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Prices vs. quantities in presence of a second, unpriced, externality.*

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Abstract

We study a situation in which two goods jointly generate an externality but only one of them is regulated. Unilateral regulation of greenhouse gas emissions and related carbon leakage is a well known example. We compare tax and quantity instruments under uncertainty *à la* Weitzman (1974). Because of the uncertainty surrounding the unregulated good, the external cost is stochastic with both instruments. Whether the unregulated good quantity is more or less variable under a tax or under a quota depends on the degree of substitutability and the correlation between uncertainties on private valuations. In case of a positive correlation and imperfect substitution, a tax better stabilize the unregulated good quantity and can therefore dominate a quota when the slope of the external cost associated to the unregulated good is large. In a specification, relevant for leakage, it is shown that if uncertainty about the unregulated good (imports) is large, a tax might be preferable to a quota, regardless of the convexity of the external cost.

Keywords: Environmental regulation; Tax ; Quotas; Multi-pollutant; Carbon leakage

JEL: D83, O33, Q55

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1 Introduction

In any economy, there are many un-corrected externalities, some are still unnoticed or debated (e.g. diet and health), others difficult to regulate (e.g. noise) or simply outside the scope of the potential regulator (e.g. foreign pollution). The correction of market failures and notably of externalities is a dynamic process during which new scientific information and a better knowledge of the economy help to oil the wheel of the economy. Along this process, the correction of an externality, its internalization via regulation, should theoretically takes into account the possible interaction with other uncorrected externalities (Lipsey and Lancaster, 1956). The objective of the present paper is to compare instruments, price and quantity, to regulate a good that generates an externality when there is an un-corrected externality associated to a second good. It applies the methodology introduced by Weitzman (1974) in his seminal work on the comparison of instruments under uncertainty.

The motivating example of the present article is the phenomenon of "carbon leakage" and the functioning of the European market for CO_2 emission permit. When a country (or a group of country) implements a policy to reduce CO_2 emissions, this regulation is likely to lead to an increase in (unregulated) emissions elsewhere in the world. Notably, there could be a substitution between domestic and foreign production of CO_2 intensive goods. Numerous studies try to evaluate the amplitude of the phenomenon and the contrasting results obtained illustrates the uncertainty surrounding it.[1] The uncertainty surrounding local demand and imports pressure is also reflected in the variations of the permit price (Chevallier, 2010). A question is whether, given the uncertainty surrounding the demand for CO_2 emitting goods and the sensitivity of EU covered sectors to foreign competitors whether a tax instrument would not be preferable.

There are many other examples, notably related to environmental regulations.[2] However, there are other fields of application. For instance, Yaniv et al. (2009) provide an interesting example that could be relevant for nutrition policy: how a tax on junk food could reduce physical activity. In that case, the externality might be related to either health-cost sharing or self-control issues (O'Donoghue and Rabin, 2006). In the latter controversial case the individual imposes a cost on his future self.

A model with two goods is considered, these goods create a private economic benefit internalized by a representative agent, and they jointly creates an external cost. The goods are either substitute or complement in the economic benefit function and can interact positively or negatively in the external damage function. The framework is quadratic and there is an

[1]The order of magnitude obtained for leakage in these studies depends on the sectors under study and on modeling assumptions; i.e. 14% for steel and mineral products in Fischer and Fox (2012), 50% in Demailly and Quirion (2008) and 70% in Ponssard and Walker (2008) for cement, up to more than 100% in Babiker (2005) because of the relocation of energy intensive producers. There is also contrasting results with computable general equilibrium simulations, the recent contribution of Elliott and Fullerton (2014) that discuss the possibility of negative leakge is a good example.

[2]Not only imports but many sources of greenhouse gas emissions are outside the scope of current regulations. For instance, the emissions associated to land-use are not regulated and the regulation of the emissions from building material (cement, steel, aluminum) is likely to lead to an increase in the use of wood, which can further decrease the total quantity of CO_2 in the atmosphere, as wood sequesters carbon (Buchanan and Levine, 1999). In an other domain, Bento et al. (2014) estimate the cost associated to higher traffic congestion induced by a regulation aiming at promoting car efficiency.

additive uncertainty on the marginal private value created by each good. The uncertainties associated to each good valuation can be correlated. One of the good is regulated either by a tax or a quota, the other good is not regulated. Without uncertainty, the optimal tax is equal to the marginal external cost plus a corrective term equal to the marginal external costs of the unregulated good times the sensitivity of this good quantity to the regulated good quantity.

In his seminal work, Weitzman (1974) shows that price and quantity instruments are not equivalent under uncertainty With a tax, regulated agents adapt their production to the realization of uncertainty, or their private information(Laffont, 1977), and this flexibility is all the more valuable that the slope of the marginal abatement cost (or foregone benefit) is large. A drawback of the tax is that the quantity produced is stochastic, and this uncertainty is costly if the marginal external cost is increasing. A quota is less flexible, in the sense that some information is left unexploited, but it guarantees a sure external cost. In a quadratic specification the comparison of instruments boils down to the comparison of the slopes of the marginal abatement cost and the marginal external cost, and the range of uncertainty does not determine the sign of the comparison.

When a second externality is present, the comparison is modified in several directions, and, in particular the range of uncertainty, and its characteristics, matters in the comparison. The main lesson is that a quota on the regulated good does not ensure the stability of the overall external cost because of the uncertainty remaining on the quantity of the unregulated good. In addition of the expected benefit and external cost associated to the regulated good, there are two additional expected external costs in the comparison. These two costs are related to the variations of the unregulated good and the correlation of the two good quantities. This correlation is costly if the two goods interact positively in the external cost, there is a “cocktail” effect. Because of these two additional costs, the tax instrument is preferred for several interesting combinations of parameter specification. In particular, if the two goods are perfect substitute in the external costs (e.g. CO2 emissions), for intermediate degree of economic substitution and a large uncertainty surrounding the demand for the unregulated good the tax instrument is preferred whatever may be the slope of the environmental damage.

This work is related to several strands of the literature. Most notably, the comparison of instruments under uncertainty and multi-pollutant regulation.

The analysis of Weitzman (1974) has been extended in various directions.[3] In particular, Stavins (1996) considers a situation in which the uncertainty on the abatement cost is correlated with an uncertainty on the external cost. The ranking of the instruments is then modified and a positive (resp. negative) correlation between the marginal abatement cost and the marginal external cost is favorable (resp. detrimental) to the quantity instrument. This phenomenon arises endogenously in the present work, from the interaction of the regulated good with the unregulated one both in the abatement cost and the external cost functions. Interestingly, Stavins (1996) mentions the existence of a complementary pollutant

[3]Some notable contributions that are not closely related to the present work include: Roberts and Spence (1976) and Weitzman (1978) who consider the use of hybrid instruments: a system of cap-and-trade permits with a price floor and a price ceiling. Hoel and Karp (2002), and Newell and Pizer (2003) compare instruments in the case of a stock pollutant in a dynamic framework. Krysiak (2008), and Weber and Neuhoff (2010) introduce innovation in abatement technologies.

as a possible explanation for a positive correlation but his analysis does not integrate the external cost associated to this second pollutant.

Several authors compare instruments in second-best setting in which, in addition to uncertainty, there is another uncorrected market failure. The second market failure in itself does not justify to discriminate among instruments, but coupled with uncertainty it influences the comparison between them. Montero (2002) compares price and quantity instrument with incomplete enforcement. As in the present work, the quantity instrument cannot stabilize the external cost, because of non compliant firms, but paradoxically, this flexibility reinforces the appeal of the quantity instrument. In a second article, Montero (2001) considers the integration of two permit markets with incomplete enforcement. Meunier (2011) analyzes whether permit markets should be linked in presence of market power. Schöb (1996) and Quirion (2004) consider pre-existing distortionary taxation. Quirion (2004) finds that the relative merit of the tax is reinforced by the presence of distortionary taxation. A difficulty faced by these two articles is to model the general equilibrium effect of distortionary taxation in a partial equilibrium setting, typical of the Weitzman (1974) type of analysis. The choices made are indeed somehow ad-hoc.[4]

The issue of multi-pollutant regulation has received some attention. Most analysis, whether static or dynamic, consider the joint regulation of several pollutants in a deterministic setting. On the theoretical side, von Ungern-sternberg (1987) determines, in a static setting with two pollutants, under which conditions the optimal strategy is to devote the whole (given) budget to the reduction of only one pollutant. Kuosmanen and Laukkanen (2011) extends this discussion in a dynamic setting. Endres (1986) considers interior situations, in a static setting, and the implementation of a multipollutant permit market. He discusses the determination of trading ratios (see also Lence et al., 1988; Lence, 1991). This discussion has been extended in a dynamic framework by Moslener and Requate (2007, 2009). There are numerous more applied studies notably related to the study of multiple greenhouse gases (see Tol, 2006, and references therein). A different perspective is taken by Fullerton and Karney (2014). They analyze the regulation of a polluting good in presence of two other, sub-optimally regulated, polluting goods.[5] They study, in a general equilibrium setting, how the regulation interacts with the two other inefficiencies and how it depends on the type of instruments used for the second pollutant.

At the crossing of the two strands of literature are the articles of Mandell (2008) and Ambec and Coria (2013). Mandell (2008) considers a single pollution (CO_2) and analyzes the optimal partitioning of a continuum of polluters between a tax regulation and a quantity regulation under uncertainty. Polluters are subject to a common shock. This work has been recently generalized and extended by Caillaud and Demange (2015). Ambec and Coria (2013) consider a two pollutants situation and analyze whether it can be optimal to use a price-instrument for one good and a quantity instrument for the other. The comparison of regulatory instruments when only one pollutant is regulated has not been analyzed. Arguably, the present work may represent a situation in which the second good is regulated by a tax, possibly suboptimal, so some of the present results already appear in Ambec and Coria

[4]Schöb (1996) introduces a linear cost of public fonds whereas Quirion (2004) considers that distortionary taxes raise the slope of the marginal abatement cost (the foregone benefit in the present framework).

[5]They analyze the regulation of CO_2 emissions by a sector when there is an uncovered CO_2 emitting sector and a second pollutant (SO_2) which is sub-optimally regulated by either a tax or a quota.

(2013). However, they restrict their attention to symmetric situations and i.i.d. shocks while we precisely focus on the characteristics related to the unregulated good: the uncertainty surrounding its valuation, the influence of correlation and the marginal damage associated to it.

Indeed, the most common example of an externality is a pollution, however the present work could be used for other externalities, even other market failures. The co-existence of market failures has been studied in numerous contexts (see Bennear and Stavins, 2007, for a review centered on environmental regulations). Whether the present contribution can be applied to a particular case depends mainly on the relevance of the separability between the private benefit and the external cost. For instance, in the case of co-existence of an environmental public good and knowledge spillovers (see Jaffe et al., 2005; Gerlagh et al., 2009, among others) the present work cannot be used if the knowledge spillovers influences the private benefit from pollution.[6] If knowledge benefits are perfectly internalize within the polluting sector but spill over other sectors, the present work could be mobilized. In such a case, the second good (clean R&D) is complementary to the regulated good (pollution), and the comparison will depend on the degree of correlation between the uncertainties surrounding each good valuation.

Another instance, in which the present work can provide some insights, is the issue of environmental taxation within a distortionary taxation scheme. The optimal taxation scheme in presence of an externality when lump-sum individualized transfers are not available has received an important attention since the seminal works of Diamond (1973) and Sandmo (1975). Bovenberg and de Mooij (1994, 1997), and Fullerton (1997) discuss how the Pigouvian rule should be modified in presence of distortionary taxes. The influence of the environmental tax on the quantities of distortionary taxed goods (e.g. labour) is crucial.[7] Instruments price and quantity are equivalent in these settings in which there is no uncertainty.[8] Even though the present work cannot be directly applied to consider distortionary taxation, it might provide some ideas of possible issues. If the regulated good and leisure are substitute, the regulation of the polluting good increases leisure and reduces public budget, if this indirect consequence is subject to a large uncertainty, a tax has the possible merit of buffering the variation of leisure time. This interpretation provides another approach to the issue than the ones used by Schöb (1996) and Quirion (2004), and suggests a path for further research.

The rest of the article is organized as follows. In Section 2 the model is presented, then, in Section 3 the case of a myopic regulator is analyzed and in Section 4 the issue of asymmetric information is considered. Section 5 concludes.

[6] The articles by Krysiak (2008), and Weber and Neuhoff (2010) on the comparison of instruments under uncertainty with innovation (or investment) do not introduce knowledge externalities.

[7] The article of Bovenberg and de Mooij (1994) frames the problem as the introduction of an environmental tax in a preexisting tax system, more recent contribution formulate the issue in a framework *à la* Mirrlees (1971), see for instance Cremer et al. (1998); Cremer and Gahvari (2001), and recently Jacobs and de Mooij (2015).

[8] The possible differences between instruments come from the revenues generated and the way they are recycled, Fullerton and Metcalf (2001) explain that the main concern is the emergence of privately-retained scarcity rents.

2 Model

We consider an economy with two goods $i = 1, 2$, the quantities of these goods are q_1 and q_2. Both goods generate an externality, but only good 1 is regulated either by a tax t or a quota $\hat{q}_1$. The economic benefit internalized by agents when producing and consuming the two goods is $B(a_1, a_2, q_1, q_2)$, in which a_1 and a_2 are two randoms variables with respective means $\bar{a}_1$ and $\bar{a}_2$. There is an external cost assumed to be separable, and denoted as $D(q_1, q_2)$. Welfare is the difference between economic benefits and the external cost:

$$W(a_1, a_2, q_1, q_2) = B(a_1, a_2, q_1, q_2) - D(q_1, q_2). \tag{1}$$

Although part of the discussion could be conducted under a general specification, most results (in particular normative ones) require the use of specific functions. In the following we consider quadratic benefits and damages:

$$B(a_1, a_2, q_1, q_2) = a_1 q_1 + a_2 q_2 - \frac{b_1}{2} q_1^2 - \frac{b_2}{2} q_2^2 - \gamma q_1 q_2, \tag{2}$$

$$D(q_1, q_2) = \frac{d_1}{2} q_1^2 + \frac{d_2}{2} q_2^2 + \delta q_1 q_2. \tag{3}$$

The marginal external damage is null if quantities are null. This assumption is innocuous since the error made by the regulator is of the second order and the comparison of instruments is only influenced by the slopes of the marginal external costs, namely, d_1, d_2 and δ. The parameter d_i, for $i = 1, 2$, will be (loosely) referred to as "the slope of the external cost with respect to good i"; and the parameter δ will be called the "external interaction parameter".

The parameter γ represents the interaction of the two goods within the private benefit, it will be referred to as "the degree of substitution". If γ is positive (resp. negative) the two goods are substitutes (resp. complements). To ensure that the economic benefit is strictly concave it is assumed that:

$$b_1 b_2 - \gamma^2 > 0. \tag{4}$$

And similarly for the external cost:

$$d_1 d_2 - \delta^2 \geq 0. \tag{5}$$

The regulator does not know the actual values of a_1 and a_2 but has a prior probability distribution, based on available information. Ex-ante, when designing the environmental policy, the regulator anticipates that $a_1 = \bar{a}_1 + \theta_1$ and $a_2 = \bar{a}_2 + \theta_2$, where θ_1 and θ_2 are two random variables with $\mathbb{E}\theta_1 = \mathbb{E}\theta_2 = 0$ and

$$\text{var}(a_1) = \sigma_1^2, \; \text{var}(a_2) = \sigma_2^2, \; \text{and } \text{cov}(a_1, a_2) = \sigma_{12} = \rho\sigma_1\sigma_2. \tag{6}$$

The stochastic variables a_i represent an uncertainty on the demand for the two goods. It could arise from imperfect knowledge of the regulator about the tastes of consumers or of production conditions, or also from the time lag between the choice made by the regulator and the market interactions that determine the outcome. In any case, it is assumed that the support of these variables is not too large to ensure that in all situations considered the two goods are produced and the quota is binding.

The regulator decides whether to use a tax or a quota and sets the tax t or the quotas $\hat{q}_1$ by maximizing expected welfare. He does not know the true value of each a_i (i=1,2), but they are known by regulated agents, and the amounts of emissions are determined accordingly. The timing is the following:

1. The regulator decides whether to use a tax on good 1 or a quota.
2. The regulator chooses the tax or the quotas by maximizing expected welfare.
3. The uncertainty is resolved, a_1 and a_2 are known and q_1 and q_2 depends upon their values and the instrument chosen:
   - **With a tax**: the quantities are $q_{1t}(a_1, a_2, t)$ and $q_{2t}(a_1, a_2, t)$ solutions of

     $$\frac{\partial B}{\partial q_1}(q_1, q_2) = t \text{ and } \frac{\partial B}{\partial q_2}(q_1, q_2) = 0 \tag{7}$$

     Expected welfare is then

     $$W_t(t) = \mathbb{E}W(a_1, a_2, q_{1t}, q_{2t}) \tag{8}$$

   - **With a quota**: the quantities are $q_{1q}(a_1, a_2, \hat{q}_1) = \hat{q}_1$ and $q_{2q}(a_1, a_2, \hat{q}_1)$ solution of:

     $$\frac{\partial B}{\partial q_2}(\hat{q}_1, q_2) = 0 \tag{9}$$

     Expected welfare is then

     $$W_q(\hat{q}_1) = \mathbb{E}W(a_1, a_2, q_{1q}, q_{2q}) \tag{10}$$

To conduct the analysis it is useful to consider the quantity of good 2 as a function of the quantity of good 1 and the parameter a_2. This is so because the regulator does not directly control the production of good 2 but only indirectly via the regulation of good 1, and the sensitivity of the quantity of good 2 to a change in the quantity of good 1 plays a key role in determining the optimal regulation. The function $f(a_2, q_1)$ is the solution of

$$\frac{\partial B}{\partial q_2}(a_1, a_2, q_1, q_2) = 0 \tag{11}$$

With the linear specification it is:

$$f(a_2, q_1) = \frac{a_2 - \gamma q_1}{b_2}. \tag{12}$$

3 Comparison of instruments

3.1 First and second best

Let us first briefly describe the first and second best regulation that would maximize welfare without uncertainty. The first-best regulation implemented by a regulator able to regulate good 2 would consist of two quantities that are the solution to the usual first order conditions:

$$\text{for i=1,2 } \frac{\partial B}{\partial q_i} = \frac{\partial D}{\partial q_i}.$$

The corresponding taxes are the Pigouvian ones.

If the regulator cannot regulate good 2 but only good 1, he implements a second-best regulation. This second-best regulation could be fully described by a quantity $q_1^*(a_1, a_2)$ of good 1 that maximizes $W(a_1, a_2, q_1, f(a_2, q_1))$ and is the solution of

$$\frac{\partial B}{\partial q_1}(a_1, a_2, q_1, f(a_2, q_1)) = \frac{\partial D}{\partial q_1} + \frac{\partial D}{\partial q_2}\frac{\partial f}{\partial q_1} \tag{13}$$

The term $\partial B/\partial q_2$ does not appear on the left-hand side because of the first order condition (11). The corresponding tax on good 1, denoted by t^* is equal to the right-hand side of (13). This tax is not equal to the direct marginal external cost because of the marginal effect on the quantity of good 2, represented by the second term of the right-hand side. This term is positive if goods are complements and negative if they are substitutes. Carbon leakage corresponds to the latter situation. In that case, the environmental damage created by a unit of good 1 is partly compensated by a reduction in the quantity of good 2. The second-best tax is less than the marginal external cost. It is even possible that good 1 should be subsidized because it may be worth increasing regulated production to reduce unregulated production.

With the linear specification, the optimal quantity is $q_1^*(a_1, a_2)$:

$$q_1^*(a_1, a_2) = \frac{a_1 - a_2\gamma/b_2 - \frac{a_2}{b_2}(\delta - d_2\gamma/b_2)}{(b_1 + d_1) + (d_2 - b_2)(\gamma/b_2)^2 - 2\delta\gamma/b_2} \tag{14}$$

3.2 Uncertainty

To compare instruments under uncertainty, we first consider their optimal choice, and then the difference between expected welfare with an optimal tax and an optimal quota. With a tax t, in a state a_1, a_2 the quantities q_{1t} and q_{2t} solve the first-order conditions (7). Under the quadratic specification, the solution values are:

$$q_{1t} = \frac{1}{b_1 b_2 - \gamma^2}\left[b_2(a_1 - t) - \gamma a_2\right] \tag{15}$$

$$q_{2t} = \frac{1}{b_1 b_2 - \gamma^2}\left[b_1 a_2 - \gamma(a_1 - t)\right] = f(a_2, q_{1t}). \tag{16}$$

Anticipating these values, in setting the tax, the regulator maximizes expected welfare. The first-order condition is

$$E\left\{\left[\frac{\partial B}{\partial q_1}-\left(\frac{\partial D}{\partial q_1}+\frac{\partial D}{\partial q_2}\frac{\partial f}{\partial q_1}\right)\right]\frac{\partial q_{1t}}{\partial t}\right\}=0. \tag{17}$$

A nice property of the linear specification is that the marginal effect of the tax on production, $-b_2/(b_1b_2-\gamma^2)$, is deterministic, so that, at the optimal tax, the expected marginal benefit equals the expected marginal external cost:

$$E\left[\frac{\partial B}{\partial q_1}\right]=\mathbb{E}\left[\frac{\partial D}{\partial q_1}+\frac{\partial D}{\partial q_2}\frac{\partial f}{\partial q_1}\right]. \tag{18}$$

With an additive uncertainty, the expectation of the marginal benefit (resp. external cost) is equal to its value at $\bar{a}_1$ and $\bar{a}_2$ and the optimal tax is independent of the distribution of the θ_i, $i=1,2$, and so is the expected quantity of good 1 at this optimal tax level.

With a quota on production 1, the amount of production 2 is:

$$q_{2q}=f(a_2,\hat{q}_1)=\frac{a_2-\gamma\hat{q}_1}{b_2} \tag{19}$$

The regulator chooses the quota $\hat{q}_1^\star$ that maximizes expected welfare. The first-order condition is similar to (18), with the difference that the value of q_1 is fixed by the quota. Thus, expected production of good 1 with the optimal tax is equal to the optimal quota, that is,

$$\mathbb{E}[q_{1t}(a_1,a_2,t^*)]=\hat{q}_1^*\ \ (=q_1^*(\bar{a}_1,\bar{a}_2)) \tag{20}$$

The comparison of expected welfare under the two instruments is made at the optimal level for each instrument, but it holds for any couple of tax and quota such that the quota is equal to the expected production of good 1 with the tax. What matters for the comparison are the benefits and costs arising from *variations* of the quantities. The benefits under a tax arise from the consistency of these variations with the actual economic values of goods 1 and 2. The costs are due to the variations of the external costs and are thus equal to the slopes of the marginal external cost times production variances.

Lemma 1 *The difference in expected welfare between a tax and a quota is:*

$$\Delta_{t,q}=_{def}\max_t \mathbb{E}W_t-\max_{\hat{q}_1}\mathbb{E}W_q=$$
$$\frac{var(q_{1t})}{2}\left[\left(b_1-\frac{\gamma^2}{b_2}\right)-d_1\right]-\frac{d_2}{2}\left[var(q_{2t})-var(q_{2q})\right]-\delta\,cov(q_{1t},q_{2t}) \tag{21}$$

The proof is shown in appendix A.1. The first term on the right-hand side of equation (21) is the difference between the expected benefits and external costs arising from the variation of the quantity of good 1 with a tax. This is similar to the result of Weitzman (1974). It is the comparison that would arise if no externality were associated with good 2. Its sign is fully determined by the comparison of the slopes of marginal economic benefit and external cost. The marginal economic benefit is $b_1-\gamma^2/b_2$, the "direct" slope of the marginal benefit from production 1 is b_1 and γ^2/b_2 is the "indirect" effect due to the adjustment of the quantity of good 2.

The second bracketed term in (21) is the difference of expected external costs due to the variations of the quantity of good 2, quantity that is variable under both instruments. The sign of this term is unclear and depends on parameter values. In particular, it will be shown that under some circumstances the quantity of good 2 varies more under a quota than under a tax. Finally, the last bracketed term in (21) represents the expected cost due to the co-movement of quantities of good 1 and 2 under a tax. An overall comparison is difficult to make because of the ambiguity of the signs of the two last terms.

The only clear-cut result is the effect on the comparison of the slope of the marginal external cost arising from good 1. This effect is negative for all risk parameters and degrees of substitutability.

Proposition 1 *There is a threshold value of d_1, the slope of the marginal external cost with respect to good 1, such that the tax is preferred if and only if d_1 is below this threshold.*

Proof. $\Delta_{t,q}$ is strictly decreasing with respect to d_1, and for a sufficiently large d_1 it is negative. ■

Concerning the regulated good, the effect of the slope of the marginal external cost is expected. If the marginal external cost is very steep, deviations from the optimal quantity of good 1 are very costly and setting a quota is optimal and the variations of the second good can be neglected. The threshold slope cannot be easily characterized because of the undetermined signs of the other two terms in (21), terms related to variations of the second good quantity and the covariance between the unregulated and regulated quantities. These ambiguities are discussed in the two following propositions.

Proposition 2 *Concerning the influence of d_2, the slope of the marginal external cost with respect to q_2, on the comparison of instruments three cases can be distinguished :*

1) If the two goods do not interact in the private benefit, i.e. $\gamma = 0$, the comparison of the instruments is independent of d_2:

$$\frac{\partial \Delta_{t,q}}{\partial d_2} (= \frac{1}{2} [var(q_{2t}) - var(q_{2q})]) = 0.$$

2) If the uncertainties on the marginal private values of the two goods are uncorrelated, i.e. $\sigma_{12} = 0$, the comparison of instruments is less favorable to the tax the larger d_2 is:

$$\frac{\partial \Delta_{t,q}}{\partial d_2} < 0.$$

3) If the two uncertainties are strictly positively (resp. negatively) correlated, i.e. $\sigma_{12} > 0$ (resp. < 0), then for small substitutability (resp. complementarity) of the two goods the comparison of instrument is more favorable to the tax the larger d_2 is.

More precisely, there is a strictly positive threshold γ_2 such that

$$\frac{\partial \Delta_{t,q}}{\partial d_2} > 0 \Leftrightarrow \gamma\sigma_{12} > 0 \text{ and } |\gamma| < \gamma_2.$$

The proof is in appendix A.2. The comparison of instruments depends upon the slope of the marginal external cost with respect to good 2, as represented by the second term of equation (21). A larger slope increases the appeal of the tax if the quantity of good 2 is less variable with the tax than with the quota. The difference between the variances of the quantity of good 2 with a tax and with a quota depends on the degree of substitutability between good 1 and good 2 and the correlation between the two uncertainties on their private valuations. If the two goods are independent in the private benefit, the quantity of good 2 is not influenced by the regulation of good 1 and does not influence the comparison of instruments.

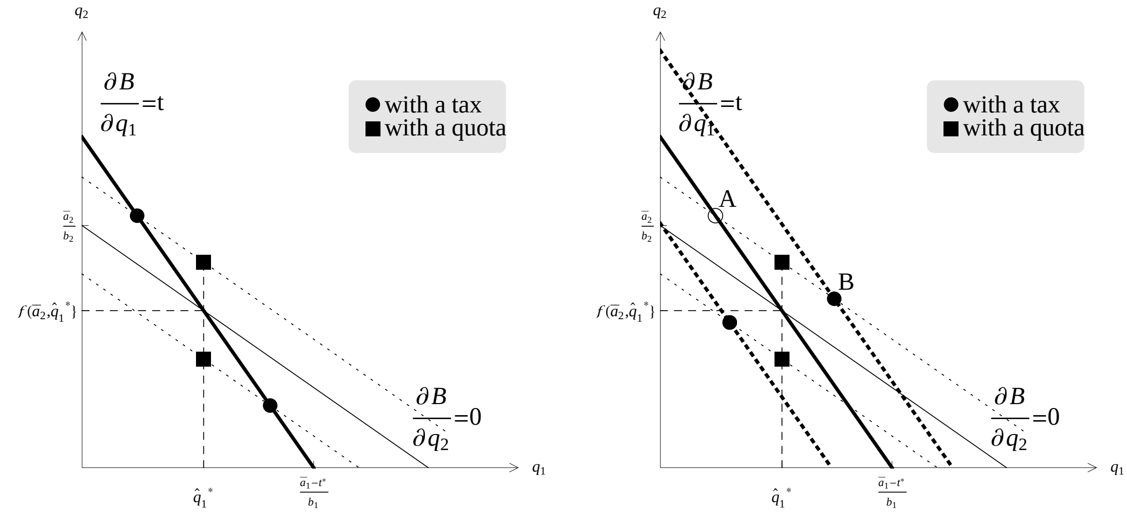
If the two good valuations, a_1 and a_2, are not correlated, the quantity of good 2 is more variable with a tax than with a quota. It is illustrated on Figure 1(a), with two substitute goods. On the Figure the two first order conditions (7) are depicted by two lines in the (q_1, q_2) space. The two equilibrium quantities are the coordinates of the intersect. The uncertainty surrounding a_2 shifts the marginal private value of good 2 and the associated curve. With a quota the change of the quantity of good 2 is directly determined by the shift of the marginal private value. With a tax, the direct effect of the shift is amplified by the indirect effect from the change of the equilibrium quantity of good 1.

If there is a correlation, it is possible that good 2 production varies less under a tax than under a quota as illustrated by Figure (1(b)). If the two goods are substitutes and the two risks are positively correlated: Whenever the valuation of good 2 is high, the valuation of good 1 is (likely to be) high. The rise of the demand for good 1 (shift of the $\partial B/\partial q_1 = t$ curve) induces a reduction in the quantity of good 2 (from point A to point B in Figure 1(b)) that counteracts the initial rise. The softening effect of correlation dominates for relatively small degree of substitution (the figure is obtained for $\gamma = 0.7\sqrt{b_1 b_2}$),[9] for larger substitutability the comparison is reversed and the quantity 2 is more variable under a tax than under a quota. An increase of the degree of substitution corresponds to a rotation (counter-clockwise) of both lines in Figure 1 and would move the tax situation (depicted by a dot) along the $\partial B/\partial q_2 = 0$ curve in either direction depending on the relative size of the shock on good 1. And for sufficiently large degrees of substitution the variations under a tax are again larger than under a quota.

Proposition 2 can then be used to obtain a result on the comparison of instruments in the particular case in which a larger slopes enhances the appeal of the tax instrument. Interestingly, it is precisely in situations in which the degree of substitution or complementarity is small, so that the connection between the two externalities might not be self-evident, that the tax induces a smaller variations of the unregulated good and is preferred if the external cost from this second good is large.

Corollary 1 *If the two uncertainties are positively (resp. negatively) correlated and the two goods are imperfect substitutes (resp. complements) with* $|\gamma| < \gamma_2$ *then there is a threshold slope of the marginal external cost with respect to good 2,* d_2*, such that the tax is preferred to a quota if and only if* d_2 *is above this threshold.*

[9] The threshold degree of substitution depends on the variances, the covariance and the market size. If the two uncertainties are perfectly correlated with $\theta_1/\sqrt{b_1} = \theta_2/\sqrt{b_2}$, the quantity is less variable with a tax than with a quota whatever the degree of substitution. Note that this parameter configuration correspond to the situation studied by Ambec and Coria (2013).



(a) Without Correlation, uncertainty shifts the $\partial B/\partial q_2 = 0$ curve.

(b) With a common shock, uncertainty shifts simultaneously both curves in the same direction.

Figure 1: The influence of the uncertainty, surrounding the private value of good 2, on the quantities of good 1 and good 2 with a tax (points) and a quota (squares) without correlation (a) and with correlation (b) for two substitute goods.
The thin plain line represents the equation $q_2 = f(\bar{a}_1, q_1)$, and the thick plain line the equation $\partial B/\partial q_1 = t$.[11]

Let us now consider the influence of the external interaction parameter of the two goods. This influence is casted by the sign of the third term in the comparison described by equation (21).

Proposition 3 *An increase of the external interaction parameter δ increases the appeal of the tax if the degree of substitution is high.*

More precisely, there is a threshold γ in $(-\sqrt{b_1 b_2}, \sqrt{b_1 b_2})$ such that, the comparison is more favorable to the tax the larger δ is, $\partial \Delta_{t,q}/\partial \delta > 0$, if and only if γ is above this threshold.

The proof is in Appendix A.3. The effect of the parameter δ is reminiscent of the work of Stavins (1996). He compares instruments price and quantity to regulate one polluting good when the marginal economic benefit (marginal abatement cost in his framework) is correlated with the marginal external cost (environmental benefit in his framework). If this correlation is positive, then the comparison is less favorable to the tax than without correlation because, with the tax, when the quantity produced is high the marginal damage is also likely to be high. The variations of the quantity produced with the tax are more costly the larger the correlation. Stavins (1996) mentions (p. 226) that a positive correlation can be due to "synergistic health effects", also commonly named "cocktail effect", between complementary pollutants. In the present model, the correlation is endogenous and arises from the variations in the quantity of good 2. The situation described by Stavins (1996) corresponds to $\delta > 0$ and

[11] The Figure is obtained for $\bar{a}_1 = \bar{a}_2 = 10$, $b_1 = b_2 = 1$, $\gamma = 0.7$ and a distribution of $a_2 = \bar{a}_2 \pm 2$ and a tax $t \approx 12.8$ which corresponds to $d_1 = d_2 = 0.3$ and $\delta = 0.1$.

$\gamma < 0$. Note that Stavins (1996) does not mention the direct effect of the second pollutant that can also influence the comparison of instruments as described by Proposition 2.

If the marginal economic benefits are independently distributed, the endogenous correlation is negative (resp. positive) if the two pollutants are substitutes (resp. complements). In the case described by Stavins (1996), the damage associated with good 1 is larger the larger is the quantity of good 2 consumed, and with complement goods the quantities are correlated which has a negative effect on expected welfare with a tax. However, if the two goods are substitute the reverse holds. Correlation in the demand for the two goods influences the threshold degree of substitution, and the more correlated the two demands are the lower the threshold is.

These results show that there is in fact no rule as simple as the one proposed by Weitzman (1974) to determine whether a tax or a quota should be used. Apart from the effects related to the variability of the regulated good 1 quantity alone, the variability of unregulated productions both with a tax and with a quota blurs the picture. It is necessary to further specify the framework to obtain additional results, in particular, to assess the effect of the risk parameters.

For the remainder of the article, it will be assumed that the external cost is a function of the sum of quantities 1 and 2 and the common slope of external cost will be labeled d ($d = d_1 = d_2 = \delta$):

$$D(q_1, q_2) = d_0(q_1 + q_2) + \frac{d}{2}(q_1 + q_2)^2. \tag{S}$$

In this particular case, the comparison of instruments could be examined from a different perspective by isolating damages from benefits in the expression (21):

$$\Delta_{t,q} = \frac{b_1 b_2 - \gamma^2}{2b_2}\text{var}(q_{1t})^2 - \frac{d}{2}\left[\text{var}\left(q_{1t} + q_{2t}\right)^2 - \text{var}\left(q_{2q}\right)^2\right] \tag{22}$$

Furthermore, it will be assumed that the risks are uncorrelated: $\sigma_{12} = 0$.

The three parameters that are scrutinized here are the slope of the external cost d, and the two variances of the private valuations σ_i, $i = 1, 2$. The effects of the two variance parameters is worth investigating because they could be interpreted as a measure of the degree of uncertainty. For instance, if good 2 is an imported good, the regulator might be less informed about the foreign industry cost than about the domestic industry cost. Whether such a configuration should favor a tax or a quota instrument depends upon the degree of substitutability between the two instruments.

Concerning the slope of the external cost, the overall picture is that the tax is more likely to be preferred the smaller is the slope of the marginal external cost. In an extreme case, for large variance on the marginal value of the unregulated good 2, the tax may dominate the quota for any value of this slope. In such situations, the variability of the total production (ie. the sum of the two productions) is higher with the quota than with the tax, which implies that the difference in expected welfare between a tax and a quota is increasing with respect to the slope of the marginal external cost (and given that it is positive for $d = 0$ it is always positive). The following Lemma describes these situations.

Lemma 2 *For a marginal external cost proportional to total production (as in S) and uncorrelated uncertainties ($\sigma_{12} = 0$), there is a threshold degree of substitution $\tilde{\gamma}(b_1, b_2) \in (0, b_2)$ such that*

$$\frac{\partial^2 \Delta_{t,q}}{\partial d \partial \sigma_2} > 0 \textit{ if and only if } 0 < \gamma < \tilde{\gamma} \tag{23}$$

The parameter $\tilde{\gamma}$ is:

$$\tilde{\gamma} = \min\left\{ b_2, \frac{b_2}{2}\left(\sqrt{1 + 8b_1/b_2} - 1\right)\right\}. \tag{24}$$

If the two goods are imperfect substitutes in the economic benefit, the cross derivative of the comparison is positive. The larger the variance of the demand for good 2, the larger the impact of the slope of the marginal external cost. Eventually, for a sufficiently large (resp. small) variance σ_2 (resp. σ_1) the comparison is increasing in the slope of the environmental damage, that is, the second term in equation (22) is negative. This monotonicity is opposite to the standard Weitzman (1974) result and is due to the fact that the variance of total production is larger under a tax than under a quota.

The comparison between the variation of the total production with a tax and a quota depends on the degree of substitutability between the two goods and the ratio of the two variances. If the two goods are complements, under a tax there is a "snowball effect": an increase of the demand for good 2 induces a increase of both quantities. With complements, controlling the quantity of good 1 by a quota limits variations not only of quantity 1 but also of quantity 2. A quota is then preferred if the demand for good 2 is highly uncertain.

If the two goods are substitutes, the picture might be reversed if the substitutability is not too large. In this case, as stated in Lemma 2, total production could be more variable under a quota than under a tax, if the uncertainty associated with good 2 is sufficiently greater than the uncertainty associated with good 1. Figure 1(a) can provide intuition: with a tax the variations of the unregulated production 2 are buffered by the variation of the regulated good 1.

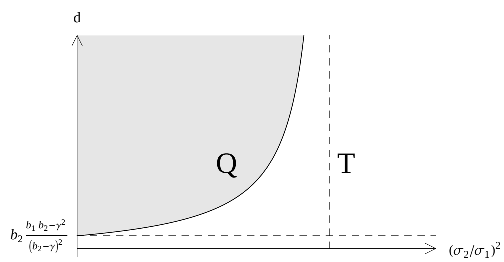
Proposition 4 *For a damage proportional to total production (as in S) and uncorrelated uncertainties ($\sigma_{12} = 0$):*

- *If the two goods are imperfect substitutes, i.e. $\gamma > 0$, with a substitution parameter lower than the threshold $\tilde{\gamma}$, and if σ_2/σ_1 is sufficiently large, the tax is preferred for all values of the slope of the marginal external cost.*

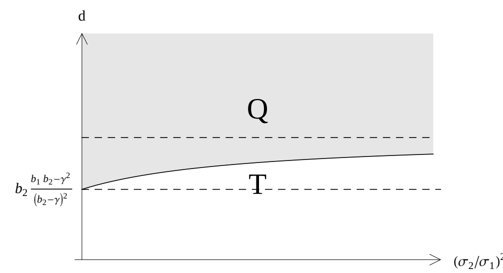
- *Otherwise, if the two goods are complements or close substitutes (i.e. $\gamma >$ gam̃ma) there is a threshold $\tilde{d}$, such that the tax is strictly preferred if and only if the slope of the marginal external cost is below this threshold ($d < \tilde{d}$).*

  This threshold increases with respect to σ_2 and decreases with respect to σ_1 if and only if $0 \leq \gamma \leq b_2$.

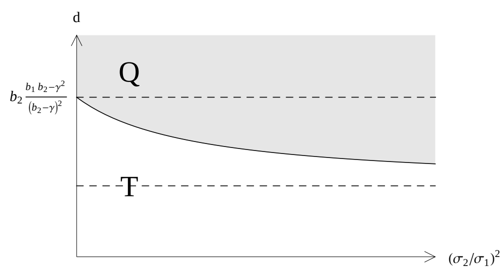
The proof is in appendix A.4. The results of this proposition are illustrated in Figure 2, which depicts the regions of preference for a tax or a quota, depending on the slope d and the ratio of the variances σ_2^2/σ_1^2. The three subfigures have been drawn for particular value of the parameters b and γ. Figure 3(b) is the most surprising as it represents a situation in which, for sufficiently large uncertainty surrounding the demand for good 2, the tax is preferred for any slope of the marginal environmental damage. Such situation might be relevant to the case of carbon leakage. The threshold degree of substitutability $\tilde{\gamma}$, described by (24), is sufficiently large so that the situation described by Figure 3b is not simply a theoretical possibility but likely to hold in many circumstances.



(a) If $0 < \gamma < \tilde{\gamma}$



(b) If $\tilde{\gamma} < \gamma < b_2$



(c) If $\gamma < 0$, or, $\gamma > b_2$

Figure 2: The preferred instrument when the damage depends on total production (specification S) and independent shocks ($\sigma_{12} = 0$), as a function of the slope of the environmental damage d (y-axis) and the ratio of the variances of the demands σ_2^2/σ_1^2 (x-axis).

4 Conclusion

The choice of a policy instrument has been analyzed in a situation with two interacting externalities. The two goods considered interact both in the private economic benefit, which is internalized via market interaction, and an external cost. When only one of the two goods is regulated by either a tax or a quota, the optimal level of the instrument is influenced by the second, uncorrected, externality. With uncertainty, the choice between instruments is also influenced by the presence of this second externality.

Weitzman (1974) shows that, under uncertainty, the advantage of a tax instrument lies in the adjustment of production to economic circumstances, but the associated variations in production can be costly if the slope of the marginal external cost is large. With a second unpriced externality, even a quota generates a stochastic external cost because of variations in the unregulated pollution. The variations of the unregulated good and the correlation between them and those of the regulated good influence the comparison. Under some circumstances, the unregulated good varies more under a quota than under tax, and if the goods are substitutes the variations are negatively correlated with a tax. Both these features are likely to reinforce the appeal of a tax instrument.

Comparisons of instruments depend not only on the slopes of the marginal external cost and economic benefit but also on the level of uncertainty surrounding the demand for the two goods. If the external damage is a function of the sum of the two productions (e.g. domestic and foreign CO2 emitting production), it has been shown that if the two goods are imperfect substitutes, total production varies more with a quota than a tax. Consequently, for sufficient large degree of uncertainty surrounding the unregulated good private benefit, the tax instrument is preferred whatever the (common) slope of the marginal external cost.

While this analysis has been developed with the situations of two pollutants in mind, it could be applied for other cases of interacting externalities or market failures. The global lesson being that in case of uncertainty and costly variations of externality generating goods, the choice of an instrument is determined by the possible amplification or compensation of variations of the two goods. The analysis of the choice of instrument in presence of distortionary taxation is an example of interacting market failures. The framework could not be directly applied to this issue, mainly because the lack of revenue recycling mechanisms, and further research is needed to bridge a gap between the so-called double-dividend literature and the comparison of instruments under uncertainty.

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APPENDIX

A Preliminaries

In these preliminaries we write expected welfare disentangling terms that are common to both instruments and terms that differ.

First, if productions are random, i.e. $q_i = \bar{q}_i + \epsilon_i$, for $i = 1, 2$ where the ϵ_i are random variables (possibly correlated with the θ_is) with $\mathbb{E}\epsilon_i = 0$, then, with the linear specification given by (2) and (3) expected welfare is

$$\mathbb{E}W(a_1, a_2, q_1, q_2) = W(\bar{a}_1, \bar{a}_2, \bar{q}_1, \bar{q}_2) + \mathbb{E}W(\theta_1, \theta_2, \epsilon_1, \epsilon_2). \tag{25}$$

The second term encompasses the effects of variations in productions and their interactions with variations in a_i.

Second, with both instruments in any state (a_1, a_2) the quantity of good 2 is $q_2 = f(a_2, q_1) = (a_2 - \gamma q_1)/b_2$. The private benefit, given by (2), can then be written as a quadratic function of q_1 by injecting $q_2 = f(a_2, q_1)$ into its expression (2). To find the resulting quadratic function, instead of brutal calculations, one can consider the derivative

$$\begin{aligned}\frac{dB}{dq_1} &= \frac{\partial B}{\partial q_1} + \frac{\partial B}{\partial q_2}\frac{\partial f}{\partial q_1} = a_1 - b_1 q_1 - \gamma f(a_2, q_1) \qquad \text{since } \partial B/\partial q_2 = 0 \\ &= a_1 - b_1 q_1 - \gamma \frac{a_2 - \gamma q_1}{b_2} = \left[a_1 - \frac{\gamma}{b_2} a_2\right] - \frac{b_1 b_2 - \gamma^2}{b_2} q_1\end{aligned}$$

So

$$B(a_1, a_2, q_1, f(a_2, q_1)) = B(a_1, a_2, 0, f(a_2, 0)) + \frac{1}{b_2}\left[(b_2 a_1 - \gamma a_2) - \frac{b_1 b_2 - \gamma^2}{2} q_1\right] q_1. \tag{26}$$

Finally, if $q_1 = \bar{q}_1 + \epsilon_1$ and $q_2 = f(a_2, q_1) = f(\bar{a}_2, \bar{q}_1) + f(\theta_2, \epsilon_1)$ then, from (25) and (26), replacing ϵ_2 by $f(\theta_2, \epsilon_1)$, expected welfare is:

$$\begin{aligned}\mathbb{E}W =& W(\bar{a}_1, \bar{a}_2, \bar{q}_1, \bar{q}_2) + \mathbb{E}[B(\theta_1, \theta_2, \epsilon_1, f(\theta_2, \epsilon_1)) - D(\epsilon_1, \epsilon_2)] \qquad \text{from (25)} \\ =& W(\bar{a}_1, \bar{a}_2, \bar{q}_1, \bar{q}_2) + \mathbb{E}\left[B(\theta_1, \theta_2, 0, f(\theta_2, 0))\right] \\ &+ \mathbb{E}\left[\left[\theta_1 - \frac{\gamma}{b_2}\theta_2 - \frac{b_1 b_2 - \gamma^2}{2b_2}\epsilon_1\right]\epsilon_1 - D(\epsilon_1, \epsilon_2)\right] \qquad \text{using (26)}\end{aligned} \tag{27}$$

The first two terms are common to both instruments since $\hat{q}_1^* = \mathbb{E}q_{1t}(t^*)$ and the last term encompasses the effects on expected welfare of variations specific to the instrument used.

A.1 Proof of Lemma 1

Under a tax the variable component is $\epsilon_1 = (b_2\theta_1 - \gamma\theta_2)/(b_1 b_2 - \gamma^2)$. Thus, with a tax,

$$\left[\theta_1 - \frac{\gamma}{b_2}\theta_2 - \frac{b_1 b_2 - \gamma^2}{2}\epsilon_1\right]\epsilon_1 = \frac{b_1 b_2 - \gamma^2}{2b_2}\epsilon_1^2 \tag{28}$$

Under a quota, the variable component ϵ_1 is zero. Injecting (28) into the expression (27), the difference of expected welfare with a tax and a quantity instrument is:

$$\begin{aligned}\Delta_{tq} =& \mathbb{E}\left[\frac{b_1 b_2 - \gamma^2}{2b_2}(q_{1t} - \bar{q}_{1t})^2\right] \\ &- \mathbb{E}\left[\frac{d_1}{2}(q_{1t} - \bar{q}_{1t})^2 + \frac{d_2}{2}(q_{2t} - \bar{q}_{2t})^2 + \delta(q_{1t} - \bar{q}_{1t})(q_{2t} - \bar{q}_{2t})\right] + \mathbb{E}\left[\frac{d_2}{2}(q_{2q} - \bar{q}_{2q})^2\right] \\ =& \left[\frac{b_1 b_2 - \gamma^2}{2b_2} - \frac{d_1}{2}\right]\text{var}(q_{1t})^2 - \frac{d_2}{2}\left[\text{var}(q_{2t}) - \text{var}(q_{2q})\right] - \frac{\delta}{2}\text{cov}(q_{1t}, q_{2t})\end{aligned} \tag{29}$$

A.2 Proof of Proposition 2

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From expression (21), the effect of d_2 on the comparison of instruments is,

$$\begin{aligned}\frac{\partial \Delta_{tq}}{\partial d_2} &= -\frac{1}{2}\left[\mathrm{var}(q_{2t}) - \mathrm{var}(q_{2q})\right] \\ &= -\frac{1}{2}\left[\frac{1}{(b_1b_2-\gamma^2)^2}\left(\gamma^2\sigma_1^2 + b_1^2\sigma_2^2 - 2b_1\gamma\sigma_{12}\right) - \frac{\sigma_2^2}{b_2^2}\right] \\ &= -\frac{1}{2}\frac{b_1\gamma^2}{(b_1b_2-\gamma^2)^2}\left[\frac{\sigma_1^2}{b_1} + (2-\frac{\gamma^2}{b_1b_2})\frac{\sigma_2^2}{b_2} - 2\frac{\sigma_{12}}{\gamma}\right] \end{aligned} \tag{30}$$

- It is zero for $\gamma = 0$.
- If $\sigma_{12} = 0$, then it is negative because the two first terms of the bracketed factor are positive (remember that $\gamma^2 < b_1b_2$).
- If $\sigma_{12} > 0$, the bracketed factor is positive for $-\sqrt{b_1b_2} < \gamma < 0$. And for $\gamma > 0$ it has the same sign than

  $$\gamma\sigma_1^2/b_1 + \gamma(2-\gamma^2/(b_1b_2))\sigma_2^2/b_2 - 2\sigma_{12}$$

  a third degree polynomial function. This function is strictly negative at $\gamma = 0$. And at $\gamma = \sqrt{b_1b_2}$ it is

  $$\sqrt{b_1b_2}\left[\sigma_1^2/b_1 + \sigma_2^2/b_2 - 2\sigma_{12}/\sqrt{b_1b_2}\right] = \mathbb{E}[\left(\theta_1/\sqrt{b_1} - \theta_2/\sqrt{b_2}\right)^2] \geq 0.$$

  Therefore, it has a unique root (its derivative cancels only once) in $(0, \sqrt{b_1b_2}]$. Let γ_2 be this root. The derivative of the comparison with respect to d_2 is strictly positive for $\gamma \in (0, \gamma_2)$, null at γ_2 and strictly negative for $\gamma \in (\gamma_2, \sqrt{b_1, b_2})$.
- If $\sigma_{12} < 0$, a similar reasoning shows that the bracketed factor is positive for $\gamma > 0$ and that it has a unique root in $(-\sqrt{b_1b_2}, 0)$; the γ_2 of the Proposition 2 is the opposite of this root. The derivative of the comparison with respect to d_2 is strictly positive for $\gamma \in (-\gamma_2, 0)$, null at γ_2 and strictly negative for $\gamma \in (-\sqrt{b_1, b_2}, -\gamma_2)$.

A.3 Proof of Proposition 3

The effect of δ on the comparison of instruments is:

$$\begin{aligned}\frac{\partial \Delta_{t,q}}{\partial \delta} &= -\mathrm{cov}(q_{1t}, q_{2t}) = \frac{-1}{(b_1b_2-\gamma^2)^2}\mathrm{cov}(b_2\theta_1 - \gamma\theta_2, b_1\theta_2 - \gamma\theta_1) \\ &= \frac{1}{(b_1b_2-\gamma^2)^2}\left[\gamma\left(b_2\sigma_1^2 + b_1\sigma_2^2\right) - \left(b_1b_2 + \gamma^2\right)\sigma_{12}\right] \end{aligned} \tag{31}$$

Let us examine the sign of the bracketed factor. It is a quadratic function of γ and

- it is positive for $\gamma = \sqrt{b_1b_2}$:

$$\sqrt{b_1b_2}\left[b_2\sigma_1^2 + b_1\sigma_2^2 - 2\sqrt{b_1b_2}\sigma_{12}\right] = \sqrt{b_1b_2} \times \mathbb{E}\left[\left(\sqrt{b_2}\theta_1 - \sqrt{b_1}\theta_2\right)^2\right] \geq 0. \tag{32}$$

- A similar calculation shows that it is negative for $\gamma = -\sqrt{b_1 b_2}$.

The quadratic function $\gamma \to \gamma(b_2\sigma_1^2 + b_1\sigma_2^2) - (b_1b_2 + \gamma^2)\sigma_{12}$ has two roots if $\sigma_{12} \neq 0$. It is negative at $-\sqrt{b_1b_2}$ and positive at $\sqrt{b_1b_2}$, so, there is a unique root in $[-\sqrt{b_1b_2}, \sqrt{b_1b_2}]$, namely

$$\frac{b_2\sigma_1^2 + b_1\sigma_2^2}{2\sigma_{12}}\left[1 - \left[1 - \frac{4b_1b_2\sigma_{12}^2}{(b_2\sigma_1^2 + b_1\sigma_2^2)^2}\right]^{1/2}\right] \tag{33}$$

This root corresponds to the threshold of the Proposition 3.

A.4 Proof of Lemma 2 and Proposition 4

Proof of Lemma 2

From the expression of the comparison of instruments (22) obtained with the specification (S):

$$2\Delta_{t,q} = \frac{b_1b_2 - \gamma^2}{b_2}\frac{b_2^2\sigma_1^2 + \gamma^2\sigma_2^2}{(b_1b_2 - \gamma^2)^2} - d[\frac{(b_2-\gamma)^2\sigma_1^2 + (b_1-\gamma)^2\sigma_2^2}{(b_1b_2-\gamma^2)^2} - \frac{\sigma_2^2}{b_2^2}]$$

So, denoting $x = \sigma_2^2/\sigma_1^2$, we have

$$2\frac{(b_1b_2-\gamma^2)^2}{\sigma_1^2}\Delta_{t,q} = \left(b_1 - \frac{\gamma^2}{b_2}\right)\left[b_2^2 + \gamma^2 x\right] - d\left\{(b_2-\gamma)^2 + \left[(b_1-\gamma)^2 - \left(b_1 - \frac{\gamma^2}{b_2}\right)^2\right]x\right\} \tag{34}$$

The sign of the cross derivative is the sign of $(b_1-\gamma)^2 - (b_1-\gamma^2/b_2)^2$ it is strictly negative if and only if (remember that $b_1b_2 - \gamma^2 > 0$)

$$\left[b_1 - \gamma > 0 \text{ and } b_1 - \gamma < b_1 - \gamma^2/b_2\right] \text{ or } \left[b_1 - \gamma < 0 \text{ and } -(b_1-\gamma) < b_1 - \gamma^2/b_2\right]$$
$$\Leftrightarrow$$
$$[\gamma < b_1 \text{ and } 0 < \gamma < b_2] \text{ or } \left[b_1 < \gamma < \frac{b_2}{2}\left(\sqrt{1 + 8b_1/b_2} - 1\right)\right]$$
$$\Leftrightarrow$$
$$\gamma \in (0, \min\{b_1, b_2\}) \text{ or } \gamma \in (b_1, \frac{b_2}{2}\left(\sqrt{1+8b_1/b_2} - 1\right)) \tag{35}$$

There are two possible sets of positive values for γ for which the cross derivative is negative. We show that either one of these sets does not exist or the two sets are joined. Let us define

$$\tilde{\gamma} = \min\left\{b_2, \frac{b_2}{2}\left(\sqrt{1+8b_1/b_2} - 1\right)\right\} \tag{36}$$

- First,
$$\frac{b_2}{2}\left[\sqrt{1+8b_1/b_2} - 1\right] \geq b_2 \Leftrightarrow 1 + 8b_1/b_2 \geq 9 \Leftrightarrow b_1 \geq b_2$$
so, $b_2 \leq b_1$ is equivalent to $\tilde{\gamma} = b_2$.

- If $b_2 \leq b_1$, since $\gamma^2 \leq b_1 b_2$ we must have $\gamma \leq b_1$ and only the first condition in (35) matters so that the cross derivative is negative if and only if $0 < \gamma < b_2 = \tilde{\gamma}$. And $\tilde{\gamma}^2 \leq b_1 b_2$ in that case.
- If $b_2 \geq b_1$, then the two sets in (35) are joined and the cross derivative is negative if and only if $0 < \gamma < \frac{b_2}{2}\left(\sqrt{1+8b_1/b_2}-1\right) = \tilde{\gamma}$. Furthermore, some calculations lead to $\tilde{\gamma}^2 \leq b_1 b_2$ in that case.

We have shown that

$$(b_1-\gamma)^2 - (b_1-\gamma^2/b_2)^2 < 0 \Leftrightarrow 0 < \gamma < \tilde{\gamma}.$$

Proof of Proposition 6

From equation (34), the sign of the comparison is the sign of

$$\left(b_1 - \frac{\gamma^2}{b_2}\right)\left[b_2^2 + \gamma^2\frac{\sigma_2^2}{\sigma_1^2}\right] - d\left\{(b_2-\gamma)^2 + \left[(b_1-\gamma)^2 - \left(b_1 - \frac{\gamma^2}{b_2}\right)^2\right]\frac{\sigma_2^2}{\sigma_1^2}\right\}$$

If $0 < \gamma < \tilde{\gamma}$, the second term is decreasing with respect to σ_2/σ_1 (cf Lemma 2). Therefore, for large σ_2/σ_1 the comparison is increasing with respect to d, and the tax is always preferred.

Otherwise, the term between braces in the above equation is positive, the threshold slope of the marginal damage is then

$$\tilde{d} = \left(b_1 - \frac{\gamma^2}{b_2}\right)\left[b_2^2 + \gamma^2\frac{\sigma_2^2}{\sigma_1^2}\right]\left\{(b_2-\gamma)^2 + \left[(b_1-\gamma)^2 - \left(b_1 - \frac{\gamma^2}{b_2}\right)^2\right]\frac{\sigma_2^2}{\sigma_1^2}\right\}^{-1} \tag{37}$$

and it is striclty increasing with respect to the ratio σ_2/σ_1 if and only if

$$\gamma^2(b_2-\gamma)^2 > b_2^2\left[(b_1-\gamma)^2 - \left(b_1 - \frac{\gamma^2}{b_2}\right)^2\right] \tag{38}$$

which is equivalent to

$$\begin{aligned}
&(b_1b_2-\gamma^2)^2 > b_2^2(b_1-\gamma)^2 - \gamma^2(b_2-\gamma)^2 \\
\Leftrightarrow &(b_1b_2-\gamma^2)^2 > (b_1b_2-\gamma^2)(b_1b_2 - 2b_2\gamma + \gamma^2) \\
\Leftrightarrow &b_1b_2 - \gamma^2 > b_1b_2 - 2b_2\gamma + \gamma^2 \\
\Leftrightarrow &0 > \gamma \leq b_2
\end{aligned}$$

Figure (2) comes from the fact that the threshold slope $\tilde{d}$ is concave if it is increasing and $(b_1-\gamma)^2 - (b_1-\gamma^2/b_2)^2 \geq 0$ (i.e. $\gamma \leq 0$ or $\gamma \geq \tilde{\gamma}$) and convex otherwise. So $\tilde{d}$ is strictly increasing and concave if $0 < \gamma < b_2$ and $\gamma > \tilde{\gamma}$, that is if $\tilde{\gamma} < \gamma < b_2$; $\tilde{d}$ is increasing and convex if $0 < \gamma < \tilde{\gamma}$ (using the fact that $\tilde{\gamma} < b_2$); and it is decreasing, and convex, if either $\gamma < 0$ or $\gamma > b_2$.