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# How Does Nondurable Consumption Respond To Transitory Income Shocks? Reconciling Natural Experiments and Structural Estimations

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# How Does Nondurable Consumption Respond To Transitory Income Shocks? Reconciling Natural Experiments and Structural Estimations

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**Abstract**

Results from natural experiments show that nondurable consumption responds strongly and significantly to transitory variations in income, such as tax rebates or tax refunds, while in estimations of life-cycle models, transitory shocks do not induce significant changes in consumption expenditures. First, I show that life-cycle estimation methods are biased in a standard framework with uncertainty because they implicitly neglect the contribution of precautionary behavior. This biases the results, as the precautionary terms induce a correlation with past shocks that undermines the estimation strategy. Second, I develop a robust estimator that allows for the presence of a correlation with past shocks, and obtain that the elasticity of consumption growth to transitory shocks is statistically significant, in accordance with the literature on tax repayments. The estimation results imply that 12% of a transitory gain in net income is consumed within the following year.

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# 1 Introduction

How do consumers respond to income shocks? Answering this question has implications for a variety of macroeconomic questions, including the economy's response to fiscal shocks, the behavior of equilibrium asset prices, or the relation between income and consumption inequalities. Yet, the response of consumption is difficult to measure because income shocks are not usually observed. In survey data, the changes in income reported reflect a mix of transitory and permanent shocks, past and contemporaneous, while, to make helpful predictions, one needs to identify separately their respective impact on consumption. Two solutions can be found in the literature. First, some papers exploit natural experiments of income shocks, such as tax refunds or fiscal stimulus, in which the income variation and its persistence are known: this technique requires little assumptions but limits the analysis to the particular shocks whose persistence can be observed, mostly transitory. Second, a more structural approach pioneered by Blundell, Pistaferri and Preston (2008) uses restrictions from life-cycle models to identify in survey data the elasticity of consumption to transitory and permanent shocks. This makes it possible to measure the impact of more general shocks, but is dependent on the validity of the modelling choices.

However, the two methods yield opposing results. In the literature on the impact of tax refunds and fiscal stimulus, transitory income shocks are found to have a strong and statistically significant impact on consumption, even though tax shifts are possibly anticipated and their impact blurred by expectations of future tax increases. This remains true when considering particular age and revenue subcategories of the population, though the magnitude of the effect varies across these subcategories[1]. In constrast, when consumption elasticities to income shocks are estimated from survey data with structural identifing restrictions, the elasticity to transitory innovations is not statistically different from zero, and both the point-estimate and the standard-error are small, suggesting that it is not due to a lack of precision in the measure. The same result holds when breaking down the population into different age and revenue subcategories[2].

In this paper, I make three contributions to the literature: (i) I prove that the approximation made in structural estimation methods that log-consumption growth is a random walk is equivalent to assuming away precautionary effects, defined as the impact of uncertainty on consumers' decisions (ii) I show that, when precautionary effects are taken into account, they raise the theoretical elasticity to transitory shocks, and they introduce a correlation between log-consumption and past shocks that

[1]Souleles (1999) finds that between 9% of a tax refund is consumed within the quarter following receipt. During recessions, the effect is even larger and between 12% and 30% of the tax rebates implemented during the fiscal stimulus episodes of 2001 and 2008 are consumed within the quarter following receipt (Parker, Johnson and Souleles (2006), Parker, Souleles, Johnson and McClelland (2013), Misra and Surico (2014)). Kan, Peng and Wang (2016) rely on the 2009 Taiwan Shopping Voucher Program, a measure implemented as part of the government's fiscal stimulus package. They estimate the marginal propensity to consume to be 25%.

[2]See Blundell, Pistaferri and Preston (2008), Heathcote, Storesletten and Violante (2014), Blundell, Pistaferri and Saporta-Eksten (2016). Remarkably, Blundell, Pistaferri and Saporta-Eksten (2016) even find that, when considering wage shocks instead of earnings shocks, transitory wage innovations are associated with a statistically significant *negative* response of consumption. They interpret the finding as resulting from a non-separability between consumption and hours worked.

undermines the identification strategy of the structural approach. This correlation holds for every consumer and at every period, thus biases the estimation for every subcategory of the population. (iii) I develop a generalized structural estimator that is robust to the presence of a correlation between log-consumption growth and past shocks. I obtain that the consumption elasticity to transitory shocks becomes statistically signficant, both on average and within different age and revenue subcategories of the population; the magnitudes are consistent with estimates from natural experiments.

My framework is the standard life-cycle model of the consumption literature, identical to that of Blundell, Pistaferri and Preston (2008)'s seminal paper, with finite-lived consumers facing uncertainty because their income is subject to transitory and permanent shocks[3]. They have isoelastic preferences and maximize their intertemporal utility subject to a budget constraint. Isoelastic preferences imply that the period utility function is increasing and concave, and that marginal utility is convex, which is to say that consumers are prudent. By assumption, they cannot default on their debt, which generates a natural credit constraint: they never borrow more than the worst possible amount they expect to earn in the future.

How does precautionary behavior arise in this framework? When future consumption is uncertain and subject to mean-zero shocks, as consumers have a convex marginal utility, the impact of bad shocks is disproportionately stronger than that of good shocks, and consumers are willing to move resources from the certain present to the uncertain future as marginal utility from consumption could be very large in case a bad shock realizes. Uncertainty also tightens the natural borrowing constraint, which prevents consumers from borrowing more than the maximum they could repay in any state of the world, because the presence of shocks lowers the worst possible income realizations. When binding, this constraint induces similar consumption transfers from the present to the future as it forces consumers who want to spend now some of their future expected resources to delay consumption and save it instead. As the constraint never binds in the absence of uncertainty, these transfers correspond to precautionary saving too.

First, I show that assuming log-consumption growth follows a random walk with an exogenous drift is equivalent to eliminating the contribution of precautionary behavior to the elasticities of consumption to income shocks. Intuitively, such an expression implies that the trend of the expected log-consumption path is exogenous, while precautionary behavior applies precisely to the value of this trend: it raises the slope of the expected log-consumption path by moving resources to the future. Therefore, the assumption of a random walk mechanically implies that the contribution of precautionary saving to log-consumption growth does not respond to shocks, thus that it does not affect the elasticity of consumption to shocks.

Second, I prove that, in this model, precautionary saving generates a correlation between log-

[3]This is consistent with microeconomic data on earnings: Attanasio and Davis (1996) show that consumers' revenue is not perfectly insured; the permanent-transitory structure of shocks is found to fit well with the observed dynamics of microeconomic of earnings (MaCurdy (1982), Abowd and Card (1989)).

consumption growth and the realizations of past shocks. As precautionary behavior implies a transfer of consumption from the present to the future, it raises log-consumption growth. The magnitude of precautionary saving depends on consumers' stock of net assets and on the level of their permanent income, which are determined by the realizations of past income shocks. In particular, I establish that, everything else equal, having experienced positive transitory shocks in the past raises consumers' current net assets without modifying future income risk, thus reduces strictly the need for precautionary saving (and conversely having experienced bad transitory shocks increases strictly the need for precautionary saving). There is therefore a direct correlation between past shocks and log-consumption growth, through the precautionary motive.

This implies that structural estimation methods are biased in this set-up: when taking precautionary behavior into account, it is not possible to use instrumental variables that also depend on the realization of past shocks to identify the response of log-consumption growth to contemporaneous shocks. Otherwise, the instruments are not exogenous and the fluctuations in log-consumption growth caused by its correlation with past shocks are mistakenly attributed to the current shock.

My third contribution is to correct for the bias by allowing for a correlation between log-consumption growth and past shocks in the estimation method. My remedy to the lack of instrument exogeneity is to replace log-consumption growth by its innovation. When excluding the expected component of log-consumption growth, which is the part that correlates with past shocks, I eliminate the bias. This substitution restores the exogeneity of the instruments whenever it fails because of a correlation with past shocks, but is innocuous in the absence of such a correlation. In effect, the innovation of log-consumption growth is the only part that is affected by current shocks, so none of their impact is being dropped out. The cost of this solution is that I need to make an assumption on the information set of consumers, to disentangle between the part of log-consumption growth that is expected and the part that is an innovation. For this reason, I check carefully the robustness of my results to variations in the variables included in the information set.

This technique is also robust to other features that have been suspected to cause a bias in the estimation, in particular borrowing constraints (Kaplan and Violante (2010). I prove that extensions such as the presence of borrowing constraints other than the natural borrowin constraint, the presence of a range of more or less risky assets yielding different interest rates, or the presence of partly illiquid assets generating wealthy-hand to mouth behavior (Kaplan and Violante (2014)), induce the same correlation between consumption growth and past shocks as precautionary behavior, thus similar problems in the identification of consumption elasticities. My generalized estimator solves these issues together with those caused by precautionary behavior. If it is likely that these additional effects are at play in the consumption decisions of population categories that are likely to be liquidity-constrained or to have little liquidity because their wealth is stored in illiquid form, precautionary behavior seems to be a better explanation for bias observed within categories with liquidities.

I implement this corrected estimator into data from the Panel Study of Income Dynamics (PSID) between 1979 and 1992 combined with imputed consumption data from the Consumer Expenditure Survey (CEX) over the same period. It is the same dataset as used in the paper of Blundell, Pistaferri and Preston (2008). The reason I do not include later periods is that some questions regarding household characteristics, in particular financial income, change after 1992. After correction, the estimated elasticity to transitory income shocks raises from 0.05 to 0.10 and become significant. The elasticity to permanent income shocks shifts from 0.66 to 0.61. The elasticity to transitory shocks on the wage rate of the male earner increases from 0.04 to 0.09 and becomes significant. The elasticity to permanent shocks on the wage rate of the male moves from 0.16 to 0.18. These findings suggest that there is indeed a correlation between log-consumption growth and past shocks that biases the traditional estimation method. The results are robust to variations in the assumption regarding the information set available to consumers and in the persistence of transitory shocks to income or to the wage rate.

**Related Literature**

This paper belongs the literature that investigates the robustness and the applicability of estimators exploiting longitudinal data to identify income shocks and the variations in consumption they produce. The most prominent method is that of Blundell, Pistaferri and Preston (2008), because it makes it possible to disentangle between shcoks of different persistence while first techniques would measure the response of consumption to total income changes (Altonji and Siow (1987), Krueger and Perri (2005), (2008)). Kaplan and Violante (2010) examine a number of biases that could be altering the predictions of the Blundell, Pistaferri and Preston method. In particular, they make the point that their identification strategy requires that log-consumption growth be independent from past income shocks, but they do not check analytically whether this condition is met in the model of Blundell, Pistaferri and Preston (I show this condition does not hold because of precautionary behavior). They also note that advanced information, mean-reverting shocks and heterogeneous income profiles could shift the estimator of Blundell, Pistaferri and Preston away from the true value of the parameters. To measure the quantitative impact of these possible biases, they implement the estimator on simulated data, and obtain that the differences with the true values are very small, except in the presence of strong borrowing constraints. The estimator of Blundell, Pistaferri and Preston may get close to the true values of the parameters when applied to these simulations and yet be substantially biased when implemented in survey data if the underlying income process differs from the one used in simulations, for example if income innovations are not normally distributed but skewed, which could bolster the precautionary motive. Blundell, Low and Preston (2013) extend the method of Blundell, Pistaferri and Preston (2008) to more general specifications of income dynamics. Blundell, Pistaferri and Saporta-Eksten (2016) incorporate endogenous labor supply and within-family insurance to account for the smaller than predicted response of consumption to permanent shocks by increasing the degree of consumption insurance available to consumers. Heathcote, Storesletten and Violante (2014) explore the same consumption insurance mechanisms, but with a model that delivers closed-form solutions for consumption and hours worked and at the cost of a few additional assumptions about the economic environment. In particular, individuals smooth shocks within the family, but households are hand-to-mouth. The bias I describe does not apply but their model is less general.

Section 2 exposes the baseline model and derives an approximation for log-consumption growth that does not ignore the precautionary correlation between log-consumption growth and past shocks. Section 3 presents an identification strategy that is robust to interactions between log-consumption growth and past shocks, and shows that ignoring them leads to a bias in the estimation of the consumption response to income shocks. Section 4 details the implementation of the estimators in panel data and the results: after correction, the response of log-consumption to transitory shocks on income or on wage rates is large and significant. The values are more in line with results from the literature on tax rebates. The overall estimation bias caused by ignoring the history dependence of log-consumption growth is significantly different from zero. Section 5 concludes.

# 2 Model

The framework I consider is standard and encompasses the model underlying the estimation of Blundell, Pistaferri and Preston (2008). Finite-lived consumers maximize intertemporally their net utility from consumption and work, subject to a budget constraint. They face a stochastic wage rate, shifted by permanent and transitory shocks at each period. Markets are incomplete and consumers only have a risk-free asset available to save and borrow. To clarify the presentation, I neglect the presence of the natural borrowing constraint, which prevents consumers from borrowing more than the maximum they could repay in any state of the world. The impact of this constraint on the response of consumption is presented in Appendix B, together with the case of exogenous borrowing constraints.

## 2.1 Income Process

The log-wage rate of household $i$ at period $t$ is modeled as a permanent-transitory process, which is to say the sum of a permanent component $p_t$ that follows a random walk, of a transitory component $\varepsilon_t$ that follows an MA(q) process, and of a term capturing the influence of individual characteristics $z_{i,t}$ (possibly time-varying):

$$ln(w_{i,t}) = p_{i,t} + \varepsilon_{i,t} + \kappa_t z_{i,t} \tag{2.1}$$

$$with \begin{cases} p_{i,t} & = p_{i,t-1} + \eta_{i,t} \\ \varepsilon_{i,t} & = \mu_{i,t} + \theta_1 \mu_{i,t-1} + ... + \theta_q \mu_{i,t-q} \end{cases}$$

The shocks $\eta_{i,t}$ and $\mu_{i,t}$ are i.i.d. across households and across periods. I don't impose a log-normal distribution; in particular, the shocks can be drawn from a mixture of log-normals to match with recent evidence of skewed log-income distribution (Busch, Domeij, Guvenen and Madera (2015)). The variable $z_{i,t}$ is a vector of income characteristics, observable and known by consumers at time t. I allow their impact $\kappa_t$ to vary over time and across cohorts. This specification encompasses models with fixed effects if some of the $z$ variables are not time-varying ($z_{i,t} = z_i$), and allows for a common time/age trend if one the variable is the year or the consumers' age ($z_{i,t} = t$).

In the reminder, I drop the consumers' index $i$. This specification implies that for $0 \leq s \leq T-t$:

$$\Delta(ln(w_t) - \kappa_t z_t) = \eta_t + \Delta\varepsilon_t \tag{2.2}$$

The number of hours worked, denoted $h_t$, is a linear combination of a fixed, exogenous, number of hours $\bar{h}$ and a number of hours chosen by the worker $\hat{h}_t$: $h_t = (1-\alpha)\bar{h} + \alpha\hat{h}_t$. A model with exogenous labor supply corresponds to the particular case where $\alpha = 0$. In that situation, income is proportional to the wage rate and can be represented as a stochastic endowment. In general, the period income of the consumers, denoted $y_t$ is the product of the number of hours they worked, their wage rate and the tax rate $\tau$ that captures both taxes and transfers: $y_t = w_t h_t \tau_t$.

## 2.2 Consumers' Problem

Consumers' intertemporal optimization problem is as follows:

$$\max_{c_t,\dots c_T} \quad E_t\left[\sum_{s=0}^{T-t} \beta^{t+s} e^{\delta_t z_t} \left(u(c_{t+s}) - g(h_{t+s})\right)\right] \tag{2.3}$$

$$s.t. \quad \sum_{s=0}^{T-t} \frac{c_{t+s}}{(1+r)^s} = (1+r)a_t + \sum_{s=0}^{T-t} \frac{y_{t+s}}{(1+r)^s} \tag{2.4}$$

Time is discrete and indexed by t = 0, 1, .... T. Consumers with discount factor $\beta < 1$ and time-separable preferences derive utility from streams of consumption $\{c_s\}_{s=t}^{T}$, and, independently, disutility from hours worked $\{h_s\}_{s=t}^{T}$. Period utility from consumption, $u(c)$ is in the Constant Relative Risk Aversion (CRRA) class of functions or their counterparts with shifted origins . Its functional form is $u(c) = \frac{c^{1-\rho} - cst}{1-\rho}$ and it is defined over $]0, +\infty[$. This implies that marginal utility is decreasing (consumers are risk-averse) and convex (consumers are prudent). Period disutility from hours worked, $g(h)$, is of the form $g(h) = \rho\frac{h^{1+\sigma}}{1+\sigma 2}$. Net utility can be influenced by a vector of individual characteristics $z_t$ whose impact is measured by coefficients $\delta_t$. They may overlap with the characteristics that shift income. Consumers face the stochastic wage rate, $w_t$, bounded below by $\underline{w_t} = 0$. There are no state-contingent securities to insure idiosyncratic wage risk, only a risk-free asset, $a_t$, which yields a constant gross interest rate $(1+r)$. Consumers can save and borrow but cannot default on their debt: $a_T \geq 0$. Together with the period budget constraints $a_{t+1} = (1+r)a_t + y_t - c_t$, this terminal wealth condition yields the intertemporal budget constraint (2.4). I present the more general case with borrowing constraints in Appendix B.

## 2.3 Consumption Allocation

Appendix A details formally the steps of the reasoning developped here. The equilibrium condition of the consumers' problem, known as the Euler equation, states that optimizing consumers equalize their expected marginal utility over time—weighted by $R_{t,t+k} = (\beta(1+r))^k e^{\delta_{t+k} z_{t+k} - \delta_t z_t}$ to capture the impact of the interest rate, the discount factor and changes in demographics :

$$u'(c_t) = E_t[u'(c_{t+1})]R_{t,t+1}$$

I denote $\varphi_t$ the equivalent precautionary premium for consumption at $t+1$. It is the counterpart of the equivalent risk premium, applied to marginal utility instead of utility: $\varphi_t$ is such that $E_t\left[u'(c_{t+1})\right] = u'(E_t[c_{t+1}] - \varphi_t)$. Under perfect foresight, defined as a situation in which income is certain and equal to its expected value, $c_{t+1} = E_t[c_{t+1}]$ and the premium $\varphi_t$ is zero. In the presence of uncertainty, however, Jensen's inequality implies that the premium $\varphi_t$ is strictly positive for prudent consumers, because their marginal utility is strictly convex.[4]. I combine this expression with the Euler equation and apply $u'(c)^{-1} = c^{-1/\rho}$ to each side:

$$c_t = (E_t[c_{t+1}] - \varphi_t)R_{t,t+1}^{-1/\rho}$$

The presence of $\varphi_t$ indicates that prudent consumers choose, not only to equalize current consumption to future expected consumption (weighted by $R_{t,t+1}^{-1/\rho}$), but to transfer additional ressources $\varphi_t R_{t,t+1}^{-1/\rho}$ from the current period to the next because of uncertainty. In effect, prudent consumers facing risk anticipate that, if an unfortunate event occurs in the future, their utility from consuming additional units of goods is going to be very high, while a good shock will not lower their marginal utility as much: they are willing to move consumption from the current, certain, period to future, uncertain, periods to have more resources in case a negative shock hits.

Iterating forward, I obtain that $c_t = E_t[c_{t+s}]R_{t,t+s}^{-1/\rho} - \sum_{k=1}^{s} E_t[\varphi_{t+k-1}]R_{t,t+k}^{-1/\rho}$, for any $0 < k < T-t$: because of uncertainty, consumers are willing to transfer an amount $\sum_{k=1}^{s} E_t[\varphi_{t+k-1}]R_{t,t+k}^{-1/\rho}$ from $t$ to each future period $t+s$. Combining these expressions with the intertemporal budget constraint (2.4), consumption writes as a constant share of consumers' expected ressources, net of the sum of these expected precautionary transfers:

$$c_t = \frac{1}{l_{t,0}}\left( \underbrace{(1+r)a_t + \sum_{s=0}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s}}_{\text{total expected ressources: } W_t} - \underbrace{\sum_{k=1}^{T-t} l_{t,k} \frac{E_t[\varphi_{t+k-1}]}{(1+r)^k}}_{\text{total expected precautionary saving: } PS_t} \right)$$

The behavior of consumers facing risk can be interpreted as a permanent-income style decision, but applied to an uncertainty-adjusted measure of their total expected ressources instead of their raw total expected resources. Intuitively, in a risky environement, prudent consumers act as if they were poorer than they actually are: they mentally discard a part of their expected ressources that they reserve for the uncertain future. The term $\frac{1}{l_{t,0}} = (\sum_{s=0}^{T-t} \frac{R_{t,t+s}^{1/\rho}}{(1+r)^s})^{-1}$ measures the share of their total uncertainty-adjusted

[4] When marginal utility $u'(c)$ is strictly convex, Jensen's inequality states that:

$$E_t\left[u'(c_{t+1})\right] > u'(E_t[c_{t+1}]) \Leftrightarrow u'(E_t[c_{t+1}] - \varphi_t) > u'(E_t[c_{t+1}]) \Leftrightarrow E_t[c_{t+1}] - \varphi_t < E_t[c_{t+1}] \Leftrightarrow 0 < \varphi_t$$

resources that consumers want to allocate to consumption at period $t$. It is exogenous and identical to the share obtained under perfect foresight. [5]. More generally, the term $\frac{1}{l_{t,k}} = (\sum_{s=0}^{T-t-k} \frac{R_{t+k,t+k+s}^{1/\rho}}{(1+r)^s})^{-1}$ is the share of ressources that consumers want to allocate to consumption between the beginning of period $t$ and the beginning of period $t+k+1$. The sum, $\frac{1}{l_{t,0}} \sum_{k=1}^{T-t} l_{t,k} \frac{E_t[\varphi_{t+k-1}]}{(1+r)^k}$, corresponds to precautionary saving at period $t$, as it coincides with the difference between what would be consumed under perfect foresight (a share $\frac{1}{l_{t,0}}$ of total expected ressources) and what is actually consumed[6]. It is the net present value sum of the expected precautionary transfers at period $t$ to all future periods $t+s$.

## 2.4 Transmission of Income Shocks to Consumption

I take the difference in (weighted) consumption between two consecutive periods:

$$c_{t+1}R_{t+1}^{-\frac{1}{\rho}} - c_t = \underbrace{\varphi_t R_{t+1}^{-\frac{1}{\rho}}}_{\text{precautionary trend}} + \underbrace{\frac{1}{l_{t+1,0}} \sum_{s=0}^{T-t-1} \frac{(E_{t+1}-E_t)[y_{t+1+s}]}{(1+r)^s}}_{\text{revision of future resources}} - \underbrace{\sum_{k=1}^{T-t-1} \frac{l_{t+1+k,0}}{l_{t+1,0}} \frac{(E_{t+1}-E_t)[\varphi_{t+k}]}{(1+r)^k}}_{\text{revision of future precautionary saving}}$$

This expression clarifies the structure of the innovation to future consumption. Unexpected shifts in consumption between two periods are driven, first, by news about future income, second, by the revisions of future precautionary saving they imply. Also, precautionary behavior generates an expected transfer of consumption from period $c_t$ to $c_{t+1}$, which raises expected consumption growth by an amount $\varphi_t R_{t+1}^{-\frac{1}{\rho}}$.

I take the logarithm of the above expression and I expand around the point where $(\varepsilon_{t+1}, \eta_{t+1}) = (0,0)$. I denote with a star the variables taken at this point. Log-consumption growth can be expressed as:

$$\Delta ln(c_{t+1}) = \underbrace{\frac{1}{\rho} ln(\beta(1+r))}_{\text{impatience}} + \underbrace{\frac{1}{\rho}\Delta(\delta_{t+1}z_{t+1})}_{\text{demographics}} + ln(1 + \underbrace{\frac{\varphi_t}{R_{t,t+1}^{1/\rho} c_t} + \frac{c_{t+1}^* - E_t\left[c_{t+1}\right]}{R_{t,t+1}^{1/\rho} c_t}}_{(1)}) \tag{2.5}$$
$$+ \varepsilon_{t+1} \frac{\left(\frac{dW_{t+1}}{d\varepsilon_{t+1}}\right)^* - \overbrace{\left(\frac{dPS_{t+1}}{d\varepsilon_{t+1}}\right)^*}^{(3)}}{W_{t+1}^* \underbrace{-PS_{t+1}^*}_{(2)}} + \eta_{t+1} \frac{\left(\frac{dW_{t+1}}{d\eta_{t+1}}\right)^* - \overbrace{\left(\frac{dPS_{t+1}}{d\eta_{t+1}}\right)^*}^{(3)}}{W_{t+1}^* \underbrace{-PS_{t+1}^*}_{(2)}} + o(\varepsilon_{t+1}, \eta_{t+1})$$

where $W_{t+1}$ denotes total expected resources at $t+1$ and $PS_{t+1}$ total expected precautionary saving at $t+1$. Let me first analyze this expression in the situation of perfect foresight, in which case the pre-

[5] When consumers are neither patient nor impatient ($\beta = \frac{1}{1+r}$) and individual characteristics are constant ($z_t = z$), $l_{t,0}$ tends toward one as T approaches infinity.

[6] Households consume less than they would under perfect foresight *at a given level of net assets* $a_t$. However, consumers that have been facing income risk for several periods might be consuming more than if they had had perfect foresight during these periods, because in the latter case they accumulate more assets than in the former ($(1+r)a_t > (1+r)a_t^{perfect foresight}$) and this additional wealth may offset the decrease in consumption caused by precautionary saving.

cautionary premium is zero so that the terms designated with numbers drop. The expected component of log-consumption growth is equal to $\frac{1}{\rho}ln(\beta(1+r))+\frac{1}{\rho}\Delta(\delta_{t+1}z_{t+1})$. It is exogenous and fully determined by the parameters of the model. The response of log-consumption to a transitory shock $\varepsilon_{t+1}$, which is a measure of the elasticity of consumption to transitory income, is simply the percentage change in future resources caused by a transitory shock, taken at the point $(\varepsilon_{t+1},\eta_{t+1})=(0,0)$. In the case of fixed hours worked, when income is exogenous, the percentage change in resources is equal to the ratio of expected future income over total expected resources, because a transitory gain of one unit increases total resources by $1\times y^*_{t+1}$ at the approximation point. This value is indeed very small, and under perfect foresight the impact of transitory shock on consumption should be practically imperceptible. Similarly, the response of log-consumption to a permanent shock is the percentage change in total resources it generates. In the case of fixed hours worked, this percentage change is equal to the ratio of total expected future income over total expected resources.

Precautionary behavior has three effects on the value of log-consumption growth, indicated with (1), (2) and (3) in equation (2.5). First, because of precautionary transfers between period $t$ and $t+1$, expected log-consumption in larger: there is an additional, strictly positive term, denoted with (1), in the expression of expected log-consumption growth[7]. As the strength of the precautionary motive depends on the level of the state variables $a_t$ and $p_t$, this term introduces some depency between consumers' history and their log-consumption growth. Second, prudent consumers spend a share of their uncertainty-adjusted resources, instead of a share of their total resources, and an income shock that raises resources without modifying precautionary saving generates a larger percentage change in uncertainty-adjusted resources than in total resources. In equation (2.5) this shows in the fact that the percentage change is computed with respect to adjusted resources, net of precautionary saving (denoted (2)). Third, wage shocks do not only cause changes in expected income, but also in expected precautionary saving. The sign of this effect depends on the persistence of the shock considered. Commault (2016) shows that, in the same model, a transitory shock reduces the need for precautionary saving while a permanent shock raises it (intuitively, because shocks are mutliplicative, a larger permanent income means that the magnitude of future shocks is increased). As a result, revisions in future precautionary saving amplifies the response to transitory shocks but mitigate the response to permanent shocks. Note that the comparison with the case of perfect foresight is made at a given level of net assets. Over time risk stimulates the accumulation of assets which would modify these conclusions.

As a result of these effects, the response of log-consumption growth to a shock does not have to coincide with the percentage change in resources caused by the shock. In the case of transitory shocks, both considering uncertainty-adjusted resources instead of total resources (2) and revising future expected precautionary transfers (3) raise the response above its perfect foresight value. In the case of permanent shocks, the impact of precautionary behavior on the response of log-consumption

[7]It is strictly positive because consumption is concave in transitory and permanent income or wage shocks (Carroll and Kimball (1996), Commault (2016)). Therefore, Jensen's inequality implies that $c_{t+1}(E_t[\varepsilon_{t+1}],E_t[\eta_{t+1}]) > E_t[c_{t+1}(\varepsilon_{t+1},\eta_{t+1})]$.

is indetermined because effects (2) and (3) have opposite directions. In all cases, the fact that the estimated elasticity of consumption to shocks differ from the percentage change in total resources caused by a shock cannot be used a test of whether the standard model with sef-insurance holds, because the standard model *does not* predict such a value for the elasticity. Incidentally, regarding permanent shocks, estimates of consumption elasticity to permanent shocks that are below the level predicted by a model with self-insurance do not necessarily reflect evidence of alternative insurance mechanisms. Finally, note that the comparison with perfect foresight is made at a given level of net assets. Over time risk stimulates the accumulation of assets which would modify these conclusions.

## 2.5 Comparison with Existing Approximations

How come that approximations derived from the same model an expression for log-consumption growth i) that is independent from past shocks and ii) in which the response of consumption to the shocks is the same as the percentage of total resources obtained under perfect foresight?

This is because the authors impose that expected log-consumption growth does not respond to past shocks. Precisely, in Blundell, Low and Preston (2013), to obtain that the difference between $t-1$ and $t$ of the Taylor expansion of the log-total consumption (equation (30)) coincides with the innovation to log-consumption growth and a term that behaves as the variance of this innovation, one needs to assume that precautionary component of log-consumption growth is unaffected by shocks[8]. Blundell, Pistaferri and Preston (2008) use the approximation derived in Blundell, Low and Preston (2013), so they rely on this hypothesis too. In Blundell, Pistaferri and Saporta-Eksten (2016), this assumption is explicitely made[9]. With the notations presented here, it amounts to assuming that the term denoted (1) in equation (2.5) is independent from past shocks.

Yet, this assumption implies the elimination of all the other contributions of precautionary behavior to log-consumption growth. Formally, the consequence of the hypothesis that $ln\left(1+\frac{\varphi_t}{R_{t,t+1}^{1/\rho}c_t}+\frac{c_{t+1}^*-E_t[c_{t+1}]}{R_{t,t+1}^{1/\rho}c_t}\right)$ does not respond to past shocks is that $\frac{\varphi_t}{R_{t+1}^{1/\rho}c_t}=k_t$, with $k_t$ an exogenous constant, and therefore that $\varphi_t=k_t(R_{t+1}^{1/\rho}c_t)$. The Euler equation is $(1+k_t)c_tR_{t,t+1}^{1/\rho}=E_t[c_{t+1}]$. As a consequence, the optimal level of consumption is a share of total expected ressources, as in the perfect foresight case but with share coefficients different from $\frac{1}{l_{t,0}}$. The approximation of log-consumption growth around small shocks is

[8] In effect, when taking the difference of equation (30) between $t-1$ and $t$, the term $\sum_{j=0}^{T-t}\theta_{it+j}\sum_{l=0}^{j}(E_t-E_{t-1})\mathscr{O}(E_{t+j-1}[|\varepsilon_{it+j}|^2])$. Setting it to zero, as the authors do, is equivalent to assuming that, for all $j$, $\mathscr{O}(E_{t+j-1}|\varepsilon_{it+j}|^2)$, which is the endogenous component of expected future log-consumption growth at $t+j-1$—it behaves like the variance of the change in marginal utility at $t+j$—, is unaffected by shocks between $t-1$ and $t$ (past shocks)

[9] "The first component[of growth of the marginal utility of wealth e.g. of log-consumption growth], $\omega_t$, is a function of the interest rate $r$, the discount factor $\delta$, and the variance in the change of marginal utility and captures the intertemporal substitution and precautionary motives for savings. Assuming that the only source of uncertainty in this setup is the idiosyncratic wage shocks, $\omega_t$ is fixed over the cross-section." (p10)

therefore identical to what would be obtained under perfect foresight:

$$\Delta ln(c_{t+1}) = \underbrace{\frac{1}{\rho} ln(\beta(1+r))}_{\text{impatience}} + \underbrace{\frac{1}{\rho}(\Delta\delta_{t+1} z_{t+1})}_{\text{demographics}} + \varepsilon_{t+1} \frac{\left(\frac{dW_{t+1}}{d\varepsilon_{t+1}}\right)^*}{W^*_{t+1}} + \eta_{t+1} \frac{\left(\frac{dW_{t+1}}{d\eta_{t+1}}\right)^*}{W^*_{t+1}} + o(\varepsilon_{t+1}, \eta_{t+1})$$

Intuitively, by imposing that past shocks do not affect the precautionary component of expected log-consumption, they mechanically assume that current shocks do not affect the future expected precautionary component of log-income growth. Also, because the variance in the change rate of marginal utility has to be constant, changes in marginal utility have to be proprotional to marginal utility, and the precautionary premium has to be proportional to the level of consumption. Thus, consumption writes as a constant share of total expected resources, not uncertainty-adjusted resources.

The response of consumption to each shock coincide with the perfect foresight ratios, yet Blundell, Pistaferri and Preston (2008) interpret these expressions as reflecting precautionary behavior[10]. This interpretation misses the fact that the authors have thrown out precautionary behavior from their model. The form of the consumption response as a ratio over total expected ressources is not, here, a result of precautionary saving but an artifact of the logarithm.

# 3 Identification

The coefficients I want to estimate are as follow:

$$\phi^{\varepsilon} = \frac{cov(\Delta ln(c_t), \varepsilon_t)}{var(ln(\varepsilon_t))}$$
$$\phi^{\eta} = \frac{cov(\Delta ln(c_t), \eta_t)}{var(\eta_t)}$$

They capture how much log-consumption growth is expected to vary with respect to a given change in $\varepsilon_t$ and $\eta_t$: they correspond to the coefficients of a linear regression of the shocks $\varepsilon_t$ and $\eta_t$ over log-consumption growth $\Delta ln(c_t)$. Because both the explanatory variable (shock to log-wage) and the dependent variable (log-consumption) are in logs, the coefficient $\phi$ can be interpreted as the percent change in consumption from a one percent change in wage, transitory or permanent, which is to say the elasticity of consumption to the transitory or permanent component of wage. If log-consumption growth is indeed a linear function of the shocks, those coefficients coincide exactly with the marginal effect of the shocks on log-consumption and thus with the elasticity; otherwise they represent a linear

[10]" For individuals who are a long time from the end of their life with the value of current financial assets small relative to remaining future labor income, $\pi_t \approx 1$, and permanent shocks pass through more or less completely into consumption, whereas transitory shocks are (almost) completely insured against through saving. Precautionary saving can provide effective self-insurance against permanent shocks only if the stock of assets built up is large relative to future labor income, which is to say $\pi_t$ is appreciably smaller than unity, in which case there will also be some smoothing of permanent shocks through self insurance." (page 1898) [$\pi_t = \left( \frac{E_t[\sum_{s=0}^{T-t-1} \frac{y_{t+1+s}}{(1+r)^s}]}{(1+r)a_t + E_t[\sum_{s=0}^{T-t} \frac{y_{t+s}}{(1+r)^s}]} \right)$ denotes the coefficient associated with permanent shocks]

approximation of this marginal effect around small shocks.

The problem is that $\varepsilon_t$ and $\eta_t$ are not directly observed. To identify the covariance and variance that compose the coefficients, Blundell, Pistaferri and Preston rely on instruments: they regress log-consumption growth and log-income growth on instrumental variables that covary with log-income growth only through the realization of the transitory or of the permanent shock.

I do not write down the contribution of demographic variables, as they are assumed to be known in advance by consumers and do not covary with anything. To clarify the exposition, I assume in this section that $\varepsilon$ follows an MA(0) process, but the spirit of the identification method is identical with an MA(1), which is the specification that best fit the data. A generalization of the method to any MA(q) process is detailed in the Appendix of Blundell, Pistaferri and Preston and can be applied to the identification presented here.

## 3.1 Transitory Shocks

An appropriate instrument to identify the impact of transitory shocks is future log-wage growth, $\Delta ln(w_{t+1})$. I use equations (2.2) and (2.5) to substitute for log-wage growth and log-consumption growth:

$$\begin{aligned}
cov(\Delta ln(w_t), \Delta ln(w_{t+1})) &= cov(\eta_t + \varepsilon_t - \varepsilon_{t-1}, \eta_{t+1} + \varepsilon_{t+1} - \varepsilon_t) \\
&= -var(\varepsilon_t) \\
cov(\Delta ln(c_t), \Delta ln(w_{t+1})) &= cov(\Delta ln(c_t), \eta_{t+1} + \varepsilon_{t+1} - \varepsilon_t) \\
&= -cov(\Delta(ln(c_t), \varepsilon_t)
\end{aligned}$$

An estimator of the transitory coefficient is:

$$\hat{\phi}^{\varepsilon} = \frac{cov(\Delta ln(c_t), \Delta ln(w_{t+1}))}{cov(\Delta ln(w_t), \Delta ln(w_{t+1}))}$$

This amounts to instrumenting the impact current log-wage growth by future log-wage growth. The reason why future log-wage growth is a good instrument here is because the current realization of the transitory shock is the only component of current log-wage growth that introduces a variation in both current log-wage growth and future log-wage growth: when a transitory shock hits, it increases current log-wage growth, but reduces it by the same amount at the next period, as the wage goes back to its initial value. On the contrary, permanent shocks last for all remaining periods, therefore they do not cause any variation in future log-wage growth; past transitory shocks affect current wage growth but not future wage growth so their impact is also eliminated by the instrumentation.

To this point, the only assumption needed regarding $\Delta ln(c_t)$ is that it is independent from future shocks but no absence of correlation with past shocks is required. Alone, this estimator is unbiased, even in the presence of precautionary effects. Yet, because the response to transitory shocks is estimated jointly with the permanent coefficient, its measure can be altered if a correlation with past shocks distorts the latter.

## 3.2 Permanent Shocks

Instrumenting by the sum of past, current and future log-wage growth eliminates the variations in current log-wage growth and log-consumption growth that are caused by contemporaneous and past transitory shocks:

$$\begin{aligned}
cov(\Delta ln(w_t), \Delta ln(w_{t-1}) + \Delta ln(w_t) + \Delta ln(w_{t+1})) &= cov(\eta_t + \varepsilon_t - \varepsilon_{t-1}, \eta_{t-1} + \eta_t + \eta_{t+1} + \varepsilon_{t+1} - \varepsilon_{t-2}) \\
&= var(\eta_t) \\
cov(\Delta ln(c_t), \Delta ln(w_{t-1}) + \Delta ln(w_t) + \Delta ln(w_{t+1})) &= cov(\Delta ln(c_t), \eta_{t-1} + \eta_t + \eta_{t+1} + \varepsilon_{t+1} - \varepsilon_{t-2}) \\
&= cov(\Delta ln(c_t), \eta_t) + \underbrace{cov(\Delta ln(c_t), \eta_{t-1}) - cov(\Delta ln(c_t), \varepsilon_{t-2})}_{\text{precautionary effects}}
\end{aligned}$$

In effect, contemporaneous transitory shocks increase the log-wage growth at one period and reduce it by the same amount at the next: they have no impact on the sum of current and future log-wage growth so they do not cause any variations in the instrument and their impact is selected out. This method also excludes variations caused by past transitory shocks because these raise past log-wage growth but then reduce current log-wage growth by the same amount and thus have no effect on their sum.

This instrument identifies the variance of the permanent shock, because it correlates with current log-wage growth only through $\eta_t$. When consumers have a precautionary motive, however, it covaries with log-consumption growth both through $\eta_t$ and through past shocks, which influence the precautionary terms in log-consumption growth. In effect, the realizations of past shocks determine the amount of net assets that consumers have at their disposal, thus their current need for precautionary saving and the steepness of their log-consumption growth. Intuitively, the estimator erroneously captures the correlation of log-consumption with past transitory shocks (through precautionary saving) as a correlation with the current shock.

This precautionary effect can be recovered and eliminated at the cost of making an assumption on the information set of consumers at $t-1$, by building $E_{t-1}[\Delta ln(c_t)]$:

$$\begin{aligned}
cov(E_{t-1}[\Delta ln(c_t)], \Delta ln(w_{t-1}) + \Delta ln(w_t) + \Delta ln(w_{t+1})) &= cov(E_{t-1}[\Delta ln(c_t)], \eta_{t-1} + \eta_t + \eta_{t+1} + \varepsilon_{t+1} - \varepsilon_{t-2}) \\
&= \underbrace{cov(\Delta ln(c_t), \eta_{t-1}) - cov(\widetilde{\varphi}_{t-1}, \varepsilon_{t-2})}_{\text{precautionary effects}}
\end{aligned}$$

An estimator of the coefficient associated with permanent shocks is:

$$\hat{\phi}^{\eta} = \frac{cov(\Delta ln(c_t) - E_{t-1}[\Delta ln(c_t)],\ \Delta ln(y_t) + \Delta ln(y_{t+1}))}{cov(\Delta ln(y_t), \Delta ln(y_{t-1}) + \Delta ln(y_t) + \Delta ln(y_{t+1}))}$$

Log-consumption growth is replaced by its innovation, which is independent from past shocks. The covariance between log-consumption and the permanent shock is identified with using $\Delta ln(w_t) + \Delta ln(w_{t+1})$ only as an instrument, because the modification eliminates any correlation with past variables: $\Delta ln(w_{t-1})$ is independent from $\Delta ln(c_t) - E_{t-1}[\Delta ln(c_t)]$ and this term has no impact on the covariance.

The hypothesis that I have to make on the information available to consumers at $t-1$ can be tested by looking into the impact of variations in the information set. I present such robustness checks in section 4. Also, if the information set I use contains less information than is available to consumers, replacing total log-consumption growth by its innnovation would still improve the estimation and reduce the bias caused by precautionary behavior. In the limit case when I assume that consumers have zero information, their expectation is a constant and innovation to log-consumption growth coincides with total log-consumption growth: the estimator is identical to one that ignores the correlation between log-consumption growth and past shocks.

When the coefficients $\phi$ are estimated independently, only the one associated with permanent innovations should be subject to lack of instrument exogeneity. Yet, Blundell, Pistaferri and Preston implement their estimator in survey data; they use more moments than required for identification and estimate the coefficients jointly. In that case, biases can affect the measure of any of the parameters that are being estimated, in particular the coefficient associated with transitory shocks, and I cannot predict their directions.

### 3.3 Empirical Implementation

The model provides more restrictions on the autocovariance of consumption growth, the autocovariance of wage growth and the covariance of the two than just those required for identification. Following Blundell, Pistaferri and Preston (2008), to take advantage of these additional moments and get a more precise estimation, I use a minimum distance estimator. I build a vector m that contains the empirical counterparts of $cov(\Delta w_t, \Delta w_{t+s})$ and $cov(\Delta ln(c_t) - E_{t-1}[\Delta ln(c_t)], \Delta w_{t+s})$ for $1 \leq t \leq T$ and $0 \leq s \leq q+1$—where $q$ is the dimension of the MA(q) transitory component of log-wage.

The estimation model is:

$$m = f(\Lambda) + \Upsilon$$

where $\Lambda$ is the vector of parameters I am interested in. It contains the variance of the transitory shock at each period $var(\varepsilon_t)$, the variance of the permanent shock at each period $var(\eta_t)$, the elasticities $\phi^{\varepsilon}$ and $\phi^{\eta}$, and the coefficient of the income process $\theta_1$ (in the case when the transitory income process is an MA(1) only). The vector $\Upsilon$ captures sampling variability. I estimate $\Lambda$ by solving:

$$\min_{\Lambda} (m - f(\Lambda))' A (m - f(\Lambda))$$

A is a weighting matrix. In the case of the diagonally weighted minimum distance estimator used here, it is a diagonal matrix. The elements in the main diagonal are given by $diag(V^{-1})$, with $V$ the variance-covariance matrix of m.

The estimator of Blundell, Pistaferri and Preston uses restrictions on the autocovariance of log-consumption growth, $cov(\Delta c_t, \Delta c_{t+s})$, that do not hold when there is a precautionary correlation between log-consumption growth and past variables. These moments generate additional estimation biases that may intensify or lessen the initial bias, depending on their direction. I do not use them in my control estimation, so that the difference I observe be entirely driven by the correlation of log-consumption growth to past shocks. I compare the results with and without these moments and find that the bias they induce is very small.

# 4 Data and Results

## 4.1 Summary of Empirical Evidence On Tax Repayments And Consumption

Before looking into the results I obtain from longitudinal data, I detail what natural experiments tell us about the response of consumption to transitory tax repayments (refunds and rebates). The advantage of these episodes is that they constitute clean measures of exogenous transitory gains while it is generally difficult to observe changes in income that are uncorrelated with the determinants of consumption growth.

Souleles (1999) exploits tax refunds between 1979 and 1990, which is roughly the same period as covered in my dataset. These refunds are commonly received each year and often large in magnitude. He estimates the marginal propensity to consume nondurable goods out of a transitory gain to be statistically significant and comprised between 5% and 9% within the quarter following receipt. Unfortunately, he does not test for the longer-run impact of refunds. Papers that investigate the impact of tax rebate obtain larger estimates of the marginal propensity to consume nondurable goods: studies converge to a value of 25% (within the quarter)[11]. These studies adequately measure the impact of a fiscal stimulus during a recession, but because the marginal propensity to consume out of windfall gains is likely to be higher when consumers are in distress, they might be overestimating the response of consumption to a typical transitory shocks. In addition, there might be some belief among taxholders that they the rebates are going to have some persistence, while refunds are undoubtedly transitory. Contrary to the transitory shocks identified in longitudinal data, tax refunds are more or less anticipated as they depend on events that occured in the previous calendar year. Thus, the results

[11] Johnson, Parker and Souleles (2006) study the response of consumption to the 2001 fiscal stimulus implemented in the U.S. They obtain that the consumption of nondurable goods increased by 38% of the rebate, within quarter following receipt. Hamilton (2008) argue that the consumption data they use are noisy and should be trimmed at the top and at the bottom, which brings the estimate down to 22%. Kaplan and Violante (2014) do a similar correction and obtain close results. Misra and Surico (2014) refine the technique to account for heterogeneity in the reponse of consumption and obtain a marginal propensity to spend on nondurable goods of 25%. Similar findings are obtained for the 2008 tax rebate (Parker, Souleles, Johnson and McClelland (2013), Misra and Surico (2014).

can be interpreted as a lower bound for the response to an unexpected transitory shock. Therefore, two main features can be deduced from these studies: i) the marginal propensity to consume out of an unexpected transitory gain is statistically significant ii) its value over the year following the gain is above 5%.

## 4.2 Data

I use the same dataset as Blundell, Pisteferri and Preston (2008). It contains observations from the Panel Study of Income Dynamics (PSID) between 1978 and 1992[12]. The part of the sample focused on low-income families (SEO sample) is excluded. The dataset selects households followed for at least two consecutive years, composed of a married couple (with or without children) whose head is between 30 and 65 years old. This is to avoid problems associated with changes in family composition (for the youngest) and changes in income process due to retirement (for the oldest). Households facing some dramatic family composition change over the sample period are dropped: the dataset contains only those with either no change, or changes in members other than the head or the wife. This is to avoid modeling the risk associated with divorce, widowhood, or other household breaking-up factors, and focus on income risk. Finally, households with missing report on race, education, and region and some income outliers are eliminated. The final sample is composed of 12,058 observations of both log-income growth and log-consumption growth.

I use alternatively income (in the case of exogenous labor supply) and the wage rate as the source of uncertainty for consumers. Net income is made of the taxable family income reported by the household, from which I remove income from financial assets, and federal taxes on nonfinancial income, and which I deflate by the Consumer Price Index (CPI). I assume that federal taxes on nonfinancial income are a proportion of total federal taxes; the proportionality coefficient is given by the ratio between nonfinancial income and total income. Raw income is the taxable family income, net of financial assets and deflated by the CPI Each earner's wage rate is built as its yearly real labor income divided by its yearly number of hours of worked. Questions on income are retrospective and refer to the previous calendar year.

Unfortunately, the PSID only reports food consumption, while it is more adequate to use a broader category of non-durable consumption for the present exercise. To overcome the problem, non-durable consumption is imputed from demographics and food consumption, with the coefficients used for the imputation computed from the Consumer Expenditure Survey (CEX) over the same period. Further details are provided in the original paper by Blundell, Pistaferri and Preston (2008). Non-durable consumption is the sum of food (at home and away from home), alcohol, tobacco, non-durable services, heating fuel, public and private transport (including gasoline), personal care, clothing and footwear.

[12] I considered including additional years after 1992, but a number of the questions used by Blundell, Pistaferri and Preston are redesigned in 1993, and the impact of these changes is difficult to measure. From 1999, the survey is remodeled again, more substantially, and is only conducted every two years.

In particular, this definition excludes expenditure on housing, health, and education. To obtain the real analog to nominal consumption, it is deflated by the CPI. The PSID survey questions on food expenditure ask about typical weekly spending: it has been argued that people report their food expenditures for an average week around March (the period of the survey), rather than for the previous calendar year as is the case for family income. Blundell, Pistaferri and Preston test this alternative assumption and find no significant effect.

I consider variables that are net of the deterministic effects of the period and their individual characteristics. More precisely, they are regressed on year and year-of-birth dummies, and on dummies for education, race, family size, number of children, region, employment status, residence in a large city, outside dependent, and presence of income recipients other than husband and wife, interacted with a cohort dummy.

## 4.3 Empirical Counterpart Of Innovation To Log-Consumption Growth

To build my corrected estimate, I need a measure of expected log-consumption. I use the fitted value of log-consumption growth (net of deterministic components) when regressing it on variables I assume constitute the information set of the consumers. My baseline information set includes lagged consumption growth and income or wage rate growth as well as the lagged value of the households' house, financial assets, food consumption (at home and away from home) and food stamps. These latter variables should capture the strength of the precautionary motive.

Table 1: Predicted $\Delta ln(c_t)$ - baseline information set

| Variable | Coefficient | p-value |
|---|---|---|
| $ln(c_{t-1})$ | $-0.486$ | 0.000 |
| $ln(y_{t-1})$ | 0.111 | 0.000 |
| wife income at $t-1$ | $-0.000$ | 0.001 |
| house at $t-1$ | 0.000 | 0.000 |
| financial income at $t-1$ | 0.000 | 0.001 |
| food at $t-1$ | $-0.000$ | 0.001 |
| food out at $t-1$ | 0.000 | 0.000 |
| food stamps at $t-1$ | $-0.000$ | 0.000 |
| constant | 0.002 | 0.815 |
| Adjusted $R^2$ | 0.241 | |
| Observations | 12,058 | |

Table 1 shows the details of the regression of $\Delta ln(c_t)$ over the variables the baseline information set. Most importantly, the $R^2$ indicates that a fourth of the volatility in log-consumption, net of demographics, is still predictable with past variables, which is at variance with the hypothesis that once the deterministic components are removed log-consumption growth is independent from past shocks. It makes the case for the necesity to account for history dependence of log-consumption growth to past shocks. Innovation to log-consumption growth is built as the residual from this regression.

## 4.4 Results

### 4.4.1 Shocks on Income

I begin with the particular case in which hours worked are exogenous, so that shocks to the wage rate can equivalently be modeled as income shocks.

Table 2: Estimates of $\phi$ - Shocks on Income

| | | Net Income | Earnings | Male Earnings |
|---|---|---|---|---|
| Corrected BPP | Transitory shocks: $\phi^{\varepsilon}$ | **0.11** (0.03) | **0.10** (0.03) | **0.10** (0.02) |
| | Permanent shocks: $\phi^{\eta}$ | 0.60 (0.09) | 0.23 (0.05) | 0.18 (0.04) |
| BPP | Transitory shocks: $\phi^{\varepsilon}$ | **0.05** (0.04) | **0.07** (0.03) | **0.05** (0.03) |
| | Permanent shocks: $\phi^{\eta}$ | 0.64 (0.09) | 0.31 (0.06) | 0.22 (0.05) |

The first two lines of Table 2, labelled *CorrectedBPP*, report the results obtained with my estimator, robust to the presence of a correlation with past income shocks. The bottom lines, labelled *BPP*, correspond to the estimator of Blundell, Pistaferri and Preston[13]. The table shows that, when accounting for precautionary behavior, the estimated elasticity of consumption to transitory shocks on net income becomes large and significant: the estimate shifts from 0.05 with the traditional estimator to 0.10 with my corrected estimator. The corrected figures are consistent with results obtained in the tax rebate literature which find significant responses of consumption to transitory shocks. The elasticity to permanent income is not modified much by the correction: the estimate decreases from 0.66 to 0.61. It is not suprising that the elasticity be below one, because most consumers finance their consumption with both their income and some net assets they have accumulated: a percentage increase in income cannot translate in a one-for-one percentage increase in consumption. The same results are obtained with raw income shocks. The estimated elasticity to transitory shocks raises from 0.06 to 0.05. The elasticity to permanent shocks decreases from 0.31 to 0.24. The comparison of the impact of shocks to net income versus shocks to raw income indicates that taxes and transfers act provide substantial insurance in particular against permanent shocks: consumers respond a little less to news about their transitory raw income than to news about their transitory net income; and much less to news about their permanent raw income than to news about their permanent net income.

[13]These results coincide almost but not exactly with those presented in the authors' original paper. This is because I exclude the autocovariance of log-consumption growth from the moments used for estimation. In effect, in the presence of correlation with past shocks, the expression for the moments of log-consumption growth used by Blundell, Pistaferri and Preston does not hold and using thoses moments generate an additional bias of undetermined direction. When I incorporate these moments, the estimates are 0.05 (0.04) for $\phi^{\varepsilon}$ and 0.64 (0.09) for $\phi^{\eta}$.

It is consistent with the model I present to observe that, together with a strong and significant bias, I find a significant response to transitory shocks. In effect, recall from the identification section that the bias is caused by the non-zero correlation between a transitory shock $\varepsilon_{t-1}$ and the contribution of precautionary behavior to expected log-consumption growth $\widehat{\varphi}_t$. If this correlation is strong, then the response of consumption to transitory shocks should be too, because the components of $\widehat{\varphi}_t$ are also in $\Delta ln(c_t)$. It would have been incompatible with my theoretical findings that the total bias be strong, but the response to transitory shocks remain small and non-significant in the corrected estimation. It is reassuring that it is not the case.

### 4.4.2 Shocks on Wage Rates

I relax the assumption that hours worked are fixed and look into the elasticity of consumption to shocks on each earner's wage rate.

Table 3: Estimates of $\phi$ - Shocks on Wage Rates

| | | Male Wage Rate | Female Wage Rate |
|---|---|---|---|
| Corrected BPP | Transitory shocks: $\phi^{\varepsilon}$ | **0.09** (0.02) | **0.03** (0.02) |
| | Permanent shocks: $\phi^{\eta}$ | 0.18 (0.05) | 0.09 (0.04) |
| BPP | Transitory shocks: $\phi^{\varepsilon}$ | **0.05** (0.03) | **0.01** (0.02) |
| | Permanent shocks: $\phi^{\eta}$ | 0.16 (0.06) | 0.11 (0.07) |

Table 3 shows that, after correction, the estimated elasticity of consumption to transitory shocks on the wage rate of the male earner is larger and becomes significant. The estimated value increases from 0.04 to 0.09 after correction. The elasticity to permanent shocks changes little: it moves from 0.16 to 0.18 after correction. Both values are close to the elasticity of consumption to raw income. This suggests that the hypothesis of fixed hours worked is not too strong of an assumption. The elasticity of consumption to the female wage rate is much smaller and not significantly different from zero: it only shifts from 0.02 to 0.03 after correction. The elasticity to permanent shocks is equally smaller: the estimate is 0.08 and is not very different from its value before correction of 0.06. This more modest response of the households' consumption to shocks on the wage rate of female should be related to the observation that female earners work on average less hours, so that a change in their wage rate has a smaller impact on their earnings. Also, the wage rate of females is below that of males, which implies that their earnings correspond to a smaller share of the household's total revenue. This is another reason why a percentage change in their labor income should not cause as big of a change in consumption as the male labor income.

## 4.5 Robustness Checks

Table 4: Estimates of $\phi$ - Variations in the information set

| Information set | $I_0 = \emptyset$ | $I_1 = (c,y)$ | $I_2 = (c,y,z^s)$ | $I_3 = (c,y,z^s,z^l)$ |
|---|---|---|---|---|
| Transitory: $\phi^{\varepsilon}$ | **0.05** (0.04) | **0.13** (0.03) | **0.11** (0.03) | **0.10** (0.03) |
| Permanent: $\phi^{\eta}$ | 0.66 (0.10) | 0.55 (0.09) | 0.60 (0.09) | 0.53 (0.09) |

Table 4 presents the impact of varying the hypothesis on consumers' information set on the estimated elasticity of consumption to shocks on net income. I need to make an assumption on the information set to build the empirical counterpart to innovations in log-consumption growth: $\Delta ln(c_t) - E_{t-1}[\Delta ln(c_t)]$. When the information set is empty, innovations to log-consumption growth coincide with log-consumption growth and the estimator is the BPP estimator. Results presented in Table 4 shows that my findings are fairly robust to variations in the composition of the information set: all elasticity estimates are within one standard deviation from another, except for the empty set. Even when only consumption and income are included ($I_1$), the elasticity to transitory income is significantly different from zero and estimated at 0.12, which is twice as large as without correction. The elasticity to permanent income decreases slightly to 0.61. The baseline information set, $I_2$, which contains consumption and income, and more detailed variables (denoted $z^s$) on consumers' financial resources—net assets, the house's value, the female wage, and the hours worked by the male earner—and the structure of their consumption—consumption of total food and of food away from home and the value of food stamps received. The elasticity to transitory income decreases a little to 0.10. The elasticity to permanent income is unchanged. Finally, I add a set of cross products variables with the idea that the impact of predictors of consumption is not linear so higher order terms would help fit more precisely future consumption growth (denoted $z^l$). I find that the estimated elasticity of consumption to transitory shocks is unaffected. The elasticity to permanent shocks decreases at 0.54. This suggests that the baseline set is a good representation of the information consumers use to predict their future expected consumption. I present here the results obtained when considering net income, but I find very similar outcomes with raw income.

Table 5: Estimates of $\phi$ - Variations in the persistence of transitory shocks

| Transitory Process | MA(0) | MA(1) | MA(2) |
|---|---|---|---|
| Transitory: $\phi^{\varepsilon}$ | **0.12** (0.04) | **0.10** (0.03) | **0.10** (0.03) |
| Permanent: $\phi^{\eta}$ | 0.43 (0.05) | 0.61 (0.09) | 0.72 (0.13) |
| First lag: $\theta_1$ | 0 (*n.a.*) | 0.11 (0.02) | 0.20 (0.03) |
| Second lag: $\theta_2$ | 0 (*n.a.*) | 0 (*n.a.*) | 0.04 (0.03) |

In Table 5 I test the robustness of the results across different assumptions on the persistence of transitory shocks (on net income). My baseline assumption is that the transitory component of log-income has MA(1) serial correlation, because some second-autocovariance are significant. Yet the evidence of serial correlation is mixed—only some second order coefficients are significant and it is worth investigating the impact of this hypothesis. The estimated coefficients of the transitory process confirm that an MA(1) fits better: $\theta_1$ is found to be significantly different from zero while $\theta_2$ is not. The response of log-consumption growth to a transitory shock is very robust to variations in their persistence. The estimates obtained for the three specifications are very close. The estimate is 0.11 with an MA(0), and 0.10 with an MA(1) and an MA(2). The estimated response to a permanent shock varies more across specifications. The estimate is 0.44 with an MA(0), 0.61 with an MA(1) and 0.73 with an MA(2).

Table 6: Estimates of $\phi$ - Variations in the estimations moments

| | | Net Income | Earnings | Male Earnings | Male Wage Rate | Female Wage Rate |
|---|---|---|---|---|---|---|
| Corrected BPP | $\phi^\varepsilon$ | **0.11** (0.03) | **0.10** (0.03) | **0.10** (0.02) | **0.09** (0.02) | **0.03** (0.02) |
| | $\phi^\eta$ | 0.60 (0.09) | 0.23 (0.05) | 0.18 (0.04) | 0.18 (0.05) | 0.09 (0.04) |
| BPPm | $\phi^\varepsilon$ | **0.05** (0.04) | **0.06** (0.03) | **0.05** (0.03) | **0.05** (0.03) | **0.02** (0.02) |
| | $\phi^\eta$ | 0.66 (0.10) | 0.31 (0.06) | 0.22 (0.05) | 0.16 (0.06) | 0.06 (0.04) |
| BPP | $\phi^\varepsilon$ | **0.05** (0.04) | **0.07** (0.03) | **0.05** (0.03) | **0.05** (0.03) | **0.01** (0.02) |
| | $\phi^\eta$ | 0.64 (0.09) | 0.31 (0.06) | 0.22 (0.05) | 0.16 (0.06) | 0.11 (0.07) |

# 5 Conclusion

In a standard consumption model with income risk, precautionary behavior raises log-consumption growth: because it is possible that a disastrous outcome materializes in the future and because additional consumption would be very valuable if this happens, consumers facing uncertainty choose to transfer more resources to the future. The size of these precautionary transfers depends on consumers' stock of net assets, which is determined by the realizations of past income shocks. There is therefore a direct correlation between past shocks and log-consumption growth through the precautionary motive.

This effect compromises the estimation of the consumption response to permanent income innovations. In effect, the technique used to identify permanent shocks is to instrument the impact income growth on consumption growth by the sum of current and future income growth, to make sure that the change is permanent and that the current increase in income does not translate into a decrease at the next periods. In the presence of a correlation between current consumption growth and past shock, this method can erroneously capture the response of-consumption growth to past transitory shocks as a response to the current permanent shock, which causes a bias in the measure of the elasticity of consumption to shocks. A solution is to replace log-consumption growth by its innovation, which is by construction independent from past shocks. This transformation is innocuous if consumers have no precautionary motive. The technique, however, requires assumptions on the amount of information consumers have at their disposal, to build an empirical counterpart to the unexpected component log-consumption growth. Yet, in the case I assume less information than consumers actually have, I still reduce part of the bias: in the limit case when I make the hypothesis that consumers have zero information, the innovation to log-consumption growth coincides with total log-consumption growth and the corrected estimator is identical to the biased estimator.

My correction generates large and significant changes in the estimates of consumption elasticity to transitory income shocks and transitory wage shocks. The estimate raises from 0.05 to 0.10 and becomes significant in the case of an income shock. It raises from 0.04 to 0.09 in the case of a shock on the wage rate of the male earner. This indicates that neglecting the precautionary component of consumption growth produces quantitatively important biases. The larger response to transitory shocks is consistent with evidence regarding the large impact of transitory tax rebates. These results are robust to a number of variations in the estimation procedure.

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# Appendix A Consumption Allocation - Without A Borrowing Constraint

## A.1 Consumption Level

The Euler equation is:

$$u'(c_t) = E_t[u'(c_{t+1})]R_{t,t+1}$$

Following Kimball (1990b), I define $\varphi_t$ the equivalent precautionary premium for consumption at $t+1$. It is the variable $\varphi_t$ such that

$$E_t\left[u'(c_{t+1})\right] = u'(E_t[c_{t+1}] - \varphi_t)$$

From Jensen's inequality, the premium $\varphi_t$ is strictly positive for strictly prudent consumers (marginal utility is strictly convex) and zero for certainty-equivalent consumers (marginal utility is linear)[14].

[14]When marginal utility is strictly convex, Jensen's inequality implies:

$$E_t\left[u'(c_{t+1})\right] > u'(E_t[c_{t+1}]) \;\Leftrightarrow\; u'(E_t[c_{t+1}] - \varphi_t) > u'(E_t[c_{t+1}]) \;\Leftrightarrow\; E_t[c_{t+1}] - \varphi_t < E_t[c_{t+1}] \;\Leftrightarrow\; 0 < \varphi_t$$

Combining this expression with the Euler equation and applying $u'(c)^{-1} = c^{-1/\rho}$ yields:

$$\begin{aligned} u'(c_t) &= u'(E_t[c_{t+1}] - \varphi_t)R_{t,t+1} \\ c_t &= (E_t[c_{t+1}] - \varphi_t)R_{t,t+1}^{-1/\rho} \end{aligned} \tag{A.1}$$

This is true at any period $t$, thus it is true at $t+1$:

$$\begin{aligned} c_{t+1} &= (E_{t+1}[c_{t+2}] - \varphi_{t+1})R_{t+1,t+2}^{-1/\rho} \\ E_t[c_{t+1}] &= (E_t[c_{t+2}] - E_t[\varphi_{t+1}])R_{t+1,t+2}^{-1/\rho} \end{aligned} \tag{A.2}$$

Plugging (A.1) in (A.2) yields:

$$\begin{aligned} c_t &= E_t[c_{t+2}](R_{t,t+1}R_{t+1,t+2})^{-1/\rho} - E_t[\varphi_{t+1}](R_{t,t+1}R_{t+1,t+2})^{-1/\rho} - \varphi_t R_{t,t+1}^{-1/\rho} \\ c_t &= E_t[c_{t+2}]R_{t,t+2}^{-1/\rho} - E_t[\varphi_{t+1}]R_{t,t+2}^{-1/\rho} - \varphi_t R_{t,t+1}^{-1/\rho} \end{aligned}$$

Iterating forward, I obtain that for any $0 < s < T-t$:

$$c_t = E_t[c_{t+s}]R_{t,t+s}^{-1/\rho} - \sum_{k=1}^{s} E_t[\varphi_{t+k-1}]R_{t,t+k}^{-1/\rho}$$

Therefore, I can express future expected consumption as a function of current consumption and the precautionary premiums:

$$E_t[c_{t+s}] = c_t R_{t,t+s}^{1/\rho} + \sum_{k=1}^{s} E_t[\varphi_{t+k-1}]R_{t+k,t+s}^{1/\rho}$$

I combine these expressions with the intertemporal budget constraint (2.4):

$$\begin{aligned} \sum_{s=0}^{T-t}\frac{E_t[c_{t+s}]}{(1+r)^s} &= (1+r)a_t + \sum_{s=0}^{T-t}\frac{E_t[y_{t+s}]}{(1+r)^s} \\ \sum_{s=0}^{T-t}\frac{c_t R_{t,t+s}^{1/\rho}}{(1+r)^s} + \sum_{s=1}^{T-t}\sum_{k=1}^{s}\frac{E_t[\varphi_{t+k-1}]R_{t+k,t+s}^{1/\rho}}{(1+r)^s} &= (1+r)a_t + \sum_{s=0}^{T-t}\frac{E_t[y_{t+s}]}{(1+r)^s} \\ \sum_{s=0}^{T-t}\frac{c_t R_{t,t+s}^{1/\rho}}{(1+r)^s} + \sum_{k=1}^{T-t}\sum_{s=k}^{T-t}\frac{E_t[\varphi_{t+k-1}]R_{t+k,t+s}^{1/\rho}}{(1+r)^s} &= (1+r)a_t + \sum_{s=0}^{T-t}\frac{E_t[y_{t+s}]}{(1+r)^s} \\ c_t\sum_{s=0}^{T-t}\frac{R_{t,t+s}^{1/\rho}}{(1+r)^s} + \sum_{k=1}^{T-t}\frac{E_t[\varphi_{t+k-1}]}{(1+r)^k}\sum_{s=k}^{T-t}\frac{R_{t+k,t+s}^{1/\rho}}{(1+r)^{s-k}} &= (1+r)a_t + \sum_{s=0}^{T-t}\frac{E_t[y_{t+s}]}{(1+r)^s} \\ l_{t,0}c_t + \sum_{k=1}^{T-t}\frac{E_t[\varphi_{t+k-1}]}{(1+r)^k}\sum_{s=0}^{T-t-k}\frac{R_{t+k,t+k+s}^{1/\rho}}{(1+r)^s} &= (1+r)a_t + \sum_{s=0}^{T-t}\frac{E_t[y_{t+s}]}{(1+r)^s} \\ l_{t,0}c_t + \sum_{k=1}^{T-t} l_{t,k}\frac{E_t[\varphi_{t+k-1}]}{(1+r)^k} &= (1+r)a_t + \sum_{s=0}^{T-t}\frac{E_t[y_{t+s}]}{(1+r)^s} \end{aligned}$$

with $l_{T-t,k} = \sum_{s=0}^{T-t-k} \frac{R^{1/\rho}_{t+k,t+k+s}}{(1+r)^s}$. Therefore, an expression for consumption is:

$$c_t = \frac{1}{l_{t,0}} \left( \underbrace{(1+r)a_t + \sum_{s=0}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s}}_{\text{total expected ressources:}W_t} - \underbrace{\sum_{s=1}^{T-t} l_{t,s} \frac{E_t[\varphi_{t+s-1}]}{(1+r)^s}}_{\text{total expected precautionary saving:}PS_t} \right) \tag{A.3}$$

Intuitively, instead of consuming a given share $l_{t,0}$ of their ressources (assets plus total future expected income), prudent consumers put aside an expected precautionary amount $\sum_{s=1}^{T-t} l_{t,s} \frac{E_t[\varphi_{t+s-1}]}{(1+r)^s}$ and consume a share of the remaining part of their ressources only.

## A.2 Consumption Growth

I consider equation (A.3), taken at period $t+1$:

$$c_{t+1} = \frac{1}{l_{t+1,0}} \left( (1+r)a_{t+1} + \sum_{s=0}^{T-t-1} \frac{E_{t+1}[y_{t+1+s}]}{(1+r)^s} - \sum_{s=1}^{T-t-1} l_{t+1,s} \frac{E_{t+1}[\varphi_{t+s}]}{(1+r)^s} \right)$$

I use the period budget constraint $a_{t+1} = (1+r)a_t + y_t - c_t$ to substitute for $a_{t+1}$ in this expression:

$$c_{t+1} = \frac{1}{l_{t+1,0}} \left( (1+r)^2 a_t + (1+r)y_t - (1+r)c_t + \sum_{s=0}^{T-t-1} \frac{E_{t+1}[y_{t+1+s}]}{(1+r)^s} - \sum_{s=1}^{T-t-1} l_{t+1,s} \frac{E_{t+1}[\varphi_{t+s}]}{(1+r)^s} \right)$$

I add $\frac{(1+r)}{l_{t+1,0}} \left( \sum_{s=0}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s} - \sum_{s=1}^{T-t} l_{t,s} \frac{E_t[\varphi_{t-1+s}]}{(1+r)^s} \right) - \frac{(1+r)}{l_{t+1,0}} \left( \sum_{s=0}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s} - \sum_{s=1}^{T-t} l_{t,s} \frac{E_t[\varphi_{t-1+s}]}{(1+r)^s} \right) = 0$ on the right-hand side.

$$\begin{aligned}
c_{t+1} = & \frac{1}{l_{t+1,0}} \left( (1+r)^2 a_t + (1+r)y_t - (1+r)c_t + \sum_{s=0}^{T-t-1} \frac{E_{t+1}[y_{t+1+s}]}{(1+r)^s} - \sum_{s=1}^{T-t-1} l_{t+1,s} \frac{E_{t+1}[\varphi_{t+s}]}{(1+r)^s} \right) \\
& + \frac{(1+r)}{l_{t+1,0}} \left( \sum_{s=0}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s} - \sum_{s=1}^{T-t} l_{t,s} \frac{E_t[\varphi_{t-1+s}]}{(1+r)^s} \right) - \frac{(1+r)}{l_{t+1,0}} \left( \sum_{s=0}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s} - \sum_{s=1}^{T-t} l_{t,s} \frac{E_t[\varphi_{t-1+s}]}{(1+r)^s} \right) \\
c_{t+1} = & \frac{1}{l_{t+1,0}} \left( (1+r) \left( (1+r)a_t + \sum_{s=0}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s} - \sum_{s=1}^{T-t} l_{t,s} \frac{E_t[\varphi_{t-1+s}]}{(1+r)^s} \right) + (1+r)y_t - (1+r)c_t \right. \\
& \left. + \sum_{s=0}^{T-t-1} \frac{E_{t+1}[y_{t+1+s}]}{(1+r)^s} - \sum_{s=1}^{T-t-1} l_{t+1,s} \frac{E_{t+1}[\varphi_{t+s}]}{(1+r)^s} - (1+r) \sum_{s=0}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s} - (1+r) \sum_{s=1}^{T-t} l_{t,s} \frac{E_t[\varphi_{t-1+s}]}{(1+r)^s} \right)
\end{aligned}$$

From equation (A.3), I can replace the expression $\left( (1+r)a_t + y_t + \sum_{s=1}^{T-t} \frac{E_t[y_{t+s} - l_{t,s}\varphi_{t+s-1}]}{(1+r)^s} \right)$ by $l_{t,0}c_t$.

$$\begin{aligned}
c_{t+1} = & \frac{1}{l_{t+1,0}} \left( (1+r)l_{t,0}c_t - (1+r)c_t + (1+r)y_t + \sum_{s=0}^{T-t-1} \frac{E_{t+1}[y_{t+1+s}]}{(1+r)^s} - (1+r)y_t - (1+r) \sum_{s=1}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^s} \right) \\
& - \frac{1}{l_{t+1,0}} \left( \sum_{s=1}^{T-t-1} l_{t+1,s} \frac{E_{t+1}[\varphi_{t+s}]}{(1+r)^s} - (1+r) \sum_{s=1}^{T-t} l_{t,s} \frac{E_t[\varphi_{t-1+s}]}{(1+r)^s} \right)
\end{aligned}$$

By construction, $l_{t,s} = l_{t+1,s-1}$.

$$c_{t+1} = \frac{1}{l_{t+1,0}} \left( (1+r)(l_{t,0}-1)c_t + \sum_{s=0}^{T-t-1} \frac{E_{t+1}[y_{t+1+s}]}{(1+r)^s} - \sum_{s=1}^{T-t} \frac{E_t[y_{t+s}]}{(1+r)^{s-1}} - \sum_{s=1}^{T-t-1} l_{t+1,s} \frac{E_{t+1}[\varphi_{t+s}]}{(1+r)^s} + \sum_{s=1}^{T-t} l_{t+1,s-1} \frac{E_t[\varphi \cdots}{(1+ \cdots} \right.$$

$$c_{t+1} = \frac{1}{l_{t+1,0}} \left( (1+r)(l_{t,0}-1)c_t + l_{t+1,0}\varphi_t + \sum_{s=0}^{T-t-1} \frac{(E_{t+1}-E_t)[y_{t+1+s}]}{(1+r)^s} - \sum_{s=1}^{T-t-1} l_{t+1,s} \frac{(E_{t+1}-E_t)[\varphi_{t+s}]}{(1+r)^s} \right)$$

Because $l_{t,0} = 1 + (R_{t,t+1}^{1/\rho}/(1+r))l_{t+1,0}$, I obtain:

$$c_{t+1} = R_{t,t+1}^{1/\rho} c_t + \underbrace{\varphi_t}_{\text{precautionary trend}} + \underbrace{\frac{1}{l_{t+1,0}}(E_{t+1}-E_t)\left[ \sum_{s=0}^{T-t-1} \frac{y_{t+1+s}}{(1+r)^s} - \sum_{s=1}^{T-t-1} \frac{l_{t+1,s}\varphi_{t+s}}{(1+r)^s} \right]}_{\text{consumption innovation} = c_{t+1} - E_t[c_{t+1}]} \tag{A.4}$$

## A.3 Log-Consumption Growth

I divide each side of equation (A.4) by $R_{t,t+1}^{1/\rho} c_t$:

$$\frac{c_{t+1}}{R_{t,t+1}^{1/\rho} c_t} = 1 + \frac{\varphi_t}{R_{t,t+1}^{1/\rho} c_t} + \frac{1}{R_{t,t+1}^{1/\rho} c_t l_{t+1,0}} (E_{t+1}-E_t)\left[ \sum_{s=0}^{T-t-1} \frac{y_{t+1+s}}{(1+r)^s} - \sum_{s=1}^{T-t-1} \frac{l_{t+1,s}\varphi_{t+s}}{(1+r)^s} \right]$$

$$\frac{c_{t+1}}{R_{t,t+1}^{1/\rho} c_t} = 1 + \frac{\varphi_t}{R_{t,t+1}^{1/\rho} c_t} + \frac{(E_{t+1}-E_t)\left[W_{t+1} - PS_{t+1}\right]}{R_{t,t+1}^{1/\rho} c_t l_{t+1,0}}$$

where $W_t$ denotes total expected resources at $t$ and $PS_t$ total expected precautionary saving at $t$. I take the logarithm:

$$\Delta ln(c_{t+1}) - \underbrace{\frac{1}{\rho} ln(R_{t,t+1})}_{\text{impatience and dem.}} = ln \left( 1 + \underbrace{\frac{\varphi_t}{R_{t,t+1}^{1/\rho} c_t}}_{\text{precaution}} + \underbrace{\frac{(E_{t+1}-E_t)\left[W_{t+1} - PS_{t+1}\right]}{R_{t,t+1}^{1/\rho} c_t l_{t+1,0}}}_{\text{consumption innovation} = \frac{c_{t+1} - E_t[c_{t+1}]}{R_{t,t+1}^{1/\rho} c_t}} \right) \tag{A.5}$$

I expand equation (A.5) around the point where $(\varepsilon_{t+1}, \eta_{t+1}) = (0,0)$, to the first order. I denote with a star the variables taken at this point.

$$
\begin{aligned}
\Delta ln(c_{t+1}) = \frac{1}{\rho} ln(R_{t,t+1}) + ln\left(1 + \frac{\varphi_t}{R_{t,t+1}^{1/\rho} c_t} + \frac{c_{t+1}^* - E_t\left[c_{t+1}\right]}{R_{t,t+1}^{1/\rho} c_t}\right) \\
+ \varepsilon_{t+1} \frac{\left(\frac{dW_{t+1}}{d\varepsilon_{t+1}}\right)^* - \left(\frac{dPS_{t+1}}{d\varepsilon_{t+1}}\right)^*}{R_{t,t+1}^{1/\rho} c_t l_{t+1,0}} \frac{1}{1 + \frac{\varphi_t}{R_{t,t+1}^{1/\rho} c_t} + \frac{c_{t+1}^* - E_t[c_{t+1}]}{R_{t,t+1}^{1/\rho} c_t}} \\
+ \eta_{t+1} \frac{\left(\frac{dW_{t+1}}{d\eta_{t+1}}\right)^* - \left(\frac{dPS_{t+1}}{d\eta_{t+1}}\right)^*}{R_{t,t+1}^{1/\rho} c_t l_{t+1,0}} \frac{1}{1 + \frac{\varphi_t}{R_{t,t+1}^{1/\rho} c_t} + \frac{c_{t+1}^* - E_t[c_{t+1}]}{R_{t,t+1}^{1/\rho} c_t}} \\
+ o(\varepsilon_{t+1}, \eta_{t+1})
\end{aligned}
$$

By construction, $R_{t,t+1}^{1/\rho} c_t + \varphi_t = E_t[c_{t+1}]$. Therefore,

$$
\begin{aligned}
l_{t+1,0}(R_{t,t+1}^{1/\rho} c_t + \varphi_t + c_{t+1}^* - E_t\left[c_{t+1}\right]) &= l_{t+1,0}(E_t[c_{t+1}] + c_{t+1}^* - E_t\left[c_{t+1}\right]) \\
&= l_{t+1,0} \frac{1}{l_{t+1,0}} \left(W_{t+1} - PS_{t+1}\right) \\
&= (W_{t+1} - PS_{t+1})
\end{aligned}
$$

I plugg this in the expression for log-consumption growth and obtain:

$$
\begin{aligned}
\Delta ln(c_{t+1}) = \frac{1}{\rho} ln(R_{t,t+1}) + ln\left(1 + \frac{\varphi_t}{R_{t,t+1}^{1/\rho} c_t} + \frac{c_{t+1}^* - E_t\left[c_{t+1}\right]}{R_{t,t+1}^{1/\rho} c_t}\right) \\
+ \varepsilon_{t+1} \frac{\left(\frac{dW_{t+1}}{d\varepsilon_{t+1}}\right)^* - \left(\frac{dPS_{t+1}}{d\varepsilon_{t+1}}\right)^*}{W_{t+1} - PS_{t+1}} + \eta_{t+1} \frac{\left(\frac{dW_{t+1}}{d\eta_{t+1}}\right)^* - \left(\frac{dPS_{t+1}}{d\eta_{t+1}}\right)^*}{W_{t+1} - PS_{t+1}} \quad + o(\varepsilon_{t+1}, \eta_{t+1})
\end{aligned}
$$

### A.3.1 Specific Case Of Exogenous Labor Supply

When the number of hours worked is exogenous ($h_t = \bar{h}$), income $y$ is proportional to the wage rate $w$ and the wage shocks can equivalently be represented as direct shocks on income. Expected future income, $E_{t+1}\left[\sum_{s=0}^{T-t-1}\frac{y_{t+1+s}}{(1+r)^s}\right]$, can be expressed as a function of $\varepsilon_{t+1}$ and $\eta_{t+1}$[15]:

$$E_{t+1}\left[\sum_{s=0}^{T-t-1}\frac{y_{t+1+s}}{(1+r)^s}\right]=E_t[y_{t+1}]ln(e^{\varepsilon_{t+1}})ln(e^{\eta_{t+1}})+\left(\sum_{s=1}^{T-t-1}\frac{E_t[y_{t+1+s}]}{(1+r)^s}\right)ln(e^{\eta_{t+1}})$$

The derivatives of total expected resources with respect to the shocks write:

$$\frac{dW_{t+1}}{d\varepsilon_{t+1}}=\sum_{s=0}^{T-t-1}\frac{\left(\frac{dE_{t+1}[y_{t+1+s}]}{d\varepsilon_{t+1}}\right)}{(1+r)^s}=E_t[y_{t+1}]e^{\varepsilon_{t+1}}e^{\eta_{t+1}}$$

$$\frac{dW_{t+1}}{d\eta_{t+1}}=\sum_{s=0}^{T-t-1}\frac{\left(\frac{dE_{t+1}[y_{t+1+s}]}{d\eta_{t+1}}\right)}{(1+r)^s}=E_t[y_{t+1}]e^{\varepsilon_{t+1}}e^{\eta_{t+1}}+\left(\sum_{s=1}^{T-t-1}\frac{E_t[y_{t+1+s}]}{(1+r)^s}\right)e^{\eta_{t+1}}$$

A the point where $(\varepsilon_{t+1},\eta_{t+1})=(0,0)$, these derivates are:

$$\left(\frac{dW_{t+1}}{d\varepsilon_{t+1}}\right)^*=E_t[y_{t+1}]$$

$$\left(\frac{dW_{t+1}}{d\eta_{t+1}}\right)^*=E_t[y_{t+1}]+\left(\sum_{s=1}^{T-t-1}\frac{E_t[y_{t+1+s}]}{(1+r)^s}\right)=\sum_{s=0}^{T-t-1}\frac{E_t[y_{t+1+s}]}{(1+r)^s}$$

I substitute for the expression of the derivatives, and for $W_{t+1}$, in my general expression for log-consumption growth:

$$\begin{aligned}\Delta ln(c_{t+1})=\frac{1}{\rho}ln(R_{t,t+1})+ln\left(1+\frac{\varphi_t}{R_{t,t+1}^{1/\rho}c_t}+\frac{c_{t+1}^*-E_t\left[c_{t+1}\right]}{R_{t,t+1}^{1/\rho}c_t}\right)\\
+\varepsilon_{t+1}\frac{E_t[y_{t+1}]-\left(\frac{dPS_{t+1}}{d\varepsilon_{t+1}}\right)^*}{(1+r)a_{t+1}+\sum_{s=0}^{T-t-1}\frac{E_{t+1}[y_{t+1+s}]}{(1+r)^s}-PS_{t+1}}\\
+\eta_{t+1}\frac{\sum_{s=0}^{T-t-1}\frac{E_t[y_{t+1+s}]}{(1+r)^s}-\left(\frac{dPS_{t+1}}{d\eta_{t+1}}\right)^*}{(1+r)a_{t+1}+\sum_{s=0}^{T-t-1}\frac{E_{t+1}[y_{t+1+s}]}{(1+r)^s}-PS_{t+1}}\quad+o(\varepsilon_{t+1},\eta_{t+1})\end{aligned}$$

---

[15]From equation (2.1)-(2.3), I have $y_{t+s}=e^{p_{t+s}}e^{\varepsilon_{t+s}}e^{\kappa_{t+s}z_{t+s}}=e^{p_t}e^{\eta_{t+1}}\times...\times e^{\eta_{t+s}}e^{\varepsilon_{t+s}}e^{\kappa_{t+s}z_{t+s}}\bar{h}$. As the shocks $\eta$ are independent from each other, independent from $\varepsilon$, and independent from the initial value of permanent income, the expected value of their product is the product of their expected value. They are drawn from exogenous distribution so that their expected value at $t+1$ is equal to their expected value at $t$. Also, shocks can be normalized so that $E_t[e^{\varepsilon_{t+1}}]=E_t[e^{\eta_{t+1}}]=1$.

for $s=1$: $E_{t+1}[y_{t+1}]=e^{p_{t+1}}e^{\varepsilon_{t+1}}e^{\kappa_{t+1}z_{t+1}}\bar{h}=e^{p_t}e^{\eta_{t+1}}e^{\varepsilon_{t+1}}e^{\kappa_{t+1}z_{t+1}}\bar{h}=\frac{e^{\varepsilon_{t+1}}}{E_t[e^{\varepsilon_{t+1}}]}\frac{e^{\eta_{t+1}}}{E_t[e^{\eta_{t+1}}]}E_t[y_{t+1}]=E_t[y_{t+1}]e^{\varepsilon_{t+1}}e^{\eta_{t+1}}$

for $s>1$: $E_{t+1}[y_{t+s}]=p_{t+1}E_t[e^{\eta_{t+2}}]\times...\times E_t[e^{\eta_{t+s}}]E_t[e^{\varepsilon_{t+s}}]e^{\kappa_{t+s}z_{t+s}}\bar{h}=\frac{e^{\eta_{t+1}}}{E_t[e^{\eta_{t+1}}]}E_t[y_{t+s}]=E_t[y_{t+s}]e^{\eta_{t+1}}$

# Appendix B Borrowing Constraints

In this section, I take into account the impact the natural borrowing constraint, which imposes that consumers cannot be indebted above the worst possible realization of their total future expected income, because they cannot die in debt. I also introduce the possibility of an exogenously imposed borrowing limit. The consumer's problem is:

$$\max_{c_t,\dots c_T} \quad E_t\left[\sum_{s=0}^{T-t}\beta^{t+s}u(c_{t+s})e^{\delta_t z_t}\right]$$

$$s.t. \quad \begin{cases} \sum_{s=0}^{T-t}\frac{c_{t+s}}{(1+r)^s} = (1+r)a_t + \sum_{s=0}^{T-t}\frac{y_{t+s}}{(1+r)^s} \\ a_{t+1} > \max\left(-\sum_{s=0}^{T-t-1}\frac{\underline{y}_{t+1+s}}{(1+r)^{s+1}}, -L_{t+1}\right) \end{cases}$$

The term $\underline{y}_{t+s}$ denotes the worst possible realization of $y_{t+s}$. The natural borrowing constraint $a_{t=1} \geq -\sum_{s=0}^{T-t-1}\frac{\underline{y}_{t+1+s}}{(1+r)^{s+1}}$ emerges from the combination of the no default condition and the requirement that consumption be positive. The exogenous borrowing constraint $a_{t+1} > L_{t+1}$ can reflect frictions on the lending market. I assume that the borrowing limit $L$ is exogenous, predictable and perfectly anticipated by consumers. The baseline model corresponds to the base where $L_t = \infty \, \forall t$

The Euler equation becomes:

$$u'(c_t) = E_t[u'(c_{t+1})]R_{t,t+1} + \lambda_t$$

where $\lambda_t \geq 0$ denotes the multiplier on constraint (3.1). In effect, when the constraint is binding, consumers cannot borrow as much as they want and they are forced to transfer consumption from period $t$, to period $t+1$: their consumption at $t$ is smaller and their marginal utility higher than it would in the absence of a borrowing limit.

With an approximation around $\lambda_t = 0$, consumption at $t$ writes:

$$c_t = (E_t[c_{t+1}] - \varphi_t)R_{t,t+1}^{-1/\rho} + \lambda_t\left(E_t[c_{t+1}] - \varphi_t\right)^{1+\rho}R_{t,t+1}^{-(1+\rho)/\rho} + o(\lambda_t)$$

Applying the same procedure as in the case of zero borrowing constraint yields that consumption at $t$ is:

$$c_t \approx \frac{1}{l_{t,0}}\left(\underbrace{(1+r)a_t + \sum_{s=0}^{T-t}\frac{E_t[y_{t+s}]}{(1+r)^s}}_{\text{total expected ressources}} - \underbrace{\sum_{k=1}^{T-t}l_{t,k}\frac{E_t[\varphi_{t+k-1}]}{(1+r)^k}}_{\text{total expected precautionary saving}} - \underbrace{\frac{1}{\rho}\sum_{k=1}^{T-t}l_{t,k}\frac{E_t[\tilde{\lambda}_{t+k-1}]}{(1+r)^k}}_{\text{total expected constrained saving}}\right)$$

with $\tilde{\lambda}_{t+k-1} = \lambda_{t+k-1}\left(E_{t+k-1}[c_{t+k}] - \varphi_{t+k-1}\right)^{1+\rho}R_{t+k-1,t+k}^{-1}$ measures the impact of the borrowing constraint at $t+k+1$ on consumption growth between $t+k+1$ and $t+k$. This term is either zero or positive: everything else equal, the forced saving generates an increase in consumption growth.

Borrowing constraint have an effect that is similar to precautionary saving: consumers take out from their expected resources the amount they expect to be constrained to save and consume a share of the remainder.

Log-consumption growth at $t$ is:

$$\Delta ln(c_{t+1}) \approx \frac{1}{\rho} ln(R_{t,t+1}) + ln\left(1 + \frac{\varphi_t}{R_{t,t+1}^{1/\rho} c_t} + \frac{\lambda_t}{R_{t,t+1}^{1/\rho} c_t} + \frac{c_{t+1}^{*} - E_t\left[c_{t+1}\right]}{R_{t,t+1}^{1/\rho} c_t}\right)$$
$$+ \varepsilon_{t+1} \frac{\left(E_t[y_{t+1}] - \left(\frac{dPS_{t+1}}{d\varepsilon_{t+1}}\right)^{*} - \left(\frac{dCS_{t+1}}{d\varepsilon_{t+1}}\right)^{*}\right)}{(1+r)a_{t+1} + \sum_{s=0}^{T-t-1} \frac{E_{t+1}[y_{t+1+s}]^{*}}{(1+r)^{s}} - PS_{t+1}^{*} - CS_{t+1}^{*}}$$
$$+ \eta_{t+1} \frac{\left(\sum_{s=0}^{T-t-1} \frac{E_t[y_{t+1+s}]}{(1+r)^{s}} - \left(\frac{dPS_{t+1}}{d\eta_{t+1}}\right)^{*} - \left(\frac{dCS_{t+1}}{d\eta_{t+1}}\right)^{*}\right)}{(1+r)a_{t+1} + \sum_{s=0}^{T-t-1} \frac{E_{t+1}[y_{t+1+s}]^{*}}{(1+r)^{s}} - PS_{t+1}^{*} - CS_{t+1}^{*}} + o(\varepsilon_{t+1}, \eta_{t+1})$$

The additional terms with respect to the perfect foreisght case are colored. Borrowing constraints have the same type of effects has precautionary behavior. Both have to be considered jointly because the presence of a borrowing constraint modifies precautionary saving and vice-versa (in particular forced saving can serve as a precautionary buffer so there is less need for additional precautionary saving).