + (\alpha-\alpha_0)\frac{d\left|\vec{\psi}(\alpha_0)\right\rangle}{d\alpha_0}\right]}_{\text{initial Cauch data}}$$

$$\alpha = \ln(a),\ \ \Omega = \frac{3}{4}\left(\hat{c}^2 - \hat{c}^{+2}\right),\ \ E = me^{3\alpha}\left(2\hat{c}^{+}\hat{c}+1\right) - ke^{4\alpha}$$

- The oscillatory behavior via squeezing with an almost constant magnitude [SPK, PRD 46 ('92), NPB Proc. Suppl 246 ('14)] is similar to the chaotic behavior of a more complex, homogeneous, anisotropic (Mixmaster) universe without matter [Belinski, Khalatnikov, Lifshitz ('70)].

# Wave Packet for FRW with a Minimal Scalar




A closed universe (k=1), m = 6, and n =120 (harmonic quantum number)
[Fig. from Kiefer, PRD 38 ('88)]

# Third Quantization

# Third Quantization in 3+1 Dimensions

| Second quantization | Third quantization |
| --- | --- |
| Particle | Universe |
| Interaction Vertex | Topology Change |
| Field | Third Quantized Field |
| Spacetime | Superspace of Three Geometries |
| Free Laplacian | Wheeler-DeWitt Operator |
| Vacuum | Void |

[Strominger, "Baby Universes," in *Quantum Cosmology and Baby Universes* edited by S. Coleman et al (World Scientific, 1991)]

# Third Quantization

- The WDW equation from the third quantized Hamiltonian

$$S = \int da d\phi \left[ -\left(\frac{\partial \Psi}{\partial a}\right)^2 + \frac{1}{a^2}\left(\frac{\partial \Psi}{\partial \phi}\right)^2 + \left(2a^4 V(\phi) - V_G(a)\right)\Psi^2 \right]$$

- A massless field is a sum of a-dependent oscillators [Banks, NPB 309 (‘88); McGuigan, PRD 38 (’88); Giddings, Strominger, NPB 321 (‘89); Hosoya, Morikawa, PRD 39 (‘89); Abe, PRD 47 (‘93); SPK, Kim, Soh, NPB 406 (‘93); Horiguchi, 48 (‘93)] and is a tachyonic state for the closed universe [SPK, NPB Proc. Suppl. 246 (‘14)].
- The third quantization of a massive field is analogous to the second quantized charged KG in a time-dependent, homogeneous, magnetic field $(\vec{A}(t,\vec{r}) = \vec{B}(t)\times\vec{r}/2)$.

# Third Quantization

- Expand the wave function by the energy eigenfunctions of Hamiltonian, $\Psi(a,\phi) = \vec{\Phi}^T(\phi,a)\cdot\vec{\psi}(a)$, for the scalar field to obtain the third quantized Hamiltonian

$$H(a) = \frac{1}{2}\vec{\pi}^T\cdot\vec{\pi} - \underbrace{\vec{\pi}^T\Omega(a)\vec{\psi}}_{\text{very early universe: O(1/a)}} + \underbrace{\frac{1}{2a^2}\vec{\psi}^T E(a)\vec{\psi}}_{\text{late universe: O}(a^{(4-2p)/(p+1)})}$$

$$\vec{\pi} = \partial\vec{\psi}(a)/\partial a + \Omega(a)\vec{\psi}(a)$$

- The massive scalar quantum cosmology can be solved in the sense that the coupling matrix Ω and the energy-eigenvalue matrix E are explicitly known.

# Intense Lasers Simulation of Quantum Universe

# WDW Eq vs KG Eq in B(t)

- Wheeler-DeWitt equation for FRW universe with a massive scalar field

$$\left[ -\pi_a^2 + \underbrace{V_G(a)}_{\text{electric field E(t)}} + \frac{1}{a^2} \underbrace{\left( \pi_\phi^2 + a^6 m^2 \phi^2 \right)}_{\text{magnetic field B(t)}} \right] \Psi(a, \phi) = 0$$

- Transverse motion of a charged scalar in a time-dependent, homogeneous, magnetic field B(t)

$$\left[ \frac{\partial^2}{\partial t^2} + \left( \vec{p}_\perp^{\,2} + \left( \frac{qB(t)}{2} \right)^2 \vec{x}_\perp^{\,2} - qB(t) L_z \right) + m^2 + k_z^2 \right] \Phi_\perp(t, \vec{x}_\perp) = 0$$

# Quantum Cosmology vs Scalar QED

| Quantum Universe | Scalar QED |
| --- | --- |
| Universes | Charged scalars |
| WDW Equation | KG Equation |
| Superspace of spacetime & matter | Electromagnetic fields |
| Massive scalar in early universe ($V_g \approx 0$) | Scalar in homogeneous magnetic field |
| Coupling of field harmonic wave functions | Coupling of Landau levels |
| Wave functions of universe | Quantum motion of charge |

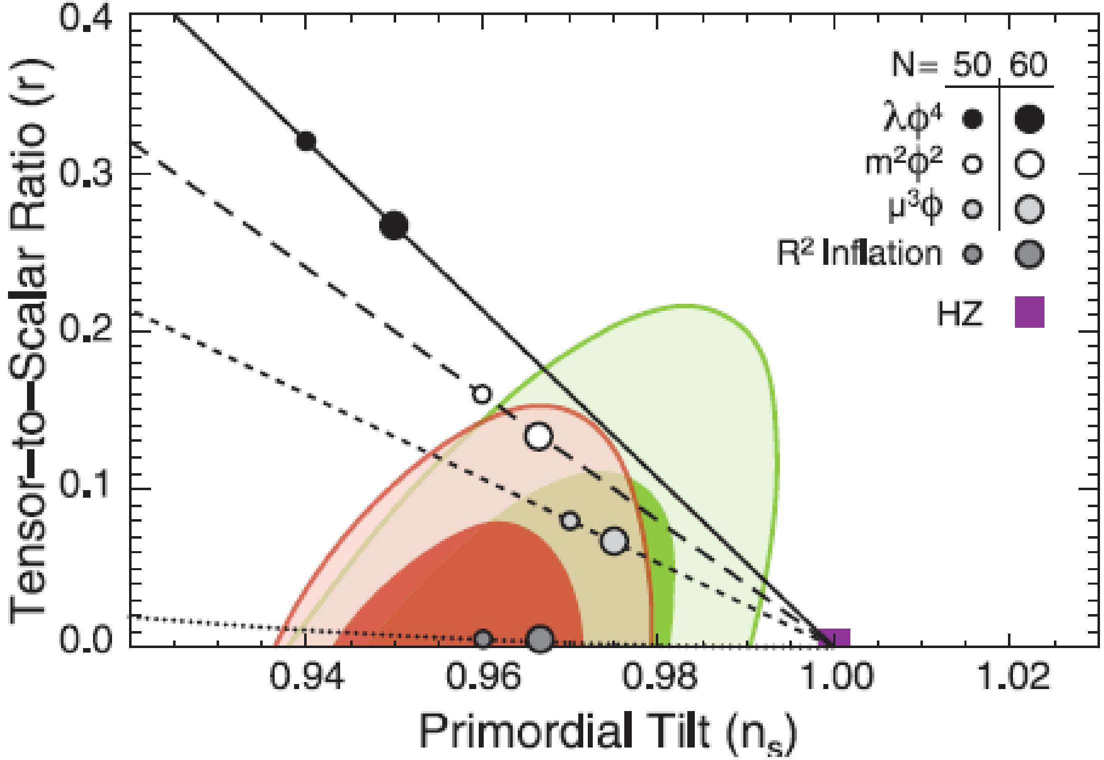
# Conclusion

- Scalar QED is an analogue of quantum cosmology with a massive scalar field
  - Homogeneous, time-dependent magnetic field $\leftrightarrow$ massive scalar
  - Homogeneous, time-dependent electric field $\leftrightarrow$ gravitational potential
- Quantum universe may be simulated by relativistic quantum motion of charged particles in a homogeneous, time-dependent magnetic and an electric fields.

# Why Massive Scalar Field Quantum Cosmology?

# 9-Year WMAP: Single-Field Inflation Models

[Astrophys. J. Suppl. 208 (‘13)]



# Planck 2013 Results

[arXiv:1303.5082v2]