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Enhanced nonperturbative pair conversion in small angle laser collisions

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Outline

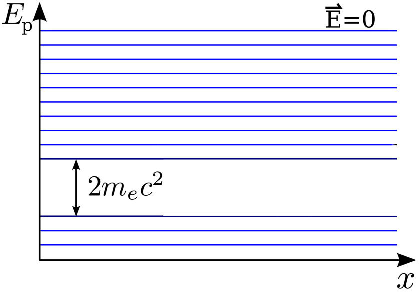
1. Nonperturbative pair production in QED
2. Toward the critical field with high intensity lasers
3. Single-photon pair-conversion: an achievable intermediate step
4. Boosting the pairs to high momentum
5. Conclusions

Instability of Electric Fields

In Dirac sea picture:
stabilize vacuum by occupying negative frequency states
$\rightarrow$ “holes” in the sea are positrons

zero electric field
energy levels are constant

separation between positive and negative frequencies is the same everywhere



Instability of Electric Fields [Heisenberg & Euler (1936), Schwinger (1951)]

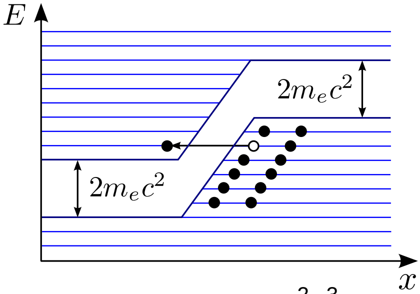
Electric fields do work on charges:
classical potential can supply pair rest energy $\Delta V = -\int E_x dx \geq 2m$

Compute effective action with classical field $F^{\mu\nu}_{\rm cl} = \partial^\mu A^\nu_{\rm cl} - \partial^\nu A^\mu_{\rm cl}$

$$\exp(iS_{\rm eff}[F_{\rm cl}]) \equiv \int \mathcal{D}\bar\psi \mathcal{D}\psi \exp\left(i\int d^4x\ \bar\psi(i\partial\!\!\!/ - eA\!\!\!/_{\rm cl} - m)\psi\right)$$

Imaginary part $\Rightarrow$ "vacuum decay"

$$|\langle 0_{\rm out}|0_{\rm in}\rangle|^2 = |\exp(iS_{\rm eff})|^2 = \exp(-L^3 t\, 2\,{\rm Im}\, V_{\rm eff})$$



Constant homogeneous field:

$$2{\rm Im}V_{\rm eff} = \frac{e^2|\vec E|^2}{4\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}e^{-n\pi m^2/|e\vec E|}$$

critical field $|\vec E_c| = \dfrac{m^2c^3}{e\hbar}$

Quasi-constant $\omega \ll m$: "spontaneous" (Schwinger) mechanism

O.K. That's cute, but why?

1. Calculable and now testable nonperturbative process in QFT,
QED is relatively "clean" case

2. Models low-energy nonperturbative aspects of QCD

► flux-tube model particle production [Casher et al PRD 20 (1979) 179]

► unstable state ⇒ positive vacuum energy $\langle F_{\rm cl}|T^\mu_\mu|F_{\rm cl}\rangle$

related to conformal anomaly [LL, Rafelski arXiv:0810.1323,0811.4467]

and condensate $-\langle\bar\psi(x)\psi(x)\rangle = \langle F_{\rm cl}|\hat{T}(\psi(x)\bar\psi(x))|F_{\rm cl}\rangle - \langle\Omega|\hat{T}(\psi(x)\bar\psi(x))|\Omega\rangle$

3. Entropy production: how does pure state (constant electric field, two hadrons) become thermalized plasma?

[QED studies: Cooper et al. hep-ph/9212206, Kluger et al hep-ph/9803372...]

4. Temperature of constant field, relation to entropy production

► Hawking-Unruh effect: "temperature of vacuum" for accelerated observer

► QED temperature in constant field

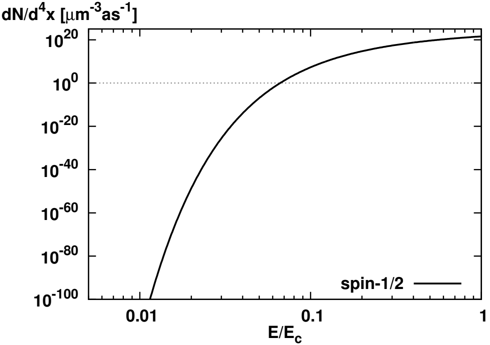
[Müller et al, Phys.Lett. A63 (1977) 181; Pauchy Hwang, arXiv:0906.3813; LL arXiv:1203.6148]

Bridging the intensity gap

$$\frac{\mathrm{I_{ELI}}}{\mathrm{I}_c} \simeq \frac{10^{25}\,\mathrm{W/cm^2}}{10^{29}\,\mathrm{W/cm^2}} = 10^{-4}$$

$$\Rightarrow \frac{|\vec{E}|_{\mathrm{ELI}}}{|\vec{E}_c|} = 10^{-2}$$

$$\exp(-\pi \tfrac{|\vec{E}_c|}{|\vec{E}|_{\mathrm{ELI}}}) \sim 10^{-137}$$

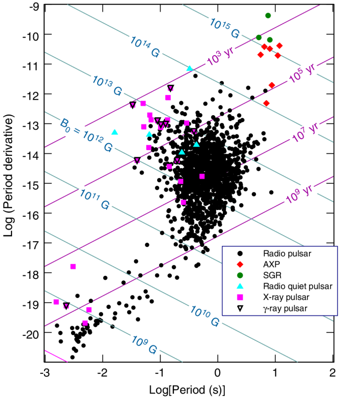


Some proposals to increase number pairs (and so discovery potential):

- ► Increase volume of strong field ($N \propto L^3 t$) [Bulanov et al arXiv:1003.2623]
- ► Frequency assist: superimpose weak, fast (classical) field [Schutzhold, Gies, Dunne, arXiv:0807.0754]
- ► Pair conversion of high energy photons ($\sim 4\times$ improvement) (disadvantage is scaling $\propto L$) [Dunne, Gies, Schutzhold, arXiv:0908.0948]

Pair conversion in astrophysics

Ultra strong magnetic fields claimed for pulsars ("magnetars")



QED magnetic field scale
$$|\vec{B}_c| = c^{-1}|\vec{E}| = 4 \times 10^9 \text{ T}$$

▶ magnetic fields do no work
($\vec{A} \to i\vec{D}$ only spatial momentum)
$\Rightarrow$ photon $\omega \geq 2m$ to pair convert

$\longleftarrow$[Harding, Lai, astro-ph/0606674]

Combined with thermal X-ray emission $\omega >$several keV
nonlinear and nonperturbative QED processes thought to be important

However, external field pair conversion not yet verified in lab

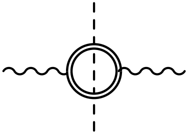
Pair conversion in external field

Key: momentum is not conserved in external field
($\vec{E}$ field does work on charges, $\vec{B}$ field curves trajectories)

Photon can absorb energy-momentum from field:
Polarization parallel to $\vec{E}$, allowed $k^\mu = (\omega, 0, 0, \omega) \longrightarrow p^\mu \simeq (2m, 0, 0, \omega)$

★ **Nonperturbative** process: coherently absorb $N \gg 1$ quanta from low frequency $\omega_{\mathrm{ext}} \ll m$ external field

All contained in photon polarization tensor:



Imaginary part exhibits "decay"→pairs (like V_{eff})

$$\mathtt{Im}\,\Pi^{\mu\nu}(k) = \mathtt{Im}\, -ie^2 \int d^4p\, \gamma^\mu G[A; k+p] \gamma^\nu G[A; k]$$

$G[A; p] \equiv$Green's function for with-field fermions

Nonperturbative à la Schwinger mechanism

$$\mathtt{Im}\Pi_{\parallel} = \frac{\alpha}{4\pi}\int_{-\infty-i\epsilon}^{+\infty-i\epsilon} ds\, s^{-1} \int_{-1}^{1} d\nu\, \frac{e^{-i\Phi s}(|\vec{E}|s)(|\vec{B}|s)}{\sinh(|\vec{E}|s)\sin(|\vec{B}|s)} \chi^2 \Big(N_0(|\vec{E}|,|\vec{B}|,s) - N_T(|\vec{E}|,|\vec{B}|,s)\Big)$$

$$\Phi(s) = m^2 + \chi^2\big(N_\Phi(i|\vec{B}|s) - N_\Phi(|\vec{E}|s)\big)$$

Low frequency Limit

$$\lim_{|k^\mu|\to 0} e^{-i\Phi s} \to e^{-m^2\pi s/|e\vec{E}|}$$

m^2 dominates,
gives Schwinger-like exponent

Integral well-approximated by saddlepoint (semi-classical)

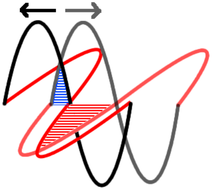
High frequency $\omega \sim m$

Invariant $\chi^2 = \dfrac{(k^\mu F_\mu^\nu)^2}{\vec{E}^2 + \vec{B}^2}$

$\chi^2 \sim m^2$, large contribution
$\Rightarrow$ enhances exponent, pair yield

$\chi^2 > 4m^2$, must do exact calculation, saddlepoint breaks down

Pair conversion, a small step



Head-on, linear-polarized pulses
$\rightarrow$ standing wave alternating field

electric$\leftrightarrow$magnetic: $\vec{E}_{\rm tot} = \vec{E}_1 + \vec{E}_2 \leftrightarrow \vec{B}_{\rm tot}$

$N_{\rm pairs} = n_\gamma(1 - \exp(-2\kappa L))$

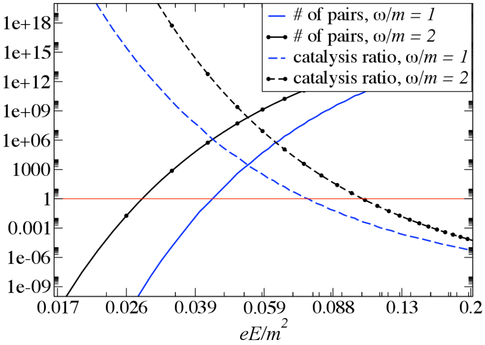
$\kappa = -\frac{1}{\omega}\Im\Pi(|\vec{E}_{\rm tot}|)$

Intensity $10^{25} - 10^{26}$ W/cm^2
$\rightarrow |\vec{E}|/|\vec{E}_c| \sim 0.01 - 0.1$

$L = 1\mu$m diffraction limit

$n_\gamma = 10^{10}$ photons
(near density limit)

86400 laser shots



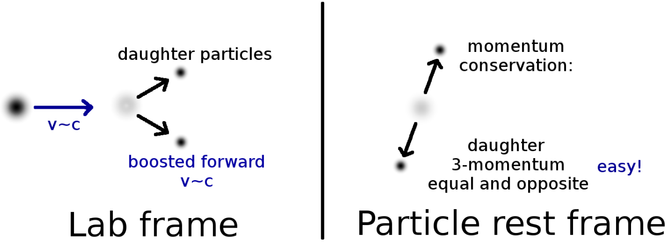
[Dunne, Gies, Schutzhold, arXiv:0908.0948]

Nonperturbative Pairs from Laser pulses

Using two pulses to setup standing wave gives up great advantage of laser fields! ▶ Pulses are Traveling field configuration

To calculate pair production,
Apply same procedure as for particle/hadron collisions:

- Decay rate, number of particles are invariant



- Motion and spectrum of decay products observer dependent

  $\Rightarrow$ Compute in rest frame of decaying field

  ...then transform to lab

Rest Frame and Mass of Fields

Rest frame defined by zero space-like momentum $\vec{P} \equiv 0$

From QED energy-momentum tensor, the 4-momentum density is

$$P^0 = u_\mu T^{\mu 0} = \frac{\varepsilon}{2}(\vec{E}^2 + \vec{B}^2) + \frac{1}{4} T^\mu_\mu \qquad \vec{P} = u_\mu T^{\mu i} = \varepsilon(\vec{E} \times \vec{B}) := \vec{S}$$

$T^\mu_\mu \equiv$ energy-momentum trace $\quad \varepsilon - 1 \equiv -\frac{\partial V_{\text{eff}}}{\partial \mathcal{S}}$ important if $|\vec{E}| \gtrsim |\vec{E}_c|$ from violation of superposition

Mass density $= \sqrt{P^\mu P_\mu} = \frac{1}{2}\left(a^2 + b^2\right) + \mathcal{O}(\alpha|\vec{E}|^2/|\vec{E}_c|^2)$

$$\mathcal{S} \equiv \frac{1}{2}(B^2 - E^2) \qquad \mathcal{P} \equiv -E \cdot B$$

$$a^2 = \sqrt{\mathcal{S}^2 + \mathcal{P}^2} - \mathcal{S} \qquad b^2 = \sqrt{\mathcal{S}^2 + \mathcal{P}^2} + \mathcal{S}$$

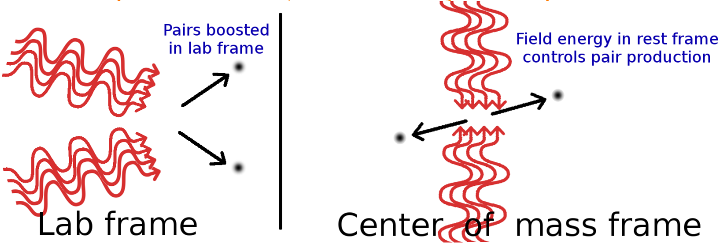
... reduce in rest frame as

$a \to |\vec{E}'|$ ‘electric’ invariant $\quad b \to |\vec{B}'|$ ‘magnetic’ invariant

Field Rest Frame and Pair production

► By momentum conservation, pairs produced in rest frame of field (vanishing 3-momentum $\vec{P}$)

[Landau & Lifshitz, Classical Theory of Fields]



Field energy available for pair production =

Invariant mass of classical field $M_{\text{field}} = \int d^3x \sqrt{P^\mu P_\mu}$

Invariant Materialization time:
(Schwinger mechanism)

$$\tau \equiv M_{\text{field}} \left(\frac{d \langle \mathcal{E}_{\text{mat}} \rangle}{dt} \right)^{-1} \simeq \tau_0 e^{\pi |\vec{E}_c| / |\vec{E}'|}$$

$$\tau_0 = 8.7\ 10^{-19}\ \text{s}$$

[arXiv:0808.0874]

Rapidity of Field Rest Frame

In quasi-constant fields with momentum density, we define quasi-local rest frame, requires coarse-graining at length scale ℓ such that $\lambda_{\rm ext} \gg \ell \gg \lambda_{\rm pair} \simeq m/|e\vec{E}|$ the "pair-formation length"

Quasi-local **Field Rapidity**:
rest frame moving relative to lab frame velocity β determined by solving $\vec{E}' \times \vec{B}' = 0$ (rest frame fields)

$$\gamma \equiv \cosh y_{\rm F} := \frac{1}{\sqrt{1-\beta^2}}$$

When creating strong fields with laser waves:

- Focusing breaks translational invariance of plane wave

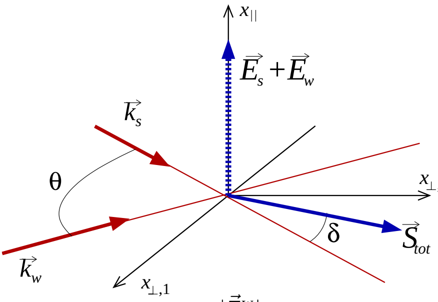
  $\Rightarrow$ converging wave vectors means momentum $|\vec{P}| < \langle N_\gamma \rangle |\vec{k}|$

  $\Rightarrow$ $(P_0)^2 - \vec{P}^2 > 0$ but small

  $\Rightarrow$ laser pulse rest frame at high rapidity

Finite field energy density divided into
field mass density ($\rightarrow$pair yield) and field momentum ($\rightarrow$pair rapidity)

Boosting Schwinger mechanism pairs



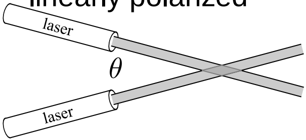
$$r = \frac{|\vec{E}^{\mathrm{w}}|}{|\vec{E}^{\mathrm{s}}|}$$

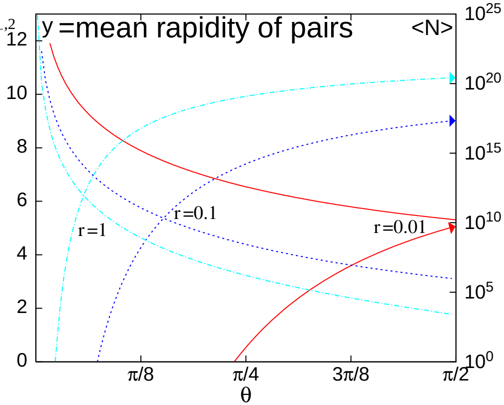
boosted in direction $\vec{S}_{\mathrm{tot}}$

Tunneling process ⇒ Gaussian distribution in transverse momentum

[arXiv:1102.5773]

linearly polarized

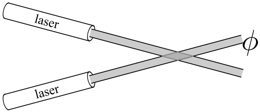




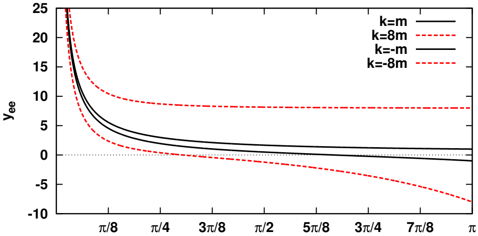
Boosted pair conversion

Similar Setup...
Inject photon co-propagating:
- ▶ $\vec{k} \| \vec{P}$ maximizes boost
- ▶ polarization $\epsilon^\mu \| \vec{E}$ maximizes yield

linearly polarized



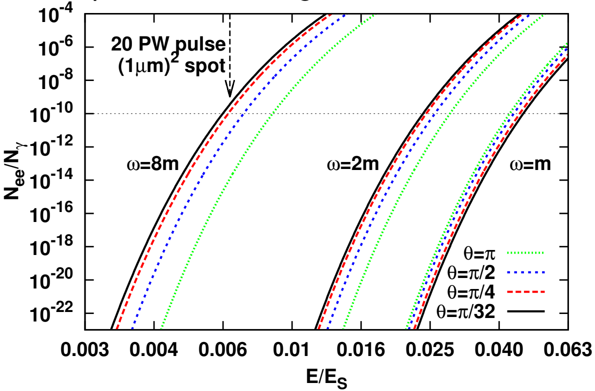
momentum conservation
+quasi-constant field
$\Rightarrow$ photon 3-momentum =
3-momentum of pair C.o.M.



$$y_{ee} = y_F + \sinh^{-1}\left(\sqrt{\frac{1-\beta^2}{1+\beta^2}}\frac{|\vec{k}|}{m}\right)$$

Closer to experimental reach

Further reduces required field strength



can use above threshold photons $|\vec{k}| > 2m$:

- ► perturbative process $\gamma\gamma \rightarrow e\bar{e}$ pairs have small momentum
- ► nonperturbative pairs identified by ϕ-dependent momentum boost
- ► Production probability increases with photon energy, must balance against backgrounds

Not the final word

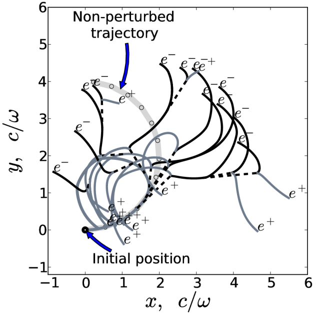
After a pair "forms", it is in strong field environment...

(semiclassical) radiation length: $\xi_{ee} = \alpha^{-1} p_0 \chi^{-2/3} \simeq 10^{-4} \gamma_{ee}^{2/3}\ \mu\mathrm{m}$

[Ritus, Nikishov, Narozhny, JETP **19**, 529 (1964)... (1965)]

single pair accelerates
→ radiates hard photon
→ photon pair-converts
→ more pairs...
...QED "cascades"

[Elkina et al, arXiv:1010.4528]



Could mitigate cascades if $\xi_{ee} > \lambda_{\mathrm{laser}}$, so pairs exit strong field before radiating hard photon, requires $\phi \sim 10^{-3}$

Cascades are natural "amplifier", but how to prove the initial pair was nonperturbative?

Conclusions

- ▶ Non-perturbative pair production is a frontier of QED
$\rightarrow$ becoming accessible through high intensity laser experiment

- ▶ Single-photon pair conversion is nonperturbative
$\rightarrow$ important to astrophysics
$\rightarrow$ nonperturbative structure of spontaneous (Schwinger) process

- ▶ Boost pair conversion in two ways:
$\rightarrow$ pairs inherit momentum from external field, so create strong field with laser pulses converging at small angle $\phi \ll \pi \rightarrow$ production probability also enhanced for small ϕ

- ▶ Creating pairs at large ϕ-dependent momentum helps identify nonperturbative origin

- ▶ Still need improved theoretical analysis to understand radiation and particle dynamics to interpret outcome

Extra Slides

Pair Creation

Potential connects positive and negative frequency states at $t \to -\infty$ and $t \to +\infty$

From explicit construction of with-field Green's function, we can calculate relative probability state p, σ occupied [Nikishov 1970]

$$w_{p,\sigma} = \sum_{p',\sigma'} |\langle \psi^{\text{out}}_{p,\sigma} | \psi^{\text{in}}_{p',\sigma'} \rangle|^2 = \frac{1}{e^{-\beta(m^2+p_\perp^2)} - 1}, \quad \beta \equiv \frac{\pi}{eE}$$

Normalized probability $P[\text{state } p, \sigma \text{ occupied}] = c_{p,\sigma} w_{p,\sigma}$

P[vacuum persists] = $P[0\,\text{pairs}] = \prod_{p,\sigma}(1 - c_{p,\sigma} w_{p,\sigma})$

$$\rightsquigarrow \exp\left(-L^3 t \sum_{p,\sigma} \ln(1 - e^{-\beta(m^2+p_\perp^2)}) \right) \stackrel{\checkmark}{=} e^{-2\,\mathrm{Im}\, S_{\text{eff}}}$$

For magnetic fields, $p_\perp^2 \to 2jeB,\ j = 0, 1, 2...$ (Landau levels)

[Kim & Page, hep-th/0301132,hep-th/0005078; LL & Rafelski, arXiv:0808.0874]

Background field methods: computing V_{eff}

Veff is gauge invariant implicit sum over eigenfunctions can be performed by alternate methods (just knowing eigenvalues or solving some equation of motion)

Define V_{eff} so that variation produces vacuum expectation of current

$$\frac{\delta V_{\mathrm{eff}}}{\delta A_\mu(x)} \equiv \langle j^\mu(x)\rangle = ie\,\mathrm{tr}\,\gamma^\mu G(x,x)$$

Functional integral $\rightarrow V_{\mathrm{eff}} = -i\,\mathrm{tr}\,\ln G^{-1} = -i\int_0^\infty \frac{du}{u}\mathrm{tr}\,e^{-iG^{-1}u}$

- ▶ Trace of operator $e^{-iG^{-1}u}$ = sum of eigenvalues
- ▶ Integral representation provides regularization: separates zero-field and charge renormalization infinities
- ▶ Complex integral defines imaginary part $m \rightarrow m + i\epsilon$

$$V_{\mathrm{eff}} = \frac{-1}{8\pi^2}\int_0^\infty \frac{du}{u^3}e^{-m^2u}\left\{\frac{eau}{\tan eau}\frac{ebu}{\tanh ebu} - 1 - \frac{u^2}{3}e^2(b^2-a^2)\right\}$$

Computational methods developed by Schwinger (1951), Casher (1979), Dunne (1998)

Imaginary part of polarization tensor

$\mathtt{Im}\,\Pi(k)$ arises from poles$\rightarrow$ resum by Cauchy theorem

$$\Im\Pi_{\|} = \frac{\alpha}{4\pi}\int_{-\infty-i\epsilon}^{+\infty-i\epsilon} ds s^{-1}\int_{-1}^{1} d\nu \frac{e^{-i\Phi s}(as)(bs)}{\sinh as \sin bs}\chi^2(N_0(a,b,s) - N_T(a,b,s))$$

$$\Phi = m^2 - \frac{v_\perp^2}{2}\frac{\cos\nu bs - \cos bs}{bs\sin bs} + \frac{v_\|^2}{2}\frac{\cosh\nu as - \cos as}{as\sinh as}$$

$$N_0(x,y,s) = \cosh\nu ys\cos\nu xs - \sinh\nu ys\sin\nu xs\coth ys\cot xs$$

$$N_T(x,y,s) = 2\cos xs\frac{\cosh ys - \cosh\nu ys}{\sinh^2 ys}$$

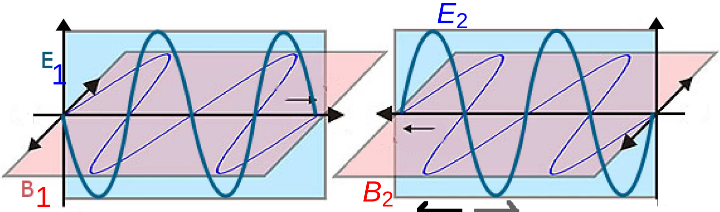
For real part, the contour is from origin $s = 0$ to $+\infty - i\epsilon$

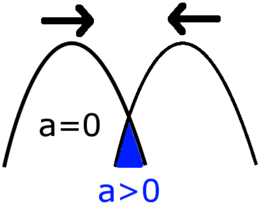
Saddlepoint satisfies: $\dfrac{1}{1+\cosh s_*} + \dfrac{2m^2}{\tilde{v}^2} = \dfrac{1}{1+\cos bs_*/a}$

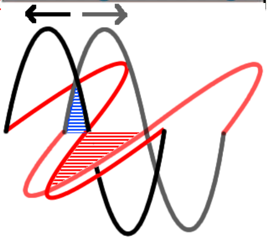
Unstable Fields from Colliding Lasers

For each plane wave $\vec{E}^2 - \vec{B}^2 = 0 = E \cdot B$,

- ► Total field $\vec{E}_1 + \vec{E}_2$ unstable only where rest frame $|\vec{E}'| = a > 0$



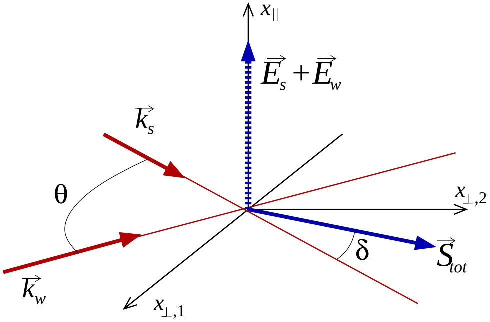




aligned, Linear Polarized:
where $\vec{E}_1 \cdot \vec{E}_2 > 0$, then $a > 0$

total invariant field strength
alternates between b and a

Spectrum: Transforming from Moving Rest Frame



Tunneling probability:

$$\Gamma = \exp\left(-\frac{\pi}{ea}(m^2 + p_\perp^2)\right)$$

Adiabatic: $p_{\|} = 0$

$p_{\|}, \vec{p}_\perp$ relative to $\vec{E}$ in moving rest frame

Gaussian distributions in $\vec{p}_\perp$ shifted by boost:

★ High rapidity along direction of $\vec{S}_{\text{tot}}$: $\langle y_p \rangle = y_S$

★ Narrow distribution in directions tranverse to $\vec{S}_{\text{tot}}$ with $\langle p_{\text{T}} \rangle = 0$

Signature of Spontaneously Produced Pairs

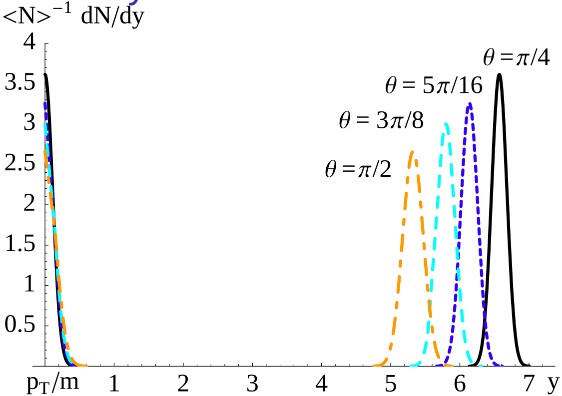
$$\langle y_p \rangle = y_S$$

$$\sinh y_S = \frac{\vec{E} \times \vec{B}}{\sqrt{P^\mu P_\mu}}$$

(controlled by θ, r)

$$\langle p_T \rangle = 0$$

[for $r = 0.01$, at right]



▶ High energy particle source achieved by choice of electromagnetic field configuration: electrons/positrons inherit high rapidity of field rest frame → No acceleration necessary!

▶ Requires large energy density to achieve high rapidity and high multiplicity in same experiment — recall $P^0 = \sqrt{P^2} \cosh y_S$

Predicting radiation in the strong field regime

Radiation is a quantum phenomenon (Einstein, 1905), so how do we get to classical radiation fields?

Quantum

Radiation only occurs in quanta (photons)

► Calculate cross-sections and rates for photon emission/absorption

► Sum (low energy, collinear) photons into classical field

Classical

(Good for low frequency, coherent fields)

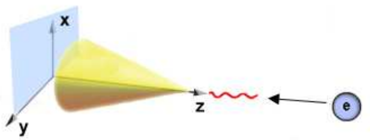
► Solve coupled Maxwell+Lorentz force equations of motion for fields and particle(s)

► Analytically include radiation momentum change perturbatively in $dp^\mu/d\tau$ (breaks down in strong fields)

However, coherent, high intensity (high occupation number) fields not available in standard QFT framework

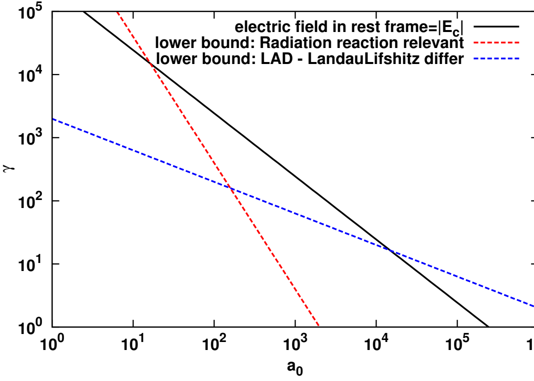
► how do we match classical and quantum regimes of radiation?

Classical to quantum regimes



electron-laser head-on

Landau-Lifshitz model:
expand LAD equation for
small acceleration
$dp^\mu/d\tau \ll m^2$



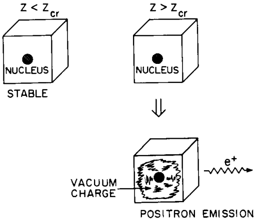
[LL, Hadad, Rafelski, arXiv:1005.3980,1204.4923]

Need consistent theory treatment bridging classical and quantum regimes – in between there is both quantum radiation (typically high energy) and high intensity, low frequency radiation (to be resummed into classical field)

Pair Formation in the Rest Frame: Example of Nuclei

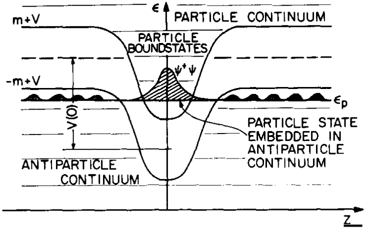
Localized nuclear potentials
(super)critical for $Z > 173$

[Rafelski, Muller, Greiner '73]



Charge appears in rest frame of potential = rest frame of nucleus

- e^- bound in potential, screens nuclear charge
- e^+ escapes to infinity, preserves global charge neutrality



[figures: Rafelski, Fulcher, Klein '78]

When supercritical potential falls apart, electron also unbound
... completing formation of electron-positron *pair*