ECOLE POLYTECHNIQUE – « INGENIEUR POLYTECHNICIEN » PROGRAM
INTERNATIONAL ADMISSIONS

Recommended knowledge in Mathematical Sciences

The recommended knowledge in Mathematical Sciences for the applicants to the " - International admissions" is detailed below.

This document is meant to give the applicants an indication on the knowledge they are likely to be interviewed. It is given for informational purposes only and cannot be considered as a basis of the programme for the second track examinations.

Ecole Polytechnique reserves the right to test an applicant's knowledge on other fields of Mathematical Sciences than those listed in this document.

An excellent level in mathematics and in physical sciences is a key to successful studies at Ecole Polytechnique.

1 - ALGEBRA

1.1 Set theory

Operations on sets, characteristic functions.
Maps, injectivity, surjectivity.
Direct and inverse image of a set.
Integer numbers, finite sets, countability.

1.2 Numbers and usual structures

Composition laws; groups, rings, fields.
Equivalence relations, quotient structures.
Real numbers, complex numbers, complex exponential.
Application to plane geometry.
Polynomials, relations between the roots and the coefficients.
Elementary arithmetics (in **Z/nZ**).

1.3 Finite dimensional vector spaces (*)

Free families, generating families, bases, dimension.
Determinant of n vectors; characterization of bases.
Matrices, operations on matrices.
Determinant of a square matrix; expansion with respect to a line or to a column; rank, cofactors.
Linear maps, matrix associated to a linear map.
Endomorphisms, trace, determinant, rank.
Linear systems of equations.

1.4 Reduction of endomorphisms

Stable subspaces.
Eigenvalues, eigenvectors of an endomorphism or a square matrix; similar matrices; geometrical interpretation.
Characteristic polynomial, Cayley-Hamilton theorem.

Reduction of endomorphisms in finite dimension; diagonalizable endomorphisms and matrices.

1.5 Euclidean spaces, Euclidean geometry

Scalar product; Cauchy-Schwarz inequality; norms and associated distances.
Euclidean spaces of finite dimension, orthonormal bases; orthogonal projections.
Orthogonal group O(E); orthogonal symmetries.
Orthogonal matrices; diagonalization of symmetric real matrices.
Properties of orthogonal endomorphisms of $\mathbf{R}^2$ and $\mathbf{R}^3$.

(*) In several countries linear algebra is studied only in $\mathbf{R}^k$ or $\mathbf{C}^k$; the candidates from these countries are strongly advised to get familiar with the formalism of abstract vector spaces.

2 - ANALYSIS AND DIFFERENTIAL GEOMETRY

2.1 Topology in finite dimensional normed vector spaces

Open and closed sets, accumulation points, interior points.
Convergent sequences in normed vector spaces; continuous mappings.
Compact spaces, images of compact sets by continuous mappings, existence of extrema.
Equivalence of norms.

2.2 Real or complex valued functions defined on an interval

Derivative at a point, functions of class C^k.
Mean value theorem, Taylor's formula.
Primitive of continuous functions.
Usual functions (exponential, logarithm, trigonometric functions, rational fractions).
Sequences and series of functions, simple and uniform convergence.

2.3 Integration on a bounded interval

Integral of piecewise continuous functions.
Fundamental theorem of calculus (expressing the integral of a function in terms of a primitive).
Integration by parts, change of variable, integrals depending on a parameter.
Continuity under the sign $\int$, differentiation under the sign $\int$.
Cauchy-Schwarz inequality.

2.4 Series of numbers, power series

Series of real or complex numbers, simple and absolute convergence.
Integral comparison criterion, product of absolutely convergence series.
Power series, radius of convergence; function that can be expanded in a power series on an interval.
Taylor series expansion of e^t, sin(t), cos(t), ln (1+t), $(1+t)^a$ where a is a real number.

2.5 Differential equations

Linear scalar equations of degree 1 or 2, fundamental systems of solutions.
Linear systems with constant coefficients.
Method of the variation of the constants.

Notions on non-linear differential equations.

2.6 Functions of several real variables

Partial derivatives, differential of a function defined on $\mathbf{R}^k$.
Chain rule.
C^1-functions; Schwarz theorem for C^2-functions.
Diffeomorphisms, inverse function theorem.
Critical points, local and global extrema.
Plane curves; tangent vector at a point, metric properties of plane curves (arc length, curvature).
Surfaces in $\mathbf{R}^3$, tangent plane to a surface defined by a Cartesian equation $F(x,y,z) = 0$.

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Recommended knowledge in Physical Sciences

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This document is meant to give the applicants directions on the knowledge they are likely to be interviewed upon. It is given for information purposes only and cannot be considered as a basis of the programme for the second track examinations.

Ecole Polytechnique reserves the right to test an applicant's knowledge on other fields of Physical Sciences than those listed in this document.

Besides, Ecole Polytechnique expects the applicants to know the numerical values of the basic constants of physics, as well as the orders of magnitude of the physical phenomena of nature.

The applicants should be able to show excellent standard mathematical skills.

An excellent level in mathematics and in physical sciences is a key to successful studies at Ecole Polytechnique.

I. MECHANICS

- Newtonian mechanics
- Mechanics of solids
- Statics and mechanics of fluids
- Applications of mechanics

II. ELECTRIC CIRCUITS

III. ELECTRICITY AND MAGNETISM

- Electrostatics
- Magnetostatics
- Electromagnetic waves

IV. OPTICS

- Geometrical optics
- Wave optics

V. THERMODYNAMICS

- Perfect gas
- First and second principles of thermodynamics

Physical constants

The values of Planck, Boltzmann and Avogadro constants, the charge and the mass of the electron, the speed of light in vacuum, the electric permittivity and the magnetic permeability of free space, in *SI* system of units (at least two significant digits are required).

Orders of magnitude

The orders of magnitude of quantities such as the magnetic field of the Earth, the radius of the Earth, the acceleration of free fall at the Earth's surface, the concentration of electrons in a typical metal, the wavelengths of the electromagnetic waves of the visible spectrum, the distance between two atoms in a solid or liquid, the Bohr radius of the fundamental state of the hydrogen atom, the size of the nucleus.

Compulsory minimal requirements of calculation skills

Mastering a certain number of calculation skills such as is compulsory

Expansions

Be able to study the behaviour of a physical quantity $A(x)$ in the neighbourhood of a given value of its argument x. The common expansions about $x \approx 0$

$$\sin x \approx x - \frac{x^3}{6}; \quad \cos x \approx 1 - \frac{x^2}{2}; \quad \tan x \approx x + \frac{x^3}{3}; \quad \cot g\, x \approx \frac{1}{x} - \frac{x}{3}$$

$$e^x \approx 1 + x + \frac{x^2}{2}; \quad \ln(1+x) \approx x; \quad (1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha)(\alpha-1)}{2} x^2$$

Derivatives and primitives of the functions of a single variable

Derivatives of the elementary functions (x^n, $\ln x$, e^x, $\sin x$, $\cos x$, $tg\, x$, $\cot g\, x$) as well as of the composition function $f(g(x))$.
Rules for the derivative of the product and the quotient of two functions of a real variable.
Primitives of the elementary functions above.
Integration by parts.
Conditions for the convergence of an integral in the cases of an infinite integration interval or the presence of points of discontinuity.

Functions of several variables. Common differential operators.

Total differential.
Partial derivatives with respect to an independent variable in the case of a function of several variables.
Nabla operator $\vec{\nabla}$. Gradient $\vec{\nabla} f$ of a function $f(\mathbf{r})$.
Curl $\vec{\nabla} \times \mathbf{A}$ of a vector field $\mathbf{A}(\mathbf{r})$. Divergence $\vec{\nabla}.\mathbf{A}$. Circulation $\oint_{(C)} \mathbf{A}.d\mathbf{l}$.

A two particle system. Central force motion, bound states, scattering states.
Expressions for the velocity and the acceleration of a material point in cylindrical and spherical co-ordinates.
Concept of potential energy. Independence on the path of the work done by a potential-derived force.
Conservation of mechanical energy of an isolated material system in the case of conservative forces.
Conservation of angular momentum in the case of central forces. First and second Kepler's laws (the law of conical sections and the law of areas).
Conservation of momentum in the case of an isolated system. Elastic and inelastic collision problems. Concept of a centre of mass of a system.
Expressions for the potential, kinetic and total energy of a particle in the case of a circular trajectory.

Mechanics of solids

Rigid bodies (non-deformable solids). Solids rotating about a fixed axis. Moment of inertia of a rigid body. Expression for the kinetic energy of a rigid body as a sum of a translational term of its centre of mass and of a rotational term referred to the centre-of-mass reference frame (Koenig's theorem). The problem of the compound pendulum.

Statics and mechanics of fluids

Euler's description (the concept of a velocity field) of a fluid. Concepts of flow density, mass flow rate and volume flow rate. Mass balance. Equation of the conservation of mass in its local form.
Definitions of a stationary flow, of an incompressible flow, of a non-rotational flow.
Perfect flows: Euler's equation, Bernoulli's relationship on incompressible and homogeneous flows.
Calculation of the resulting force of the pressure forces exerted upon an object, in fluid statics.
The Archimedes' principle (the buoyancy force applied to an object immersed in a fluid).

Applications of mechanics

Lorentz force (force exerted on a charged particle in constant electric and magnetic fields).
Trajectory of a charged particle in a static and uniform magnetic field.
Linear oscillations; damped harmonic oscillations. Forced oscillations, resonance.

II. ELECTRIC CIRCUITS

Electric voltage. Kirchoff's laws of knots and meshes. Electric current. Ohm's law. Superposition theorem.

Basic circuit components: resistor, capacitor, coil. Their impedances in sinusoidal regime.

Transient regime of charging and discharging a capacitor.
Sinusoidal currents and voltages. Maximum value, rms (root mean square) value. Impedances in series and in parallel.
Study of resonances in circuits in sinusoidal regime. *RLC* circuit. Relation to resonance in mechanics.

III. ELECTRICITY AND MAGNETISM

Electrostatics

Coulomb's law. The concept of electric field. Electrostatic field **E**. Circulation and flow of **E**. Gauss' theorem. Symmetry properties of **E**.

Electrostatic potential ϕ and Poisson's equation.
Calculation of **E** and ϕ for a simple charge distribution ρ. Electrostatic potential between the plates of a planar capacitor.
Concept of electric dipole, field created by a dipole at large distances, interaction energy of a permanent dipole with the electric field. Definition of the electric polarization vector.
Electric field in a conductor at equilibrium. Equipotential surfaces.
Electric field in the vicinity of a metal surface.
Coulomb's law between two charges immersed in a homogenous linear and isotropic dielectric medium.

Magnetostatics

Magnetic field **B**. Symmetry properties of **B**.
Magnetic field created by a thin wire carrying a current (Biot-Savart law), the two Maxwell equations (the divergence of **B** and Ampère's law), vector potential **A**.
Non-unicity of the electrostatic potential ϕ and the vector potential **A**, unicity of the electric field **E** and the magnetic field **B**.
Circulation of **B**. Relationship between the circulation of **B** and the encircled currents (theorem of the total current).
Calculation of **B** created by straight wires and circular loops. Field along the axis of a circular loop and of a coil (solenoid) having a circular cross-section.
Magnetic dipole and magnetic moment **M**. Expression for the interaction energy between a magnetic moment and a magnetic field **B**.
Flux of **B**. Electromagnetic induction phenomenon, Faraday's law, Lenz' rule.

Electromagnetic waves

Electromagnetic waves in vacuum.
Maxwell's equations in vacuum. Progressive harmonic plane waves as solutions of the Maxwell's equations in vacuum. Frequency, wavelength, wave vector. The concept of phase velocity.
Transversality of the electric and magnetic fields.
The state of polarization state an electromagnetic wave. Linear and circular polarizations.
Volume density of the electromagnetic energy, Poynting vector.
Concept of wave packet. Group velocity.

Electromagnetic waves in matter (linear and isotropic medium).
Macroscopic **E** and **B** fields. Constitutive relationships complementing Maxwell's equations.
Frequency-dependent complex dielectric constant $\varepsilon(\omega)$.
Concepts of complex refraction index, dispersion and absorption.
Microscopic models describing the material polarization of the medium: Drude model, model of the elastically bound electron (Lorentz model).

IV. OPTICS

Geometric optics

Concept of light ray. Reflection and refraction by a plane mirror. Snell-Descartes' laws. Limit angle. The total reflection phenomenon.
Spherical mirrors, lenses, conjugation and magnification relations.

Wave optics

Reflection and refraction of a harmonic progressive polarized plane wave at the interface between two dielectric media. Proof of Snell-Descartes laws.
Concept of optical path. Interference between two totally coherent waves. Michelson's interferometer. Thin slabs. Fabry-Pérot cavity.
Diffraction at infinity. Huyghens-Fresnel principle. Diffraction by a rectangular slit. Diffraction at infinity by two slits (Young's slits), by a row of slits.

V. THERMODYNAMICS

Thermodynamic state functions: internal energy, entropy, enthalpy, free energy, free enthalpy, as well as their differentials.
Extensive and intensive variables, thermodynamic equilibrium.
Heat capacities at a constant volume and at a constant pressure.

Perfect gas

Perfect monoatomic gas model. Maxwell-Boltzmann distribution of velocities for a monoatomic perfect gas. The equipartition theorem.
Collisions against a wall. Relationship between pressure and mean square velocity.
Perfect gas in a field of forces having a potential energy $V(\mathbf{r})$. The barometric formula.
Limitations of the perfect gas model. Real gases. The van der Waals gas.

First and second principles of thermodynamics

First principle. Internal energy U. Heat transfer. Work exchanged by a system. The work of pressure forces. Enthalpy and Joule-Thomson expansion. The enthalpy of a perfect gas.
Second principle. The entropy S. Entropy balance. Reversible and irreversible processes. Thermodynamic definition of temperature.
The entropy of a perfect gas (for a condensed and idalatable phase).
Heat machines. Dithermal cycle. Efficiency. Carnot's theorem.
Equilibrium between the phases of a pure substance. Triple point, critical point, enthalpy and entropy of phase changes. Clapeyron's formula.
Free energy and free enthalpy: definitions and differentials. Chemical potential. The perfect gas case. Equilibrium between two phases. Generalization, Gibbs' phase rules.