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Abstract

We characterize the design of insurance schemes when policyholders face several insurable risks in a context of adverse selection. Splitting risks emerges as a feature of second-best Pareto-optimal allocations. This may take the form of risk-specific contracts, or of contracts where risks are bundled, but subject to differential coverage rules such as risk specific copayments, combined with a deductible, an out-of-pocket maximum or a cap on coverage.

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1 Introduction

Most people are simultaneously affected by several insurable risks, including property, casualty, liability and health risks. It is a fact that these various categories of risks require specific underwriting and claim handling skills that are not uniformly shared by all insurers. This is a sufficient reason why we may cover different risks through different insurers, and thus, for instance, why, for instance, many people purchase automobile and medical insurance from different insurers.

Understanding why similar risks incurred by the same economic unit (e.g., a family) are not covered by the same insurance contract is less obvious. To take a concrete example, why is it the case that, most of the time, the automobile risks of both members of a couple who drive different cars are covered by separate contracts, with specific premium and deductibles. Similarly, why the health care risks of these two persons may be covered by independent contracts?[1]

These very simple questions do not have simple answers. When there are no transaction costs, risk averse individuals would optimally purchase full coverage with actuarially fair insurance premium, and, obviously, in such a case, it does not matter whether the risks of both members of a couple are covered through specific contracts or through a unique contract. This is no more true when insurers charge loaded premiums and households react by purchasing partial coverage. A variant of the wellknown "theorem of the deductible" says that, under proportional loading, households should be covered by an "umbrella policy" with full marginal coverage of the total losses beyond an aggregate deductible.[2] The fact that they may choose to be covered by separate insurance policies is inconsistent with this conclusion.

[1] These questions are legitimate insofar as both members of the couples have identical access rights to the same set of contracts, and if they jointly manage their budget. Matters should be seen differently in a collective approach to family decision making, where a household is described as a group of individuals, with their own preferences and among whom there is a coperative or non-cooperative decision process; see Bourguignon and Chiappori (1992).

[2] Umbrella insurance usually refers to coverage provided as a complement to other policies, particularly automobile and homeowners liability policies. As others before us, we use this terminology because the principle of umbrella policies is to globally protect policyholders against uncovered risks, without reference to the specificity of these risks. After Eeckhoudt et al. (1991) who considered the particular case of a binomial distribution and, more generally, Gollier and Schlesinger (1995) established the optimality of an umbrella

A similar question arises with respect to the coverage of various risks incurred by an individual. This is particularly the case in medical insurance. In an umbrella policy approach, the insurance indemnity depends on total medical expenses measured on a per period basis. However, much more frequently, various medical services (e.g., visits to general practitioners, visits to specialists, inpatient care, dental care...) are bundled in a unique contract, but the insurance indemnity depends on service specific copayments, coinsurance rates or policy limits, frequently combined with an aggregate deductible, an out-of-pocket maximum or a cap on the indemnity. Here also, this casual observation of medical insurance contracts is at odds with the optimality of umbrella policy. Likewise, homeowner insurance usually provides bundled coverage for both property and liability risks, with different deductibles and policy limits for these risks.

The fact that risk specific policies and differential coverage of risks bundled in a contract are so frequent suggests that splitting risks is often an optimal answer to the insurance choice problem. Umbrella policies may be suboptimal for various reasons that are ignored in the standard model of insurance demand, and that would be worth further exploring. Most of them are probably related with asymmetries of information between insurer and insured, for instance when the two members of a couple do not react to financial incentives in the same way under moral hazard, or when medical expenses can be more or less easily monitored by insurers, depending on the type of health care service (e.g., visit to general practionners vs inpatient care).[3]

In this paper, we will analyze how the splitting of risks is a rational answer to the adverse selection problem. We will consider a setting where households face several risk exposures (only two, for the sake of simplicity), with hidden information about the type of each one: they may be high risk or low risk. For instance, in the case of health insurance, the two risk exposures

policy under linear transaction costs. As shown by Gollier (2013), this property as well as the optimality of a straight deductible contract hold when policyholders dislike any zero-mean lottery that would be added to their final wealth, which is more general than the case of risk aversion under the expected utility criterion.

[3] Cohen (2006) considers a related issue. She analyzes the determinants of deductible in insurance contracts that cover a risk that may materialize more than once during the life of thee policy. She shows that aggregate deductibles may produce higher verifiction costs and moral hazard costs than per-loss deductibles. In a similar perspective, Li. et al. (2007) analyze the case of automobile insurance in Taiwan, and they provide evidence of the incentive effects of increasing per-claim deductibles on the policyholders' behavior.

may correspond to the medical expenses of each member of a couple, or to different types of medical services. We will have high risk households with two high risk exposures, medium risk households with a high risk and a low risk (with two subgroups according to identity of the high and low risk exposures), and low risk households with two low risks. An umbrella policy provides coverage for the aggregate losses that may result from the two risk exposures, while a risk specific policy only covers the loss of a particular risk. In this setting, the coverage of risks may be splitted by purchasing insurance through two risk specific policies, such as two automobile insurance policies for the two cars owned by the household, or separate health insurance contracts for each member of a couple. Bearing in mind the case of health insurance, we may also consider that policyholders are individuals who face two additive risks, each of them corresponding to a specific service, such as visits to general practitionners and dental care. In that case, splitting risks refers to the bundling of insurance coverage with service specific coverage, possibly combined with a deductible, an out-of-pocket maximum or a cap on indemnity.

Crocker and Snow (2011) study a related but different problem. They analyze how the bundling of distinct perils (seen as factors that may be at the origin of a loss) in a unique contract with differential deductibles improves the efficiency of insurance markets under adverse selection. They consider a setting with two types of policyholders (high risks and low risks) where a loss may result from various perils, and they show that bundling the coverage of these perils in a unique contract facilitates the separation of high risks and low risks. If the probabilities that the loss results from different perils (conditionnally on a loss occurring) differ between high risk and low risk individuals, then bundling perils with differential deductibles allows insurer to implement a multidimensional screening that yields a Pareto efficiency gain and makes the existence of a Rothschild-Stiglitz equilibrium more likely.

Our perspective is different. We consider a setting in which each policyholder faces two risks that may be high or low, and potentially one loss per risk. We thus have four types of policyholders and 0,1 or 2 losses. The issue we consider is whether the aggregate loss that may result from multiple risk exposures should be covered through an umbrella policy, and if not, whether risk splitting takes the form of risk specific policies where risk exposures are unbundled, or of a unique policy where risks are bundled but different indemnity rules apply for each risk.

We will not try here to analyze the market equilibrium that may arise when there is adverse selection about multiple insurable risks, although this should be an ultimate research objective. Our objective will be more modestly limited to the characterization of efficient allocations. Efficiency will be in a second-best sense, meaning that allocations should satisfy the incentive compatibility conditions inherent in an hidden information setting. Implicitly, we presume that either competition forces or market regulation leads to such a second-best Pareto optimal allocation.

The main lesson learned from the characterization of efficient allocations will be that risk splitting facilitates the separation of risk types. The intuition for this result is as follows. As in the Rothschild-Stiglitz (1975) model, unsurprisingly, at any second-best Pareto-optimal allocation, high risk households are fully covered, which may correspond to an umbrella policies or to two risk specific policies. They are detered from choosing the risk coverage of the "upward adjacent" type of household, which is the medium risk in the present model, by providing partial coverage to this type. In our multiple risk setting, partial coverage may correspond to several insurance schemes, such as, for instance, a uniform proportional coinsurance for the two risk exposures. It turns out that full coverage for the high risk exposure and partial coverage for the low risk exposure is the most efficient way to separate high and medium risk households, since the existence or the absence of such a low risk exposure is the only difference between these two categories of households. This requires that medium risk households are covered by risk specific insurance policies, with full coverage for their high risk and partial coverage for their low risk.

The situation is different for low risk households, since their two risk exposures are homogeneous (both are low risks), and their insurance policy should include decreasing coverage at the margin for both risk exposures to facilitate the separation from medium risk households. Risk specific policies are not optimal for low risk household, but splitting the coverage of risk exposures is again optimal, because of the (possibly uneven) pressure of the two types of medium risk policyholders. It takes a form that will sound very familiar in a health economics perspective: risk specific contributions of the policyholder in the case of a loss (i.e., risk specific copayment in the terminology of health insurance) should be combined with an aggregate deductible, an out-of-pocket maximum or a cap on the insurance indemnity.

We will also consider the case when actuarial premiums are loaded up because of transaction costs. This extension is of particular interest because of the theorem of the deductible that states the optimality of an aggregate deductible when there is proportional loading and no asymmetry of information. Taking into account the insurance loading will modify our conclusions about the optimality of full coverage for high risk households and of risk specific insurance contracts for medium risk households, but splitting risks will remain an optimal answer to the adverse selection problem. In particular, high risk households should be protected by a straight deductible contract, while medium and low risk households should be covered by a policy that includes risk specific copayments, combined either with an aggregate deductible, an out-of-pocket maximum or a cap on the insurance indemnity.

The rest of the paper is arranged as follows. Section 2 presents our model of insurance market with multiple risks and hidden information about risk types. Section 3 shows why and how risk splitting may be an answer to adverse selection. Section 4 analyzes the effect of transaction costs on insurance schemes. Section 5 concludes. The proofs are in an appendix.

2 Model

2.1 Notations

We consider an economy where households are affected by two additive financial risk exposures (more briefly, risks), indexed by $r \in \{1, 2\}$ and identified by observable characteristics.[4] We can think of the health care expenses incurred by both members of a couple, hence the "household" terminology. The model may also be interpreted as the case of a single person who faces two risks, such as doctors' visit and inpatient care in the case of health insurance, or accident and theft risk in the case of automobile risk. For the sake of notational simplicity, we assume that each risk r may lead to the same loss L, with probability p_ℓ or p_h, with $0 < p_\ell < p_h < 1$, and that the occurrence of these losses are independent events. Hence, r is a high risk when the corresponding probability of loss is p_h and it is a low risk when the probability is p_ℓ. Thus, there are four household types characterized by a double index ij, with $i, j \in \{h, \ell\}$, where i and j are the types of risk exposures $r = 1$ and 2, respectively. Types hh and $\ell\ell$ are high risk and low risk

[4] Hence, households cannot present their risk exposure 1 as belonging to category 2, and vice versa. The analysis would be different if both risk exposures could be substituted, as for instance in the case of property owners seeking fire insurance for a group of two buildings without special features enabling to distinguish them.

households because their two risks are either high or low. Types $h\ell$ and ℓh are medium risk households, because one of their risk exposures is high and the other one is low. We denote λ_{ij} the proportions of type ij households with $\lambda_{hh}+\lambda_{h\ell}+\lambda_{\ell h}+\lambda_{\ell\ell}=1$. Households have private information on their type and all of them have the same initial wealth w_0.

The state-contingent final wealth of a household is written as $w=(w^{00},w^{01}, w^{10},w^{11})$, where w^{xy} denotes its final wealth in state $(x,y)\in\{0,1\}^2$, where $x=0$ when risk $r=1$ does not lead to a loss and $x=1$ when it does, with a similar interpretation of index y for risk $r=2$. For instance, w^{10} is the household's final wealth when only risk 1 is at the origin of a loss.

Households are decision units: they are risk averse and they maximize the expected utility of their aggregate final wealth w_f, with utility function $u(w_f)$, such that $u'>0,u''<0$. To obtain more specific results, we may also assume that households are downward risk averse (or prudent), which corresponds to the additional assumption $u'''>0$, but we will mention it explicitly when this assumption is needed. The expected utility of a type ij household is denoted $U_{ij}(w)$ for all $i,j\in\{h,\ell\}$, with

$$\begin{aligned}
U_{hh}(w) &\equiv (1-p_h)^2u(w^{00})+p_h(1-p_h)[u(w^{01})+u(w^{10})]+p_h^2u(w^{11}),\\
U_{h\ell}(w) &\equiv (1-p_h)(1-p_\ell)(w^{00})+p_\ell(1-p_h)u(w^{01})\\
&\quad +p_h(1-p_\ell)u(w^{10})+p_hp_\ell u(w^{11}),\\
U_{\ell h}(w) &\equiv (1-p_h)(1-p_\ell)(w^{00})+p_h(1-p_\ell)u(w^{01})\\
&\quad +p_\ell(1-p_h)u(w^{10})+p_hp_\ell u(w^{11}),\\
U_{\ell\ell}(w) &\equiv (1-p_\ell)^2u(w^{00})+p_\ell(1-p_\ell)[u(w^{01})+u(w^{10})]+p_{\ell\ell}^2u(w^{11}).
\end{aligned}$$

The state-contingent final wealth of a type ij household is denoted by $w_{ij}=(w_{ij}^{00},w_{ij}^{01},w_{ij}^{10},w_{ij}^{11})$, with expected utility $U_{ij}(w_{ij})$. An allocation $\mathcal{A}=\{w_{hh},w_{h\ell},w_{\ell h},w_{\ell}\}$ specificies the state-contingent final wealth for each type of household.

Insurance schemes[5] are characterized by the non-negative indemnity T^{xy} paid by the insurer to the household when there are x and y losses, in risk exposures $r=1$ and 2, respectively, with $(x,y)\in\{0,1\}^2$ and $T^{00}=0$, and by the premium P paid in all states by the policyholder to the insurer. If type ij households are protected by an insurance scheme with coverage schedule

[5] We use the terminology "insurance scheme" instead of "insurance policy or contract" because, in what follows, an insurance scheme may correspond to the combination of two risk specific insurance policies.

$T_{ij} \equiv (T_{ij}^{01}, T_{ij}^{10}, T_{ij}^{11})$ and premium P_{ij}, then we have

$$w_{ij}^{xy} = w - (x+y)L - P_{ij} + T_{ij}^{xy} \quad \text{for all } x, y \in \{0,1\}, \tag{1}$$

with $T_{ij}^{00} = 0$.

In what follows, for the sake of simplicity and realism, we will restrict attention to insurance schemes where the indemnity does not increase more than losses, and thus we assume[6]

$$\begin{aligned} &0 \leq T_{ij}^{01}, T_{ij}^{10} \leq L, \\ &T_{ij}^{11} - T_{ij}^{01} \leq L, \\ &T_{ij}^{11} - T_{ij}^{10} \leq L, \end{aligned}$$

or, equivalently

$$\begin{aligned} w_{ij}^{00} - L &\leq w_{ij}^{01} \leq w_{ij}^{00}, \\ w_{ij}^{00} - L &\leq w_{ij}^{10} \leq w_{ij}^{00}, \\ w_{ij}^{10} - L &\leq w_{ij}^{11} \leq w_{ij}^{10}, \\ w_{ij}^{01} - L &\leq w_{ij}^{11} \leq w_{ij}^{01}, \end{aligned}$$

for all $i, j \in \{h, \ell\}$.

2.2 Typical multirisk insurance schemes

Risk specific policies and *umbrella policies* are particular forms of insurance schemes that restrict the set of feasible allocations. A risk specific insurance policy provides coverage for a particular risk. A policy that covers risk 1 is defined by an indemnity I^1 paid to the household, should risk 1 lead to a loss, and by an insurance premium Q^1, with similar notations I^2, Q^2 for a policy that covers risk 2. Hence, if type ij households are covered by two risk specific policies (I_{ij}^1, Q_{ij}^1) and (I_{ij}^2, Q_{ij}^2), then we have

$$\begin{aligned} T_{ij}^{10} &= I_{ij}^1, \\ T_{ij}^{01} &= I_{ij}^2, \\ T_{ij}^{11} &= I_{ij}^1 + I_{ij}^2, \\ P_{ij} &= Q_{ij}^1 + Q_{ij}^2, \end{aligned}$$

[6] In addition to the fact that overinsurance may be prohibited by law, policyholders may be incentivized to deliberately create damages (e.g., by commiting arson in the case of fire insurance) if coverage were larger than losses.

which induces an allocation that satisfies

$$w_{ij}^{11} = w_{ij}^{01} + w_{ij}^{10} - w_{ij}^{00} \text{ for all } i, j \in \{h, \ell\}. \tag{2}$$

This characterization can be reversed: any allocation that satisfies (2) is induced by an insurance scheme where type ij households are covered by two risk-specific policies.

Under an umbrella policy, the coverage is a function of the household's total loss, and the policy may be written as (J^1, J^2, P), where J^1 and J^2 denote the indemnity in the case of one or two losses, respectively, and, as before, P is the total premium paid for the coverage of the two risks. Thus, when type ij households are covered by an umbrella policy $(J_{ij}^1, J_{ij}^2, P_{ij})$,we have

$$T_{ij}^{10} = T_{ij}^{01} = J_{ij}^1,$$
$$T_{ij}^{11} = J_{ij}^2.$$

The corresponding allocation satisfies

$$w_{ij}^{01} = w_{ij}^{10} \text{ for all } i, j \in \{h, \ell\}, \tag{3}$$

with, here also, an inverse relationship: any allocation that satisfies (3) is induced by an umbrella policy for type ij households.

If an umbrella policy $(J_{ij}^1, J_{ij}^2, P_{ij})$ provides the same state-contingent wealth w_{ij} as two risk specific policies (I_{ij}^1, Q_{ij}^1) and (I_{ij}^2, Q_{ij}^2), then we have

$$w_{ij}^{11} - w_{ij}^{01} = w_{ij}^{10} - w_{ij}^{00} = w_{ij}^{01} - w_{ij}^{00},$$

from (2) and (3), and thus

$$J_{ij}^2 = 2J_{ij}^1.$$

Hence, the indemnity is doubled when the loss is doubled, which means that linear coinsurance is an umbrella policy equivalent to two separate risk specific policies, both of them providing linear coinsurance at the same rate.

Risk specific policies and umbrella policies are far from being the only forms of multirisk insurance schemes. Consider the case of *risk specific copays* c_{ij}^1 and c_{ij}^2 combined with a *deductible* D_{ij}, with $c_{ij}^1 + D_{ij} < L$ and $c_{ij}^2 + D_{ij} < L$. Hence the insurance indemnity is $L - c_{ij}^1 - D_{ij}$ or $L - c_{ij}^2 - D_{ij}$ if a loss affects risk exposure 1 or 2, and it is $2L - c_{ij}^1 - c_{ij}^2 - D_{ij}$ if losses occur for both risk exposures.[7] Thus, we have

$$
\begin{aligned}
T_{ij}^{10} &= L - c_{ij}^1 - D_{ij}, \\
T_{ij}^{01} &= L - c_{ij}^2 - D_{ij}, \\
T_{ij}^{11} &= 2L - c_{ij}^1 - c_{ij}^2 - D_{ij},
\end{aligned}
$$

and (1) gives

$$
\begin{aligned}
& c_{ij}^1 = w_{ij}^{01} - w_{ij}^{11}, \\
& c_{ij}^2 = w_{ij}^{10} - w_{ij}^{11}, \\
& D_{ij} = w_{ij}^{00} + w_{ij}^{11} - (w_{ij}^{10} + w_{ij}^{01}).
\end{aligned}
$$

Hence, w_{ij} can be sustained by non-negative copays c_{ij}^1, c_{ij}^2 and deductible D_{ij} if

$$w_{ij}^{00} + w_{ij}^{11} \geq w_{ij}^{10} + w_{ij}^{01}. \tag{4}$$

Consider now the case where the insurance scheme of type ij households includes risk specific copays c_{ij}^1, c_{ij}^2 and an *out-of-pocket maximum* M_{ij} instead of an aggregate deductible. Assume $c_{ij}^1 + c_{ij}^2 \geq M_{ij}$ and $c_{ij}^1, c_{ij}^1 < M_{ij}$, which means that the out-of-pocket maximum is reached in the case of two losses. In that case, we have

$$
\begin{aligned}
T_{ij}^{10} &= L - c_{ij}^1, \\
T_{ij}^{01} &= L - c_{ij}^2, \\
T_{ij}^{11} &= 2L - M_{ij},
\end{aligned}
$$

and (1) yields

$$
\begin{aligned}
& c_{ij}^1 = w_{ij}^{00} - w_{ij}^{10}, \\
& c_{ij}^2 = w_{ij}^{00} - w_{ij}^{01}, \\
& M_{ij} = w_{ij}^{00} - w_{ij}^{11}.
\end{aligned}
$$

[7] In health insurance, a *copay* (or *copayment*) is a set out-of-pocket amount paid by the insured for health care services. We use this terminology although our approach is not limited to health insurance. We assume, as in most health insurance contracts, that the amounts paid in copays do not count toward meeting the deductible. In other words, the insurance indemnity is equal to the difference between out-of-pocket costs above copays and the deductible if this difference is positive, which is what we assume here. See section 4 for a more general formulation, in which the insurance indemnity may be nil in the case of a unique loss.

Hence, the type ij state-dependent wealth can be sustained by non-negative copays c_{ij}^1, c_{ij}^2 and an out-of-pocket maximum M_{ij} here also if (4) holds.

Finally, consider the case of risk specific copays c_{ij}^1 and c_{ij}^2 combined with an *upper limit on coverage* UL_{ij} reached in the case of two losses. Thus, we assume $L - c_{ij}^1 \leq UL_{ij}, L - c_{ij}^2 \leq UL_{ij}$ and $UL_{ij} \leq 2L - c_{ij}^1 - c_{ij}^2$. In that case, we have

$$\begin{aligned} T_{ij}^{10} &= L - c_{ij}^1, \\ T_{ij}^{01} &= L - c_{ij}^2, \\ T_{ij}^{11} &= UL_{ij}, \end{aligned}$$

and (1) gives

$$\begin{aligned} &c_{ij}^1 = w_{ij}^{00} - w_{ij}^{10}, \\ &c_{ij}^2 = w_{ij}^{00} - w_{ij}^{01}, \\ &UL_{ij} = 2L + w_{ij}^{11} - w_{ij}^{00}, \end{aligned}$$

with $UL_{ij} \leq 2L - c_{ij}^1 - c_{ij}^2$ if

$$w_{ij}^{00} + w_{ij}^{11} \leq w_{ij}^{10} + w_{ij}^{01}, \tag{5}$$

which is the condition under which w_{ij} can be sustained by non-negative copays c_{ij}^1, c_{ij}^2 and an upper limit on coverage UL_{ij}.

2.3 Incentive compatibility and feasibiity

The incentive compatibility of an allocation $\mathcal{A} = \{w_{hh}, w_{h\ell}, w_{\ell h}, w_{\ell\ell}\}$ requires that any type ij household weakly prefers w_{ij} to $w_{i'j'}$ if $i \neq i'$ and/or $j' \neq j$, i.e.,

$$U_{ij}(w_{ij}) \geq U_{ij}(w_{i'j'}), \tag{6}$$

for all $i, j, i', j' \in \{h, \ell\}$ such that $i \neq i'$ and/or $j \neq j'$.

Let $C_{ij}(w)$ be the expected wealth of a type ij household, for state-dependent wealth $w = (w^{00}, w^{01}, w^{10}, w^{11})$, with

$$\begin{aligned}
C_{hh}(w) &\equiv (1-p_h)^2 w^{00} + p_h(1-p_h)(w^{01}+w^{10}) + p_h^2 w^{11},\\
C_{h\ell}(w) &\equiv (1-p_h)(1-p_\ell)w^{00} + p_\ell(1-p_h)w^{01}\\
&\quad + p_h(1-p_\ell)w^{10} + p_h p_\ell w^{11}\ ,\\
C_{\ell h}(w) &\equiv (1-p_h)(1-p_\ell)w^{00} + p_h(1-p_\ell)w^{01}\\
&\quad + p_\ell(1-p_h)w^{10} + p_h p_\ell w^{11}\\
C_{\ell\ell}(w) &\equiv (1-p_\ell)^2 w^{00} + p_\ell(1-p_\ell)(w^{01}+w^{10}) + p_\ell^2 w^{11},
\end{aligned}$$

By definition, the allocation $\mathcal{A}$ is *budget-balanced* if the average wealth in the population is lower or equal to the average wealth in the absence of any redistribution between households. This is written as

$$\sum_{i,j\in\{h,\ell\}} \lambda_{ij} C_{ij}(w_{ij}) \leq \overline{w}, \tag{7}$$

where

$$\overline{w} = w_0 - L[2\lambda_{hh}p_h + (\lambda_{h\ell}+\lambda_{\ell h})(p_h+p_\ell) + 2\lambda_{\ell\ell}p_\ell].$$

Finally, we say that the allocation $\mathcal{A}$ is *feasible* if it is incentive compatible and budget-balanced.

Any feasible allocation can be induced by an insurance scheme where insurers offer menus of insurance schemes and make non-negative expected profit when households choose their best scheme. Indeed, assume that each type ij household chooses a scheme (T_{ij},P_{ij}), among the offers available in the insurance market. Then, the induced allocation $\mathcal{A} = \{w_{hh}, w_{h\ell}, w_{\ell h}, w_{\ell\ell}\}$ is defined by Equation (1) for all $i,j \in \{h,\ell\}$ and this allocation is incentive compatible. Conversely, any incentive compatible allocation $\mathcal{A}$ is induced by a menu of insurance contracts with coverage $T_i \equiv (T_{ij}^{01}, T_{ij}^{10}, T_{ij}^{11})$ and premium P_{ij} such that $P_{ij} = w_{ij}^{00} - w$ and $T_{ij}^{xy} = w_{ij}^{xy} - w_{ij}^{00} + (x+y)L$ if $(x,y) \neq (0,0)$.

Assume that insurers offer the menu of insurance schemes $\{(T_{ij},P_{ij}), i,j = h,\ell\}$ and that (T_{ij},P_{ij}) is chosen by type ij households in this menu. Assume moreover that policyholders are evenly spread among insurers. Then insurers make non-negative expected profit if

$$
\begin{aligned}
&\lambda_{hh}\left[P_{hh} - p_h(1-p_h)(T^{01}_{hh} + T^{10}_{hh}) - p_h^2 T^{11}_{hh}\right] \\
&+\lambda_{h\ell}\left[P_{h\ell} - p_\ell(1-p_h)T^{01}_{h\ell} - p_h(1-p_\ell)T^{10}_{h\ell} - p_h p_\ell T^{11}_{h\ell}\right] \\
&+\lambda_{\ell h}\left[P_{\ell h} - p_h(1-p_\ell)T^{01}_{\ell h} - p_\ell(1-p_h)T^{10}_{\ell h} - p_h p_\ell T^{11}_{\ell h}\right] \\
&+\lambda_{\ell\ell}\left[P_{\ell\ell} - p_\ell(1-p_\ell)(T^{01}_{\ell\ell} + T^{10}_{\ell\ell}) - p_\ell^2 T^{11}_{\ell\ell}\right] \\
&\geq 0
\end{aligned} \tag{8}
$$

$\mathcal{A}$ satisfies (7) if and only if (8) holds, which shows the correspondence between the insurers' break-even condition and the budget balance of allocations induced by insurance schemes. The results obtained sofar are summarized in the following proposition.

Proposition 1 *Conditions (6) and (7) define the set of feasible allocations that can be induced by insurance schemes offered by insurers that break-even. Under condition (2) type ij households can be covered by risk specific policies. Under condition (3) they can be covered by an umbrella policy. Under condition (4) - respect. condition (5) - they can be covered by an insurance policy that combines risk-specific copays with either a deductible or an out-of-pocket maximum - respect. an upper limit on indemnity -.*

2.4 Second-best Pareto-optimality

By definition, an allocation is second-best Pareto-optimal if it is feasible and if there does not exist another feasible allocation with expected utility at least as large for all types, and larger for at least one type. More explicitly, let us consider a feasible allocation $\mathcal{A}^* = \{w^*_{hh}, w^*_{h\ell}, w^*_{\ell h}, w^*_{\ell\ell}\}$, where $w^*_{ij} = (w^{00*}_{ij}, w^{01*}_{ij}, w^{10*}_{ij}, w^{11*}_{ij})$ for $i, j \in \{h, \ell\}$. $\mathcal{A}^*$ is second-best Pareto-optimal if, for all i, j, $U_{ij}(w_{ij})$ reaches its maximum at $\mathcal{A}^*$ in the set of feasible allocations $\mathcal{A} = \{w_{hh}, w_{h\ell}, w_{\ell h}, w_{\ell\ell}\}$ such that $U_{i'j'}(w_{i'j'}) \geq U_{i'j'}(w^*_{i'j'})$ for all $i', j' \in \{h, \ell\}, i' \neq i$ and/or $j' \neq j$.

Investigating the competitive interactions that could lead to a particular second-best Pareto-optimal allocation is out of the scope of the present paper. In what follows, we will limit ourselves to analyzing the insurance contracts that sustain second-best Pareto-optimal allocations. Thus, we implicitly presume that, in the absence of trade restraint (such as barriers to entry or exogenous constraints on contracts), competitive forces should lead to a second-best Pareto optimal allocation, which is of course a debatable assumption. We may also have in mind a regulated insurance market where second-best Pareto-optimality results from public intervention, for instance through taxation of contracts.

3 Risk splitting as an answer to adverse selection

A second-best Pareto-optimal allocation $\mathcal{A}^* = \{w^*_{hh}, w^*_{h\ell}, w^*_{\ell h}, w^*_{\ell\ell}\}$ maximizes $U_{\ell\ell}(w_{\ell\ell})$ in the set of feasible allocations $\mathcal{A} = \{w_{hh}, w_{h\ell}, w_{\ell h}, w_{\ell\ell}\}$ with expected utility at least equal to $u^*_{ij} \equiv U_{ij}(w^*_{ij})$ for $i, j \in \{h, \ell\}, i \neq \ell$ and/or $j \neq \ell$, and with $u^*_{\ell\ell} \equiv U_{\ell\ell}(w^*_{\ell\ell})$. Incentive compatibility implies that lower risk types reach weakly higher expected utility.[8] We will assume

$$u^*_{hh} < \min\{u^*_{h\ell}, u^*_{\ell h}\} \leq \max\{u^*_{h\ell}, u^*_{\ell h}\} \leq u^*_{\ell\ell}, \tag{9}$$

where the strict inequality $u^*_{hh} < \min\{u^*_{h\ell}, u^*_{\ell h}\}$ is made to exclude allocations where high risk households are pooled with medium risk households, i.e. where they have the same insurance coverage.[9]

Thus, $\mathcal{A}^*$ maximizes $U_{\ell\ell}(w_{\ell\ell})$ with respect to $w_{hh}, w_{h\ell}, w_{\ell h}$ and $w_{\ell\ell}$, subject to

- *Incentive compatibility constraints*:

$$U_{ij}(w_{ij}) \geq U_{ij}(w_{i'j'}), \tag{$IC^{i'j'}_{ij}$}$$

for all $i, j, i', j' \in \{h, \ell\}$, with $i \neq i'$ and/or $j \neq j'$,

- *Individual rationality constraints*:

$$U_{ij}(w_{ij}) \geq u^*_{ij}, \tag{IR_{ij}}$$

[8] This is true because overinsurance is excluded. In the Rothschild-Stiglitz (2016) setting, with only one risk exposure per type, there exist second-best Pareto-optimal allocations where high risk individuals are overcovered and reach a higher expected utility than low risk individuals, and such allocations could also exist in the present setting if overinsurance were allowed; See Crocker and Snow (1985).

[9] Such pooling allocations may be second-best Pareto-optimal. For instance, in the standard Rothschild-Stiglitz model, the pooling allocation with full coverage at average actuarial price is a second-best Pareto-optimum. Our results can be extended to the case $u^*_{hh} = \min\{u^*_{h\ell}, u^*_{\ell h}\}$. Types hh and $h\ell$ (respect. hh and ℓh) would be pooled with the same coverage if $u^*_{hh} = u^*_{h\ell}$ (respect. if $u^*_{hh} = u^*_{\ell h}$). See the comments on Proposition 2.

for all $i, j \in \{h, m\}$ with $i \neq \ell$ and/or $j \neq \ell$,

- *Break-even constraint*:

$$\sum_{i,j \in \{h,\ell\}} \lambda_i C_i(w_i) \leq \overline{w}, \tag{BC}$$

and sign and no-overinsurance constraints.

Propositions 2 and 3 characterize the optimal solution to this problem, and the associated insurance schemes. The proof of Proposition 2 is in two stages. At the first stage, we characterize the optimal solution of the relaxed problem where only the "upward" incentive compatibility constraints $IC_{hh}^{h\ell}, IC_{hh}^{\ell h}, IC_{hh}^{\ell\ell}, IC_{h\ell}^{\ell\ell}, IC_{\ell h}^{\ell\ell}$ are taken into account. At the second stage, we show that the optimal solution of the relaxed problem satisfies the incentive compatibility constraints that have been ignored at stage 1, and thus that it is the solution of the complete problem.[10] This leads us to the following results:

$$w_{hh}^{00} = w_{hh}^{10} = w_{hh}^{01} = w_{hh}^{11}, \tag{10}$$
$$w_{h\ell}^{00} = w_{h\ell}^{10} > w_{h\ell}^{01} = w_{h\ell}^{11}, \tag{11}$$
$$w_{\ell h}^{00} = w_{\ell h}^{01} > w_{\ell h}^{10} = w_{\ell h}^{11} \tag{12}$$
$$w_{\ell\ell}^{00} > w_{\ell\ell}^{10} > w_{\ell\ell}^{11}, \tag{13}$$
$$w_{\ell\ell}^{00} > w_{\ell\ell}^{01} > w_{\ell\ell}^{11}, \tag{14}$$

with correspondence in terms of insurance contracts stated in Proposition 2.

Proposition 2 *Any second-best Pareto-optimal allocation is sustained by insurance schemes such that : high risk households are fully covered, medium risk households are covered by risk-specific policies with full coverage for the large risk and partial coverage for the low risk, and low risk households are covered by a policy with partial coverage at the margin.*

Proposition 2 states that households are covered by insurance contracts that depend on their risk type, at the same time regarding the size and contractual form of coverage. Equation (10) shows that, as in the Rothschild-Stiglitz (1976) model, high risk households are fully covered, which means

[10] This approach is usual in adverse selection problems with the single-crossing condition and an arbitrary number of types. It is less obvious here, because there are four possible outcomes for each household, and not only two as in more standard models.

that $T^{01}_{hh} = T^{10}_{hh} = L$ and $T^{11}_{hh} = 2L$. This is compatible either with an umbrella policy $(J^1_{hh}, J^2_{hh}, P_{hh})$ such that $J^1_{hh} = L$ and $J^2_{hh} = 2L$, or with two risk specific policies, (I^1_{hh}, Q^1_{hh}) and (I^2_{hh}, Q^2_{hh}), for risks $r = 1$ and 2, respectively, with $I^1_{hh} = I^2_{hh} = L$ and $Q^1_{hh} + Q^2_{hh} = P_{hh}$.

Using (2) and equations (11) and (12) shows that medium risk households are covered by two risk specific policies $(I^1_{h\ell}, Q^1_{h\ell})$ and $(I^2_{h\ell}, Q^2_{h\ell})$ such that $I^1_{h\ell} = L$ and $I^2_{h\ell} < L$, and $(I^1_{\ell h}, Q^1_{\ell h})$ and $(I^2_{\ell h}, Q^2_{\ell h})$ such that $I^1_{\ell h} < L$ and $I^2_{\ell h} = L$. Hence, type $h\ell$ households have partial coverage policy for their low risk $r = 2$, while full coverage is maintained for their high risk $r = 1$, an symmetrically for type ℓh. This is not an astonishing conclusion, if we have in mind the standard Rothschild-Stiglitz model where the contract chosen by low risk individuals provides partial coverage, so that it is not attractive for high risk individuals. However, it is striking, and of course logical, that the same objective of reducing insurance compensation should lead insurers to offer risk specific policies, where the low coverage is for the low risk exposure only. Reducing the compensation for the high risk exposure would just be a waste of risk protection, without any effect on the separation of type $h\ell$ (or ℓh) from hh, since both are equally affected by this high risk.[11]

Finally, from equation (13), low risk households are covered by an insurance scheme that provides partial coverage at the margin, meaning that the household's final wealth decreases with the number of accidents.

A more technical remark should be made at this stage, in order to highlight the determinants of the low risk household coverage. In the present setting, households' indifference curves are 3-dimensional manifolds, in the $w = (w^{00}, w^{01}, w^{10}, w^{11})$ 4-dimensional space, and their intersections are either empty or they correspond to a continuum of points. Hence, indifference curves cannot satisfy a single-crossing property as in more usual settings, like the Rothshild-Stiglitz (1976) model, or its n-type extension analyzed by Spence (1978). In adverse selection problems (including many screening or signalling models), the single-crossing property guarantees that all incentive constraints are satisfied when adjacent incentive constraints hold. In the present setting, considering $hh - h\ell, hh - \ell h, h\ell - \ell\ell$ and $\ell h - \ell\ell$ as couples of "upward" adjacent types, this would mean that $IC^{\ell\ell}_{hh}$ is strongly satisfied (i.e., satisfied and not binding) when $IC^{h\ell}_{hh}, IC^{\ell\ell}_{h\ell}$ and/or $IC^{\ell h}_{hh}, IC^{\ell\ell}_{\ell h}$ hold.

[11] It can be checked that the strict inequalities in (11) and (12) become equalities if $u^*_{h\ell} = u^*_{hh}$ and $u^*_{h\ell} = u^*_{\ell h}$, respectively. In that case, medium risks are pooled with high risk policyholders and they have full coverage.

When preferences satisfy this property, by an abuse of terminology, we say that "local preferences are global", meaning that the strong incentive compatibility between non-adjacent household types follows from the incentive compatibility between adjacent types.

Proposition 3 *At a second-best Pareto-optimal allocation, low risk households should be covered by an insurance scheme that combines risk specific copays $c^1_{\ell\ell}, c^2_{\ell\ell}$ with a deductible $D_{\ell\ell}$, an out-of-pocket maximum $M_{\ell\ell}$ or an upper limit on coverage $UL_{\ell\ell}$. Assuming that households are downward risk averse (or prudent, i.e., $u''' > 0$) and that local preferences are global are sufficient conditions for combining risk specific copays with a deductible or with an out–of-pocket maximum to be optimal.*

We know from Proposition 2 that low risk households should be partially covered at the margin, and this may go through positive copays $c^1_{\ell\ell}$ and $c^2_{\ell\ell}$. The state dependent wealth $w_{\ell\ell}$ should reach a compromise between providing risk coverage to type $\ell\ell$ households, and satisfying the incentive compatibility constraints of higher risk types, namely $IC^{\ell\ell}_{hh}, IC^{\ell\ell}_{h\ell}$ and $IC^{\ell\ell}_{\ell h}$. Intuitively, $IC^{\ell\ell}_{h\ell}$ will be all the easier without affecting too much the risk protection of type $\ell\ell$ households, when the coverage of risk exposure $r = 1$ is lower, i.e., when $c^1_{\ell\ell}$ is larger, because $r = 1$ is a high risk for type $h\ell$ and a low risk for type $\ell\ell$. The conclusion is reversed for $IC^{\ell\ell}_{\ell h}$, with here a reason for increasing $c^2_{\ell\ell}$. Which copay should be the larger depends on the relative importance of types $h\ell$ and ℓh, including, among other things, their weights $\lambda_{h\ell}, \lambda_{\ell h}$ and expected utility levels $u^*_{h\ell}, u^*_{\ell h}$. The non-adjacent incentive compatibility constraint $IC^{\ell\ell}_{hh}$ does not justify treating differently risks $r = 1$ and $r = 2$. Under the joint effect of adjacent and non-adjacent constraints, two cases are generically possible. If

$$w^{00}_{\ell\ell} + w^{11}_{\ell\ell} > w^{10}_{\ell\ell} + w^{01}_{\ell\ell}, \tag{15}$$

then we know from condition (4) that risk-specific copays combined with either a deductible $D_{\ell\ell}$ or an out-of-pocket maximum $M_{\ell\ell}$ are optimal. On the contrary, if

$$w^{00}_{\ell\ell} + w^{11}_{\ell\ell} < w^{10}_{\ell\ell} + w^{01}_{\ell\ell}, \tag{16}$$

then risk-specific copays combined with an upper limit on idemnity $UL_{\ell\ell}$ are optimal. Proposition 3 shows that we are in the first case when local preferences are global - so that $IC_{hh}^{\ell\ell}$ is not binding - and the policyholders are downward risk averse.[12]

4 On the role of transaction costs

Let us now consider how transaction costs affect our conclusions. We assume that insurance premium should also cover such transaction costs, under the form of proportional loading at rate σ. Hence, insurers make non-negative expected profit if

$$\begin{aligned}
&\lambda_{hh}\left\{P_{hh}-(1+\sigma)\left[p_h(1-p_h)(T_{hh}^{01}+T_{hh}^{10})-p_h^2T_{hh}^{11}\right]\right\}\\
&+\lambda_{h\ell}\left\{P_{h\ell}-(1+\sigma)\left[p_h(1-p_\ell)T_{h\ell}^{01}-p_\ell(1-p_h)T_{h\ell}^{10}-p_hp_\ell T_{h\ell}^{11}\right]\right\}\\
&+\lambda_{\ell h}\left\{P_{\ell h}-(1+\sigma)\left[p_\ell(1-p_h)T_{\ell h}^{01}-p_h(1-p_\ell)T_{\ell h}^{10}-p_hp_\ell T_{\ell h}^{11}\right]\right\}\\
&+\lambda_{\ell\ell}\left\{P_{\ell\ell}-(1+\sigma)\left[p_\ell(1-p_\ell)(T_{\ell\ell}^{01}+T_{\ell\ell}^{10})-p_\ell^2T_{\ell\ell}^{11}\right]\right\}\\
&\geq 0,
\end{aligned} \tag{17}$$

instead of (8). Using (1) shows that the budget balance condition may still be written as equation (7), after rewriting the households' expected wealth as

$$\begin{aligned}
C_{hh}(w) \equiv\ & [1-(1+\sigma)p_h(2-p_h)]w^{00}+(1+\sigma)p_h(1-p_h)(w^{01}+w^{10})\\
&+(1+\sigma)p_h^2w^{11},\\
C_{h\ell}(w) \equiv\ & [1-(1+\sigma)(p_h+p_\ell-p_hp_\ell)]w^{00}+(1+\sigma)[p_\ell(1-p_h)w^{01}\\
&+p_h(1-p_\ell)w^{10}]+(1+\sigma)p_hp_\ell w^{11}\ ,\\
C_{\ell h}(w) \equiv\ & [1-(1+\sigma)(p_h+p_\ell-p_hp_\ell)]w^{00}+(1+\sigma)[p_h(1-p_\ell)w^{01}\\
&+p_\ell(1-p_h)w^{10}]+(1+\sigma)p_hp_\ell w^{11}\\
C_{\ell}(w) \equiv\ & [1-(1+\sigma)p_\ell(2-p_\ell)]w^{00}+(1+\sigma)p_\ell(1-p_\ell)(w^{01}+w^{10})\\
&+(1+\sigma)p_\ell^2w^{11}.
\end{aligned}$$

[12] These are sufficient conditions, not necessary ones.

With this new definition of insurance costs, second-best Pareto-optimal allocations are such that

$$w_{hh}^{00} > w_{hh}^{10} = w_{hh}^{01} \geq w_{hh}^{11}, \quad (18)$$
$$w_{h\ell}^{00} > w_{h\ell}^{10} > w_{h\ell}^{01} = w_{h\ell}^{11}, \quad (19)$$
$$w_{\ell h}^{00} > w_{\ell h}^{01} > w_{\ell h}^{10} = w_{\ell h}^{11}, \quad (20)$$
$$w_{\ell\ell}^{00} > w_{\ell\ell}^{10} > w_{\ell\ell}^{11}, \quad (21)$$
$$w_{\ell\ell}^{00} > w_{\ell\ell}^{01} > w_{\ell\ell}^{11}, \quad (22)$$

with insurance contracts characterized in Proposition 4.

Proposition 4 *If insurance pricing includes a positive loading, then any second-best Pareto-optimal allocation is sustained by insurance schemes such that: high risk households are covered by a straight deductible policy, medium risk households are covered by a policy that combines a deductible or an out-of-pocket maximum with a copayment for the low risk, and low risk households are covered with a deductible and partial coverage at the margin, that may be sustained by risk specific copays and either a deductible, an out-of-pocket maximum or an upper limit on coverage.*

The weak inequality in condition (18) is binding when the loading factor σ is large, so that not providing positive coverage in the case of a single loss is optimal. Indeed, the sign constraints $T_{hh}^{01} \geq 0, T_{hh}^{10} \geq 0$ are equivalent to $w_{hh}^{00} - w_{hh}^{10} \leq L$ and $w_{hh}^{00} - w_{hh}^{01} \leq L$, and these inequalities may be binding for σ large. Thus, (18) can be more precisely rewritten as

$$\begin{array}{lll} w_{hh}^{00} & > & w_{hh}^{10} = w_{hh}^{01} = w_{hh}^{11} \text{ if } w_{hh}^{00} - w_{hh}^{10} = w_{hh}^{00} - w_{hh}^{01} < L, \\ w_{hh}^{00} & > & w_{hh}^{10} = w_{hh}^{01} > w_{hh}^{11} \text{ if } w_{hh}^{00} - w_{hh}^{10} = w_{hh}^{00} - w_{hh}^{01} = L. \end{array}$$

This gives

$$T_{hh}^{xy} = \sup\{0, (x+y)L - D_{hh}\},$$

where D_{hh} is the deductible of type hh households. Hence, full coverage is just replaced by a straight deductible as in usual insurance demand model with proportional loading. There is nothing astonishing here, since incentive compatibility constraints do not justify distorting the high risk state-dependent wealth.

(4) and (19),(20) show that medium risk households should be covered by an insurance policy that combines a copay $c_{h\ell}^2$ or $c_{\ell h}^1$ for the low risk exposure

(i.e., $r = 2$ for type $h\ell$, and $r = 1$ for type ℓh) and no copay for the high risk exposure, with either a deductible $D_{h\ell}$ or $D_{\ell h}$, or an out-of-pocket maximum $M_{h\ell}$ or $M_{\ell h}$. When the loading factor σ is large, providing no coverage for the high risk may be optimal, and thus the optimal indemnity schedule is written as

$$\begin{aligned} T_{h\ell}^{xy} &= \sup\{0, xL + y(L - c_{h\ell}^2) - D_{h\ell}\}, \\ T_{\ell h}^{xy} &= \sup\{0, x(L - c_{\ell h}^1) + yL - D_{h\ell}\} \end{aligned}$$

Hence, as in Proposition 2, the high risk exposure should be more extensively covered than the low risk, but because of insurance loading, partial coverage of the high risk becomes optimal, and this takes the form of a copay.

Finally, from conditions (22) and (23), types $\ell\ell$ should still be covered by a mixture of risk specific copays $c_{\ell\ell}^1, c_{\ell\ell}^2$,and either a deductible, an out-of-pocket maximum or an upper limit on coverage.[13] For instance, in the case of a deductible, this is written as

$$T_{\ell\ell}^{xy} = \sup\{0, x(L - c_{\ell\ell}^1) + y(L - c_{\ell\ell}^2) - D_{\ell\ell}\}.$$

Hence, loading affects the size of the deductible, but not the structure of the indemnity schedule, with specific indemnity rules for each risk exposure.

5 Conclusion

Although risk splitting is ubiquitous in insurance markets, economic theory has lacks of arguments to explain why, more often than not, policyholders are not covered by umbrella policies. In this paper, we have pursued the idea that, under adverse selection, risk splitting facilitates the separation of risk types, and thus that risk specific indemnity rules should be part and parcel of insurance policies, except for high risk households. By doing so, we have established that either risk specific insurance policies or the combination of risk specific copays with a deductible, an out-of-pocket maximum, or an upper limit on coverage are in fact optimal answers to the adverse selection challenge.

Much work remains to be done to reach a fully satisfactory understanding of the structure of insurance contracts when policyholders face multiple

[13] As in Proposition 3, a deductible or an out-of-pocket maximum is optimal if the policyhoder is downward risk averse and if local preferences are global.

risks. Future research in this field could consider a setting that would be less restrictive than the one contemplated in this paper, with more than two risk exposures and two risk types. Moreover, it would be worth exploring alternative explanations of risk splitting, be they associated with other forms of asymmetry of information between insurer and insured or with managerial constraints. Finally, and most importantly, we have only explained why risk splitting is inherently linked with second-best efficiency, but we have not analyzed how market strategies may actually lead to the implemention of such insurance contracts. Understanding how competition forces interact under adverse selection when policyholders face multiple risks is of prime importance. It has to be noted that the analysis of insurance markets under adverse selection has not reached a consensus in the case of one single risk per policyholder,[14] and undoubtly, considering multiple risk exposures is an additional challenge for this line of research.

[14] See the survey of Mimra and Wambach (2014).

Appendix 1: Proofs

Proof of Proposition 2

Consider a second-best Pareto optimal allocation with expected utility u_{ij}^* for type ij households, with $i, j \in \{h, \ell\}$. This allocation maximizes

$$(1-p_\ell)^2 u(w_{\ell\ell}^{00}) + (1-p_\ell)p_\ell[u(w_{\ell\ell}^{01}) + u(w_{\ell\ell}^{10})] + p_\ell^2 u(w_{\ell\ell}^{11}) \tag{23}$$

with respect to $w_{hh}, w_{h\ell}, w_{\ell h}, w_{\ell\ell}$, subject to:

- **Incentive compatibility constraints**:

$$\begin{aligned}&(1-p_h)^2 u(w_{hh}^{00}) + p_h(1-p_h)[u(w_{hh}^{01}) + u(w_{hh}^{10})] + p_h^2 u(w_{hh}^{11})\\ &\geq\\ &(1-p_h)^2 u(w_{h\ell}^{00}) + p_h(1-p_h)[u(w_{h\ell}^{01}) + u(w_{h\ell}^{10})] + p_h^2 u(w_{h\ell}^{11}),\end{aligned} \tag{$IC_{hh}^{h\ell}$}$$

with a similar inequation for $(IC_{hh}^{\ell h})$,

$$\begin{aligned}&(1-p_h)^2 u(w_{hh}^{00}) + p_h(1-p_h)[u(w_{hh}^{01}) + u(w_{hh}^{10})] + p_h^2 u(w_{hh}^{11})\\ &\geq\\ &(1-p_h)^2 u(w_{\ell\ell}^{00}) + p_h(1-p_h)[u(w_{\ell\ell}^{01}) + u(w_{\ell\ell}^{10})] + p_h^2 u(w_{\ell\ell}^{11}),\end{aligned} \tag{$IC_{hh}^{\ell\ell}$}$$

$$\begin{aligned}&(1-p_h)(1-p_\ell)u(w_{h\ell}^{00}) + p_\ell(1-p_h)u(w_{h\ell}^{01}) + p_h(1-p_\ell)u(w_{h\ell}^{10}) + p_h p_\ell u(w_{h\ell}^{11})\\ &\geq\\ &(1-p_h)(1-p_\ell)u(w_{hh}^{00}) + p_\ell(1-p_h)u(w_{hh}^{01}) + p_h(1-p_\ell)u(w_{hh}^{10})] + p_h p_\ell u(w_{hh}^{11}),\end{aligned} \tag{$IC_{h\ell}^{hh}$}$$

with a similar inequation for $(IC_{\ell h}^{hh})$,

$$\begin{aligned}&(1-p_h)(1-p_\ell)u(w_{h\ell}^{00}) + p_\ell(1-p_h)u(w_{h\ell}^{01}) + p_h(1-p_\ell)u(w_{h\ell}^{10}) + p_h p_\ell u(w_{h\ell}^{11})\\ &\geq\\ &(1-p_h)(1-p_\ell)u(w_{\ell h}^{00}) + p_\ell(1-p_h)u(w_{\ell h}^{01}) + p_h(1-p_\ell)u(w_{\ell h}^{10})] + p_h p_\ell u(w_{\ell h}^{11}),\end{aligned} \tag{$IC_{h\ell}^{\ell h}$}$$

with a similar inequation for $(IC_{\ell h}^{h\ell})$,

$$(1-p_h)(1-p_\ell)u(w_{h\ell}^{00}) + p_\ell(1-p_h)u(w_{h\ell}^{01}) + p_h(1-p_\ell)u(w_{h\ell}^{10}) + p_h p_\ell u(w_{h\ell}^{11})$$
$$\geq$$
$$(1-p_h)(1-p_\ell)u(w_{\ell\ell}^{00}) + p_\ell(1-p_h)u(w_{\ell\ell}^{01}) + p_h(1-p_\ell)u(w_{\ell\ell}^{10})] + p_h p_\ell u(w_{\ell\ell}^{11}), \qquad (IC_{h\ell}^{\ell\ell})$$

with a similar inequation for $(IC_{\ell h}^{\ell\ell})$,

$$(1-p_\ell)^2 u(w_{\ell\ell}^{00}) + (1-p_\ell)p_\ell[u(w_{\ell\ell}^{01}) + u(w_{\ell\ell}^{10})] + p_\ell^2 u(w_{\ell\ell}^{11})$$
$$\geq$$
$$(1-p_\ell)^2 u(w_{hh}^{00}) + (1-p_\ell)p_\ell[u(w_{hh}^{01}) + u(w_{hh}^{10})] + p_\ell^2 u(w_{hh}^{11}), \qquad (IC_{\ell\ell}^{hh})$$

$$(1-p_\ell)^2 u(w_{\ell\ell}^{00}) + (1-p_\ell)p_\ell[u(w_{\ell\ell}^{01}) + u(w_{\ell\ell}^{10})] + p_\ell^2 u(w_{\ell\ell}^{11})$$
$$\geq$$
$$(1-p_\ell)^2 u(w_{h\ell}^{00}) + (1-p_\ell)p_\ell[u(w_{h\ell}^{01}) + u(w_{h\ell}^{10})] + p_\ell^2 u(w_{h\ell}^{11}), \qquad (IC_{\ell\ell}^{h\ell})$$

with a similar inequation for $(IC_{\ell\ell}^{\ell h})$,

- **Individual rationality constraints**:

$$(1-p_h)^2 u(w_{hh}^{00}) + (1-p_h)p_h[u(w_{hh}^{01}) + u(w_{hh}^{10})]$$
$$+p_h^2 u(w_{hh}^{11}) = u_{hh}^*, \qquad (IR_{hh})$$
$$(1-p_h)(1-p_\ell)u(w_{h\ell}^{00}) + p_\ell(1-p_h)u(w_{h\ell}^{01})$$
$$+p_h(1-p_\ell)u(w_{h\ell}^{10})] + p_h p_\ell u(w_{h\ell}^{11}) = u_{h\ell}^*, \qquad (IR_{h\ell})$$

with a similar equation for $(IR_{h\ell})$, and subject to the break-even constraint (BC), and to non-over insurance constraints:

$$w_{ij}^{00} \geq w_{ij}^{01}, \qquad (NOI_{ij}^{00/01})$$
$$w_{ij}^{00} \geq w_{ij}^{10}, \qquad (NOI_{ij}^{00/10})$$
$$w_{ij}^{01} \geq w_{ij}^{11}, \qquad (NOI_{ij}^{01/11})$$
$$w_{ij}^{10} \geq w_{ij}^{11}, \qquad (NOI_{ij}^{10/11})$$

for $i, j \in \{h, \ell\}$. The rest of the proof is in two stages.

Stage 1

Let us characterize the optimal solution to a relaxed problem, in which we delete the downward incentive compatibility constraints $IC_{\ell\ell}^{h\ell}, IC_{\ell\ell}^{\ell h}, IC_{h\ell}^{hh}$, $IC_{\ell h}^{hh}$ *and* $IC_{\ell\ell}^{hh}$ *and the no-overinsurance constraints, except* $NOI_{h\ell}^{00/10}, NOI_{h\ell}^{01/11}$, $NOI_{\ell h}^{00/01}$ *and* $NOI_{\ell h}^{10/11}$. Let

$$\begin{aligned}&\alpha_{hh}^{h\ell}, \alpha_{hh}^{\ell h}, \alpha_{hh}^{\ell\ell}, \alpha_{h\ell}^{\ell h}, \alpha_{h\ell}^{\ell\ell}, \alpha_{\ell h}^{h\ell}, \alpha_{\ell h}^{\ell\ell} \geq 0\\ &\delta_{h\ell}^{00/10}, \delta_{h\ell}^{01/11}, \delta_{\ell h}^{00/01}, \delta_{\ell h}^{10/11}, \gamma \geq 0,\\ &\beta_{hh}, \beta_{h\ell}, \beta_{\ell h},\end{aligned}$$

be Lagrange multipliers associated with

$$\begin{aligned}&IC_{hh}^{h\ell}, IC_{hh}^{\ell h}, IC_{hh}^{\ell\ell}, IC_{h\ell}^{\ell h}, IC_{h\ell}^{\ell\ell}, IC_{\ell h}^{h\ell}, IC_{\ell h}^{\ell\ell},\\ &NOI_{h\ell}^{00/10}, NOI_{h\ell}^{01/11}, NOI_{\ell h}^{00/01}, NOI_{\ell h}^{10/11}, BC,\\ &IR_{hh}, IR_{h\ell}, IR_{\ell h},\end{aligned}$$

respectively. Let $\mathcal{L}$ be the Lagrangian of the relaxed problem. The first-order optimality conditions are written as:

$$\frac{\partial \mathcal{L}}{\partial w_{hh}^{00}} = (1-p_h)^2[(\alpha_{hh}^{h\ell} + \alpha_{hh}^{\ell h} + \alpha_{hh}^{\ell\ell} + \beta_{hh})u'(w_h^{00}) - \gamma\lambda_{hh}] = 0, \tag{24}$$

$$\frac{\partial \mathcal{L}}{\partial w_{hh}^{01}} = p_h(1-p_h)[(\alpha_{hh}^{h\ell} + \alpha_{hh}^{\ell h} + \alpha_{hh}^{\ell\ell} + \beta_{hh})u'(w_h^{01}) - \gamma\lambda_{hh}] = 0, \tag{25}$$

$$\frac{\partial \mathcal{L}}{\partial w_{hh}^{10}} = p_h(1-p_h)[(\alpha_{hh}^{h\ell} + \alpha_{hh}^{\ell h} + \alpha_{hh}^{\ell\ell} + \beta_{hh})u'(w_h^{10}) - \gamma\lambda_{hh}] = 0, \tag{26}$$

$$\frac{\partial \mathcal{L}}{\partial w_{hh}^{11}} = p_h^2[(\alpha_{hh}^{h\ell} + \alpha_{hh}^{\ell h} + \alpha_{hh}^{\ell\ell} + \beta_{hh})u'(w_{hh}^{11}) - \gamma\lambda_{hh}] = 0, \tag{27}$$

$$\frac{\partial \mathcal{L}}{\partial w_{h\ell}^{00}} = (1-p_h)(1-p_\ell)[(\alpha_{h\ell}^{\ell h} + \alpha_{h\ell}^{\ell\ell} + \beta_{h\ell})u'(w_{h\ell}^{00}) - \gamma\lambda_{h\ell}]$$
$$- [(1-p_h)^2\alpha_{hh}^{h\ell} + (1-p_h)(1-p_\ell)\alpha_{\ell h}^{h\ell}]u'(w_{h\ell}^{00}) + \delta_{h\ell}^{00/10} = 0, \quad (28)$$
$$\frac{\partial \mathcal{L}}{\partial w_{h\ell}^{01}} = p_\ell(1-p_h)[(\alpha_{h\ell}^{\ell h} + \alpha_{h\ell}^{\ell\ell} + \beta_{h\ell})u'(w_{h\ell}^{01}) - \gamma\lambda_{h\ell}]$$
$$- [p_h(1-p_h)\alpha_{hh}^{h\ell} + p_h(1-p_\ell)\alpha_{\ell h}^{h\ell}]u'(w_{h\ell}^{01}) + \delta_{h\ell}^{01/11} = 0, \quad (29)$$
$$\frac{\partial \mathcal{L}}{\partial w_{h\ell}^{10}} = p_h(1-p_\ell)[(\alpha_{h\ell}^{\ell h} + \alpha_{h\ell}^{\ell\ell} + \beta_{h\ell})u'(w_{h\ell}^{10}) - \gamma\lambda_{h\ell}]$$
$$- [p_h(1-p_h)\alpha_{hh}^{h\ell} + p_\ell(1-p_h)\alpha_{\ell h}^{h\ell}]u'(w_{h\ell}^{10}) - \delta_{h\ell}^{00/10} = 0, \quad (30)$$
$$\frac{\partial \mathcal{L}}{\partial w_{h\ell}^{11}} = p_h p_\ell[(\alpha_{h\ell}^{\ell h} + \alpha_{h\ell}^{\ell\ell} + \beta_{h\ell})u'(w_{h\ell}^{11}) - \gamma\lambda_{h\ell}] = 0,$$
$$- [p_h^2\alpha_{hh}^{h\ell} + p_\ell p_h\alpha_{\ell h}^{h\ell}]u'(w_{h\ell}^{11}) - \delta_{h\ell}^{01/11} = 0, \quad (31)$$

with symmetric conditions for $w_{\ell h}$, and

$$\frac{\partial \mathcal{L}}{\partial w_{\ell\ell}^{00}} = (1-p_\ell)^2[u'(w_{\ell\ell}^{00}) - \gamma\lambda_{\ell\ell}]$$
$$- [(1-p_h)(1-p_\ell)(\alpha_{h\ell}^{\ell\ell} + \alpha_{\ell h}^{\ell\ell}) + (1-p_h)^2\alpha_{hh}^{\ell\ell}]u'(w_{\ell\ell}^{00}) = 0, \quad (32)$$
$$\frac{\partial \mathcal{L}}{\partial w_{\ell\ell}^{01}} = p_\ell(1-p_\ell)[u'(w_{\ell\ell}^{01}) - \gamma\lambda_{\ell\ell}]$$
$$- [p_\ell(1-p_h)\alpha_{h\ell}^{\ell\ell} + p_h(1-p_\ell)\alpha_{\ell h}^{\ell\ell} + p_h(1-p_h)\alpha_{hh}^{\ell\ell}]u'(w_{\ell\ell}^{01}) = 0, \quad (33)$$
$$\frac{\partial \mathcal{L}}{\partial w_{\ell\ell}^{10}} = p_\ell(1-p_\ell)[u'(w_{\ell\ell}^{10}) - \gamma\lambda_{\ell\ell}]$$
$$- [p_h(1-p_\ell)\alpha_{h\ell}^{\ell\ell} + p_\ell(1-p_h)\alpha_{\ell h}^{\ell\ell} + p_h(1-p_h)\alpha_{hh}^{\ell\ell}]u'(w_{\ell\ell}^{10}) = 0, \quad (34)$$
$$\frac{\partial \mathcal{L}}{\partial w_{\ell\ell}^{11}} = p_\ell^2[(u'(w_{\ell\ell}^{11}) - \gamma\lambda_{\ell\ell}]$$
$$- [p_h p_\ell(\alpha_{h\ell}^{\ell\ell} + \alpha_{\ell h}^{\ell\ell}) + p_h^2\alpha_{hh}^{\ell\ell}]u'(w_\ell^{11}) = 0. \quad (35)$$

We have $\gamma > 0$, because equations (32)-(35) would lead to a contradiction if $\gamma = 0$.[15] Equations (24)-(27) then give $\alpha_{hh}^{h\ell} + \alpha_{hh}^{\ell h} + \alpha_{hh}^{\ell\ell} + \beta_{hh} > 0$. Using $u'' < 0$ yields

$$w_{hh}^{00} = w_{hh}^{01} = w_{hh}^{10} = w_{hh}^{11}. \quad (36)$$

[15] Equations (32)-(35) may be rewritten as (43)-(46) below, which cannot hold if $\gamma = 0$.

Equations (28)-(31) may be rewritten as

$$u'(w_{h\ell}^{00})\left[\alpha_{h\ell}^{\ell h}+\alpha_{h\ell}^{\ell\ell}+\beta_{h\ell}-\alpha_{hh}^{h\ell}\frac{1-p_h}{1-p_\ell}-\alpha_{\ell h}^{h\ell}\right]=\gamma\lambda_{h\ell}-\frac{\delta_{h\ell}^{00/10}}{(1-p_h)(1-p_\ell)},\tag{37}$$

$$u'(w_{h\ell}^{01})\left[\alpha_{h\ell}^{\ell h}+\alpha_{h\ell}^{\ell\ell}+\beta_{h\ell}-\alpha_{hh}^{h\ell}\frac{p_h}{p_\ell}-\alpha_{\ell h}^{h\ell}\frac{p_h(1-p_\ell)}{p_\ell(1-p_h)}\right]=\gamma\lambda_{h\ell}-\frac{\delta_{h\ell}^{01/11}}{p_\ell(1-p_h)},\tag{38}$$

$$u'(w_{h\ell}^{10})\left[\alpha_{h\ell}^{\ell h}+\alpha_{h\ell}^{\ell\ell}+\beta_{h\ell}-\alpha_{hh}^{h\ell}\frac{1-p_h}{1-p_\ell}-\alpha_{\ell h}^{h\ell}\frac{p_\ell(1-p_h)}{p_h(1-p_\ell)}\right]=\gamma\lambda_{h\ell}+\frac{\delta_{h\ell}^{00/10}}{p_h(1-p_\ell)}\tag{39}$$

$$u'(w_{h\ell}^{11})\left[\alpha_{h\ell}^{\ell h}+\alpha_{h\ell}^{\ell\ell}+\beta_{h\ell}-\alpha_{hh}^{h\ell}\frac{p_h}{p_\ell}-\alpha_{\ell h}^{h\ell}\right]=\gamma\lambda_{h\ell}+\frac{\delta_{h\ell}^{01/11}}{p_hp_\ell}.\tag{40}$$

Suppose $w_{h\ell}^{10}<w_{h\ell}^{00}$, with $\delta_{h\ell}^{00/10}=0$. Using $u''<0$ gives $u'(w_{h\ell}^{10})>u'(w_{h\ell}^{00})$, which contradicts (37) and (39). Thus, we have $w_{h\ell}^{10}=w_{h\ell}^{00}$. Similarly, $w_{h\ell}^{11}<w_{h\ell}^{01}$, with $\delta_{h\ell}^{01/11}=0$, would contradict (38) and (40), and thus $w_{h\ell}^{11}=w_{h\ell}^{01}$. We have $w_{h\ell}^{11}\leq w_{h\ell}^{00}$ from (37) and (40), or from the no-overinsurance constraints. If $w_{h\ell}^{11}=w_{h\ell}^{00}$, then (36) gives $u_{hh}^{*}=u_{h\ell}^{*}$, which contradicts (9). Overall, we have

$$w_{h\ell}^{11}=w_{h\ell}^{01}<w_{h\ell}^{10}=w_{h\ell}^{00},\tag{41}$$

and symmetrically

$$w_{\ell h}^{11}=w_{\ell h}^{01}<w_{\ell h}^{10}=w_{\ell h}^{00}.\tag{42}$$

Equations (32)-(35) yield

$$u'(w_{\ell\ell}^{00})\left[1-(\alpha_{h\ell}^{\ell\ell}+\alpha_{\ell h}^{\ell\ell})\frac{1-p_h}{1-p_\ell}-\alpha_{hh}^{\ell\ell}\frac{(1-p_h)^2}{(1-p_\ell)^2}\right]=\gamma\lambda_{\ell\ell},\tag{43}$$

$$u'(w_{\ell\ell}^{01})\left[1-\alpha_{h\ell}^{\ell\ell}\frac{1-p_h}{1-p_\ell}-\alpha_{\ell h}^{\ell\ell}\frac{p_h}{p_\ell}-\alpha_{hh}^{\ell\ell}\frac{p_h(1-p_h)}{p_\ell(1-p_\ell)}\right]=\gamma\lambda_{\ell\ell},\tag{44}$$

$$u'(w_{\ell\ell}^{10})\left[1-\alpha_{h\ell}^{\ell\ell}\frac{p_h}{p_\ell}-\alpha_{\ell h}^{\ell\ell}\frac{1-p_h}{1-p_\ell}-\alpha_{hh}^{\ell\ell}\frac{p_h(1-p_h)}{p_\ell(1-p_\ell)}\right]=\gamma\lambda_{\ell\ell},\tag{45}$$

$$u'(w_{\ell\ell}^{11})\left[1-(\alpha_{h\ell}^{\ell\ell}+\alpha_{\ell h}^{\ell\ell})\frac{p_h}{p_\ell}-\alpha_{hh}^{\ell\ell}\frac{p_h^2}{p_\ell^2}\right]=\gamma\lambda_{\ell\ell}.\tag{46}$$

Let $\Phi(z) = u'^{-1}(\gamma\lambda_{\ell\ell}/1 - z)$, with $z < 1$. $u'' < 0$ gives $\Phi' < 0$. Furthermore, we have $\Phi'' > 0$ if $u''' > 0$. Equations (43)-(46) give $w_{\ell\ell}^{xy} = \Phi(z^{xy})$ for $(x, y) \in \{0, 1\}$, with

$$\begin{aligned}
z^{00} &= (\alpha_{h\ell}^{\ell\ell} + \alpha_{\ell h}^{\ell\ell})\frac{1-p_h}{1-p_\ell} + \alpha_{hh}^{\ell\ell}\frac{(1-p_h)^2}{(1-p_\ell)^2}, \\
z^{01} &= \alpha_{h\ell}^{\ell\ell}\frac{1-p_h}{1-p_\ell} + \alpha_{\ell h}^{\ell\ell}\frac{p_h}{p_\ell} + \alpha_{hh}^{\ell\ell}\frac{p_h(1-p_h)}{p_\ell(1-p_\ell)}, \\
z^{10} &= \alpha_{h\ell}^{\ell\ell}\frac{p_h}{p_\ell} + \alpha_{\ell h}^{\ell\ell}\frac{1-p_h}{1-p_\ell} + \alpha_{hh}^{\ell\ell}\frac{p_h(1-p_h)}{p_\ell(1-p_\ell)}, \\
z^{11} &= (\alpha_{h\ell}^{\ell\ell} + \alpha_{\ell h}^{\ell\ell})\frac{p_h}{p_\ell} + \alpha_{hh}^{\ell\ell}\frac{p_h^2}{p_\ell^2},
\end{aligned}$$

and thus

$$z^{00} < \min\{z^{01}, z^{10}\} \leq \max\{z^{01}, z^{10}\} < z^{11},$$

which gives

$$w_{\ell\ell}^{00} > \max\{w_{\ell\ell}^{01}, w_{\ell\ell}^{10}\} \geq \min\{w_{\ell\ell}^{01}, w_{\ell\ell}^{10}\} > w_{\ell\ell}^{11}.$$

Stage 2

Let us show that the optimal solution of the relaxed problem satisfies the incentive constraints that have been ignored at stage 1.

1: Constraints $IC_{\ell\ell}^{h\ell}$ and $IC_{\ell\ell}^{\ell h}$.

If $IC_{\ell\ell}^{h\ell}$ were not satisfied, then we would have $U_{\ell\ell}(w_{\ell\ell}) < U_{\ell\ell}(w_{h\ell})$. We have

$$U_{\ell\ell}(w_{h\ell}) = (1 - p_\ell)u(w_{h\ell}^{00}) + p_\ell u(w_{h\ell}^{11}) = U_{h\ell}(w_{h\ell}) = u_{h\ell}^*,$$

and thus $U_{\ell\ell}(w_{\ell\ell}) < u_{h\ell}^*$. At the optimal solution of the relaxed problem, the expected utility of type $\ell\ell$ households is at least equal to $u_{\ell\ell}^*$, which is its value at the optimal solution of the complete problem. Thus, we have $u_{\ell\ell}^* \leq U_{\ell\ell}(w_{\ell\ell})$, which implies $u_{\ell\ell}^* < u_{h\ell}^*$, which contradicts (9). The same argument applies for $IC_{\ell\ell}^{\ell h}$.

2: Constraints $IC_{h\ell}^{hh}$ and $IC_{\ell h}^{hh}$.

If $IC_{h\ell}^{hh}$ were not satisfied, then we would have $U_{h\ell}(w_{h\ell}) < U_{h\ell}(w_{hh})$. Using $u_{h\ell}^* = U_{h\ell}(w_{h\ell})$ and $U_{h\ell}(w_{hh}) = U_{hh}(w_{hh}) = u_{hh}^*$, would then give $u_{h\ell}^* < u_{hh}^*$, which contradicts (9). The same argument holds for $IC_{\ell h}^{hh}$.

3: Constraint $IC_{\ell\ell}^{hh}$.

We may write

$$\begin{aligned} U_{\ell\ell}(w_{\ell\ell}) &\geq (1-p_\ell)u(w_{h\ell}^{00}) + p_\ell u(w_{h\ell}^{11}) && (47)\\ &> (1-p_h)u(w_{h\ell}^{00}) + p_h u(w_{h\ell}^{11}) && (48)\\ &\geq U_{hh}(w_{hh}) && (49)\\ &= U_{\ell\ell}(w_{hh}) && (50) \end{aligned}$$

where (47),(48),(49) and (50) respectively result from $IC_{\ell\ell}^{h\ell}$, from $p_h > p_\ell$ and $w_{h\ell}^{00} \geq w_{h\ell}^{11}$, from $IC_{h\ell}^{hh}$ and from (36). Thus, $IC_{\ell\ell}^{hh}$ is satisfied.

Proof of Proposition 3

The first part of the proposition has been established in section 3. It remains to show that $w_{\ell\ell}^{00}+w_{\ell\ell}^{11} > w_{\ell\ell}^{10}+w_{\ell\ell}^{01}$ when $u''' > 0$ and local preferences are global. Observe first that $\Phi'' > 0$ if $u''' > 0$. Furthermore, when local preferences are global, $IC_{hh}^{\ell\ell}$ is not binding, and thus $\alpha_{hh}^{\ell\ell} = 0$, which yields

$$z^{00} + z^{11} = z^{01} + z^{10}.$$

In that case, lottery $(z^{00}, 1/2; z^{11}, 1/2)$ is obtained from lottery $(z^{01}, 1/2; z^{10}, 1/2)$ by a mean-preserving spread. Using $\Phi'' > 0$ gives

$$\begin{aligned} w_{\ell\ell}^{00} + w_{\ell\ell}^{11} &= \Phi(z^{00}) + \Phi(z^{11}) \\ &> \Phi(z^{01}) + \Phi(z^{10}) \\ &= w_{\ell\ell}^{01} + w_{\ell\ell}^{10}, \end{aligned}$$

which completes the proof.

Proof of Proposition 4

The proof is in two stages, as for Proposition 2.

Stage 1

We consider the relaxed problem as in the proof of Proposition 2, with unchanged notations for Lagrange multipliers. The first-order optimality conditions become

$$\frac{\partial \mathcal{L}}{\partial w_{hh}^{00}} = (1-p_h)^2 \left[(\alpha_{hh}^{h\ell} + \alpha_{hh}^{\ell h} + \alpha_{hh}^{\ell\ell} + \beta_{hh})u'(w_{hh}^{00}) - \gamma\lambda_{hh}\frac{1-(1+\sigma)p_h(2-p_h)}{(1-p_h)^2}\right] = 0, \tag{51}$$

$$\frac{\partial \mathcal{L}}{\partial w_{hh}^{01}} = p_h(1-p_h)[(\alpha_{hh}^{h\ell} + \alpha_{hh}^{\ell h} + \alpha_{hh}^{\ell\ell} + \beta_{hh})u'(w_{hh}^{01}) - \gamma\lambda_{hh}(1+\sigma)] = 0, \tag{52}$$

$$\frac{\partial \mathcal{L}}{\partial w_{hh}^{10}} = p_h(1-p_h)[(\alpha_{hh}^{h\ell} + \alpha_{hh}^{\ell h} + \alpha_{hh}^{\ell\ell} + \beta_{hh})u'(w_{hh}^{10}) - \gamma\lambda_{hh}(1+\sigma)] = 0, \tag{53}$$

$$\frac{\partial \mathcal{L}}{\partial w_{hh}^{11}} = p_h^2[(\alpha_{hh}^{h\ell} + \alpha_{hh}^{\ell h} + \alpha_{hh}^{\ell\ell} + \beta_{hh})u'(w_{hh}^{11}) - \gamma\lambda_{hh}(1+\sigma)] = 0, \tag{54}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w_{h\ell}^{00}} &= (1-p_h)(1-p_\ell)\left[(\alpha_{h\ell}^{\ell h} + \alpha_{h\ell}^{\ell\ell} + \beta_{h\ell})u'(w_{h\ell}^{00}) - \gamma\lambda_{h\ell}\frac{1-(1+\sigma)(p_h+p_\ell-p_hp_\ell)}{(1-p_h)(1-p_\ell)}\right] \\
&\quad - [(1-p_h)^2\alpha_{hh}^{h\ell} + (1-p_h)(1-p_\ell)\alpha_{\ell h}^{h\ell}]u'(w_{h\ell}^{00}) + \delta_{h\ell}^{00/10} = 0, && (55) \\
\frac{\partial \mathcal{L}}{\partial w_{h\ell}^{01}} &= p_\ell(1-p_h)[(\alpha_{h\ell}^{\ell h} + \alpha_{h\ell}^{\ell\ell} + \beta_{h\ell})u'(w_{h\ell}^{01}) - \gamma\lambda_{h\ell}(1+\sigma)] \\
&\quad - [p_h(1-p_h)\alpha_{hh}^{h\ell} + p_h(1-p_\ell)\alpha_{\ell h}^{h\ell}]u'(w_{h\ell}^{01}) - \delta_{h\ell}^{01/11} = 0, && (56) \\
\frac{\partial \mathcal{L}}{\partial w_{h\ell}^{10}} &= p_h(1-p_\ell)[(\alpha_{h\ell}^{\ell h} + \alpha_{h\ell}^{\ell\ell} + \beta_{h\ell})u'(w_{h\ell}^{10}) - \gamma\lambda_{h\ell}(1+\sigma)] \\
&\quad - [p_h(1-p_h)\alpha_{hh}^{h\ell} + p_\ell(1-p_h)\alpha_{\ell h}^{h\ell}]u'(w_{h\ell}^{10}) + \delta_{h\ell}^{00/10} = 0, && (57) \\
\frac{\partial \mathcal{L}}{\partial w_{h\ell}^{11}} &= p_hp_\ell[(\alpha_{h\ell}^{\ell h} + \alpha_{h\ell}^{\ell\ell} + \beta_{h\ell})u'(w_{h\ell}^{11}) - \gamma\lambda_{h\ell}(1+\sigma)] = 0, \\
&\quad - [p_h^2\alpha_{hh}^{h\ell} + p_\ell p_h\alpha_{\ell h}^{h\ell}]u'(w_{h\ell}^{11}) - \delta_{h\ell}^{01/11} = 0, && (58)
\end{aligned}$$

with symmetric conditions for $w_{\ell h}$, and

$$\frac{\partial \mathcal{L}}{\partial w_{\ell\ell}^{00}} = (1-p_\ell)^2 \left[u'(w_\ell^{00}) - \gamma\lambda_\ell \frac{1-(1+\sigma)p_h(2-p_h)}{(1-p_h)^2}\right]$$

$$- [(1-p_h)(1-p_\ell)(\alpha_{h\ell}^{\ell\ell} + \alpha_{\ell h}^{\ell\ell}) + (1-p_h)^2\alpha_{hh}^{\ell\ell}]u'(w_{\ell\ell}^{00}) = 0, \tag{59}$$

$$\frac{\partial \mathcal{L}}{\partial w_{\ell\ell}^{01}} = p_\ell(1-p_\ell)[u'(w_\ell^{01}) - \gamma\lambda_\ell(1+\sigma)]$$

$$- [p_\ell(1-p_h)\alpha_{h\ell}^{\ell\ell} + p_h(1-p_\ell)\alpha_{\ell h}^{\ell\ell} + p_h(1-p_h)\alpha_{hh}^{\ell\ell}]u'(w_{\ell\ell}^{01}) = 0, \tag{60}$$

$$\frac{\partial \mathcal{L}}{\partial w_{\ell\ell}^{10}} = p_\ell(1-p_\ell)[u'(w_\ell^{10}) - \gamma\lambda_\ell(1+\sigma)]$$

$$- [p_h(1-p_\ell)\alpha_{h\ell}^{\ell\ell} + p_\ell(1-p_h)\alpha_{\ell h}^{\ell\ell} + p_h(1-p_h)\alpha_{hh}^{\ell\ell}]u'(w_{\ell\ell}^{10}) = 0, \tag{61}$$

$$\frac{\partial \mathcal{L}}{\partial w_{\ell\ell}^{11}} = p_\ell^2[(u'(w_\ell^{11}) - \gamma\lambda_\ell(1+\sigma)]$$

$$- [p_h p_\ell(\alpha_{h\ell}^{\ell\ell} + \alpha_{\ell h}^{\ell\ell}) + p_h^2\alpha_{hh}^{\ell\ell}]u'(w_\ell^{11}) = 0. \tag{62}$$

Conditions (51)-(54), (55)-(58) and (59)-(62) respectively give (19)-(23), with intermediate steps similar to the proof of Proposition 2.

Stage 2

The proof is straighforwardly adapted from the proof of Proposition 2.

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