

**EXAMPLES OF EXERCISES GIVEN TO THE ORAL EXAM FOR
THE BACHELOR PROGRAM**

Exercise 1. Let $p \in \mathbb{R}$. Find the number of real solutions of the equation $x^5 - 5x = p$.

Exercise 2. Show that $\sum_{k=0}^n k^3 = (\sum_{k=0}^n k)^2$. Compute $\sum_{k=0}^n k^2$.

Exercise 3. Your dog Alfred is sitting in the corner A of a square $ABCD$. A bone is placed in the center O of the square. We assume that Alfred moves randomly, following the segments created by $OABCD$. We call p_n the probability for Alfred to reach the bone in the center O for the first time after n moves.

- (1) Compute p_1 and then p_n .
- (2) We call E_n the event "Alfred reached the bone at least once after n moves" and S_n its associated probability. Find the link between S_n and p_n .
- (3) Compute S_n . Could you have predicted this result more easily?
- (4) Compute $\lim_{n \rightarrow \infty} S_n$ and comment the result.

Compute $\lim_{n \rightarrow \infty} S_n$ and comment the result.