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Abstract: In this paper, we study the link between choices derived from monotonous set-dependent preferences and sequential rationalizability. This link is quite natural since choices derived from monotonous set-dependent preferences (introduced in [Houy, 2008b]) are characterized by a strong axiom of revealed preferences whereas sequentially rationalizable choice functions (introduced in [Manzini and Mariotti, 2007]) are characterized by a weak axiom of revealed preferences.

Key Words : Weak/Strong axioms of revealed preferences, sequential rationalizability.

Classification JEL: D0

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1 Introduction

Classical rational choice theory is based on the axiom of revealed preferences. In short, we say that an individual reveals that he prefers an alternative a to an alternative b if there exists a choice set S such that b is in S and a is chosen from S. One of the most important results of classical choice theory is that a choice function is rationalized by a linear order if and only if the revealed preferences derived from it are asymmetric (weak axiom of revealed preferences, introduced by [Samuelson, 1938]). Moreover, in this framework, the asymmetry of the reavealed preferences is equivalent to their acyclicity (strong axiom of revealed preferences, introduced by [Houthakker, 1950]).

However, many experiments tend to show that the traditional rational choice theory is violated by most of individuals (see [Loomes *et al.*, 1991], [Roelofsma et Read, 2000] and [Tversky, 1969] for instance). Then, in order to study the choice patterns of the observed individuals, one need to weaken the axioms and equivalently representations in the theory of rational choices. A few recent articles have studied rational choices in this line of analysis. First, [Manzini and Mariotti, 2007] studied sequentially rationalizable choice functions.[1] In order to characterize those choice functions, an weak axiom of revealed preferences is used. This axiom still requires the revealed preferences to be asymmetric but it defines a revealed preference as follows: a is said to be revealed preferred to b if there exist two sets S and S' such that $a, b \in S \subseteq S'$ and such that b is chosen in S whereas a is chosen in S'. A second approach has been studied by [Tyson, 2008] and [Houyb, 2008a]. In those study, the authors weaken the representation by remarking that traditional choice theory corresponds to some choice set in-

[1]For more studies in this line, see [Ballester and Apesteguia, 2008], [Houy, 2007], [Houy and Tadenuma, 2007] and [Tadenuma, 2002].

dependent preferences. Obviously, just considering that preferences could be choice set dependent would not be binding since any choice function can be rationalized by choice set dependent preferences. Hence, the authors set some structure on the dependence of the preferences on the choice set. [Tyson, 2008] considers a nested structure whereas [Houyb, 2008a] considers an anti-nested structure. In [Houy, 2008b], a monotonous structure is imposed and it is shown that then, the choice functions obtained are characterized by the strong axiom of revealed preferences applied to the revealed preferences introduced in [Manzini and Mariotti, 2007]. Hence, even though the results and representation are very different, there is a close relation between [Manzini and Mariotti, 2007] and [Houy, 2008b]. In this article, we study these relations.

2 Formal framework

Let X be a finite set of alternatives. Let $\mathcal{X}$ be the set of non-empty subsets of X, $\mathcal{X} = 2^X \setminus \emptyset$. A choice function is a function $C : \mathcal{X} \to \mathcal{X}$ such that $\forall S \in \mathcal{X}, C(S) \subseteq S$ and $\#C(S) = 1$.

Let $S \in \mathcal{X}$. A binary relation on S is a subset of $S \times S$. A binary relation $P \subseteq S \times S$ is asymmetric if $\forall a, b \in S$ with $a \neq b$, $(a, b) \in P \Rightarrow (b, a) \notin P$. It is complete if $\forall a, b \in S$ with $a \neq b$, $(a, b) \in P$ or $(b, a) \in P$. It is transitive if $\forall a, b, c \in S$, $(a, b), (b, c) \in P \Rightarrow (a, c) \in P$. It is acyclic if $\forall n \in N \setminus \{1\}$, $\forall a_1, ..., a_n \in S$, $(a_i, a_{i+1}) \in P$ for all $i \in \{1, ..., n-1\}$ implies $a_1 \neq a_n$. A preference relation is an asymmetric binary relation. A linear order is an asymmetric, transitive and complete binary relation. For a set $S \in \mathcal{X}$ and a binary relation P, we write $M(S, P) = \{a \in S, \forall b \in S, (b, a) \notin P\}$ and $J(S, P) = \{a \in S, \forall b \in S, (a, b) \in P\}$.

$\pi : \mathcal{X} \to 2^{X \times X}$ is a binary relation (resp. preference relation, linear order) function if for all $S \in \mathcal{X}$, $\pi(S)$ is a binary relation (resp. preference relation, linear order) on S. We say that π is monotonous if $\forall S, S', S'' \in \mathcal{X}$ such that $S \subseteq S' \subseteq S''$, $\forall a, b \in S$, $[(a,b) \in \pi(S) \cap \pi(S'') \Rightarrow (a,b) \in \pi(S')]$ and $[(a,b) \notin \pi(S) \cup \pi(S'') \Rightarrow (a,b) \notin \pi(S')]$. We say that π is expansive if $\forall S, S' \in \mathcal{X}$, $\forall a \in S \cap S'$, $[\forall b \in S \cup S', (b,a) \notin \pi(S) \cup \pi(S') \Rightarrow \forall b \in S \cup S', (b,a) \notin \pi(S \cup S')]$.

Let π be a binary relation function. A choice function C is rationalized by π if $\forall S \in \mathcal{X}, C(S) = M(S, \pi(S))$. A choice function C is justified by π if $\forall S \in \mathcal{X}, C(S) = J(S, \pi(S))$. Finally, if C is a choice function and $P_1, P_2 \subseteq X \times X$ are two binary relations, we say that C is sequentially rationalized (S-rationalized) by the ordered pair (P_1, P_2) if $\forall S \in \mathcal{X}, C(S) = M(M(S, P_1), P_2)$.

The main difference between our study and classical rational choice theory lies in the next definition. As we said in the Introduction, we consider cases where there can be preference reversals. However, in our study, we will consider only "one way" reversals. Hence, it is not because we observe that the individual chooses an alternative a whereas a second alternative b is available that we can deduce that the individual intrinsicly prefers a to b. It could be the case that the individual lacks information from the choice set. Then, in order to assess that the individual intrinsicly prefers a to b, it must be the case that the individual chooses b rather than a in a set S and b rather than a in a larger set (with respect to inclusion) S'. The main assumption of this paper is that then, there will not be any set S'' such that S' is included in S'' and such that individual chooses b again from it. In some sense, the information the individual gets from the choice set is progressive and makes the preferences of the individual closer to some "fundamental" preferences. Then, for a choice function C, we define the progressive knowledge revealed preferences binary relation on X, P^r_C, by: $\forall a, b \in X, (a,b) \in P^r_C$ if and only if $\exists S, S' \in \mathcal{X}$ such that $a, b \in S \subseteq S'$, $\{b\} = C(S)$ and $\{a\} = C(S')$.

The first two axioms are statements of the usual weak and strong axioms of revealed preferences in the case of progressive knowledge revealed preferences. PKSARP imposes the acyclicity of P^r_C whereas PKWARP imposes its asymmetry. Obviously PKSARP is stronger than PKWARP.

Axiom 1 (PKWARP, Progressive Knowledge WARP)
A choice function C satisfies Progressive Knowledge WARP if P^r_C is asymmetric.

Axiom 2 (PKSARP, Progressive Knowledge SARP)
A choice function C satisfies Progressive Knowledge SARP if P^r_C is acyclic.

The next axiom is the usual Expansion Consistency Axiom. It is sometimes referred to as Axiom γ.

Axiom 3 (Expansion)
A choice function C satisfies Expansion if $\forall S, S' \in \mathcal{X}$ and $\forall a \in X$, $a \in C(S) \cap C(S') \Rightarrow a \in C(S \cup S')$.

3 Results

Theorem 1 has been given in [Houy, 2008b] and characterizes the choice functions satisfying PKSARP as being the only ones rationalized by monotonous linear order functions.

Theorem 1 (Houy)
Let C be a choice function. C satisfies PKSARP if and only if it is rationalized by some monotonous linear order function.

Theorem 2 has been given in [Manzini and Mariotti, 2007] and characterizes the choice functions satisfying PKWARP and Expansion as being the only ones S-rationalized by pairs of preference relations.

Theorem 2 ([Manzini and Mariotti, 2007])
Let C be a choice function. C satisfies PKWARP and Expansion if and only if it is S-rationalized by some pair of preference relations (P_1, P_2).

Even though the two previous theorems use similar concepts, it seems that the choice functions they characterize have very different representations in terms of preferences. In the following, we study the relationships between the previous theorems and combine the different axioms needed for them.

First, Theorem 3 shows that if we weaken PKSARP to PKWARP in Theorem 1, rationalization is not the right concept to use. Justification is better adapted. The proof of this theorem is given in Appendix A.

Theorem 3
Let C be a choice function. C satisfies PKWARP if and only if it is justified by some monotonous preference relation function.

In Appendix C, an example is given that shows that Theorems 1 and 3 are not redundant, that is that PKWARP does not imply PKSARP.

The next two theorems characterize the choice functions that satisfy PKWARP/PKSARP and Expansion. The proofs are quite straightforward and omitted.

Theorem 4
Let C be a choice function. C satisfies PKWARP and Expansion if and only if it is justified by some monotonous expansive preference relation function.

Theorem 5
Let C be a choice function. C satisfies PKSARP and Expansion if and only if it is rationalized by some monotonous expansive linear order function.

The following example shows that Theorems 4 and 5 are not redundant with Theorems 1 and 3. Indeed, PKWARP does not imply Expansion.

Example 1
Let $X = \{a, b, c\}$. Let C be the choice function defined by: $C(\{a\}) = \{a\}$, $C(\{b\}) = \{b\}$, $C(\{c\}) = \{c\}$, $C(\{a, b\}) = \{b\}$, $C(\{a, c\}) = \{c\}$, $C(\{b, c\}) = \{b\}$ and $C(X) = \{a\}$. Obviously, C satisfies PKSARP (and then PKWARP). However, it does not satisfy Expansion since $C(\{a, b\}) = C(\{b, c\}) = \{b\}$ whereas $C(X) = \{a\}$.

Finally, we give a theorem showing a representation in terms of sequential rationality for the same choice functions as the ones considered in Theorem 5.

Theorem 6
Let C be a choice function. C satisfies PKSARP and Expansion if and only if it is S-rationalized by some pair of preference relations (P_1, P_2) with P_2 acyclic.

In Appendix C, we show a case of a choice function that satisfies PKWARP and Expansion but not PKWARP. This shows that theorems 2 and 6 are not redundant. Said differently, there exist choice functions that can be S-rationalized and such that if (P_1, P_2) rationalizes it, P_2 is cyclic.

A Proof of Theorem 3

A.1 Only If:

Let C be a choice function satisfying PKWARP.

Let $S \in \mathcal{X}$. Let us define the irreflexive binary relations $P_1(S)$ and $P_2(S)$ as follows: $\forall a, b \in S$,

- $(a, b) \in P_1(S)$ if and only if $a \neq b$ and $\exists S', S'' \in \mathcal{X}$ such that $a, b \in S' \subseteq S'' \subseteq S$, $\{b\} = C(S')$ and $\{a\} = C(S'')$.
- $(a, b) \in P_2(S)$ if and only if $a \neq b$ and $[(a, b), (b, a) \notin P_1(S)$ and $\exists S', S'' \in \mathcal{X}$ such that $a, b \in S' \subseteq S \subseteq S''$, $\{a\} = C(S')$ and $\{a\} = C(S'')]$.

Let us define the binary relation function π as $\forall S \in \mathcal{X}, \pi(S) = P_1(S) \cup P_2(S)$.

I. Let us show that $\forall S \in \mathcal{X}$, $\pi(S)$ is asymmetric.

1) $P_1(S)$ is asymmetric. Indeed, by definition, $P_1(S) \subseteq P_1(X) = P_C^r$. Hence, $P_1(S)$ is asymmetric by PKWARP.

2) $P_2(S)$ is asymmetric. Assume on the contrary that $P_2(S)$ is symmetric. Then, there exist $a, b \in X$ with $a \neq b$ such that $(a, b), (b, a) \in P_2(S)$. Then, by definition, $[(a, b), (b, a) \notin P_1(S)$ and $\exists S_1', S_1'', S_2', S_2'' \in \mathcal{X}$ such that $S_1' \subseteq S$, $S_2' \subseteq S$, $S \subseteq S_1''$, $S \subseteq S_2''$, $\{a\} = C(S_1')$, $\{a\} = C(S_1'')$, $\{b\} = C(S_2')$ and $\{b\} = C(S_2'')]$. Hence, $\{a\} = C(S_1')$ and $\{b\} = C(S_2'')$ imply $(b, a) \in P_C^r$. $\{b\} = C(S_2')$ and $\{a\} = C(S_1'')$ imply $(a, b) \in P_C^r$. This contradicts PKWARP.

3) Assume that there exist $a, b \in X$ with $a \neq b$ such that $(a, b) \in P_1(S)$ and $(b, a) \in P_2(S)$. This cannot be by definition of $P_2(S)$.

II. Let us show that C is justified by π. Let $S \in \mathcal{X}$, let $\{a\} = C(S)$ and let $b \in S \setminus \{a\}$.

1) Assume that there exists $S' \subseteq S$ such that $a, b \in S'$ and $\{b\} = C(S')$. Then, by definition, $(a, b) \in P_1(S) \subseteq \pi(S)$.

2) Assume that $\forall S' \in \mathcal{X}$ such that $a, b \in S' \subseteq S$, $\{b\} \neq C(S')$. Then, by definition, $(a, b), (b, a) \notin P_1(S)$. Hence, by definition, $(a, b) \in P_2(S) \subseteq \pi(S)$.

III. Let us show that π is monotonous.

Let $S, S', S'' \in \mathcal{X}$ be such that $S \subseteq S' \subseteq S''$. Let $a, b \in S$ be such that $(a, b) \in \pi(S) \cap \pi(S'')$. Let us show that $(a, b) \in \pi(S')$. i) Assume that $(a, b) \in P_1(S)$. Then, by definition, $(a, b) \in P_1(S') \subseteq \pi(S')$. ii) Assume that $(a, b) \in P_2(S'')$. Then, there exists $S_1 \subseteq S''$ such that $\{a\} = C(S_1)$. Moreover, by definition, $(a, b) \notin P_1(S'')$ which implies by definition, $\{a\} = C(\{a, b\})$ and then, $\forall S_2 \in \mathcal{X}$ such that $a, b \in S_2 \subseteq S'', \{b\} \neq C(S_2)$. Moreover, there exists $S_3 \in \mathcal{X}$ such that $S_3 \supseteq S''$ and $\{a\} = C(S_3)$. Hence, $(a, b) \in P_2(S')$. iii) Assume that $(a, b) \in P_1(S'')$ and $(a, b) \in P_2(S)$. The same proof as ii) shows that $(a, b) \in P_2(S)$ implies $\{a\} = C(\{a, b\})$. Moreover, $(a, b) \in P_1(S'')$ implies that $\exists S_1, S_2 \in \mathcal{X}$ with $a, b \in S_1 \subseteq S_2 \subseteq S''$ such that $\{b\} = C(S_1)$, $\{a\} = C(S_2)$. Hence, $\{a\} = C(\{a, b\})$ and $\{b\} = C(S_1)$ imply $(b, a) \in P_1(S_1) \subseteq P_1(S'')$. This contradicts the asymmetry of $P_1(S'')$. Hence, this case cannot be.

A.2 If:

Let C be a choice function justified by π, monotonous preference relation function. Assume that C does not satisfy PKWARP. Let $a, b \in X$ be such that $(a, b), (b, a) \in P^r_C$. Then, there exist $S_1, S'_1 \in \mathcal{X}$ be such that $a, b \in S_1 \subseteq S'_1$ with $\{b\} = C(S_1)$ and $\{a\} = C(S'_1)$ and there exist $S_2, S'_2 \in \mathcal{X}$ be such that $a, b \in S_2 \subseteq S'_2$ with $\{a\} = C(S_2)$ and $\{b\} = C(S'_2)$. Then, by C being justified by π, $(b,a) \in \pi(S_1)$, $(a,b) \in \pi(S_1')$, $(a,b) \in \pi(S_2)$, $(b,a) \in \pi(S_2')$. Hence, by monotonicity of π, $(a,b) \in \pi(X)$ and $(b,a) \in \pi(X)$. This contradicts the assumption that π is a preference relation function.

B Proof of Theorem 6

Here, we prove Theorem 6 for the sake of completeness, however, our proof is straightforward from [Manzini and Mariotti, 2007].

B.1 Only If:

Let C be a choice function satisfying PKSARP and Expansion.

Let $S \in \mathcal{X}$. Let us define the irreflexive binary relations P_1 and P_2 as follows: $\forall a, b \in X$,

- $(a,b) \in P_1$ if and only if $a \neq b$ and $\forall S \in \mathcal{X}$ $a \in S \Rightarrow \{b\} \neq C(S)$.
- $(a,b) \in P_2(S)$ if and only if $a \neq b$, $\{a\} = C(\{a,b\})$ and $\exists S \in \mathcal{X}$, $a \in S$ and $\{b\} = C(S)$.

I. Let us show that P_1 and P_2 are preference relations, *i.e.* are asymmetric. P_1 and P_2 are irreflexive by definition. Let $a, b \in S$ with $a \neq b$.

1) Let us show that P_1 is asymmetric. Assume $(a,b) \in P_1$. By definition, $\{a\} = C(\{a,b\})$. Then, by definition, $(b,a) \notin P_1$.

2) Let us show that P_2 is asymmetric. Assume $(a,b) \in P_2$. By definition, $\{a\} = C(\{a,b\})$. Then, by definition, $(b,a) \notin P_2$.

II. Let us show that P_2 is acyclic. Assume on the contrary, that P_2 is cyclic. Then, there exist $n \in \mathcal{N} \setminus \{1\}$ and $a_1, ..., a_n \in P_2$ such that $\forall i \in \{1, ..., n-1\}$, $(a_i, a_{i+1}) \in P_2$ and $a_1 = a_n$. Then, $\forall i \in \{1, ..., n-1\}$, $\{a_i\} = C(\{a_i, a_{i+1}\})$ and $\exists S_i \in \mathcal{X}$, $a_i \in S_i$ and $\{a_{i+1}\} = C(S_i)$. Then, by definition,

$(a_{i+1}, a_i) \in P_C^r$. Hence, $\forall i \in \{1, ..., n-1\}$, $(a_{i+1}, a_i) \in P_C^r$ and $a_1 = a_n$. Then, P_C^r is cyclic which contradicts PKSARP.

III. Let us show that C is S-rationalized by (P_1, P_2). Let $S \in \mathcal{X}$ and $a \in S$.

1) Assume that $a \notin M(M(S, P_1), P_2)$. Then, by definition, there exists $b \in S$ such that $(b, a) \in P_1$ or $[b \in M(S, P_1)$ and $(b, a) \in P_2]$. If $(b, a) \in P_1$, then, by definition, $a \notin C(S)$. If $b \in M(S, P_1)$ and $(b, a) \in P_2$, then, $\{b\} = C(\{a, b\})$ and $\forall c \in S \setminus \{b\}, (c, b) \notin P_1$ which implies $\exists S_c \in \mathcal{X}$ such that $\{b\} = C(S_c)$. Hence, by Expansion, $\{b\} = C(\bigcup_{c\in S\setminus\{b\}} S_c \bigcup\{a\})$. Obviously, $\bigcup_{c\in S\setminus\{b\}} S_c \bigcup\{a\} \supseteq S$. Hence, by PKSARP, $\{b\} = C(\bigcup_{c\in S\setminus\{b\}} S_c \bigcup\{a\})$ and $\{b\} = C(\{a, b\})$ imply $\{a\} \neq C(S)$.

2) Assume that $a \notin C(S)$ and $a \in M(M(S, P_1), P_2)$. Let $b \in S \setminus \{a\}$ be such that $C(S) = \{b\}$. By what we showed above, $b \in M(M(S, P_1), P_2)$. Obviously, since by definition $P_1 \cup P_2$ is complete, we cannot have $a, b \in M(M(S, P_1), P_2)$.

B.2 If:

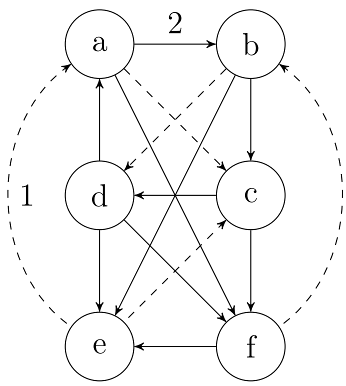
Let C be a choice function S-rationalized by some pair of preference relations (P_1, P_2) with P_2 acyclic.

By Theorem 2, C satisfies Expansion. Let us show that C satisfies PKSARP.

Assume on the contrary that C does not satisfy PKSARP. Then, P_C^r is cyclic. Then, $\exists n \in \mathbb{N} \setminus \{1\}$, $\exists a_1, ..., a_n \in X$ such that $\forall i \in \{1, ..., n-1\}, (a_i, a_{i+1}) \in P_C^r$ and $a_n = a_1$. Then, by definition, $\forall i \in \{1, ..., n-1\}$, $\exists S_i, S_i' \in \mathcal{X}$ such that $a_i, a_{i+1} \in S_i \subseteq S_i'$, $\{a_{i+1}\} = C(S_i)$ and $\{a_i\} = C(S_i')$. By Theorem 2, C satisfies PKWARP. Hence, obviously, $\forall i \in \{1, ..., n-1\}$, $\{a_{i+1}\} = C(\{a_i, a_{i+1}\})$. Let $i \in \{1, ..., n-1\}$. $\{a_{i+1}\} = C(\{a_i, a_{i+1}\})$ implies that $(a_{i+1}, a_i) \in P_1$ or $[(a_i, a_{i+1}), (a_{i+1}, a_i) \notin P_1$ and $(a_{i+1}, a_i) \in P_2]$. If $(a_{i+1}, a_i) \in P_1$, by definition, we cannot have $\{a_i\} = C(S_i')$. Hence, $\forall i \in \{1, ..., n-1\}$, $(a_i, a_{i+1}), (a_{i+1}, a_i) \notin P_1$ and $(a_{i+1}, a_i) \in P_2$ which implies the cyclicity of P_2.

C PKSARP $\wedge$ Expansion $\nRightarrow$ PKWARP $\wedge$ Expansion

Let us have $X = \{a, b, c, d, e, f\}$. Let P_1 and P_2 be given by the following graph.



P_1 be given by the dashed arrows. Hence, there is a dashed arrow from a to b if and only if $(a, b) \in P_1$. P_2 be given by the plain arrows. Hence, there is a plain arrow from a to b if and only if $(a, b) \in P_2$. P_1 and P_2 given in the graph above are such that $M(M(S, P_1), P_2)$ is a choice function. Hence, we define the choice function C such that $\forall S \in \mathcal{X}, C(S) = M(M(S, P_1), P_2)$. By Theorem 2, C satisfies Expansion and PKWARP. Let us show that C does not satisfy PKSARP. We have $C(\{a, b\}) = \{a\}$ and $C(\{a, b, e\}) = \{b\}$ which by definition imply that $(b, a) \in P_C^r$. We have $\{b\} = C(\{b, c\})$ and $\{c\} = C(\{b, c, f\})$ which by definition imply that $(c, b) \in P_C^r$. We have $\{c\} = C(\{c, d\})$ and $\{d\} = C(\{a, c, d\})$ which by definition imply that $(d, c) \in P_C^r$.

We have $\{d\} = C(\{a, d\})$ and $\{a\} = C(\{a, b, d\})$ which by definition imply that $(a, d) \in P_2'$. Hence, C does not satisfy PKSARP.

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