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UNDER IMPERFECT INSURANCE

Olivier ALLAIS
Yann ALGAN
Edouard CHALLE
Xavier RAGOT

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DEPARTEMENT D'ECONOMIE

Route de Saclay
91128 PALAISEAU CEDEX
(33) 1 69333033
<http://www.economie.polytechnique.edu/>
<mailto:chantal.poujouly@polytechnique.edu>

The Welfare Cost of Inflation Risk Under Imperfect Insurance*

Olivier Allais[†] Yann Algan[‡] Edouard Challe[§] Xavier Ragot[¶]

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Abstract

What are the costs of inflation fluctuations and who bears those costs? In this paper, we investigate this question by means of a quantitative incomplete-market, heterogeneous-agent model wherein households hold real and nominal assets and are subject to both idiosyncratic labor income shocks and aggregate inflation risk. A key feature of our analysis is a nonhomothetic specification for households' preferences towards money and consumption goods. Unlike traditional specifications, ours allows the model to reproduce the broad features of the distribution of monetary assets (in addition to being consistent with the distribution of nonmonetary assets). Inflation risk is found to generate significant welfare losses for most households, i.e., between 1 and 1.5 percent of permanent consumption. The loss is small or even negative for households at the very top of the productivity and/or wealth distribution.

JEL codes : E21; E32; E41

Keywords : Money-in-the-utility; incomplete markets; inflation risk; welfare.

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[†]INRA; olivier.allais@ivry.inra.fr.

[‡]Sciences Po and CEPR; yann.algan@sciences-po.fr.

[§]CNRS, Ecole Polytechnique and CREST; edouard.challe@gmail.com.

[¶]CNRS, Paris School of Economics and OFCE; ragot@pse.ens.fr.

1 Introduction

What are the costs of inflation fluctuations and who bears those costs? In this paper, we investigate this question by means of a quantitative incomplete-market, heterogenous-agent model wherein households hold real and nominal assets and are subject to both idiosyncratic labor income shocks and aggregate inflation risk. To be more specific, the model has the following three main features:

First, households solve an optimal portfolio choice between money and capital claims, subject to an occasionally binding borrowing constraint. Both assets can coexist in household portfolios because the lower real return paid by money balances is compensated by the liquidity services that it provides – as captured by the fact that real money balances affect current utility, in the money-in-the-utility, or “MIU” tradition.¹ This feature is in contrast with existing incomplete-market models with aggregate shocks, which either abstract for portfolio choices (by considering a single asset or a menu of perfectly substitutable assets), or focuses on the choice between two *real* assets (so that inflation risk does not affect the relative returns of the assets).²

Second, the model is designed to match the cross-sectional distribution of money holdings, in addition to matching the cross-section of nonliquid wealth. Because our investigation is primarily quantitative, a prerequisite for a meaningful analysis of the impact of inflation risk on the welfare of different types of households is that the model match the cross-sectional distribution of monetary wealth in the first place. We show that this requirement requires departing from the functional form commonly used to parameterize money demand in monetary models with representative agent.³ As discussed extensively in Ragot (2014), the empirical cross-sectional distribution of money holdings in the US is close to that of financial assets and hence very different from that of consumption.⁴ This property cannot follow from the usual MIU specification, which features a constant elasticity of substitution between money and goods and hence strict proportionality between the demands

¹This tradition goes back at least to Patinkin (1956) and Sidrauski (1967). See Walsh (2010) for a survey of the flexible-price MIU model under *complete* markets.

²In such models, when the borrowing constraint is not zero, private bonds are traded next to capital claims, but the two are usually assumed to be perfect substitutes (see, e.g., Krusell and Smith, 1998). Heathcote (2005) introduces government bonds next to the capital stock, but again makes the necessary assumptions ensuring that the two are perfect substitutes. Krusell and Smith (1997) consider the portfolio choice between a real riskless bond and capital claims. Krusell et al. (2011) and Challe et al. (2013) analyze the portfolio choice between real bonds (differing by their maturity).

³See, e.g., Chari et al. (1996, 2000).

⁴For example, Ragot (2014) finds that in the US Gini coefficient for individual consumption levels is 0.54, while that for money balances is 0.85. In Italy the corresponding figures are 0.30 and 0.68, respectively.

for money and consumption goods. If this were the case then inequalities in money holdings would simply mirror inequalities in consumption, which is strongly against the data. We overcome this limitation of the standard MIU specification by introducing a more general utility function that nests the constant-elasticity case but also accommodates a non-constant elasticity of substitution, thereby allowing individual money holdings to vary more than proportionally with individual consumption levels. This utility function allows us to reproduce the broad features of the distribution of monetary wealth in the US economy, whilst at the same time being consistent with the observation that, at the individual level, higher wealth is associated with greater absolute money holdings but lower money holdings relative to total wealth.

Third, the model incorporates as part of the solution households' rational portfolio response to the inflation risk that they face. As far as we are aware, existing work on the effect of inflation on welfare in heterogenous-agent models has focused on the welfare impact of either *mean* inflation or *unexpected shocks* to inflation, leaving aside (by construction) households' optimal portfolio response to their expectation of future inflation shocks.⁵ This dimension is crucial in the present study for at least two reasons. First, households are likely to respond *ex ante* to the inflation risk they are facing, i.e., we expect different levels of inflation risk to generate different mean holdings of nominal assets; and second, allowing significant redistributive effects of inflation but ignoring the *ex ante* portfolio response to inflation risk is likely to overestimate the welfare cost of inflation shocks.

The model features both aggregate (inflation) and idiosyncratic (labor productivity) shocks and is calibrated accordingly. Regarding the time-series dimension, we feed the model with an exogenous process for the money growth rate so that the equilibrium response of inflation matches the volatility properties of actual inflation over the post-Volcker, pre-Great Recession period. Treating equilibrium inflation (rather than the money supply) as the forcing term in our analysis ensures that the inflation risk that households face in the model is consistent with the historical inflation risk. Concerning the cross-sectional dimension, we calibrate the wage income process as well as

⁵Imrohoroglu (1992) and Erosa and Ventura (2002) study the impact of mean inflation on welfare in incomplete-market environments. Cao et al. (2012) examine the redistributive and welfare effects of changes in mean inflation within an overlapping generations model. Similarly, Doepke and Schneider (2006a, 2006b) attempt to evaluate the redistributive and welfare effects of a moderate but persistent inflation episode in the US economy. They find these effects to be large, due to the significant degree of heterogeneity in nominal asset positions across US households and the large fraction of foreign holders of US assets. In the same spirit, Sterk and Teynero (2014) use an overlapping-generation model to evaluate the impact of open market operations on durables consumption.

household’s preferences towards money and consumption goods so as to match the joint cross-sectional distributions of monetary *and* nonmonetary assets. Our analysis can thus be interpreted as way to discipline preferences towards money and goods on the basis of the statistical moments of cross-sectional data on monetary positions.

To evaluate the cost of inflation fluctuations and the way it is distributed across households, we compute the ex ante welfare of an agent conditional on a particular idiosyncratic state and portfolio, in economies with and without inflation risk. To summarize, we find significant welfare costs of inflation fluctuations for all households except for those at the top end of the productivity scale and/or the wealth distribution. For example, for a low-productivity household holding the average portfolio of the households with similar productivity (where the “average” is computed from the invariant distribution of the economy with aggregate risk), full elimination of inflation risk is equivalent to a 1.5 percent increase in lifetime consumption. We find similar welfare costs for intermediate productivity households starting with either the portfolio of the average household with similar productivity or the average portfolio of low-productivity households. While inflation risk generates no welfare loss (or even a small benefit) for households at the top of the productivity and/or wealth distribution, the welfare loss for the average household is close to 1 percent of permanent consumption. Such welfare losses are large: they are one order of magnitude larger than those computed by Lucas (1987, 2003) in a representative-agent context;⁶ and about five times the welfare cost of the business cycle as computed by Imrohoroglu (1989) within a (nonmonetary) incomplete-market, heterogenous-agent model. The magnitude of the welfare costs that we find is similar to that in models with incomplete insurance wherein the extent of idiosyncratic risk is systematically related to the size of aggregate productivity shocks, a channel that is absent from our model.⁷

The next section presents the model. Section 3 discusses our parameterization. Section 4 presents our main results on the welfare costs of inflation fluctuations, and Section 5 concludes.

⁶Lucas’ calculations only rely on the assumption of complete market and a specification for the representative agent’s preferences. While maintaining complete markets and suitably choosing preferences can generate large welfare costs of the business cycle, Ostrok (2001) shows that this argument does not survive in general equilibrium wherein households optimally choose (and effectively smooth) consumption.

⁷See, e.g., Storsletten et. al. (2001) and Krusell et al. (2009).

2 The model

The model is a MIU, heterogenous-agent model augmented with i) aggregate shocks to inflation (driven by underlying changes in money growth), and ii) a nonhomothetic utility functional designed to accommodate significant dispersion in holdings of monetary assets.

2.1 Preferences

Households are infinitely-lived and in constant mass equal to 1. They share identical and additively time-separable preferences over sequences of consumption, $c \equiv \{c_t\}_{t=0}^{\infty}$, and real money balances, $m \equiv \{m_t\}_{t=0}^{\infty}$. Thus, they maximize

$$U(c, m) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, m_t), \quad (1)$$

where $\beta \in (0, 1)$ is the subjective discount factor, \mathbb{E}_t denotes expectations conditional on the information set at date t , and u is the instant utility function. This function is assumed to take the following parametric form:

$$u(c_t, m_t) = \frac{1}{1-\lambda} \left(\omega c_t^{1-\rho} + (1-\omega) m_t^{\theta(1-\rho)} \right)^{\frac{1-\lambda}{1-\rho}}, \rho, \theta, \lambda > 0. \quad (2)$$

When $\theta = 1$, equation (2) becomes a standard homothetic utility function similar to that used by Chari et al. (1996, 2000) and Algan and Ragot (2010), among others. In this case, the interest-rate elasticity of real money demand is $1/\rho$, while the intertemporal elasticity of substitution between consumption and real balances is $(1-\rho)/(\lambda-\rho)$. As discussed above, an important limitation of the homothetic specification is that it implies a strict proportionality between individual real money holdings and individual consumption levels (for any given values of the nominal interest rate and the inflation rate). This makes it impossible to reproduce the highly unequal distribution of money holdings that is observed in US data. Our baseline calibration will thus have $\theta \neq 1$.

2.2 Idiosyncratic uncertainty

In every period, households are subject to idiosyncratic labor income shocks. Labor productivity can take three different values: $e_t \in E$, $E = \{e^l, e^m, e^h\}$ with $e^l < e^m < e^h$, and where e^h stands for ‘high productivity’, e^m for ‘medium productivity’, and e^l for ‘low productivity’. Each household’s productivity evolves according to a first-order Markov chain with the 3×3 transition matrix F .

We denote by p^* the vector of stationary ergodic probabilities and normalize productivity levels so that the mean of the invariant distribution is one, i.e., $\sum p_i^* e_i = 1$. Given a population of measure one, we can interpret p^* as describing the distribution of the population across productivity states. It follows that the effective aggregate labor supply is equal to $\sum p_i^* e_i \bar{n} = \bar{n}$, where \bar{n} stands for labour hours per period.

2.3 Production

Markets are competitive. In every period t , the representative firm uses aggregate capital $K_t \in \mathbb{R}_+$ and households' labor to produce $Y_t \in \mathbb{R}_+$ units of a single good with the aggregate technology

$$Y_t = f(K_t, \bar{n}) = K_t^\alpha \bar{n}^{1-\alpha}.$$

Capital depreciates at the constant rate $\delta \in (0, 1)$ and accumulates according to the law of motion

$$K_{t+1} = I_t + (1 - \delta)K_t. \quad (3)$$

where I_t denotes aggregate investment. Perfect competition in the markets for the representative firm's inputs implies that the real interest rate, r_t , and the real wage, w_t , can be written as:

$$r_t = \alpha K_t^{\alpha-1} \bar{n}^{1-\alpha} - \delta, \quad w_t = (1 - \alpha) K_t^\alpha \bar{n}^{-\alpha}.$$

2.4 The household's problem

We assume that markets are incomplete, so that households cannot write insurance contracts contingent on their labor income. Moreover, they face borrowing constraints and are thus prevented from using private loans to fully smooth out individual income fluctuations. Each household i maximizes its expected lifetime utility (1) subject to the following constraints:

$$c_t^i + k_{t+1}^i + m_t^i = a_t^i + (1 - \tau_t^l) w_t e_t^i \bar{n}, \quad (4)$$

$$k_{t+1}^i \geq 0, \quad m_t^i \geq 0, \quad \text{and} \quad c_t^i \geq 0, \quad (5)$$

where k_{t+1}^i and m_t^i denote the claims to the capital stock and the real money balances held by household i at the end of date t , and where

$$a_t^i = (1 + (1 - \tau_t^c) r_t) k_t^i + \frac{m_{t-1}^i}{1 + \pi_t}$$

is the household's financial wealth at the beginning of date t . In (4) and (5), τ_t^l and τ_t^c are proportional taxes on labor and capital, respectively. In (5), the presence of the borrowing constraint is reflected in the fact that capital and money holdings must be nonnegative at all times, while no other assets (i.e., private bonds) can be issued by the households.

From the households' objective and constraints, we find that their optimal asset demands, m_t^i and k_{t+1}^i , must satisfy the following first-order conditions:

- Money:

$$u_c(c_t^i, m_t^i) - u_m(c_t^i, m_t^i) = \beta E_t \left[\frac{u_c(c_{t+1}^i, m_{t+1}^i)}{1 + \pi_{t+1}} \right]. \quad (6)$$

- Capital:

$$\begin{aligned} \text{Either } u_c(c_t^i, m_t^i) &= \beta E_t [(1 + (1 - \tau_t^c) r_{t+1}) u_c(c_{t+1}^i, m_{t+1}^i)] \text{ and } k_{t+1}^i > 0, \\ \text{or } u_c(c_t^i, m_t^i) &> \beta E_t [(1 + (1 - \tau_t^c) r_{t+1}) u_c(c_{t+1}^i, m_{t+1}^i)] \text{ and } k_{t+1}^i = 0. \end{aligned} \quad (7)$$

The instant utility function (2) implies that $u_m(c_t^i, 0) = \infty$, so the demand for real balances is always interior. In contrast, the demand for capital may be corner (i.e., $k_{t+1}^i = 0$), in which case the household would like to raise current consumption by borrowing against future income but is prevented from doing so (by a binding borrowing constraint). The solution to the households' problem provides sequences of policy functions $m_t(a, e)$, $k_t(a, e)$ and $c_t(a, e)$, $(a, e) \in \mathbb{R}_+ \times \{e_1, e_2, e_3\}$, where a and e denote individual beginning-of-period asset wealth and productivity, respectively.

To better understand the implications of our assumed period utility function with non-constant elasticity of substitution (i.e., (2)), consider the optimal trade-off between consumption and real money holdings by an unconstrained household (so that $k_{t+1}^i > 0$ in (7)) and abstract from aggregate shocks momentarily. From (6)–(7) and the functional form (2), we find the relation between money holdings and consumption to be:

$$m_t^i = A((1 - \tau_t^c) r_{t+1}, \pi_{t+1}) (c_t^i)^{\frac{\rho}{1-\theta(1-\rho)}}, \quad (8)$$

where $A((1 - \tau_t^c) r_{t+1}, \pi_{t+1})$ is a coefficient whose value depends on the returns on the two assets and the deep parameters of the utility function. In the constant-elasticity case (i.e., $\theta = 1$), we have $m_t^i = A((1 - \tau_t^c) r_{t+1}, \pi_{t+1}) c_t^i$, that is, real money demand is strictly proportional to consumption, so that cross-sectional inequalities in these two variables exactly mirror each other. For individual

money holdings to increase more than proportionally following an increase in individual consumption, so that money be more unequally distributed than consumption (as is observed in the data), one needs $\rho / (1 - \theta(1 - \rho)) > 1$ (whether θ must lie above or below 1 for this inequality to hold depends on the value of ρ .)

2.5 Market clearing

Define $\mu_t : \mathbb{R}_+ \times \{e^h, e^m, e^l\} \rightarrow \mathbb{R}_+$ as the joint cross-sectional distribution of wealth and individual productivity at the beginning of period t . The market-clearing conditions for real balances and capital claims are given by:

$$\int m_t^i(a_t, e_t) d\mu(a_t, e_t) = \Omega_t^s, \quad (9)$$

and

$$\int k_t(a_t, e_t) d\mu(a_t, e_t) = K_{t+1}, \quad (10)$$

where Ω_t^s is the total quantity of real balances at date t . By Walras law, the goods market clear when both the money and the capital markets clear.

2.6 Fiscal and monetary policy

Fiscal and monetary policies interact here because monetary policy generates seigniorage revenues that determine government income jointly with tax collections. We specify both as follows.

First, we assume that inflation follows a two-state Markov chain: the inflation rate transits between π^L and $\pi^H > \pi^L$ according to the transition matrix T . The money growth rate that produces this inflation process as an equilibrium outcome is determined endogenously. The two inflation levels as well as the transition probabilities between the two can then be calibrated according to historical data. This ensures that the amount of inflation risk faced by the households in the model is consistent with its historical counterpart.⁸

For a given inflation process, the growth rate of the quantity of money γ_t is determined as follows. First, let Δ_t denote the nominal quantity of newly issued money at date t (relative to the stock of money at the end of date $t - 1$) and by $\Pi_t = P_t/P_{t-1} = 1 + \pi_t \in \{\Pi^L, \Pi^H\}$ the gross

⁸Technically, we first estimate a two-state Markov process for inflation and then impose the constraint that the inflation series generated by our model be consistent with the estimated process. We can then recover the underlying process for money growth that has generated the equilibrium response of inflation. Calibrating a two state Markov process for money growth rather than would result in inflation overshooting following regime changes and would thereby overestimate actual inflation risk.

inflation rate between date $t - 1$ and date t . In real term, the quantity of newly issued money can be expressed as:

$$\Delta_t/P_t = \gamma_t P_{t-1} \Omega_{t-1}^s / P_t = \gamma_t \Omega_{t-1}^s / \Pi_t, \quad (11)$$

and the dynamics of total real money balances by:

$$\Omega_t^s = (1 + \gamma_t) \Omega_{t-1}^s / \Pi_t. \quad (12)$$

Second, given Ω_{t-1}^s and current gross inflation $\Pi_t \in \{\Pi^L, \Pi^H\}$ we determine γ_t so that Ω_t^s clears the money market (i.e., so that (9) holds), given aggregate money demand (which is a function of the aggregate state, including the inflation factor Π_t).

It is assumed that the newly issued money is given to the government as part of its resources. This assumption captures the fact that in practice money is created by open market operations which, once the budget of the government and the Central Bank are consolidated, are effectively transfers to the government (see, e.g., Walsh, 2010, chapter 5). Our specification allows us to incorporate spending and taxes without introducing public debt as a third asset, and thereby to keep the model within the current computational limits. Under this monetary/fiscal arrangement the budget of the government is balanced in every period and such that:

$$G_t = \gamma_t \Omega_{t-1}^s / \Pi_t + \tau_t^l w_t \bar{n} + \tau_t^c r_t K_t,$$

We assume that $G_t = G > 0$ is constant over time and then let the tax rates on capital and labor adjust to satisfy the government budget constraint in every period. This ensures that inflation risk does not generate wealth effects “in the aggregate” (which would somewhat artificially amplify the welfare cost of inflation fluctuations). For simplicity we assume that the ratio of the two taxes $\psi = \tau_t^c / \tau_t^l$ is constant over time, so that the taxes on labor and capital are given by, respectively:

$$\tau_t^l = \frac{G - \gamma_t \Omega_{t-1}^s / \Pi_t}{w_t L + \psi r_t K_t}, \quad \tau_t^c = \psi \tau_t^l.$$

2.7 Recursive problem and equilibrium

We are considering a recursive equilibrium in which the aggregate state, which includes the aggregate stock of capital, changes over time. Note that the way we have specified monetary policy implies that households need not keep track of the aggregate money supply, because inflation (the inverse of the return on money) enters as an exogenous forcing term here (with the rate of money

growth adjusting to make a particular inflation path happen). It follows that the recursive problem of a household can be written as:

$$v(a_t, e_t; \pi_t, K_t) = \max_{m_t, c_t, k_{t+1}} u(c_t, m_t) + \beta E_t [v(a_{t+1}, e_{t+1}; \pi_{t+1}, K_{t+1}) | e_t, \pi_t], \quad (13)$$

subject to (4)–(5). Following Krusell and Smith (1998) and much of the subsequent literature, we posit that households are able to successfully forecast the dynamics of the endogenous aggregate state K_t by means of (log-) linear laws of motion (one per value of the exogenous aggregate state π_t) involving K_t and Ω_t^s :

$$\ln(K_{t+1}) = b_1(\pi_t) + b_2(\pi_t) \ln(K_t) + b_3(\pi_t) \ln(\Omega_t^s). \quad (14)$$

The solution to (13) produces individual decision rules for consumption as well as holdings of real balances and claims to the capital stock, which we denote by $g_c(a_t, e_t; \pi_t, K_t)$, $g_m(a_t, e_t; \pi_t, K_t)$ and $g_k(a_t, e_t; \pi_t, K_t)$, respectively. The law of motion of the distribution for beginning-of period total wealth is denoted by H . For a given set of individual policy rules, this law of motion can be written as

$$\mu_t = H(\mu_{t-1}, \pi_t),$$

i.e., the cross-sectional distribution of wealth μ_t depends on its value in the previous period μ_t as well as the current realization of the exogenous aggregate state π_t .

Definition of the recursive equilibrium. *A recursive equilibrium is defined by a law of motion H , a set of optimal individual policy functions and value function $\{g_c, g_m, g_k, v\}$, a set of price functions $\{\pi, r, w\}$, and a law of motion for K such that:*

1. *Given $\{\pi, r, w\}$, the law of motion for K_t and the transition matrices F and T , the policy functions $\{g_c, g_m, g_k\}$ solve the household's problem;*
2. *The money and capital markets clear;*
3. *The law of motion of H is generated by the optimal decisions $\{g_c, g_m, g_k\}$, and the transition matrices F and T .*

We solve for the recursive equilibrium using the same approach as in Krusell and Smith (1997, 1998). However, rather than using Monte Carlo simulations to generate an updated cross-sectional

distribution, we use the grid-based simulation procedure proposed by Young (2010), which keeps track of the mass of households at a fine grid of wealth levels. This allows us to get rid of the cross-sectional sampling variations in the Monte Carlo simulation procedure. A detailed description of the algorithm is provided in the Appendix.

3 Parameterization

Table 1: Parameter values

<i>Preferences</i>	λ	1.00	<i>Production</i>	α	0.360
	β	0.99		δ	0.025
	ρ	20.00		\bar{n}	0.300
	ω	0.06			
	θ	0.21			
<i>Productivity</i>	e^l	3.940			
	e^m	0.849			
	e^h	0.213			
	$\pi_{l,l}$	0.975			
	$\pi_{m,m}$	0.992			

Table 1 reports the parameters of the model. The time period is a quarter. Following Chari et al. (2000), our benchmark value for the utility parameter λ is 1. The capital share is set to $\alpha = 0.36$, and the depreciation rate to 0.025. Finally, labor supply is constant for all households and set to 0.3. The individual productivity states and the transition probabilities across states are calibrated as follows. Following Domjei and Heathcote (2004), we use a Markov chain with three states, zero probabilities to transit between extreme states (i.e., $F_{h,l} = F_{l,h} = 0$), and an equal probability to reach any of the extreme states when in the intermediate state (i.e., $F_{m,h} = F_{m,l}$). The transition matrix is then fully identified once $F_{l,l}$, $F_{m,m}$ and $F_{h,h}$ are set, and we set $F_{l,l} = F_{h,h} = 0.9750$, $F_{m,m} = 0.9925$. Finally, the ratios of productivity levels are set to $e^h/e^m = 4.64$ and $e^m/e^l = 3.99$. This process yields an autocorrelation of the real wage equal to 0.91 and a standard deviation of the innovation term equal to 0.22 at annual frequency, in line with the data.

Table 1 also provides the preference parameters that best match key targets such as the Gini of the money distribution; the money to GDP ratio; the interest rate elasticity of money; and the capital-output ratio. The monetary aggregate we use is M1. It is measured as the quantity of money in checking accounts in the Survey of Consumer Finances. The Gini coefficient of this

distribution is as high as 0.85. Broader monetary aggregates have similar dispersion (Ragot, 2014), but the narrow aggregate that we use is more conservative. The quartely value of money over GDP over the 1982-2005 period is 0.32.

We model the dynamics of monetary conditions between 1982Q1 and 2005Q4 as a two-state, first-order Markov chain. More specifically, we estimate this chain using CPI-inflation and extract the inflation levels that prevail in the ‘high-inflation’ versus ‘low-inflation’ regimes, as well as the probabilities to transit between those regimes. This gives $\pi^L = 0.64\%$, $\pi^H = 1.17\%$, and probabilities to stay in the same state $\Pr(\pi^L|\pi^L) = 0.944$ and $\Pr(\pi^H|\pi^H) = 0.889$.

We specify the fiscal policy parameters G and ψ as follows. According to Domeij and Heathcote (2004), for the period 1990-1996 the capital income tax rate averaged 39.7 percent, while the labor income tax rate averaged 26.9 percent. We thus set $\psi = 39.7/26.9$. Moreover, applying those rates to a version of our model without aggregate risk (and with inflation equal to the unconditional mean of inflation in the baseline model) gives $G = 0.25$ as a residual; we thus calibrate G to this value in the baseline economy.

4 Results

4.1 Equilibrium distributions and laws of motion

We first check that our calibration reproduces the broad features of the US wealth distribution. Unlike earlier studies, we seek to match the distributions of two components of total wealth: money wealth and nonmonetary wealth. Given our focus on the portfolio structure chosen by the households, our empirical counterparts from the Survey of Consumer Finances (SCF 2004) are the following. First, we report the empirical distribution of money across U.S. households, as measured in households’ checking accounts. Second, we compute the distribution of nonmonetary wealth by removing monetary assets from the financial assets held by the households in the SCF. Nonmonetary wealth refers to bonds, stocks, life insurance, retirement plans and other managed financial assets, and other liquid assets. Table 2 compares the properties of those two distributions with those generated by the model, under the parameter configuration specified in the previous section. We also report the model-generated wealth distributions under homothetic period utility case, just to illustrate its failure at getting anywhere close to the empirical distribution of monetary wealth.

Given our parametric utility function, all households hold some monetary assets in our model,

even though the amount effectively held may be very small. However, many households are not wealthy enough to hold both money and nonmonetary assets: they are borrowing-constrained and endogenously choose not to hold capital claims.

The benchmark model predicts a fairly high Gini index for the distribution of nonmonetary assets (0.81), as is consistent with the data (0.82). Moreover, the model does a reasonable job in matching the lower tail of the distribution of nonmonetary assets. Perhaps unsurprisingly, the model underestimates the nonmonetary wealth share of the top 1%, which is predicted to be 9.03% while it is 34.30% in the data. This flaw is common to many models that only use idiosyncratic income risk to generate wealth dispersion and ignore, for example, entrepreneurship (see, e.g., Quadrini, 2000.)

Our model predicts that 6.6% of the households on average face a binding borrowing limit. The range of available estimates for this share is notoriously large. Using information on the number of borrowing requests which were rejected in the SCF, Jappelli argued that up to 19% of families are liquidity-constrained. However, using updated SCF data, Budria Rodriguez et al. (2002) reported that only 2.5% of the households have zero wealth, which might better correspond to our theoretical borrowing limit.

Finally, we find the following laws of motion for the capital stock:

$$\text{low inflation} \quad : \quad \ln K_{t+1} = 0.4723 + 0.7870 \ln K_t + 0.1140 \ln \Omega_t^s, \quad R^2 = 0.9982.$$

$$\text{high inflation} \quad : \quad \ln K_{t+1} = 0.4421 + 0.8085 \ln K_t + 0.1155 \ln \Omega_t^s, \quad R^2 = 0.9981.$$

Thus, just as in Krusell and Smith (1998), we find the first-order moments of the distributions of capital and real balances to yield an almost perfect prediction of future capital (hence of the return on capital claims).

4.2 Individual policy rules

Figure 1 displays the individual policy rules when the money growth rate is π^L (the policy rules when $\pi = \pi^H$ have a very similar shape) and for the average levels of capital. The policy rules for total wealth, nonmonetary assets, money and consumption are decomposed for each level of productivity e^h , e^m and e^l . For example, the third panel of Figure 1 reports the individual policy rules for nonmonetary assets. The policy rule lies above the 45-degree line for the most productive household

Table 2: Wealth distribution

	Data	Homothetic utility*	Nonhomothetic utility*
Distribution of nonmonetary assets			
Gini	0.82	0.74	0.81
Share of constrained households	[2.5%, 20%]	8.76%	6.6%
Fraction of total asset held by			
Bottom 20%	0.00	0.03	0.12
Bottom 40%	0.20	0.08	0.26
Top 20%	84.70	76.85	78.22
Top 10%	71.20	51.68	52.89
Top 1%	34.30	8.68	9.03
Distribution of money holdings			
Gini	0.85	0.31	0.81
Fraction of total money held by			
Bottom 20%	0.001	12.50	0.84
Bottom 40%	0.90	23.50	1.66
Top 20%	88.20	41.33	87.69
Top 10%	78.10	24.62	67.34
Top 1%	33.0	3.39	15.55
Capital/GDP	12.00	10.26	10.37
Money/GDP	0.32	0.33	0.32

* The model properties are averages over a 10,000 period simulation.

(with productivity e^h) and below the 45-degree line for the other two types (with productivity e^m or e^l). This implies that the former accumulate nonmonetary assets for self-insurance purposes whereas the latter dis-save to smooth individual consumption. For medium- and low-productivity households, the policy rule displays a kink at low levels of wealth; this is because at such wealth levels these households hold money but no nonmonetary assets. The second and fourth panels show the policy rules for money holdings and consumption, which roughly display the same pattern as the policy rule for nonmonetary assets. The more productive the household (holding wealth constant), or the wealthier the household (holding productivity constant), the higher are individual consumption and money holdings. The close connection between the policy rules for consumption and money holdings stems from the complementarity between the two, a direct implication of our assumed nonhomothetic utility function.

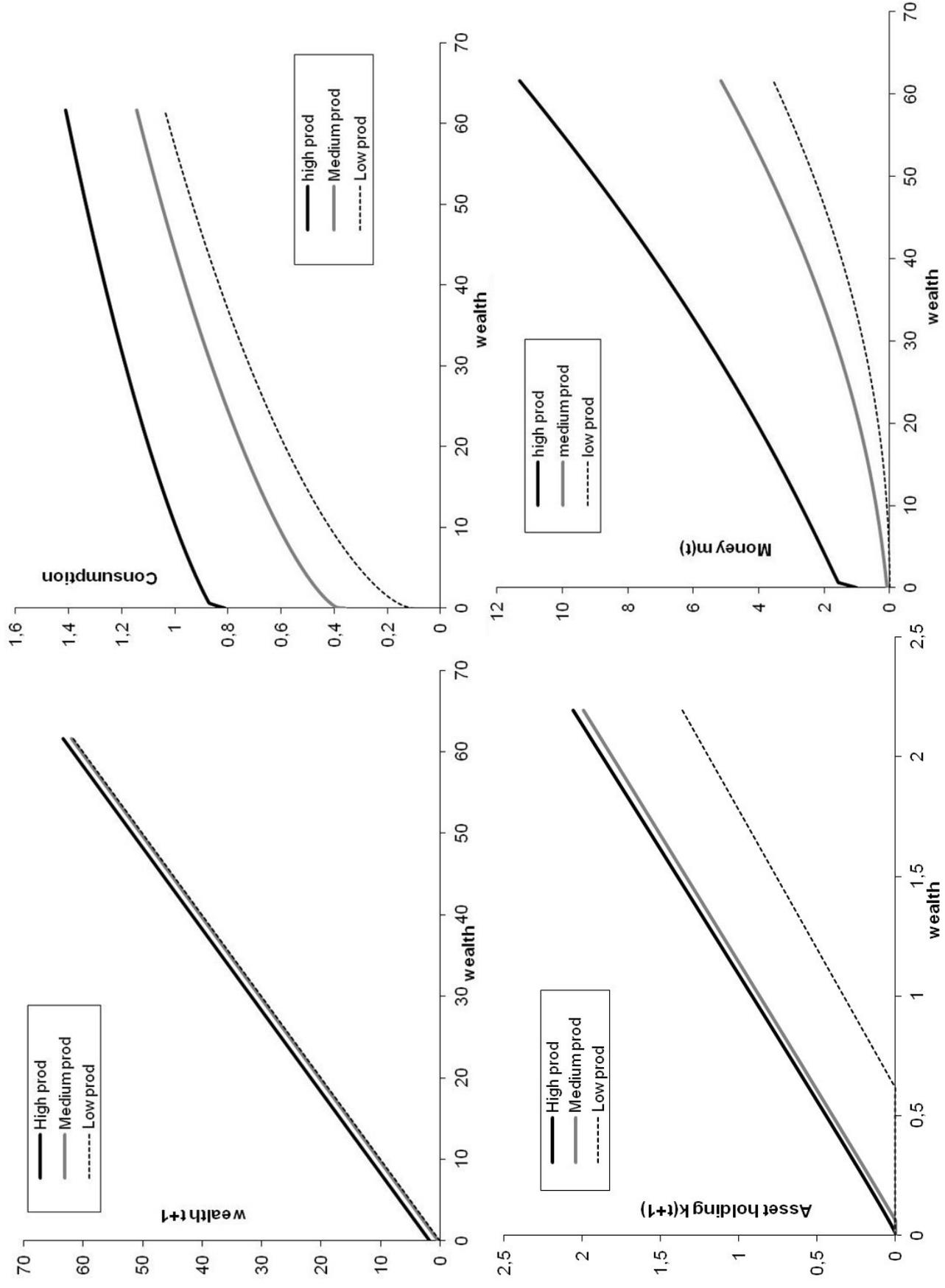


Figure 1: Individual policy rules as a function of total wealth for $\pi = \pi^L$.

4.3 Welfare

We now evaluate the welfare cost of inflation fluctuations within our incomplete-market economy. We do so by performing ex ante welfare comparison conditional on a particular value of the individual state vector (productivity and wealth). The benchmark that we use for comparison is an economy without inflation risk, where the inflation rate is constant and equal to the unconditional mean of inflation (as computed from the baseline model with inflation risk). We wish to understand who are the losers (or winners) from fluctuating inflation and thus proceed as follows.

First, we compute the ergodic distribution of households over productivity e and beginning-of-period wealth a generated by our baseline model, and extract from the ergodic distribution the average beginning-of-period wealth over all households (denoted by \bar{a}) as well as the corresponding averages for low-, medium- and high-productivity households (denoted by a^l , a^m and a^h , respectively).

Second, we compute the ex ante welfare of typical households in the baseline economy. For example, to compute the ex ante welfare of a low-productivity (i.e., e^l) household, we first compute his ex post intertemporal welfare ($\sum_{t=0}^{\infty} \beta^t u(c_t, m_t)$) under a particular history of idiosyncratic and aggregate shocks after date 0, letting him start his life with the average portfolio of households with similar productivity (i.e., a^l). We then repeat the same computation for ten thousands productivity histories, but under the same initial conditions (a and e) and history of aggregate shocks. Averaging over all idiosyncratic histories then gives us the ex ante welfare of this typical household (i.e., $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, m_t)$). We perform similar conditional welfare computations for the typical medium- (e^m, a^m) and high-productivity (e^h, a^h) households. To broaden the picture, we also compute the ex ante welfare of medium-productivity households (e^m) starting his life with the typical portfolio of a high- or a low-productivity household (ie. a^l or a^m). This would be the ex ante welfare level of a household who would have just transited into the medium-productivity class, after having stayed highly (or little) productive for a while. We also compute the ex ante welfare levels by initial productivity types for households starting their life with the economywide average beginning-of-period wealth \bar{a} .

The welfare cost of inflation fluctuations, expressed in terms of equivalent life-time proportional

consumption loss, is then calculated by finding the δ such as the following equality is satisfied:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(\delta g_c(a_t, e_t; \bar{\pi}, \bar{K}), g_m(a_t, e_t; \bar{\pi}, \bar{K})) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(g_c(a_t, e_t; \pi_t, K_t), g_m(a_t, e_t; \pi_t, K_t))$$

where the left hand side of the equation is the ex ante welfare in the benchmark economy without aggregate inflation shocks, i.e., wherein inflation and capital stay constant at the unconditional mean inflation rate $\bar{\pi}$ and capital stock \bar{K} of the baseline model (using the same individual histories as in the economy with inflation risk). In table 3 we report the welfare costs of inflation $1 - \delta$ (%). For example, for a currently mid-productivity household (e^m) holding the typical portfolio of this category (a^m), switching from the constant-inflation benchmark to an economy with inflation risk is as costly as staying in the constant inflation economy but experiencing a permanent 1.23 percent drop in individual consumption.

Table 3: The welfare cost of inflation risk (%)

Productivity	Initial wealth	Welfare cost*
e^h	a^h	-0.36
e^m	a^h	-0.80
e^m	a^m	1.23
e^m	a^l	1.77
e^l	a^l	1.24
e^h	\bar{a}	0.87
e^m	\bar{a}	0.89
e^l	\bar{a}	0.93

*The welfare costs are averages over a 10,000 productivity histories, but under the same initial conditions (e^x, a^x) and history of aggregate shocks.

Differentiating households by productivity types only – that is, considering the welfare loss of individuals with different beginning-of-life productivity – but holding the same initial wealth – the economywide average \bar{a} – illustrates the fact that on average individuals tend to suffer from inflation fluctuations, with an equivalent permanent consumption loss close to 1%.

Importantly, the welfare cost of inflation fluctuations that we find are very unevenly distributed across households. In particular, while most households incur large welfare losses (up to 1.77% of permanent consumption), the most productive and/or wealthy households incur no loss or even a gain. Such is the case both for the typical high-productivity individual (starting with the average wealth of individuals with same productivity, a^h) and for a medium-productivity household having been lucky enough in the past to have accumulated a^h . To understand why this is the case, one

must bear in mind how inflation risk affects households’ utility under our preference specification. More specifically, the impact of inflation risk on individual welfare can be traced back to two effects: a direct effect due to fluctuations in the marginal utility of money holdings, and an indirect effect working through the distribution of aggregate capital income across households. The first effect implies that inflation risk, which mechanically generates volatility in real money holdings, hurts all households holding cash (since utility is concave in real balances). The second effect is related to the complementarity between money and consumption. Because the marginal utility of real balances is concave and consumption and real balances are complement, an increase in the volatility of future real money balances (as implied by greater inflation risk) lowers the average marginal utility of future consumption and hence discourages current savings.⁹ This is true for all types of savings, including holdings of capital claims, and for all households, i.e., at all productivity levels. Hence, inflation risk lowers the capital stock and raises the return on capital claims, which tilts the distribution of total capital income towards high-productivity households (who holds greater capital holdings on average). Indeed, the quarterly after-tax rates of return on capital claims are on average 0.60% and 0.59% in the high and the low inflation states of the baseline economy, respectively, while the same return is time-invariant at 0.55% in the economy without inflation risk. The indirect effect turns out to (mildly) dominate the direct effect for high productivity or mid-productivity/high wealth households, so that they ultimately benefit from inflation risk in equilibrium.

5 Conclusion

Existing analyses of the welfare cost of fluctuations in general equilibrium (i.e., with endogenous consumption) have focused on the impact of business cycles driven by real factors, namely changes in total factor productivity and/or the extent of idiosyncratic income risk. As far as we are aware, our analysis is the first one that explicitly examines the welfare impact of another key source of aggregate fluctuations, namely, inflation risk. In so doing, we have used the simplest monetary framework (i.e., Sidrauski’s MIU, flexible price model), combined with the assumption that house-

⁹Note that the lower future average marginal utility of consumption is accompanied by a greater *volatility* of future marginal utility of consumption. Since the marginal utility of consumption is convex at all levels of real balances, a “prudence” effect will limit, but not overturns, the impact of inflation risk on the average marginal utility of consumption (formally, the impact of prudence on current consumption-saving choices operates at one order of magnitude below the effect due to the complementarity between consumption and money).

holds face uninsured idiosyncratic income risk (in the tradition of Bewley, Hugget, Aiyagari and Krusell and Smith). We have then used the moments of the empirical cross-sectional distribution of monetary assets to discipline preferences towards money and consumption goods, which led us to adopt the nonhomothetic preference structure. Our main result is that a moderate amount of inflation risk (i.e., of magnitude equal to its empirical measure over the Volcker to pre-Great Recession period) generates substantial welfare losses for the average households (about one percent of permanent consumption), and is most harmful for households towards to the lower end of the distribution of income and wealth.

A Numerical Algorithm

A.1 Overview of the Algorithm

The algorithm used to obtain the solution of the model is as follows.

1. Given the law of motion for capital, defined by (14), solve the individual problem given by equations (4), (5), (13), with $k_{t+1} > 0$.¹⁰
2. Simulate the economy to approximate the equilibrium law of motion for K . We use the grid-based simulation procedure proposed by Young (2010).
 - (a) Set an initial wealth/employment-efficiency distribution $\mu_0(a, e)$ that provides $p_0^{i,e}$, i.e. the mass of agents of employment-efficiency type e with wealth a_i at the i th wealth grid point for, $i = 1, \dots, N_{grid}$.
 - (b) Given individual decision rules $g_k(a_t, e_t; \pi_t, K_t)$ and $g_m(a_t, e_t; \pi_t, K_t)$ found in step 1, the wealth/employment-efficiency distribution $\mu(a_t, e_t)$, and a draw for π_t , compute

$$\int g_m(a_t, e_t; \pi_t, K_t) d\mu(a_t, e_t) = M_t$$

and

$$\int g_k(a_t, e_t; \pi_t, K_t) d\mu(a_t, e_t) = K_{t+1}$$

- (c) Repeat steps (b) and (c) to get a long time series for K and M , of which the first part is discarded.

¹⁰The next sub-section describes the algorithm.

3. Use the time series obtained in step (2) to get the new equilibrium law of motion for K .
4. Compare the new equilibrium law of motion for K with that used in step (1). If they are similar, stop. Otherwise, update the coefficients of the laws of motion, and go to step (1).

A.2 Details on the Resolution of the Individual Problem

We solve the individual problem defined by the following FOC:

$$u_c(c_t, m_t) - u_m(c_t, m_t) = \beta E_t \left[\frac{v_a(a_{t+1}, e_{t+1}; \pi_{t+1}, K_{t+1})}{1 + \pi_{t+1}} \right] \quad (15)$$

$$u_c(c_t, m_t) = \beta E_t \left[(1 + (1 - \tau_{t+1}^c) r_{t+1}) v_a(a_{t+1}, e_{t+1}; \pi_{t+1}, K_{t+1}) \right] \text{ if } k_{t+1} > 0 \quad (16)$$

and the budget constraint (4) by iterating on the derivative of the value function with respect to a , $v_a(\cdot)$. We stop the iteration process if the new derivative of the value function is sufficiently close to that in the previous step. Otherwise, we update $v_a(\cdot)$ with the new derivative of the value function.

We have two distinct cases, whether the demand for asset is constrained or not:

1) If $k_{t+1} = 0$, we solve equation (15) to get m_t with $c_t = a_t + (1 - \tau_t^l) w_t e_t \bar{n} - m_t$, knowing $v_a(a_{t+1}, e_{t+1}; \pi_{t+1}, K_{t+1})$ and K_{t+1} from the aggregate law of motion.

2) If $k_{t+1} > 0$, we find the solution for g_c , g_m , g_k , and v_a using nested bisection methods. First, we solve for m and k given a certain level of consumption using equation (15) and the budget constraint. Second, we solve equation (16) for c , where the m and k are given by the previous step, knowing $v_a(a_{t+1}, e_{t+1}; \pi_{t+1}, K_{t+1})$ and K_{t+1} from the aggregate law of motion.

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