

Linear Algebra

1. Linear Equations
 - a. Gaussian Elimination and Matrices ,
 - b. Gauss–Jordan Method,
 - c. Rectangular Systems and Echelon Forms .
2. Matrix Algebra
 - a. Addition, multiplication, transposition,
 - b. Properties of Matrix Multiplication,
 - c. Matrix Inversion,
 - d. The LU and QR factorizations.
3. Vector Spaces
 - a. Spaces and Subspaces,
 - b. linear independence,
 - c. Basis and Dimension.
4. Norms, Inner Products, and Orthogonality :
 - a. Vector Norms, Matrix Norms,
 - b. Inner-Product Spaces, Orthogonal Vectors, Gram–Schmidt Procedure,
 - c. Unitary and Orthogonal Matrices, Range-Nullspace Decomposition,
 - d. Orthogonal Decomposition, orthogonal basis,
 - e. Singular Value Decomposition .
5. Eigenvalues and Eigenvectors :
 - a. Elementary Properties of Eigensystems,
 - b. Diagonalization by Similarity Transformations ,
 - c. Functions of Diagonalizable Matrices,
 - d. Positive Definite Matrices, spectral theorems.

Recommended reading: Introduction to Linear Algebra, Gilbert Strang, MIT Press, 2009 (videolectures are available).

Analysis

1. Differentiable calculus for functions of one variable
 - a. Functions and limits
 - b. Continuity
 - c. Differentiable functions of one variable
 - d. Taylor’s theorem
2. Integral calculus for functions of one -variable
 - a. Definition and existence of the integral,
 - b. Properties of the integral
3. Infinite sequences and series
 - a. Convergence of Sequences of real numbers
 - b. Subsequences and Cauchy Sequences
 - c. Sequences and series of functions
 - d. Power series
4. Real valued functions of several variables
 - a. Structure of \mathbb{R}^n
 - b. Continuous real-valued functions of n variables
 - c. Partial derivatives and the differential
 - d. The chain rule and Taylor’s theorem

5. Vector valued functions of several variables (optional but an introduction is a plus)
 - a. Linear transformation and matrices,
 - b. Continuity and differentiability of transformations,
 - c. Lagrange multipliers
 - d. Absolute Minimum and Maximum
 - e. Local linear and Quadratic Approximations
6. Integral of functions of several variables
 - a. Definition and existence of the multiple integral
 - b. Change of variables
7. Optimization
 - a. Unconstrained optimization / constrained optimization,
 - b. Equality / Inequality constraints,
 - c. Gradient-descent algorithms,
 - d. Newton-Raphson methods,

Suggested reading:

- Numerical Analysis for Statisticians, Kenneth Lange, Chapters 1-12
- Introduction to real analysis, William Trench, 2013,
<http://digitalcommons.trinity.edu/mono/7/>

Probability

1. Combinatorial analysis
2. Axiom of probability (Sample space, events)
3. Conditional probability and independence
 - a. Elementary conditional probability
 - b. Bayes's formula
 - c. Independent event
 - d. Conditional probability, conditional expectations
4. Random variables
 - a. Expected values, expected values of a function of a random variable
 - b. Moments and cumulants
 - c. Discrete random variables examples: Bernoulli random variables, Poisson random variables
 - d. Continuous random variables examples: uniform, normal, exponential, Gamma random variable
5. Random vectors
 - a. Joint distribution
 - b. Independent random variables
 - c. Sum of independent random variables
 - d. Conditional distribution (discrete and continuous case)
 - e. Order statistics (min, max, median)
 - f. Covariance and correlation matrices
6. Properties of expectation
 - a. Expectation of sum of random variable
 - b. Covariance, variance of sums, correlations
 - c. Additional properties of Normal random vectors

Mandatory:

- A first course in probability, Sheldon Ross, Pearson publishing

Recommended reading

- Probability for statistics and Machine learning: fundamentals and advanced topics, A. Dasgupta, Chapter 1-10.

Statistics

1. Exploratory statistics, data visualization
2. Statistical inference principles
3. Parametric inference
 - a. Maximum likelihood
 - b. Methods of moments
4. Hypothesis testing
 - a. Neyman-Pearson lemma
 - b. Some classical tests in the Gaussian models
 - c. p-value of a test
5. Bayesian inference
6. Analysis of Variance, linear regression

Recommended reading

- An introduction to mathematical statistics and its applications, R. Larsen and M. Marx, Prentice Hall (Chapters 1-10)
- All of Statistics, a concise course of statistical inference, L. Wasserman, Springer text in Statistics.