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ANTITRUST VERSUS INDUSTRIAL POLICIES, ENTRY AND
WELFARE

Guy MEUNIER
Jean-Pierre PONSSARD
Francisco RUIZ-ALIZEDA

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DEPARTEMENT D'ECONOMIE

Route de Saclay

91128 PALAISEAU CEDEX

(33) 1 69333033

<http://www.economie.polytechnique.edu/>

<mailto:chantal.poujouly@polytechnique.edu>

Antitrust versus industrial policies, entry and welfare^{*}

Guy Meunier^{a,b} Jean-Pierre Ponsard^b Francisco Ruiz-Aliseda^b
guy.meunier@ivry.inra.fr jean-pierre.ponsard@polytechnique.edu francisco.ruiz-aliseda@polytechnique.edu

^a INRA–UR1303 ALISS

^b Department of Economics, Ecole Polytechnique

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Abstract

In industries with large sunk costs, the investment strategy of competing firms depends on the regulatory context. We consider ex-ante industrial policies in which the sunk cost may be either taxed or subsidized, and antitrust policies which could be either pro-competitive (leading to divestiture in case of high ex-post profitability) or lenient (allowing mergers in case of low ex-post profitability). Through a simple entry game we completely characterize the impact of these policies and examine their associated dynamic trade-offs between the timing of the investment, the ex-post benefits for the consumers, and the possible duplication of fixed costs. We find that merger policies are dominated by ex-ante industrial policies, whereas the latter are dominated by divestiture policies only under very special circumstances.

JEL-Classification: L1, L4, L5

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1 Introduction

In 2013, the UK government struck a deal with the French utility EDF to build the country's first new nuclear plant in a generation (Financial Times, October 21, 2013). The deal guaranteed a price in real terms for 35 years that roughly doubled the current price. In 2014, the UK Competition Commission required that, notwithstanding the late introduction of a new player in 2013 (Mittal), the cement market structure remained too concentrated, and that the dominant players in UK (Lafarge-Tarmac and Hanson) should divest plants to a newcomer (The Telegraph, October 9, 2014). The first example illustrates that industrial policies, and in particular those targeted to mitigate climate change, aim to promote investment possibly to the detriment of the consumers' future welfare due to limited future competition. The second example illustrates that antitrust policies aim to increase current consumer welfare, possibly to the detriment of future investment (and future welfare).

This paper explores the interaction between investment and regulatory policies. In particular, we aim to capture a fundamental ingredient in the underlying welfare analysis, the interdependencies between such regulatory policies that may often be shortsighted, as well as their possible feedback on the long term investment efforts of firms. To this end, we build upon a simple framework used by Cabral (2004) that is known in the literature as the grab-the-dollar game (see Fudenberg and Tirole, 1991, p.127 for a textbook introduction). Grab-the-dollar games are the stationary variant of Fudenberg and Tirole's (1985) seminal work on preemptive strategic investment when the industry exhibits natural monopoly features.¹ In a grab-the-dollar game such as the one we analyze, a limited number of firms repeatedly decide whether or not to invest in a market. Investment is costly and irreversible. After some random time which may eventually be very long and is endogenously determined to some extent, no outsider further invests, and the market structure remains stationary. The limited size of the market implies that ex-post market structure may be either profitable or not.

This model provides a stylized and analytically tractable framework to discuss the welfare impact of entry dynamics. Indeed, Cabral (2004) discusses whether some structural characteristics may lead to a situation of over-investment or under-investment. In case of over-investment (under-investment), the total discounted social welfare would increase (decrease) if investment were taxed (subsidized). We revisit this welfare analysis whenever the

¹As in Fudenberg and Tirole (1985), most of the literature has analyzed games with complete information. In particular, natural extensions of their duopoly setting to one of oligopoly have been studied by Vettas (2000) and Shen and Villas-Boas (2010) (in stationary settings) as well as Argenziano and Schmidt-Dengler (2014) (in nonstationary settings). It is worth noting though that Levin and Peck (2003) have analyzed a stationary variant of Fudenberg and Tirole (1985) when the sunk cost of entry borne by each firm constitutes private information.

social planner commits to some antitrust policy before firms choose their investment behavior. We consider three antitrust policies: no intervention, i.e., the base case for the model; a "lenient" policy in which mergers are allowed whenever the ex-post market structure generates a profitability deemed too low; a "pro-competitive" policy in which divestiture is forced whenever the ex-post market structure generates a profitability considered high. The impact of such an active antitrust policy will always be favorable ex-post. The model allows for exploring the feedback of investment on ex-ante welfare, which constitutes the focus of the analysis.

There is already a significant amount of literature on the interaction of certain competition policies and R&D incentives, pioneered by Segal and Whinston (2007). Shapiro (2012) provides a useful and recent discussion of the interaction of merger policy and R&D, as opposed to the main focus of Segal and Whinston (2007) on exclusionary behavior. Gans and Persson (2013) extend Segal and Whinston's (2007) framework to show that innovation policy and antitrust policy complement each other when firms can engage in cooperative forms of R&D such as licensing. See also Baker (2007) for various economic arguments of when antitrust interventions are most likely to foster innovation. As for R&D, antitrust policy concerns the extent of market power that should be allowed ex-post but the public good nature of R&D implies that both the welfare analysis and the regulatory instruments differ from the ones we highlight.

In contrast with the R&D literature, there has been much less research on the interdependence of entry incentives and antitrust policy. To the best of our knowledge, Gowrisankaran (1999) is the first one to point out this issue relative to merger policy. Typically, the welfare analysis of mergers is done in a static framework (see e.g. Salant, Switzer and Reynolds, 1983, and Farrell and Shapiro 1990), but Gowrisankaran (1999) develops a dynamic computational model along the lines of Ericson and Pakes (1995) in which firms endogenously decide whether or not to merge at each period. Allowing for mergers has an impact on the entry probability, the long term market structure, and eventually the associated welfare. This model is intended to illustrate that it could be feasible to carry out a welfare analysis in a concrete antitrust case. More recently, Mason and Weeds (2013) have examined the effect of merger policy on entry incentives when an incumbent is already active in the market. They analyze some determinants of the socially optimal merger policy, and then compare it with an entry subsidization policy. Their paper and ours differ in a number of respects: policies in our setting are set before the market structure forms, and our parameters of interest are the size of the sunk costs of entry and the intensity of competition, whereas they study changes in other parameters; we also consider a larger range of policies, such as forced divestiture. Despite these differences, it is worth noting that our results confirm their find-

ing that merger policy is dominated by ex-ante industrial policy when firm subsidies involve negligible distortions at the economy-wide level.²

Our model allows for an extensive analysis of the impact of an antitrust policy with respect to three key aspects: ex-post welfare (consumer surplus and industry profits per period once the market structure becomes stationary), the delay to achieve the stationary market structure, and cost duplication. We shall not only consider allowing or not for mergers, but also analyze an antitrust policy which forces divestiture in case that a highly concentrated market arises. We obtain a complete characterization of the conditions under which the ex-ante discounted welfare increases when the feedback on investment is taken into account. Moreover, these antitrust policies will be compared to an ex-ante industrial policy which would either subsidize or tax investment.

We shall proceed as follows. In our framework, for analytical tractability, the number of competing firms is limited to two, as in Cabral (2004) and Mason and Weeds (2013). In a static context, the investment cost and the market size would be such that only a monopolist is profitable. We consider an infinite-horizon setting in which firms decide their investment actions simultaneously. In each period, the situation is one of the following three cases: (i) no firm invested in the past (in which case firms need to decide again whether or not to invest); (ii) there is a monopoly, and; (iii) there is a duopoly. The latter two cases can be seen as absorbing states of the game: the firm(s) active in the market remain in place forever, and the other firm never enters. Following Cabral (2004), it is convenient to formalize the industry characteristics through two key parameters: the ratio of the investment cost to the monopoly profit, and the ratio of the duopoly profit to the monopoly profit. The higher the first parameter, the higher is the potential inefficiency in case of entry cost duplication. The lower the second parameter, the more intense is the ex-post competition in case of duopoly.

Our main contribution is to shed light on the effects of various commonly used policies on the ex-ante incentives to invest in a socially optimal manner given that the industry may end up exhibiting substantial ex-post market power. Thus, we explicitly determine the unregulated probability of entry for all combinations of our two key parameters, and then study how such a probability is affected by the various policies examined (i.e., no regulation, ex-ante industrial policy and ex-post antitrust policies). This allows for an explicit ranking of all policies in terms of welfare. We show that allowing mergers is always dominated by an appropriate ex-ante industrial policy, even though it would almost achieve the same welfare in those cases in which the probability of entry is small. The industrial policy also dominates

²This result follows from their fourth proposition assuming no extra cost of public funds ($R = 1$), a welfare function that puts equal weight on consumer surplus and industry profit ($\lambda = 1$), and the lack of "ineffective entry" ($\rho = 0$).

the divestiture policy, except whenever the probability of entry is close to one: in such cases, it would be beneficial to divest a monopoly into a duopoly. As soon as entry becomes less likely, however, we find that a divestiture policy would completely discourage entry. To the best of our knowledge, the current work provides the first welfare comparison of policies that are widely used by government agencies, so we can provide some guidance about their relative pros and cons.

The remainder of the paper is organized as follows. Section 2 introduces the model, solves it for the base case (unregulated equilibrium), and characterizes the optimal industrial policy. Section 3 analyzes the impact of the antitrust policies (whether to allow mergers or force divestiture). Section 4 discusses the policy relevance of the results using a numerical illustration. Section 5 concludes by summing up the results, and suggesting how this work could be extended.

2 The model

We consider a market for a homogeneous good and two potential entrants. A firm is active if it has already entered, or else it is inactive. Any inactive firm can enter the market and immediately become active by paying a fixed sunk cost $K > 0$. Time is discrete and infinite, and at each time $t = 0, 1, \dots$ inactive firms simultaneously decide whether to enter or not. We denote π_n for the individual per-period profit if $n \in \{1, 2\}$ firms are active in the market, and we assume it is collected at the end of the corresponding period (this is just a normalization). Firms discount future payoffs at rate $r > 0$, with $\delta \equiv (1 + r)^{-1}$ denoting the discount factor. We assume that

$$0 \leq \pi_2 < rK < \pi_1, \tag{A1}$$

so the market has the potential to be profitable for at most one firm and exit never takes place (voluntarily).

The per-period consumer welfare is denoted by s_n if $n \in \{1, 2\}$ firms are active. Also, $w_n \equiv s_n + n\pi_n$ denotes the per-period social welfare generated by $n \in \{1, 2\}$ active firms. We use capital letters for discounted streams of surplus: $\Pi_n = \sum_{t=1}^{+\infty} \delta^t \pi_n = \pi_n/r$ for a firm's profit, $S_n = s_n/r$ for consumers' surplus and $W_n = w_n/r$ for the social welfare if $n \in \{1, 2\}$ firms are active from the current date on. For convenience, define $A_n \equiv W_n - nK$ for $n \in \{1, 2\}$.

Note that the product homogeneity assumption allows us to write consumer surplus and social welfare as a function of industry profit: $w(n\pi_n)$ and $s(n\pi_n)$ ($n = 1, 2$), respectively. Thus, $w_n \equiv w(n\pi_n) = s(n\pi_n) + n\pi_n$. Welfare is decreasing with respect to industry profit

because of the reduction in (allocative) efficiency, so consumer surplus is decreasing in such a profit with a slope smaller than -1 . We shall assume that the per-period social welfare is weakly increasing and concave in the number of active firms:

$$w_2 \geq w_1 \text{ and } w_1 \geq w_2 - w_1. \quad (\text{A2})$$

The second condition in (A2) asserts that the entry of a single firm creates more gross welfare than the addition of a second firm; alternatively, a monopoly achieves more than half the gross welfare gains of a duopoly. A direct implication is that if a monopoly does not create net welfare (i.e., $w_1 < rK$), then neither will a duopoly.³

Our model can be seen as depending on two key parameters: on the one hand, π_2/π_1 represents the intensity of competition, ranging from 0 (pure Bertrand competition) to $1/2$ (collusion); on the other hand, rK/π_1 represents the profitability of the market for a monopoly: as rK/π_1 increases from π_2/π_1 to 1, the profitability declines from the limit profitability needed to sustain a duopoly to the limit profitability needed to sustain a monopoly.

2.1 Unregulated equilibrium

Given our assumptions on profits, there are two stationary equilibria in pure strategies, with one firm entering in every period and the other staying out. These equilibria are asymmetric, so each firm should anticipate perfectly its role as an entrant or as a firm that remains inactive. From now on, we follow Cabral (2004) and consider symmetric equilibria in stationary strategies. In subgames in which one firm is already active, the other firm chooses not to enter with probability one. The discounted payoff of an active firm when $n \in \{1, 2\}$ firms have entered is then Π_n , because no further entry will take place. And, when $n \in \{1, 2\}$ firms have entered, the expected payoff of an inactive firm is zero.

In subgames with no active firm, symmetric behavior can be easily seen to require randomization. In particular, the two inactive firms must be indifferent between entering and waiting one more period to make the entry decision. Suppose that any one of the inactive firms chooses to enter with probability $p_0 \in (0, 1)$. Denote the other one's expected payoff if it waits by V_w , and let V_e denote its payoff if it immediately enters, so that stationarity

³Assumption (A2) holds for all $\pi_2 \in [0, \pi_1/2]$ if and only if it holds for $\pi_2 = 0$ (perfect competition), that is, a monopoly should achieve more than 50% of the maximal welfare. It always holds for the market of a homogeneous product with $p(q) = a - bq^\alpha$ in which $\alpha > -1$ and $b\alpha > 0$. In that case, the percentage welfare loss associated with a monopoly is $1 - (2 + \alpha)/(1 + \alpha)^{(\alpha+1)/\alpha}$, which is smaller than 50% (for an extensive discussion of welfare losses in Cournot oligopoly, see Corchón, 2008). Furthermore, if the demand function is concave, Johari and Tsitsiklis (2005) show that the percentage welfare loss is at most 33% for a monopoly.

yields

$$V_e = p_0\Pi_2 + (1 - p_0)\Pi_1 - K$$

and

$$V_w = (1 - p_0)\delta \max(V_w, V_e).$$

Because $V_w = V_e$ in a mixed-strategy equilibrium with stationary strategies, we have that $V_w = V_e = 0$, that is, the equilibrium mixing probability p_0^* solves:

$$p_0^* \frac{\pi_2}{r} + (1 - p_0^*) \frac{\pi_1}{r} - K = 0. \quad (1)$$

It readily follows that the equilibrium probability as a function of π_1 , π_2 and rK is

$$p_0^*(\pi_1, \pi_2, rK) = (\pi_1 - rK)/(\pi_1 - \pi_2), \quad (2)$$

a positive value not exceeding one because $\pi_1 > rK > \pi_2$. The expected payoff of an inactive firm in these subgames is $E_0^* = 0$. That all profits from entry are dissipated in expectation because of aggressive entry behavior is clear given that there is room for only one firm in the market. The symmetric equilibrium probability of entry increases as either π_1 or π_2 grow and as the (per-period) cost of capital rK falls, all of them intuitive results as well.

2.2 Ex-ante industrial policy

Suppose now that a social planner could indirectly control the (nondiscriminatory) entry probability of firms by somehow taxing or subsidizing the sunk cost. Since a higher (lower) sunk cost decreases (increases) the entry probability, taxation (subsidization) could be referred as a situation of “excessive” (“insufficient”) entry. This is indeed the approach followed by Cabral (2004): he studies the net marginal welfare gain relative to the sunk cost of entry. Even though he simply focuses on limit cases within the parameter space, we will focus on the entire parameter space, extending his pioneering analysis. To this end, we shall directly derive the optimal entry probability and then obtain the related tax or subsidy as well as the associated welfare.

Recall that W_n denotes the discounted social welfare when $n \in \{1, 2\}$ firms are active and that $A_n \equiv W_n - nK$ for $n \in \{1, 2\}$. If the planner chooses an entry probability of p , then the equation of motion for expected welfare given that no firm is yet active is

$$W_0(p) = 2p(1 - p)A_1 + p^2A_2 + (1 - p)^2\delta W_0(p), \quad (3)$$

which gives

$$W_0(p) = \frac{p[2(1-p)A_1 + pA_2]}{1 - (1-p)^2\delta}. \quad (4)$$

For $p \in (0, 1)$, an increase in the probability of entry on welfare has the following effect (by differentiation of (3)):

$$[1 - \delta(1-p)^2] \frac{\partial W_0}{\partial p} = 2p(A_2 - A_1) + 2(1-p)(A_1 - \delta W_0). \quad (5)$$

An increase in the probability of entry makes it more likely that a duopoly results rather than a monopoly (first term on the right hand side), but also makes it more likely that a monopoly results as opposed to having no entry at all (second term). Which aspect dominates depends upon the probability of entry. The following proposition makes this precise.

Proposition 1 *The discounted social welfare stream is (globally) maximized*

- at $p = 0$ if $w_1 \leq rK$;
- at $p = 1$ if $rK \leq w_2 - w_1$;
- at

$$p = \hat{p} \equiv \frac{\sqrt{(2w_1 - w_2)^2 + 4(w_1 - rK)(w_1 + rK - w_2)/r} - (2w_1 - w_2)}{2(w_1 + rK - w_2)/r} \in (0, 1)$$

otherwise.

Proof. See Appendix. ■

The social planner trades-off the social cost of capital (rK) against how much competition it would like to generate, keeping also in mind that entry delays may be socially costly. Given these aspects that the planner takes into account, the socially optimal probability of entry captures intuitive features. Indeed, the first two cases are clear: no entry is pursued when the per-period social cost of capital exceeds the per-period welfare generated by a monopoly; also, the immediate entry of the two firms is optimal if discounted welfare with a duopoly relative to a monopoly is higher than the per-period social cost of having one more firm active in the market. In the third case, that is, in circumstances in which a monopoly would be the social planner's preferred market structure if it could directly enforce it, the optimal probability of entry trades-off the probability of excessive entry with the risk of delay. In such a case, the optimal probability of entry can be shown to grow as the per-period cost of capital falls or as the intensity of competition increases, which constitutes our next result.

Proposition 2 Lowering either π_2 or K lead the social planner to pursue a larger extent of entry.

Proof. See Appendix. ■

Based on this proposition and on the fact that the equilibrium probability grows with π_2 , it is clear that, for any $K \in (0, \pi_1/r)$, there exists a value $\pi_2^C(K) \in (0, \pi_1]$ such that there is excess entry if $\pi_2 > \pi_2^C(K)$ and insufficient entry otherwise.⁴ There is a clear tension related to the intensity of competition: if competition is more intense, entry is more desirable but less likely in equilibrium. From a social welfare standpoint, the unregulated equilibrium results in insufficient entry for low π_2 , and in excessive entry for high π_2 . As for the entry cost, an increase in K reduces the equilibrium probability of entry as well as the socially optimal one. The monotonicity of the comparison is ambiguous. Still, if the fixed cost is large (close to Π_1) the probability of entry is close to zero, and (by Proposition 1, since $w_1 < rK$ in such cases) the unregulated equilibrium probability of entry leads to insufficient entry because of the cost of delaying an investment that would create some consumer surplus. In that case, a monopoly is much more likely than a duopoly and the intensity of competition has a negligible effect on the comparison. In Figure 1, we plot on the $(rK/\pi_1, \pi_2/\pi_1)$ -plane the areas in which there is socially insufficient entry and socially excessive entry. The shapes of the areas illustrates the preceding discussion. In figure 1, we use a linear demand and a constant marginal production cost and $r = 0.1$ (Cabral, 2004, Fig. 2 depicts $r = 0.01$). More on this numerical illustration in section 4.

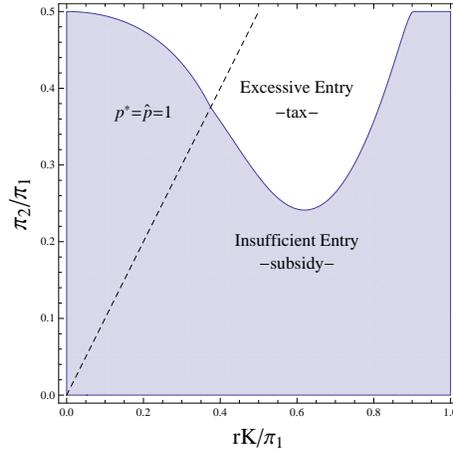


Figure 1: The ex-ante industrial policy as a function of the fixed cost (rK/π_1) and the intensity of competition (π_2/π_1) with a linear demand and $r = 0.1$. The dotted line corresponds to $\pi_2 = rK$.

⁴Note that Cabral (2004) simply gives sufficient conditions for excessive entry based on π_2 , whereas our conditions are both necessary and sufficient.

It is worth stressing that, in our framework, maximizing expected total welfare is equivalent to maximizing consumer surplus net of the expected subsidy (provided there is no extra cost of public funds, an aspect that is easy to incorporate into the analysis). The point is that all the subsidies received by firms are dissipated due to the more intense competition for the market (in equilibrium, expected net profits are null). More formally, for a subsidy $\sigma \in \mathfrak{R}$ given upon entry, the equilibrium probability of investment becomes $p^*(\pi_1, \pi_2, r(K - \sigma))$, written p^* as a shorthand in the remainder of this subsection. Social welfare in expression (4) could be rewritten by isolating expected consumers surplus and firm profits. Then with the aid of expression (1) (having replaced K with $K - \sigma$), we can eliminate the profits so that the following holds:

$$\begin{aligned} W_0(\sigma) &= \frac{p^*}{1 - (1 - p^*)^2\delta} [2(1 - p^*)(S_1 - \sigma) + p^*(S_2 - 2\sigma) \\ &\quad + 2(1 - p^*)(\Pi_1 - (K - \sigma)) + p^*(2\Pi_2 - 2(K - \sigma))] \\ &= \frac{p^*}{1 - (1 - p^*)^2\delta} [2(1 - p^*)(S_1 - \sigma) + p^*(S_2 - 2\sigma)] \\ &= \frac{p^*}{1 - (1 - p^*)^2\delta} [2(1 - p^*)S_1 + p^*S_2] - \bar{\sigma}, \end{aligned}$$

where $\bar{\sigma} \equiv 2p^*\sigma/[1 - (1 - p^*)^2\delta]$ denotes the discounted expected public cost of an ex-ante policy that implements a probability of entry equal to p^* through a subsidy of size σ . Treating $\bar{\sigma}$ as a function of p^* helps to clarify the trade-off generated by the ex-ante industrial policy:⁵ the benefit for consumers owing to a greater probability of entry should be compared with the larger expected budget of the policy.

3 Ex-post antitrust policies

Two antitrust interventions are considered in this section: allowing duopolists to merge or forcing a monopolist to divest. Intuitively, allowing mergers would be appropriate in case of insufficient entry, whereas divestiture would be desirable in case of excessive entry. However, each antitrust intervention modifies the probability of entry through a modification of the ex-post profit of firms. Our analysis sheds light on the relationship.

⁵Note that the subsidy could be written as a function of the equilibrium probability of entry: $\sigma = s(p^*) \equiv (\pi_1/r - \pi_2/r)p^* - (\pi_1/r - K)$, which is strictly increasing. The expected subsidy $\bar{\sigma}$ then becomes a monotone function of p^* (differentiation shows that it is strictly increasing), so it is equivalent for the planner to choose the probability directly or to work with $\bar{\sigma}$.

3.1 Decentralized entry with merger in duopoly

We first consider a situation where the social planner allows firms to merge if the market structure happens to be a duopoly. When firms are allowed to merge, there is a welfare loss in the duopoly case relative to the unregulated equilibrium. The possible gain arises from the greater incentive to enter the market, though. If there is already excessive entry, allowing merger would both increase the inefficiencies associated to cost duplication and those associated to market power. Therefore, it can only be welfare enhancing to allow mergers in situations in which entry is insufficient. This leads to the following proposition.

Proposition 3 *Allowing mergers can only be welfare enhancing in a situation of insufficient entry.*

Further clarification of the relationship between this antitrust policy and insufficient entry can be obtained on the basis of Proposition 2 and Figure 1. Entry might be insufficient because of intense competition (low π_2) or a large fixed cost. If entry is insufficient because competition is too intense, softening competition by allowing merger cannot increase welfare. However, if entry is insufficient because of a large fixed cost that is hardly covered by monopoly profit, it could be welfare enhancing to promote entry by allowing mergers. In such a case, a monopoly is much more likely than a duopoly and still better than nothing, so sacrificing welfare in the (unlikely) duopoly outcome is worthwhile. This reasoning explains why, in Figure 1, merger dominates on the right (large fixed cost) of the insufficient entry area. Even though the linear case is further described in Section 4, the following proposition formally states this result in the general case.

Proposition 4 *For all π_2 there is a threshold entry cost $\tilde{K}(\pi_2)$ below π_1/r such that mergers should be allowed for $K > \tilde{K}(\pi_2)$.*

Proof. See Appendix. ■

3.2 Decentralized entry with asset divestiture in monopoly

Asset divestitures only concern monopoly situations, and they are modeled in a simple way: if only one firm happens to enter the industry, the regulator (instantaneously) forces it to resell half of its assets. Two cases need be distinguished: resale to an outsider firm or resale to the firm that remains inactive. The latter case is more complicated since a waiting motive may arise, so we first discuss the former case, and then extend the results to the latter one.

We assume that purchasing divested assets allows a firm to avoid having to pay K . In these conditions, the divestiture results in a duopoly without duplication of the fixed costs,

so s_1 is equal to s_2 and A_1 is equal to $A_2 + K$. Even though the regulator cannot directly control the entry probability, it can indirectly do so via the value of the assets divested. More precisely, we assume that the regulator can choose the share $\alpha \in [0, 1]$ of the value of divested assets that the divested monopoly obtains, so it holds that $\pi_1 = (1 + \alpha)\pi_2$, and α can be interpreted as a parameter that captures the incumbent's bargaining power. If the planner set institutional rules that induced intense competition for the divested assets, α would be close to 1; in a situation close to asset expropriation, α would be close to 0, however. Even if the regulator might not have a large control over α in some occasions, providing the planner with this lever is useful from an abstract point of view, since it enjoys an additional instrument to implement its desired outcome. As we shall see, the social planner cannot improve upon ex-ante industrial policies in many situations even in the best case scenario in which it has this extra lever at its disposal. And even if it can, this requires a very precise knowledge of industry features, since small mistakes in their assessment may have dramatic consequences. As we argue in the next section, this lack of robustness, coupled with the possibility that it may be hard for the regulator to commit ex-ante to a specific value of α , suggests that it is hard to defend divestitures as an approach preferred over ex-ante industrial policy.⁶

Turning to the formal analysis, when a firm foresees that a monopolist will be (immediately) forced to divest, the equilibrium entry probability becomes

$$p_0^* = \frac{(1 + \alpha)\pi_2 - rK}{\alpha\pi_2}, \quad (6)$$

which is an increasing function of α , as one would expect. By choosing α , the regulator can modify the probability of entry without affecting the ex-post efficiency. However, the regulator is constrained by the total value of the assets. Recalling that π_2 represents the extent of competition when in duopoly, one can then prove the following result.

Proposition 5 *Suppose that the planner can fully control the price of divested assets. Then there exists $\bar{\pi}_2 \in (rK/2, rK)$ such that*

- *if $\pi_2 > \bar{\pi}_2$, then the planner's optimal choice is $\alpha^* < 1$.*
- *if $\pi_2 < \bar{\pi}_2$, then the planner's optimal choice is $\alpha^* = 1$.*
- *if $\pi_2 < rK/2$, entry never occurs and the expected welfare is null.*

⁶Note also that divestiture has no ex-post social costs (e.g., no share of K needs to be paid by the firm that gets the divested assets), so the scenario we consider should be highly favorable towards divestiture.

Proof. See Appendix. ■

The proposition highlights the fact that if competition is anticipated to be mild, entry is not highly desirable, and in addition the economic value of divested assets is high. Consequently, the regulator can implement the optimal extent of entry given the ex-post duopoly structure by giving only a share of the value of divested assets to the divested monopoly. If competition is intense, entry is highly desirable but not very profitable. Taking into account that the regulator is constrained by the low value of divested assets, the firm that initially entered as monopolist gets the full economic value of its divested assets, even though this will still induce insufficient entry from a welfare perspective, given the allocative efficiency of a duopoly. In the extreme case in which competition is very fierce and the fixed cost of entry is relatively high, asset divestiture completely eliminates the incentive to enter and the expected welfare is null. In such a case, a duopoly does not generate enough profit to cover the fixed cost.

Since the planner can implement the optimal extent of entry for $\pi_2 > \bar{\pi}_2$, it should come as no surprise using a continuity argument that asset divestiture is socially useful even when π_2 is below $\bar{\pi}_2$. In particular, the following holds.

Proposition 6 *There is a threshold $\tilde{\pi}_2 \in (rK/2, \bar{\pi}_2)$ such that expected welfare is strictly higher with asset divestiture than without it if and only if $\pi_2 > \tilde{\pi}_2$.*

Proof. See Appendix. ■

This proposition can be compared to Proposition 3. When competition is mild (i.e., π_2 has a high value), if there is excessive entry in an unregulated equilibrium, divestiture will be welfare enhancing as long as the regulator controls the resale value of the assets. In that case, divestiture achieves both a short-term gain by enhancing competition in the product market and a long-term one by reducing the extent of entry. However, as will be shown shortly in our numerical illustration in the next section, it may well hold that divestiture is welfare enhancing even if there is insufficient entry. This situation arises if the gain in the post-entry welfare offsets the lower probability of entry. Our illustration will also show that this gain rapidly turns into a loss as duopoly competition becomes more intense. With fierce competition, the full value of a duopoly is so low that firms are reluctant to enter and welfare is reduced. At the extreme, asset divestiture fully discourages entry. To sum up, while we could expect that divestiture would be an appropriate policy whenever there is excessive entry, a detailed analysis shows that the situation is not clear-cut.

Suppose now that the assets up for resale are (immediately) bought by the other potential entrant. If only one firm enters, the other firm obtains a profit $(1 - \alpha)\pi_2$ from the divested assets. This profit should be taken into consideration by a firm contemplating the

profitability of entry. Suppose that a firm enters with probability p_0 . Then, if the other firm enters with probability p_0 as well, the firm must expect to gain

$$V_e = p_0\Pi_2 + (1 - p_0)(1 + \alpha)\Pi_2 - K.$$

Staying out yields

$$V_w = p_0(1 - \alpha)\Pi_2 + (1 - p_0)\delta \max(V_e, V_w).$$

In equilibrium, $V_w = V_e$ must hold, so the equilibrium entry probability p_0^* solves

$$p_0^*\Pi_2 + (1 - p_0^*)(1 + \alpha)\Pi_2 - K = p_0^*(1 - \alpha)\Pi_2 \frac{r + 1}{r + p_0^*}.$$

The left-hand side is the value of entering, which is decreasing with respect to the probability that the rival enters, and the right-hand side is the value of waiting and possibly obtaining the divested assets, which increases with the probability of the rival's entry. From this equation, one can conclude that there is a unique equilibrium probability of entry, it is positive if $(1 + \alpha)\Pi_2 > K$ and null otherwise.

In that case, the equilibrium probability of entry is still increasing with respect to α : for $\alpha = 1$, it is equal to the probability obtained with a third party given by equation (6); otherwise, it is strictly lower if it is positive (i.e., when $\alpha > (K - \Pi_2)/\Pi_2$) or obviously it is null. Similar results as those in Propositions 5 and 6 could be obtained, but the value of the divested assets that a monopoly obtains should be larger in that case, that is, the regulator should anticipate that the perspective of obtaining the divested assets will soften the incentive to enter.

4 An illustration

4.1 A linear specification

To illustrate our findings, let us assume that the inverse demand is $P(q) = a - bq$ for a total production q and that the marginal cost production is constant, in which case we can set it equal to zero without loss of generality. The unregulated monopoly profit is obtained for a quantity of $q_m = a/(2b)$ and it is equal to $\pi_m = bq_m^2 = a^2/(4b)$. Welfare can be written as a function of the total production q : $w(q) = (a - bq/2)q = s(q) + \pi^*(q)$, where $s(q) = bq^2/2$ stands for consumer surplus and $\pi^*(q)$ for industry profit. Note that Assumptions (A1) and (A2) are satisfied. Both consumer surplus and welfare could be written as a function of the industry profit π^* as follows: the profit obtained with a production

q is $\pi^*(q) = (a - bq)q = \pi_m - b(q - q_m)^2$, so that the quantity $q(\pi^*) > q_m$ corresponding to profit π^* is $q(\pi^*) = q_m + [(\pi_m - \pi^*)/b]^{1/2}$, and consumer surplus and total welfare are $s(q(\pi^*))$ and $w(q(\pi^*))$, respectively.

The results can be displayed in a two-dimensional diagram. The horizontal axis represents rK/π_1 varying from 0 to 1, whereas the vertical axis represents π_2/π_1 varying from 0 to .5 (that is, as q goes from a/b to q_m). In the area in which $\pi_2/\pi_1 < rK/\pi_1$, Assumption (A1) is satisfied, i.e. the equilibrium probability of entry is smaller than one. Otherwise, it is equal to one and the market structure is a duopoly. In the former area, the entry probability declines as rK/π_1 increases and becomes null exactly when $rK/\pi_1 = 1$. In the latter area, our model should be extended to allow for more competitors.

Without loss of generality, we take $a = 1$ and $b = 1/4$ so that $\pi_1 = 1$; the discount rate is $r = 0.1$, so $\Pi_1 = 10$. The gross welfare associated with a monopoly is $W_1 = 15$, whereas the maximum gross welfare obtained with a Bertrand duopoly is $W_2 = 20$.

4.2 Comparison of the policies

Given this specification, we first derive the areas in which there is insufficient entry or excessive entry, respectively (see Figure 2(a)). In case of insufficient entry, the darker area depicts a configuration in which the optimal subsidy leads to an entry probability of one, extending the duopoly area to the right. Conversely for the case of excessive entry, the duopoly area is restricted on the left.

In Figure 2(b), we depict the optimal antitrust policy. As expected, merger policy dominates only in the area in which there is insufficient entry (Proposition 3). Divestiture may be optimal whenever there is excessive entry, but this is neither necessary nor sufficient. We can gain further insights from these results by drawing the welfare as a function of π_2/π_1 for two given values of rK/π_1 , namely 0.6 (high monopoly profit) and 0.9 (low monopoly profit). This gives Figure 3 and Figure 4. In these figures, the vertical axis units are measured as a percentage of the welfare obtained with a regulated monopoly, which we denote by W_{RM} and note that it is independent from the horizontal axis (W_{RM} is obtained by setting the price of the good so that the per-period profit of the firm equals rK).⁷ This shows that the policies at hand induce a large welfare loss, in particular in the case of low monopoly profit (Figure 4). We shall return to this aspect shortly, but consider first the comparison between ex-ante and ex-post policies. Observe, on the one hand, that an ex-ante industrial policy always dominates merger, although the difference gets smaller as rK/π_1 becomes close to 1

⁷We consider a first-best regulation (having the fixed investment cost be directly paid by consumers while selling the product at marginal cost) to be irrelevant for our comparison. For an extensive presentation of the theory of the regulation of a natural monopoly, see Train (1991).

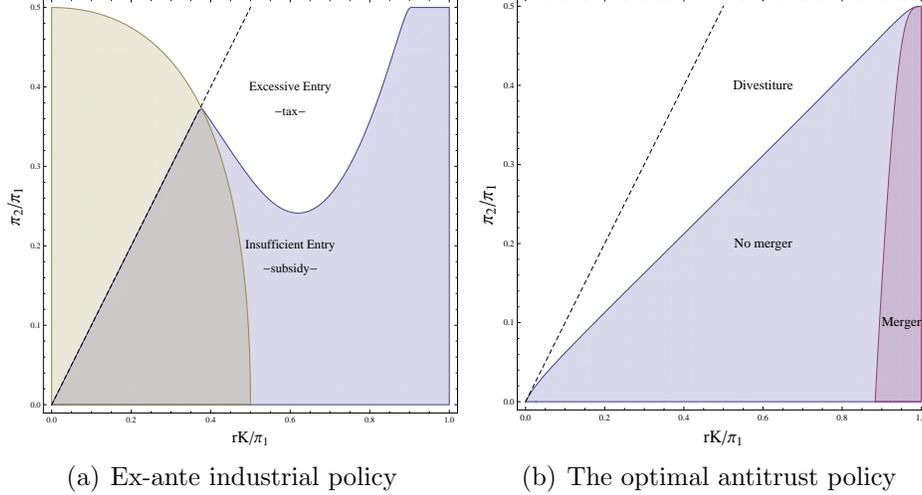


Figure 2: Optimal policy as a function of the intensity of competition and sunk cost. The dotted line corresponds to $\pi_2 = rK$ above it the equilibrium probability of entry is 1. In the brown area the optimal probability of entry is 1.

(Figure 3). On the other hand, divestiture may be better than an ex-ante industrial policy, but it may also be much worse; in case of intense duopolistic competition (low π_2/π_1), a divestiture policy might totally discourage entry.

We return now to the welfare loss. It is conceivable to design a policy that combines both ex-ante and ex-post features though its implementation is unlikely since it would require coordination between currently independent regulatory agencies. Such a “hybrid” policy would encourage investment ex-ante through subsidies but discourage market power ex-post through divestiture of monopoly. It is a simple matter to prove that this hybrid policy dominates both a pure ex-ante industrial policy and a pure divestiture policy,⁸ and it significantly reduces the welfare loss relative to the case of the regulated monopoly (see Figures 3 and 4).

It is also important to stress that the policies under consideration differ in terms of commitment. An industrial policy is a once-and-for-all commitment. For antitrust policies they are typically implemented after the fact, that is, ex-post and rarely explicit ex-ante. In our framework, we assume that the authorities can commit ex-ante. To emphasize the role of this assumption, consider for instance a situation in which the authorities are supposed to implement an industrial policy and no antitrust policy, and then, after the fact, “hold up”

⁸With this hybrid policy, the regulator subsidizes entry and divests a monopoly into a duopoly. Writing $W_0(w_1, w_2, r, K, p)$ in expression (4), it holds that expected welfare with the hybrid policy is then $\max_p W_0(w_2, w_2, r, K, p)$, which is higher than welfare with the ex-ante industrial policy (since $W_0(w_2, w_2, r, K, p) \geq W_0(w_1, w_2, r, K, p)$ for all $p \in [0, 1]$), and higher than welfare with divestiture (it is equal if $\alpha < 1$ and strictly higher if $\alpha = 1$).

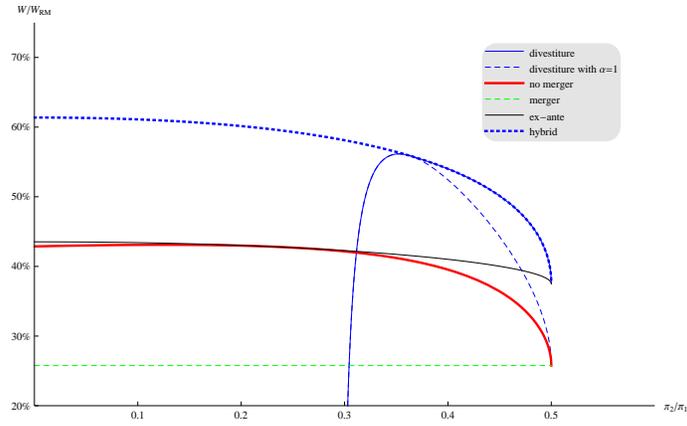


Figure 3: Welfare relative to welfare with a regulated monopoly ($W_{RM} = 19.3\$$) as a function of the degree of competition for $K = 0.6\pi_1/r$.

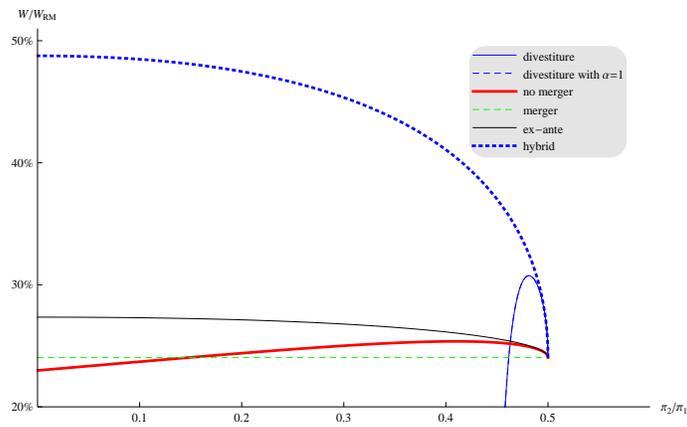


Figure 4: Welfare relative to welfare with a regulated monopoly ($W_{RM} = 17.6\$$) as a function of the degree of competition for $K = 0.9\pi_1/r$.

a resulting monopolist by forcing it to divest. This may be more realistic than our hybrid policy.⁹ It may reduce the welfare loss as long as the firms are myopic. We leave the analysis of such out-of-equilibrium scenarios for further research.

4.3 How each policy balances the trade-offs

Each policy trades-off three aspects: (i) the ex-post market structure (a duopoly being more favorable to the consumers; more generally, the probability of getting a monopoly versus that of getting a duopoly); (ii) the delay in obtaining such ex-post market structure (the earlier the better, so, everything else equal, a higher probability of entry would be better), and; (iii) the expected discounted investment costs (a duopoly duplicates fixed costs). Figure 5 and Figure 6 explicit the trade-offs associated to each policy.

Take the no-merger policy as the benchmark (section 2.1). In a configuration such as $\pi_2/\pi_1 = 0.4$ and $rK/\pi_1 = 0.6$, we have excessive entry (see Figure 2(a)). An ex-ante policy would tax investment, increase the delay, make a monopoly situation more likely and reduce the overall expected investment cost. A divestiture policy would increase the delay even further, make sure that a duopoly prevails, and reduce the expected fixed cost even more. As can be seen in Figure 3, divestiture dominates the ex-ante policy; alas, as soon as $\pi_2/\pi_1 < 0.3$, divestiture leads to no entry. For $rK/\pi_1 = 0.9$, the range on which divestiture dominates shrinks to approximately $[0.46, 0.5]$ (see Figure 4).

As can be seen in Figures 5 and 6, a merger policy arbitrates in favor of reducing the delay, at the cost of guaranteeing that a monopoly prevails and at the expense of increasing the entry cost. It is only appropriate in case of insufficient entry due to a very high investment cost relative to the size of the market (i.e. $rK/\pi_1 = 0.9$). Then its sensitivity relative to π_2/π_1 is low. The interested reader will note in Figure 7 that differences among policies persist in the neighborhood of $\pi_2/\pi_1 = 0.5$ for $rK/\pi_1 = 0.9$.

4.4 Why a dynamic structure matters

The linear specification can also help to understand the effect of the dynamic structure of our framework. To do so, it is worth comparing our dynamic game to a “static game” in which firms first decide whether to enter and then produce to earn the stream Π_i for $i = 1, 2$. This is the typical framework of a two-stage entry game, and Mason and Weeds (2013) analyze merger control in such a framework. The difference with the dynamic game is that entry,

⁹In some sectors such as electricity, we have rather observed a sequence of divestiture of monopolies followed by investment subsidies for capacity investments. See for instance Busnello (2014) for the Italian case.

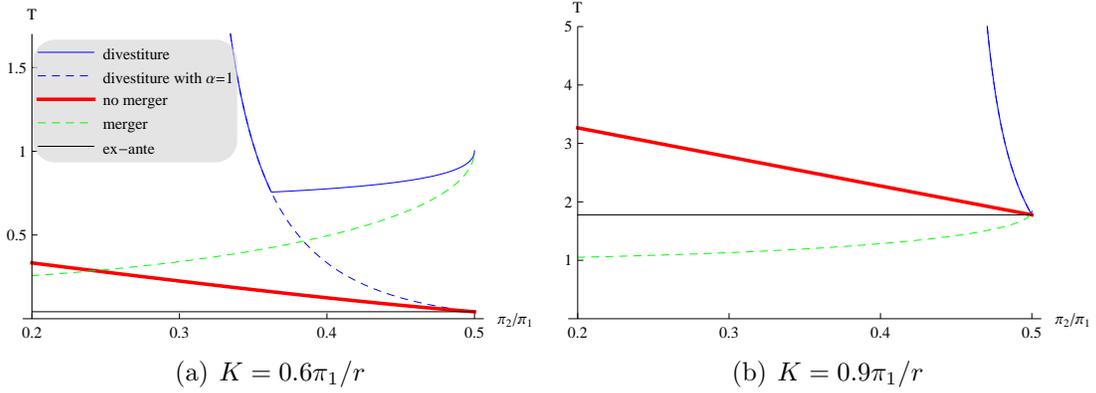


Figure 5: Expected delay before entry as a function of the degree of competition.

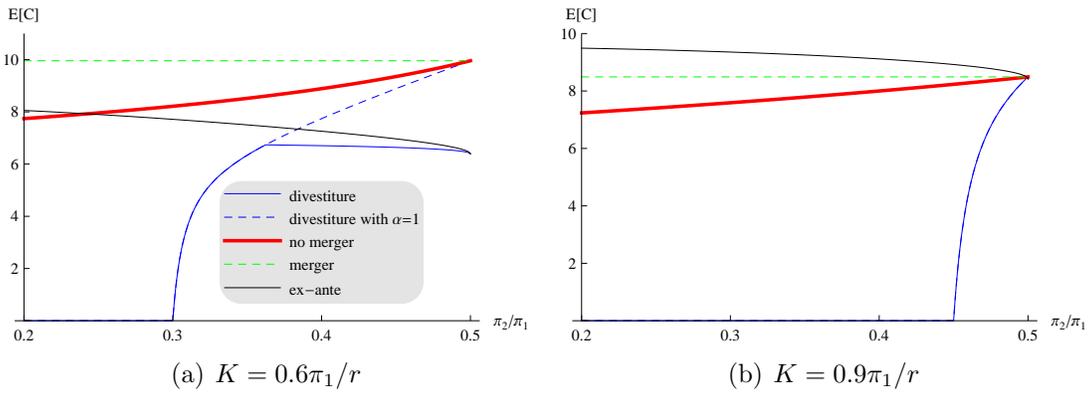


Figure 6: Expected cost as a function of the degree of competition.

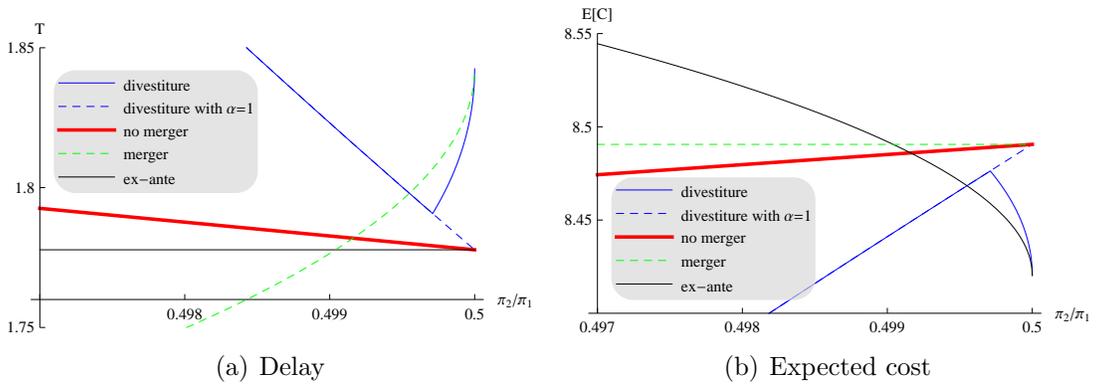


Figure 7: Zoom of Delay and Expected cost as a function of the degree of competition for $K = 0.9\pi_1/r$

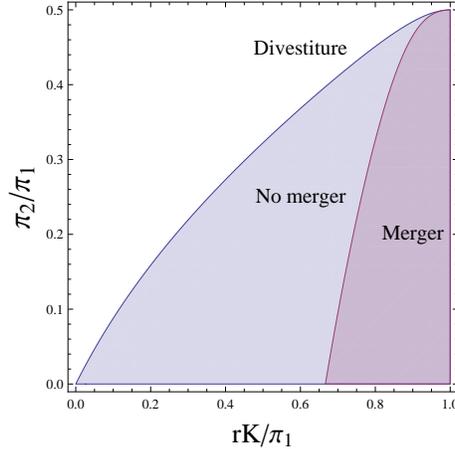


Figure 8: Comparison of the antitrust policies in a two-stage game

in the static game, occurs only once and if a firm does not enter it will never do so. From the firms' point of view, the value of entering and staying out are the same in the dynamic and the static game, so that the equilibrium probabilities of entry are identical in the two games. However, from a welfare perspective, entry is more valuable in the static game than in a dynamic game because entry cannot be postponed to the next period in the former case. The implications are that insufficient entry is more frequent and hence the ex-ante industrial policy consists of a larger entry subsidy in the static game than in the dynamic one. Any antitrust intervention that decreases the probability of entry has a larger long-term welfare cost in the static game. Consequently, the dynamic framework is more favorable to no merger and divestiture than the static one. To illustrate and quantify the difference, the optimal antitrust policy in the static game for our linear specification is depicted in Figure 8.

5 Conclusion

We have proposed a model in which industrial and antitrust policies can be compared taking into account the dynamic feedback that these policies generate on the firms' investment strategies. In our model, two firms repeatedly compete (through incurring or not a sunk fixed cost) for a natural monopoly situation: in case only one firm invests, it will secure the market; in case both firms invest, the discounted stream of duopoly profit will not compensate the sunk cost of entry. An ex-ante industrial policy takes the form either of a tax or a subsidy for the investment that allows the regulator to decrease or increase the extent of entry. An ex-post antitrust policy may either (if pro-competitive) divest a monopoly into a duopoly or

(if lenient) allow a duopoly to merge into monopoly.

We compare these policies in terms of their discounted welfare, which captures consumers' surplus, entry delays and the possible duplication of fixed costs. Usually, the industrial policy dominates the antitrust policies. A lenient policy is relatively good whenever the sunk cost is so large that the entry probability is very small: in such cases, the loss in consumers' surplus of allowing a duopoly and the duplication of fixed cost are more than compensated by the reduction in the entry probability; still, an ex-ante industrial policy would achieve a somewhat higher welfare. A pro-competitive policy is better than an industrial policy in specific circumstances, that is, whenever the probability of entry is close to one, but as this probability decreases such a policy would quickly destroy all investment incentives. Moreover, in order to be really better than the ex-ante policy, the bargaining power of the monopolist when reselling its assets may have to be closely monitored by the regulator.

We now discuss both limitations and possible extensions. Our model is limited to two firms, which is a strong limitation, so it would be useful to extend our results to the case of an oligopolistic structure involving $n > 2$ firms. However, this will make the model more complicated. For instance, assume that the market size is such that it can profitably accommodate 4 firms and say that $n = 7$. Without any policy, the long term market structure may be 4, 5, 6, 7 (assuming that market structures with 0, 1, 2 or 3 firms would be transient). The impact of an industrial policy is easily extended, but modeling the impact of antitrust policies is more elaborate. The targeted market structures for the authorities need to be defined; this depends on the capacity of the regulator to fully appreciate the structural characteristics of the industry. Also, the merger process needs to be precisely designed, eventually giving up to the convenient assumption of symmetry among firms. Finally, the nature and the degree of differentiation on the market would need to be more specific, cost functions and capacity constraints may need to be introduced, etc.

The boundary between an analytical model and a simulation model such as the one in Gowrisankaran (1999) is easily crossed at the expense of tractability. Despite its simplicity, we believe that the current model already delivers some qualitative policy recommendations. Then, on a case by case approach, a more refined model should be made to precisely discuss the benefits or pitfalls associated with implementations of policies in a real world situation. We can think of two sectors in which this approach would seem worthwhile. The first one would require extending the model so that investment leads to a new (green) technology, whereas firms that do not invest would remain stuck to an old (dirty) technology. Delay would then be associated not so much with a loss in consumer surplus but rather with the environmental damage due to the persistence of the old technology. Such an extension looks both feasible and interesting even in the case of a two-firm model. The competition

between various powertrains (electric, hydrogen, hybrids) for clean transportation could provide motivation for this extension, and it could deliver some insights for the analysis of the relative efficiency of various public policies intended to promote their deployment (see for instance Rösler et al., 2014).

A second context in which this approach could be both interesting and relevant concerns the cement sector. Indeed, this seems quite feasible since Ryan (2012) precisely provides a numerical dynamic model of this industry for the US using the approach of Ericson and Pakes (1995). He shows that the 1990 Clean Air Act Amendment increased the setup cost for new entrants by 20%, leading to higher market concentration, lower profit and loss of consumer surplus. Building on the same approach, Perez-Saiz (2014) and Fowlie et al. (2012) investigate how various regulatory policies may indirectly affect the market structure. However, in these applied studies antitrust policies are not explicitly considered in spite of their empirical relevance for cement.

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APPENDIX

Proof of Proposition 1. Note that $A_1 = w_1/r - K \leq 0$ implies that $A_2 \leq 2A_1 \leq 0$ because we have assumed that $w_2 - w_1 \leq w_1$. So $A_1 \leq 0$ implies the expression in (4) is nonpositive and clearly maximized at $p = 0$.

Turn now to the case in which $A_1 > 0$. Using expression (4) after multiplying through by δ yields that

$$A_1 - \delta W_0 = \frac{A_1[1 - \delta(1 - p)^2] - \delta p^2 A_2 - 2p(1 - p)\delta A_1}{1 - \delta(1 - p)^2},$$

so expression (5) can be rewritten as

$$\frac{\partial W_0}{\partial p} = 2 \frac{p[1 - (1 - p)\delta](A_2 - A_1) + (1 - p)(1 - \delta)A_1}{[1 - \delta(1 - p)^2]^2}. \quad (7)$$

If $A_2 \geq A_1$ (i.e. $w_2 - w_1 \geq rK$) then $\partial W_0/\partial p > 0$ for all $p \in [0, 1]$ and W_0 is maximized by choosing $p = 1$.

If $A_2 < A_1$ (i.e. $w_2 - w_1 < rK$), from (7) the derivative is positive for $p = 0$ and negative for $p = 1$, so that (the numerator being a second order polynomial) both first-order and second-order conditions are satisfied at (and only at)

$$\hat{p} = \frac{r}{2(A_1 - A_2)} \left[\left[(2A_1 - A_2)^2 + 4A_1(A_1 - A_2)/r \right]^{1/2} - (2A_1 - A_2) \right].$$

Hence, $W_0(p)$ is maximized at $p = \hat{p} \in (0, 1)$. ■

Proof of Proposition 2. Since the welfare under duopoly, w_2 , is a decreasing function of π_2 (recall that $w_2 \equiv w(2\pi_2)$), it follows (from (7)) that

$$\frac{\partial^2 W_0}{\partial \pi_2 \partial p} = \frac{4p[1 - (1 - p)\delta]w'}{[1 - \delta(1 - p)^2]^2} < 0$$

and

$$\frac{\partial^2 W_0}{\partial K \partial p} = - \frac{2\{p[1 - (1 - p)\delta] + (1 - p)(1 - \delta)\}}{[1 - \delta(1 - p)^2]^2} < 0.$$

The strict concavity of $W_0(p)$ whenever $w_1 \geq rK > w_2 - w_1$ yields the desired result. ■

Proof of Proposition 4. In order to analyze the situation, it is worth considering the expected consumers surplus:

$$S_0(s_1, s_2, r, p) = \frac{2p(1 - p)s_1/r + p^2 s_2/r}{1 - \delta(1 - p)^2}.$$

Expected firms profits are null at equilibrium, so expected welfare is equal to expected consumer surplus. Consumer surplus only depends indirectly, via the entry probability, on the fixed cost K , property that will be exploited in what follows.

In particular, we show that the difference between the expected consumer surplus with merger and the expected consumer surplus without merger is positive for large K . First, note that the difference in expected consumers surplus is continuous and null for $K = \pi_1/r$. Second, the effect of K on expected consumer surplus is (by analogy with equation (7)):

$$\frac{dS_0}{dK} = \frac{\partial S_0}{\partial p} \frac{p_0^*}{\partial K} = -\frac{2\{p_0[1 - (1 - p_0)\delta](s_2 - s_1) + (1 - p_0)(1 - \delta)s_1\}}{(\pi_1 - \pi_2)[1 - \delta(1 - p_0)^2]^2}.$$

When K goes to π_1/r from below, the equilibrium probability goes to 0, and the derivative of consumer surplus goes to $-2(1 - \delta)s_1/(\pi_1 - \pi_2)$. Given that $\pi_2 = \pi_1/2$ with merger, while $\pi_2 < \pi_1/2$ without merger, the derivative of the difference of consumers surplus is negative for large K . Finally, the difference of expected consumers surplus with and without merger is decreasing for large K and null at $K = \pi_1/r$, so there must be a range of K such that the difference is positive. ■

Proof of Proposition 5. Welfare is quasi-concave with respect to the entry probability (c.f. proof of Proposition 1). The equilibrium probability $p_0^*((1 + \alpha)\pi_2, \pi_2, rK)$ is increasing with respect to α . Therefore, welfare is first increasing then decreasing w.r.t. α , and it is maximized either at an interior solution such that $p_0^* = \hat{p}$, or at $\alpha = 1$. The former arises if and only if $p^*(2\pi_2, \pi_2, rK) > \hat{p}$.

The equilibrium probability $p_0^*(2\pi_2, \pi_2, rK)$ is increasing w.r.t. π_2 . The optimal probability \hat{p} is increasing with respect to the duopoly welfare w_2 (from the first order condition (7)), so it is decreasing w.r.t. to the duopoly profit. Furthermore, for $\pi_2 \geq rK/2$, $p^*(2\pi_2, \pi_2, rK) = 0$ while $\hat{p} > 0$. Thus, there is $\bar{\pi}_2 > 0$ such that $\alpha^* = 1$ if and only if $\pi_2 \leq \bar{\pi}_2$.

■

Proof of Proposition 6. Let us respectively denote $W_0^D(\pi_2)$ and $W_0(\pi_2)$ for the expected welfare with divestiture (and optimal α) and the expected welfare without divestiture as functions of π_2 .

If $\pi_2 \geq \bar{\pi}_2$, then asset divestiture induces the optimal probability of entry given that there is a post-entry duopoly. Without asset divestiture, the ex-post welfare is lower if one firm enters, so the expected welfare is lower whatever the entry probability is. Therefore, the expected welfare without divestiture is lower than the optimal expected welfare with divestiture: $W_0(\pi_2) < W_0^D(\pi_2)$. However, if $\pi_2 < \bar{\pi}_2$ the expected welfare with divestiture is sub-optimal, in the sense that the probability of entry is sub-optimal. The probability of

entry is null for $\pi_2 = rK/2$, and W_0^D is null for lower π_2 while $W_0(\pi_2) > 0$ in such cases. By continuity, there must exist $\tilde{\pi}_2 \in (rK, \bar{\pi}_2)$ such that $W_0^D(\pi_2) > W_0(\pi_2)$ if and only if $\pi_2 > \tilde{\pi}_2$.

■