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AND CARBON LEAKAGE**

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Capacity decisions with demand fluctuations and carbon leakage *

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Abstract

Leakage is an important issue to evaluate unilateral climate policies designed to mitigate carbon emissions. This issue is ordinarily addressed in short term models which neglect demand fluctuations and the existence of multinational firms which optimize their regional production through home and foreign plants. The impact of relaxing these assumptions is explored in a simple model which builds on the large literature concerned by the comparative statics properties for decisions under uncertainty. In our model a unilateral change in the CO₂ price of country A will affect capacities of the plants in that country. It is proved that there would be long term leakage while there would be none if uncertainty were neglected. It is also proved that the pass through rate would be lower in the short term than the one obtained if there were no uncertainty. The key to these results is the strategic behavior of the multinationals which only imports in high demand states. The relevance of the results for the EU-ETS is discussed.

JEL Classification: D81, D92, Q56, L13

Keywords: carbon leakage, demand fluctuations, capacity decisions, relocation.

1 Introduction

When a country A implements a unilateral climate policy to mitigate carbon emissions leakage refers to the fact that the reduction of emissions in country A may be partly offset by the increase of emissions in the rest of the world. This issue plays a major role in the evaluation and design of alternative climate policies (Droege and Cooper, 2009).

In the literature leakage originates from a combination of two main channels (Reinaud, 2005). Firstly, the reduction of the demand for energy sources in country A generates a decrease in the world price of fossil fuels, which generates an increase in the consumptions of these fuels in the rest of the world. Secondly, the reduction in the production of carbon intensive products in country A may be partially compensated by an increase in the imports of these products from the rest of the world. One commonly refers to the first channel as the *energy-price-driven* leakage and to the second channel as to the competitiveness-driven or *sectoral* leakage.

The paper focuses on the sectoral leakage in the simple case in which the climate policy consists in an increase of the CO₂ price. It leaves aside the combination of both channels, best discussed in the context of computable general equilibrium models.

The quantification of sectoral leakage is an important topic for *sensitive* sectors. A sector is sensitive under two conditions: (Grubb and Neuhoff, 2006) the impact of the CO₂ price is high relative to its value added (*value at stake*), it is highly exposed to international trade (*import intensity*). If both conditions are satisfied sectoral leakage is high. Typical sensitive sectors are:

cement, steel, basic chemicals, aluminium... The quantification of leakage in such sectors through economic modeling has been actively used to analyze climate policies and, in particular, the role of free allocations and/or border adjustment mechanisms.

Ordinarily sectoral leakage is quantified through partial equilibrium models featuring home producers competing in a product market with foreign producers (for a survey see chapter 1 pp 13:15 in Hourcade et al., 2008). Two forms of competition are explored: either Cournot competition or pure competition using the Armington framework. Most of the models are essentially short term and ignore the implications for international trade of the existence of multinationals, a typical characteristic of sensitive sectors. These firms globally optimize their production in a large network of plants to face many regional fluctuating demands. If country A unilaterally increases its CO₂ price, in the long term, multinationals can be expected to adjust their capacities in country A. Our model will show that whenever this is relevant there would a higher long term leakage rate than if uncertainty were neglected. Our model also allows for the analysis of uncertainty on the pass through rate, i.e. the ratio of the increase in the product price relative to the increase of the carbon cost. It will be shown that the short term pass through rate should be expected to be lower than the long term one. The strategic behavior of the multinationals is key in explaining these results. In the short term these firms only imports when the demand is high and saturates their local capacities. In the long term they relocate their investments taking into account the relative CO₂ prices in the various countries.

These results are in line with two sets of empirical observations. Firstly,

the crucial role of capacity constraints to explain international trade flows relative to regional economic cycles has been recognized for a number of carbon intensive sectors and in particular for cement (see section 3.1.1 in chapter 3 pp 60:69 in Hourcade et al., 2008). Secondly, there is so far very little empirical evidence that the introduction of the EU-ETS (2005-2008) supports the high sectoral leakage rate derived from the existing analytical models (see chapter 9 pp 260:287 in Ellerman et al., 2010). However, these authors suggest that the current observations cannot yet capture long run effects.

There are some specific models in the cement sector that do introduce capacity constraints. Demailly and Quirion (2008) built a world model in which capacity decisions (and constraints) are introduced over a 30 year time horizon. Capacity expansion follows deterministic demand trends and firms do not optimize their sources depending on the economic cycle. Ponsard and Walker (2008) analyze the role of exogenous capacity constraints in a spatial model of the cement sector. The Cement Sustainability Initiative (WBCSD, 2009) explores scenarios over the period 2005 to 2030. These scenarios assume future demand in the various world regions, available technologies, including present and future abatement opportunities (e.g. energy efficiency, fuel switching, increased blending, carbon capture), production and transport costs. In any given region, investment decisions are made in the year when capacity is needed in that region—the model does not look ahead. Excess capacity may then be use to export from that region if profitable. The sectoral leakage rates derived from these models are respectively 50%, 70% and 56%. Neither of these approaches do recognize the existence

of multinational firms with its implications for international trade.

To make our point on the influence of multinationals on carbon leakage we construct a model based on a number of simplifications. Country A market is taken as one unique regional market. The demand in this market is subject to uncertain fluctuations over business cycles. The competing producers have local plants and they can also import to face demand fluctuations. Competition is of the Cournot type. Because of the significance of transport costs country A market is served in priority by home plants, unless these plants are saturated. Investment in home plants is irreversible. Under these conditions, the long term decisions concern the evolution of capacity in country A; and the short term decisions concern production in country A and import from abroad. Each firm has to take into consideration the variability of country A demand and the total cost of producing in country A compared to the cost of importing.

The analysis is carried on assuming linearity in the long run average cost function (investment and production) and in the demand function. Import costs are also assumed to be linear. The demand function includes an additive random parameter uniformly distributed over a given range. Without uncertainty, capacity is equal to home production and there are no imports. With uncertainty, the firm imports when the demand is high and saturates its capacity. We analyze the influence of uncertainty on the optimal capacity, and on the effect of the implementation of a CO₂ price. We prove that if the CO₂ price is below some threshold, the optimal capacity increases as the range of uncertainty increases, while it decreases if it is above. Furthermore, the negative effect of the CO₂ price on home capacity is amplified by uncer-

tainty. This monotonicity result makes the relevance of our model to study leakage quite important. For instance, the long term leakage is shown to be increasing with respect to the range of uncertainty.

From a theoretical standpoint our modeling approach falls in the large literature initiated by Rothshild and Stiglitz (1970, 1971) on the comparative statics properties for decisions under uncertainty. This literature investigates the existence of general conditions for monotonicity on the current decision when the risk increases. In our model, monotonicity does not hold. Still we exhibit a simple condition which says that if the foreign production cost is high relative to the domestic costs (production and investment) then capacity increases with the range of uncertainty, and vice versa. This condition is easily understandable in economic terms.

The rest of the paper is organized as follows. The model is described in section 2 and analyzed in section 3. Section 4 gives a numerical illustration based on the cement sector. The concluding section discusses limitations, extensions and further research.

2 The model

2.1 Assumptions

Let us consider the market for a homogeneous good. The price function is assumed to be linear and random : $p = a + \lambda\theta - bq$

- in which p is the price, q the quantity on the market, a and b two positive parameters;

- uncertainty is introduced through the random variable θ assumed to be uniformly distributed on the interval: $[-1, +1]$ with density $1/2$;
- the parameter λ measures the range of demand variations, the case of no uncertainty corresponds to $\lambda = 0$.

A firm is in a monopoly situation on the market (the model is further extended to oligopoly in section 2.3). It has access to two technologies: a home one and a foreign one. The cost function for the home technology consists of two terms:

- a linear investment cost c_k relative to a capacity choice denoted k ;
- a linear production cost c_h which includes the impact of the CO₂ regulation.

The cost function for the foreign technology involves a linear production cost c_f and no investment cost. The production cost should be interpreted as an average delivered cost to the home market from foreign plants that have excess capacity relative to their own home markets. Assuming that the home market is small relative to the foreign market explains that there is no capacity constraint. It is explicitly assumed that the monopoly firm has either direct or indirect control on imports to its home market. This latter assumption may be more or less realistic depending on the sector under analysis.

In case of no uncertainty the home technology would be preferred to the foreign one,

$$c_h + c_k < c_f$$

and the demand would be high enough to make production worthwhile,

$$a > c_h + c_k.$$

Furthermore, the range of demand variations is limited so that in all demand states, in the short term, it is worth producing with the home technology:

$$0 \leq \lambda \leq a - c_h.$$

The decision process takes place in three steps. First, the firm decides its home capacity k . Second, uncertainty unfolds, the realized value of θ is revealed to the firm. Third, the production decisions (q_h, q_f) using respectively the home and foreign technologies are made by the firm.

Denote $k^*(\lambda, c_h)$ the optimal monopoly capacity. The question under study concerns the dependence of k^* on λ as c_h varies.

2.2 The solution

The monopoly long term profit $\pi(k)$ for a given capacity choice k is given by:

$$\pi(k) = \frac{1}{2} \int_{-1}^{+1} \max_{(q_h < k, q_f)} [pq - c_h q_h - c_f q_f] d\theta - c_k k. \quad (1)$$

The integrand represents the firm's short-term profit once k has been

chosen. In each state θ , the firm selects q_h and q_f to maximize its short-term profit $pq - c_h q_h - c_f q_f$ with $q = q_h + q_f$ and subject to $q_h \leq k$. In the short-term, three situations can occur depending on the level of the demand. There are two thresholds θ^- and θ^+ in $[0, 1]$ with $\theta^- < \theta^+$ (see appendix A1) such that

- (i) if $\theta < \theta^-$, the firm has excess capacity, it produces the unconstrained monopoly quantity using c_h as its marginal cost: $q_h = (a - c_h + \lambda\theta)/2b$ and $q_f = 0$,
- (ii) if $\theta^- < \theta < \theta^+$, the capacity constraint is binding, $q_h = k$, and the foreign production is null, $q_f = 0$,
- (iii) if $\theta^+ < \theta$, the total production is the unconstrained monopoly quantity using c_f as its marginal cost: $q = (a + \lambda\theta - c_f)/2b$ with $q_h = k$ and $q_f = q - k$.

The occurrence of these situations depends on the amplitude of demand variation and the level of capacity chosen by the firm. Figure 1 illustrates a case in which all three situations occur. The marginal revenue of the firm is represented in three demand states, the two extreme ones ($\theta = -1, 1$) and the average one ($\theta = 0$). In the low demand state ($\theta = -1$), the capacity constraint is not binding, it is situation (i). In the average demand state ($\theta = 0$), the capacity constraint is binding but it is not worth importing for $a - 2bk < c_f$, it is situation (ii). In the high demand state ($\theta = 1$) the firm imports and its aggregate production (capacity and imports) is the unconstrained monopoly production with marginal cost c_f , it is situation (iii).

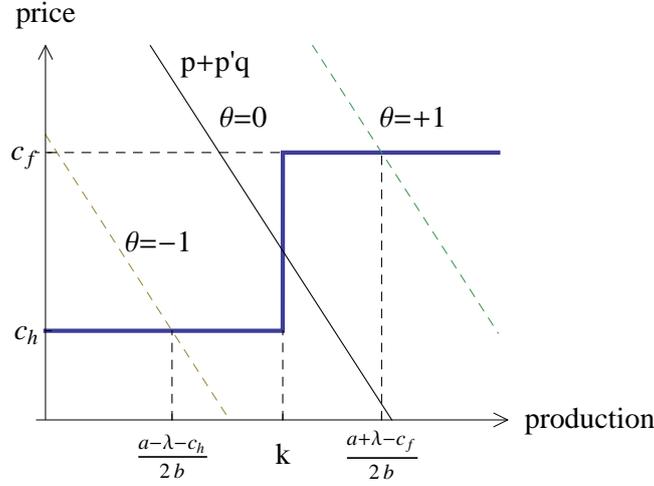


Figure 1: Marginal revenue ($a + \theta - 2bq$) and marginal cost as functions of total production, the short term production is the intercept of the two functions.

In the long term, the firm chooses k to maximize its profit (1). The first order condition is

$$\frac{1}{2} \left[\int_{\theta^-}^{\theta^+} (p + p'k - c_h) d\theta + \int_{\theta^+}^1 (c_f - c_h) d\theta \right] - c_k = 0. \quad (2)$$

The firm should equalize the marginal cost of a capacity with the expected short term marginal profit which is the shadow price of the capacity constraint (i.e. $q_h \leq k$). This flow of revenue is constituted of two integrals: the first one is the integral of the usual difference $p + p'k - c_h$ obtained when the capacity sets the price (situation (ii)), the second term is the integral of $c_f - c_h$ the short term cost reduction when the firm imports (situation (iii)).

Lemma 1 *The optimal capacity $k^*(\lambda, c_h)$ writes:*

- *Case 1: for $c_h \leq c_f - 2c_k$:*

- If $0 \leq \lambda \leq c_k$ then $k^* = k_1$;
- if $c_k \leq \lambda \leq (c_f - c_h)^2/4c_k$: $k^* = k_2$;
- if $(c_f - c_h)^2/(4c_k) \leq \lambda \leq a - c_h$, then $k^* = k_4$.

• *Case 2: for $c_h \geq c_f - 2c_k$*

- if $0 \leq \lambda \leq c_f - (c_h + c_k)$, then $k^* = k_1$;
- if $c_f - (c_h + c_k) \leq \lambda \leq (c_f - c_h)^2/4(c_f - c_h - c_k)$, then $k^* = k_3$;
- if $(c_f - c_h)^2/4(c_f - c_h - c_k) \leq \lambda \leq a - c_h$, then $k^* = k_4$.

where:

$$k_1 = [a - (c_h + c_k)] / 2b, \quad (3)$$

$$k_2 = [a - c_h + \lambda - 2(\lambda c_k)^{1/2}] / 2b, \quad (4)$$

$$k_3 = (a - c_f - \lambda) / 2b + [\lambda(c_f - c_k - c_h)]^{1/2} / b, \quad (5)$$

$$k_4 = [a - (c_h + c_f)/2 + \lambda(1 - 2c_k/(c_f - c_h))] / 2b. \quad (6)$$

The proof is in Appendix A. The result of the lemma is illustrated Figure 2. Each area corresponds to an expression of the equilibrium capacity. For case 1, as λ increases one goes through areas 1, 2 and 4. For case 2, as λ increases one goes through the areas 1, 3 and 4. In area 1, the firm produces at full capacity in all demand states and never imports. In area 2, the firm never imports but it has excess capacity in low demand states. In area 3, the firm always produces at full capacity and imports in high demand states. In area 4, both situations occur, the firm has excess capacity in low demand states and imports in high ones.

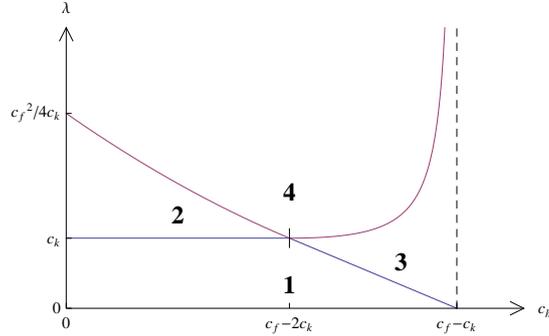


Figure 2: The typology of situations as a function of λ and c_h .

Let us now consider the effect of a change in the amplitude of demand variations on the optimal capacity.

Proposition 1 *The optimal monopoly capacity is increasing (resp. decreasing) with respect to λ if $c_h \leq c_f - 2c_k$ (resp. $c_h \geq c_f - 2c_k$).*

The proof is in Appendix B1. The optimal capacity is either increasing or decreasing with respect to uncertainty depending on the cost structure. The two situations corresponds to the two cases of Lemma 1. For small c_h it is increasing and for large c_h it is decreasing. This result could be interpreted in terms of options. In our model, there are two options: an option to import and an option to produce. The option to produce is related to the possibility to produce less than the capacity. Without this possibility the capacity would be decreasing with uncertainty, which corresponds to the standard result of the option-value literature (Dixit and Pindyck, 1994). Similarly, without the option to import, capacity would be increasing with respect to uncertainty. An additional unit of capacity creates an option to produce but deletes an option to import. An increase of uncertainty increases the value of these two options and the overall effect on investment depends on the comparison

of these changes. Given the linearity of our framework we get a clearcut distinction between the two cases based on the comparison of $c_f - (c_h + c_k)$ the opportunity cost to import and c_k the cost to invest more than necessary.

In addition to its theoretical content this result has practical consequences for environmental policy. As the CO₂ price increases the average production cost increases, say, from c_h to $c_h + \Delta c_h$. One may go from the first case to the second case. That is,

$$k^*(\lambda, c_h + \Delta c_h) < k^*(0, c_h + \Delta c_h) < k^*(0, c_h) < k^*(\lambda, c_h) \quad (7)$$

The central inequality captures the impact of the CO₂ price on capacity when there is no uncertainty. The first and third inequalities capture the impact of uncertainty on the choice of capacity. The introduction of uncertainty can amplify the impact of the regulation on investment. This relationship is formally stated in the following corollary.

Corollary 1 *When c_h increases the optimal capacity decreases. This effect is larger the larger the demand variability λ :*

$$\frac{\partial k^*}{\partial c_h} \leq 0; \quad \frac{\partial^2 k^*}{\partial c_h \partial \lambda} \leq 0. \quad (8)$$

The proof is in Appendix B2. The implication of this corollary is that a model that does not account for demand variability or uncertainty will underestimate the effect of the CO₂ regulation on investment, and the larger the variability is the larger the mistake will be.

2.3 Oligopoly

The previous analysis is carried on under a monopoly situation. The results can be extended to a Cournot oligopoly of n firms assuming that firms simultaneously select their capacity and production plan contingent on the state of demand. There is a unique equilibrium and this equilibrium is symmetric.¹

Proposition 2 *Under Cournot competition with n firms, there is a unique symmetric Nash equilibrium. The aggregate equilibrium quantity $k_n^*(\lambda, c_h)$ is:*

$$k_n^* = \frac{n}{n+1} 2k^*$$

and the condition given in proposition 1 for monotonicity of k^ applies.*

This proposition is proved in Appendix C1. Taking the limit in n , proposition 4 can be extended to the case of perfect competition.

Proposition 3 *With perfect competition, the aggregate equilibrium capacity $k_{+\infty}^*$ is:*

$$k_{+\infty}^* = 2k^*$$

and the condition given in proposition 1 for monotonicity of k^ applies.*

This proposition is proved in appendix C2.

¹Preemption and entry issues are left for further research to be carried in a Markov model. These issues are important for growing markets which is certainly not the case for the EU.

2.4 Leakage and pass through

Let us briefly discuss the effect of the implementation of a carbon policy on two common indicators: the leakage rate and the pass through rate. We consider the value of these indicators in the short and long term. In this section we describe formally these indicators and establish some analytical results on their properties. These results are further discussed and detailed in the numerical application provided in Section 3.3.

We consider that in the short term the capacity is fixed at $k_n^*(\lambda, c_h)$ and that in the long term this capacity varies according to the carbon policy to become $k_n^*(\lambda, c_h + \Delta c_h)$.

To precisely describe the various indicators we need to introduce some notations. Let us denote $q_h(k, c_h, n, \theta)$ and $q_f(k, c_h, n, \theta)$ the short term aggregate production of an oligopoly of n firms in which each firm has a capacity k/n . The expressions of these quantities are given by (15) in Appendix C1.

The leakage is the increase of foreign emissions that follows the implementation of a carbon policy. The leakage rate is the ratio between the rise of emissions abroad and the decrease of home emissions. If foreign production and home production have the same emission rate, the short term leakage rate is:

$$L_{ST} = \frac{\mathbb{E} [q_f(k_n^*(c_h), c_h + \Delta c_h, n, \theta) - q_f(k_n^*(c_h)_n, c_h, n, \theta)]}{\mathbb{E} [q_h(k_n^*(c_h)_n, c_h, n, \theta) - q_h(k_n^*(c_h), c_h + \Delta c_h, n, \theta)]}. \quad (9)$$

And the long term leakage rate is:

$$L_{LT} = \frac{\mathbb{E} [q_f(k_n^*(c_h + \Delta c_h), c_h + \Delta c_h, n, \theta) - q_f(k_n^*(c_h)_n, c_h, n, \theta)]}{\mathbb{E} [q_h(k_n^*(c_h)_n, c_h, n, \theta) - q_h(k_n^*(c_h + \Delta c_h), c_h + \Delta c_h, n, \theta)]}. \quad (10)$$

Corollary 2

- *The short term leakage rate is null.*
- *The long term leakage rate is*
 - (i) independent of n ;*
 - (ii) strictly positive if and only if Δc_h is larger than a threshold that is decreasing with respect to λ ;*
 - (iii) it is increasing with respect to λ , for small Δc_h .*

The proof is in Appendix D1. We also defined *observed* short term and long term leakage rates. The observed rates are the rates in a particular state θ . The formal definition of these indicators is similar to (9) and, (10) without the expectation operator. Note that the short term and long term rates L_{ST} and L_{LT} are the ratio of the expected changes of productions and not the expectation of the observed rates, which, in our sense, does not have a sensible economic interpretation. We do not formally analyzed these indicators but they are computed for the numerical simulations below.

Concerning the output price, the impact of the carbon price on the output price is measured by the pass through rate. This is the ratio between the output price change and the cost change. The short term pass through rate is:

$$PT_{ST} = \frac{\mathbb{E} [p(q(k_n^*(c_h), c_h + \Delta c_h, n, \theta), \theta) - p(q(k_n^*(c_h), c_h, n, \theta), \theta)]}{\Delta c_h} \quad (11)$$

and the long term one:

$$PT_{LT} = \frac{\mathbb{E} [p(q(k_n^*(c_h + \Delta c_h), c_h + \Delta c_h, n, \theta), \theta) - p(q(k_n^*(c_h), c_h, n, \theta), \theta))]}{\Delta c_h}. \quad (12)$$

Without uncertainty, the model corresponds to a standard Cournot oligopoly with linear demand and constant marginal cost, there is no imports and production equals capacity. In this case—to be referred as the *standard Cournot model*—the short term pass through rate is null and the long term one is $n/(n + 1)$.

Corollary 3

- *The short term pass through rate is*
 - (i) *strictly positive if and only if $c_h < c_f - 2\lambda[1 + (1 - c_k/\lambda)^{1/2}]$ and $\lambda > c_k$;*
 - (ii) *smaller than $n/(n + 1)$;*
 - (iii) *for small Δc_h it is increasing with respect λ .*
- *The long term pass through rate is:*

$$\frac{n}{n + 1}.$$

The proof is in Appendix D2.

As for the leakage rate, we also define the *observed* pass through rate for each value of theta. This indicator could also be computed in the short term (with unchanged capacity) and in the long term (with adjusted capacity).

The expressions of these rates would be similar to (11) and (12) without the expectation operator.² All these indicators are discussed in the numerical illustration below.

3 An illustration based on the cement sector

3.1 Calibration of the model

The data corresponding to the numerical illustration is given in Table 1. This data is illustrative of the EU cement industry and originates from Ponsard and Walker (2008). The demand function is calibrated such that at the Cournot equilibrium with 6 firms without uncertainty, each firm produces 1 Mt, the market price is 100€/t and the price elasticity at the equilibrium is .27. The demand fluctuations captured through the parameter λ/a corresponds to the average demand fluctuations of a European country over the last 20 years. The variable costs in € per ton of cement are: for investment $c_k = 15$ (annualized over a 40 years life duration for a plant), for home production $c_h = 75$, for import $c_f = 25$ (involving sea transport, terminal cost and further inland transportation by road; it is suggestive of an inland region; for a coastal region it would be more around 35€/t and, everything being equal, such a plant should close immediately for a 40€/tCO₂ price), and for the increase in variable cost $\Delta c_h = 25$ (which is consistent with a CO₂ price of 40€/tCO₂ and an emission rate of .65 tCO₂ per ton of cement).

The home technology is preferred to imports since $c_f = 75 > c_k + c_h = 40$.

²Contrary to the leakage rate the short term (resp. long term) pass through rate is the expectation of the observed short term (resp. long term) pass through rate.

Further more, the optimal capacity is increasing with uncertainty when the price of CO₂ is zero and it is decreasing when it is 40€/t CO₂ since

$$2c_k + c_h = 55 < c_f = 75 < c_k + c_h + \Delta c_h = 80.$$

Cost parameters		Demand Parameters	
c_k	15	a	470
c_h	25	b	61.7
c_f	75	λ/a	15%
Δc_h	25		

Table 1: Parameters for the calibration of the model

3.2 The impact on the capacity decisions

As the CO₂ price increases from zero up to 40€/t the optimal capacity drops by 12%. This may be compared to a 6% drop in case of no uncertainty (see Figure 3).

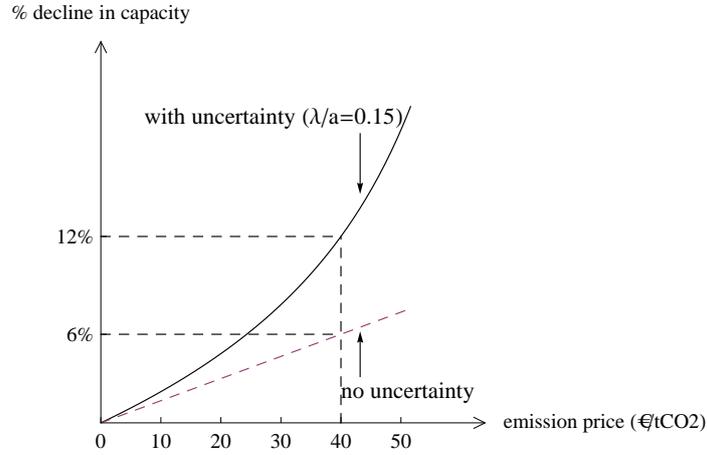


Figure 3: percent decline in capacity with respect to the CO₂ price, with and without uncertainty

This can be understood by considering how the optimal capacity depends on one side on the CO₂ price for a given value of uncertainty at 15% (Figure 4a) and on the other side on uncertainty for a given value of the CO₂ price taken at 40€/t (Figure 4b). Figure 4(a) shows that there is a critical value of the CO₂ price such that the optimal capacity is larger (resp. lower) with uncertainty than without uncertainty if the CO₂ price is lower (resp. larger) than this critical value (Proposition 1). With 0 and 40€/t we are on both sides so all inequalities (7) are satisfied. Figure 4(b) illustrates the result of Corollary 1, it shows that the effect of the CO₂ price on capacity is larger the higher level of uncertainty.

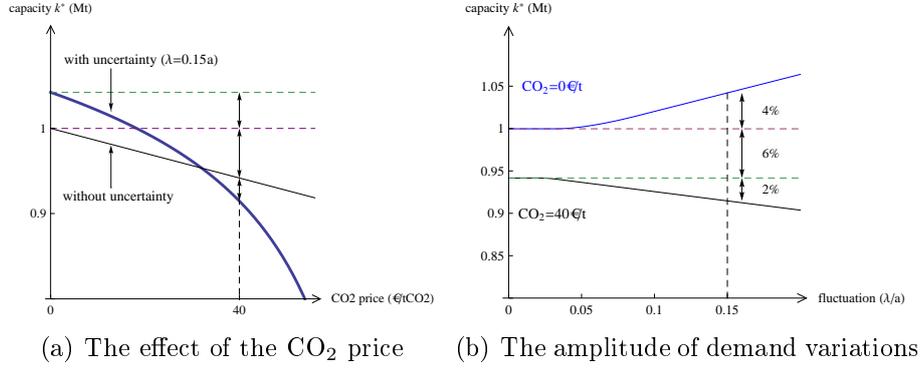


Figure 4: Individual capacity as a function of the CO_2 price (a) and demand variation (b).

3.3 Leakage and pass through rates

Consider first the leakage rate (Corollary 2). In the short term case, imports are determined by the capacity constraint, there is no leakage. In the long term, capacity is reduced taking into consideration uncertainty and the CO_2 price. Ex post demand is observed; the firm decides to import or not given the new capacity. The situation is depicted in Figure 5(a). Suppose the observed demand corresponds to $\theta = .2$. The consumption with the zero CO_2 price corresponds to point A; with the 40€/t CO_2 price it is at B; the level of imports corresponds to the segment BC. The observed leakage rate corresponds to the ratio $BC/AC = 28\%$.

Figure 5(b) gives the evolution of the observed long term leakage rate as a function of θ . For completeness the long term leakage rate (defined by 10) is also depicted.

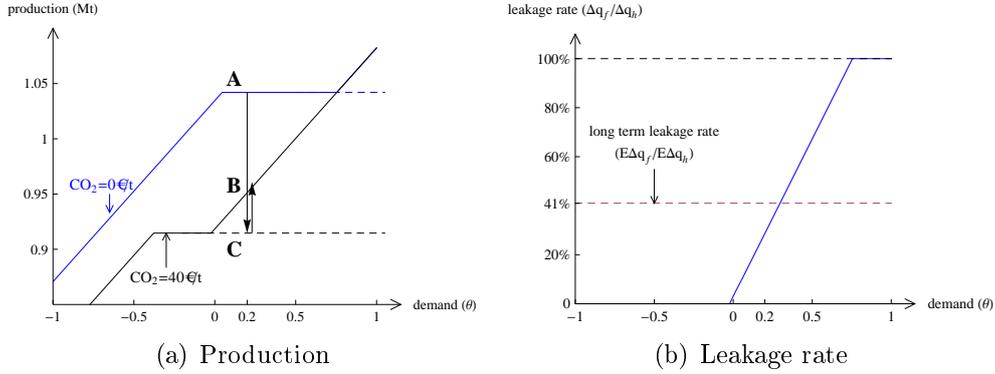


Figure 5: Production and the observed long term leakage rates for each θ .

A sensitivity analysis of the long term leakage rate with respect to the CO₂ price and the level of uncertainty is made. The results are given in Table 2. It is increasing in both dimensions. Recall that the short term leakage rate is zero. Recall also that, without uncertainty, by construction there would not be any leakage in our model.

		CO ₂ price			
		20	30	40	50
Demand Uncertainty λ/a	10%	13%	20%	29%	42%
	15%	24%	31%	41%	54%
	20%	32%	39%	49%	62%
	25%	38%	46%	55%	67%
	30%	43%	51%	60%	71%

Table 2: long term leakage rate (10)–the short term one is null.

Consider now the cement price and the observed pass through rates, both short term and long term. They are depicted in Figure 6. Three zones emerge depending on the value of θ . Firstly, for $\theta < \theta^-(40)$, capacity is not

a constraint. The effect of the CO₂ price on the short and long term rates is similar, the pass through in these states is $n/(n+1)$ as in a standard Cournot model. Secondly, for $\theta > \theta^+(0)$, the price is set by the import cost and the pass through is null. In both cases there is no difference between short term and long term.

What happens in the median zone can be inferred from the graphs in Figure (6).

The prices are depicted in Figure(6)a. It corresponds to

- **abcd** for CO₂ = 0€/t,
- **ehcd** (CO₂ = 40€/t) in the short term,
- **efgcd** (CO₂ = 40€/t) in the long term.

The pass through rates are depicted in Figure 6(b). The short term pass through rate is 52% and the long term one is 86%=6/7. In Figure 6(b) the observed pass through rates in each demand state are also depicted. The observed short term pass through rate remains at 6/7 until $\theta^-(0)$ and then progressively decreases to zero at $\theta = 0.4$ (the value of θ at which capacity is constrained) and then remains at zero.

The observed long term pass through rate increases from 6/7 for $\theta^-(40) < \theta < \theta^+(40)$. It remains constant for $\theta^+(40) < \theta < \theta^-(0)$. It decreases to zero for $\theta^-(0) < \theta < \theta^+(0)$. The peak can be computed to be at 171%, which is well above 100%!

The introduction of uncertainty has a major impact on the pass through rates. The observed pass through rates, both the short term and the long

term rates, may be either higher or lower than the one in a standard Cournot model. In particular it will be lower in high demand states such as the period corresponding to the first EU-ETS (2005-2008). The short term pass through rate is %52 which is much lower than the rate in a standard Cournot model i.e. $6/7=86\%$. Recall that, due to the linearity of our model, the long term pass through rate is equal to the rate in a standard Cournot model (cf Corollary 3).

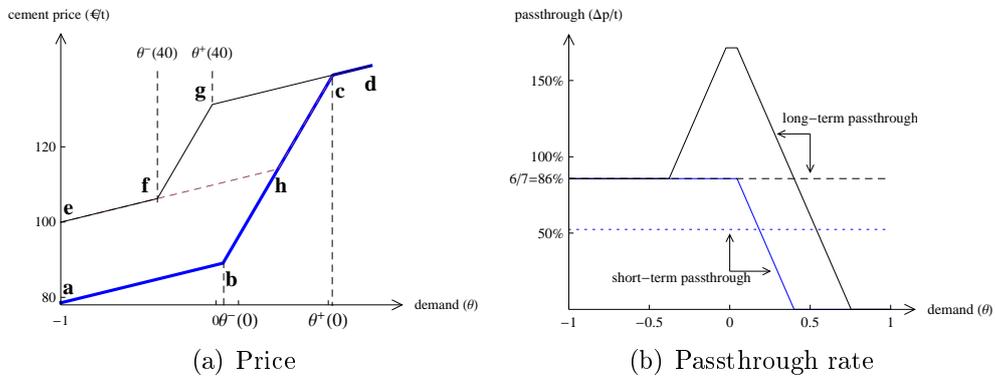


Figure 6: Changes in output prices and the passthrough rates in each demand state (θ).

A sensitivity analysis of the short term pass through rate with respect to the CO_2 price and the level of uncertainty is made. The results are given in Table 3. It is slightly increasing in both dimensions, from 49% to 57%. Recall that the standard Cournot rate is $6/7 = 86\%$, which is much higher.

		CO ₂ price			
		20	30	40	50
Demand	10%	43%	46%	49%	51%
Uncertainty	15%	49%	51%	52%	54%
λ/a	20%	51%	53%	54%	56%
	25%	53%	54%	55%	57%
	30%	54%	55%	56%	57%

Table 3: Short term pass through rate (11).

Table 4 summarizes the relative positioning of the three leakage rates (respectively pass through rates): standard Cournot model, short term and long term. For instance read Table 4 as follows: the short term pass through rate (in column) is smaller than the standard Cournot model rate (in line). It is clear that when equality stands this comes from the linearity of the model. It would be interesting to extend these relations to more general assumptions.

Leakage	Standard Cournot	Short term	Pass through	Standard Cournot	Short term
Short term	=		Short term	<	
Long term	>	>	Long term	=	>

Table 4: Summary of qualitative results.

4 Conclusion

We have developed a simple model to analyze the role of demand fluctuations on sectoral leakage. Competition is assumed to be dominated by multinationals which optimize their current production through their international network of plants. To meet the regional demand in country A home plants are used in priority to imports, unless the capacity of these plants is saturated. As a consequence imports only occur in high demand states in country A. In the long term capacities are adjusted to a unilateral change in the CO₂ price in country A. The leakage and pass through rates are derived both in the short term (without capacity adjustments) and in the long term (with capacities adjustments).

We show that the leakage rates and the pass through rates are much more significant in the long term than in the short term. Furthermore, we emphasize the role played by uncertainty and capacity constraint. We show in particular that with uncertainty there is leakage in the long term while there would be none without uncertainty and that the short term pass through rate is much lower with uncertainty than the one without uncertainty.

These qualitative results are in line with the observations reported in (Ellerman et al., 2010). They are also relevant for the evaluation of the free allocation mechanism to be introduced in the EU-ETS post 2013 for sensitive sectors. This mechanism is based on capacities, and not on production (European Commission, 2011). Our model could be used to explore the economic properties of such mechanisms to mitigate leakage. In this perspective it would be interesting to extent our results to more general assumptions,

such as the introduction of firms with no plants in country A, and explicitly introduce assumptions about the rest of the world (correlation in regional business cycles and capacity decisions). One should also allow for more general demand and cost functions, and possibly dynamics based on a Markov framework rather than on a two stage one.

Exploring whether the monotonicity results obtained in our model could be generalized would also be an interesting contribution to the literature on the comparative statics properties for decisions under uncertainty.

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Appendix

A Proof of Lemma 1

A.1 Expression of the thresholds

The two threshold states are related to the capacity of the firm and the amplitude of demand variation. If demand varies sufficiently, at the low (resp. high) threshold state θ^- (resp. θ^+) the unconstrained monopoly production with marginal cost c_h (resp. c_f) is precisely equal to the capacity. For small λ the thresholds are -1 or 1 respectively. This gives:

$$\theta^- = \max \{(2bk - a + c_h) / \lambda, -1\} \quad (13)$$

$$\theta^+ = \min \{(2bk - a + c_f) / \lambda, 1\} \quad (14)$$

A.2 Proof

The monopoly long term profit is a strictly concave function of $k \in [0, (a + \lambda - c_h) / 2b]$. There is a unique profit maximizing capacity k^* that solves the first order condition (2). Four situations can arise whether at k^* : $\theta^- = -1$ or not and $\theta^+ = 1$ or not.

We determine the solution of (2) for each expressions of thresholds and corresponding inequalities satisfied by λ . The lemma follows.

1. For $\theta^- = -1$ and $\theta^+ = 1$: the solution of (2) is:

$$k_1 = (a - (c_h + c_k))/2b, \text{ and}$$

$$(2bk_1 - a + c_h)/\lambda \leq -1 \Leftrightarrow \lambda \leq c_k,$$

$$(2bk_1 - a + c_f)/\lambda \geq 1 \Leftrightarrow \lambda \leq c_f - c_h - c_k.$$

2. For $\theta^- = (2bk - a + c_h)/\lambda$, and $\theta^+ = 1$: injecting expressions of θ^-, θ^+ into (2) gives the equation: $2c_k = \lambda \int_{\theta^-}^1 (\theta - \theta^-) d\theta$ i.e. $4c_k = \lambda (1 - \theta^-)^2$ hence, $\lambda\theta^- = \lambda - 2(\lambda c_k)^{1/2}$ so:

$$k_2 = \left[a - c_h + \lambda - 2(\lambda c_k)^{1/2} \right] / 2b, \text{ and}$$

$$(2bk_2 - a + c_h)/\lambda > -1 \Leftrightarrow \lambda > c_k,$$

$$(2bk_2 - a + c_f)/\lambda \geq 1 \Leftrightarrow \lambda \leq (c_f - c_h)^2 / 4c_k.$$

3. For $\theta^- = -1$ and $\theta^+ = 2bk - a + c_f$, equation (2) is: $2c_k = \int_{-1}^{\theta^+} (a - c_h + \lambda\theta - 2bk) d\theta + (1 - \theta^+) (c_f - c_h)$. Injecting $a - 2bk = c_f - \lambda\theta^+$ gives $2c_k = 2(c_f - c_h) - \lambda(1 + \theta^+)^2 / 2$ and the solution is:

$$k_3 = \left[a - c_f - \lambda + 2[\lambda(c_f - (c_k + c_h))]^{1/2} \right] / 2b,$$

$$(2bk_3 - a + c_h)/\lambda \leq -1 \Leftrightarrow \lambda \geq (c_f - c_h)^2 / 4(c_f - c_h - c_k),$$

$$(2bk_3 - a + c_f)/\lambda < 1 \Leftrightarrow \lambda > c_f - c_h - c_k.$$

4. For $\theta^- = (2bk - a + c_h)/\lambda$, and $\theta^+ = (2bk - a + c_f)/\lambda$ equation (2) is: $2c_k = \lambda(\theta^+ - \theta^-)^2 / 2 + (1 - \theta^+) (c_f - c_h)$ and $\theta^+ - \theta^- = (c_f - c_h)/\lambda$ so

$\theta^+ = 1 + (c_f - c_h) / 2\lambda - 2c_k / (c_f - c_h)$ and replacing θ^+ by its expression gives:

$$k_4 = [a - (c_f + c_h) / 2 + \lambda(1 - 2c_k / c_f - c_h)] / 2b, \text{ and}$$

$$(2bk_4 - a + c_h) / \lambda > -1 \Leftrightarrow \lambda > (c_f - c_h)^2 / 4(c_f - c_h - c_k),$$

$$(2bk_4 - a + c_f) / \lambda < 1 \Leftrightarrow \lambda > (c_f - c_h)^2 / 4c_k.$$

And finally, as λ increases from 0 to $a - c_h$: if $c_f \geq 2c_k + c_h$ (resp. $c_f \leq 2c_k + c_h$) the optimal capacity is successively k_1, k_2 , (resp. k_3, k_4).

B Proof of Proposition 1 and Corollary 1.

B.1 Proof of Proposition 1

We use the expression established in Lemma 1.

- $c_h \leq 2c_k - c_f$:
 - For $\lambda \leq c_k$ the parameter λ has no effect on k^* .
 - For $c_k \leq \lambda \leq (c_f - c_h)^2 / 4c_k$ the monopoly capacity is $k^* = k_2$ and derivation gives $[1 - (c_k / \lambda)^{1/2}] / 2b$ which is positive for $c_k \leq \lambda$.
 - For $(c_f - c_h)^2 / 4c_k \leq \lambda$, the derivative of monopoly capacity with respect to λ is $[1 - 2c_k / (c_f - c_h)] / 2b$ positive in that case.
- For $c_h \geq 2c_k - c_f$:
 - For $h \leq c_f - (c_h + c_k)$ the parameter λ has no effect on k^*

- For $c_f - (c_h + c_k) \leq \lambda \leq (c_O - c_I)^2 / 4(c_f - c_h - c_k)$, the derivative of the monopoly capacity is $\left[[(c_f - c_k - c_h) / \lambda]^{1/2} - 1 \right] / 2b$ which is negative for $c_f - (c_h + c_k) \leq \lambda$.
- For $(c_f - c_h)^2 / 4(c_f - c_h - c_k) \leq \lambda$, the derivative of monopoly capacity with respect to λ is $[1 - 2c_k / (c_f - c_h)] / 2b$

B.2 Proof of Corollary 1

We use the expressions of k_1 , k_2 , k_3 and k_4 established in Lemma 1.

- With the expressions (3) of k_1 : $\partial k_1 / \partial c_h = -1/2b$ and $\partial^2 k_1 / \partial \lambda \partial c_h = 0$.
- From the expression (4), $\partial k_2 / \partial c_h = -1/2b$ and $\partial^2 k_2 / \partial \lambda \partial c_h = 0$.
- From the expression (5),

$$\frac{\partial k_3}{\partial c_h} = \frac{-\lambda}{b(c_f - c_h - c_k)^{1/2}} < 0; \quad \frac{\partial^2 k_3}{\partial \lambda \partial c_h} = \frac{-1}{b(c_f - c_h - c_k)^{1/2}} < 0.$$

- And from (6),

$$\frac{\partial k_4}{\partial c_h} = \frac{-1}{b} \left[\frac{1}{4} + \frac{\lambda c_k}{(c_f - c_h)^2} \right] < 0; \quad \frac{\partial^2 k_4}{\partial \lambda \partial c_h} = \frac{-c_k}{b(c_f - c_h)^2} < 0.$$

C Proof of Proposition 2 & 3

C.1 Proof of proposition 2

In order to limit the introduction of notations only a brief sketch of the proof is provided here, a more detailed one can be obtained by request to the

authors.

Let assume that there are n firms with $n \in \mathbb{N}^*$. Each Firm simultaneously chooses its capacity and a production plan. At an equilibrium: on the short term, in each demand state firms play a constraint Cournot game with two technologies available, and, in the long term, each firm capacity is a solution of a first order equation that equalizes the capacity cost c_k with expected short term marginal profit. Any equilibrium is symmetric because the expected marginal short term profit of two firms is equal if and only if their capacity are equal. Then the only possible equilibrium is symmetric and the aggregate equilibrium capacity k_n^* is the unique solution of equation:

$$\int_{\theta^-(n,k)}^{\theta^+(n,k)} \left(a - c_h + \lambda\theta - \frac{n+1}{n}bk \right) d\theta + \int_{\theta^+(n,k)}^1 (c_f - c_h) d\theta - 2c_k = 0 \quad (15)$$

where $\theta^-(n, k)$ and $\theta^+(n, k)$ are :

$$\begin{aligned} \theta^- &= \max \left\{ \left((n+1)bk/n - a + c_h \right) / \lambda, -1 \right\}, \\ \theta^+ &= \min \left\{ \left((n+1)bk/n - a + c_f \right) / \lambda, +1 \right\}, \end{aligned}$$

and aggregate equilibrium productions $q_h(k, c_h, n, \theta)$ and $q_f(k, c_h, n, \theta)$ are constrained Cournot one:

$$\begin{aligned} 0 \leq \theta \leq \theta^- & : q_h = n(a + \lambda\theta - c_h)/(n+1) \text{ and } q_f = 0 \\ \theta^- \leq \theta \leq \theta^+ & : q_h = k \text{ and } q_f = 0 \\ \theta^+ \leq \theta \leq 1 & : q_h = k \text{ and } q_f = n(a + \lambda\theta - c_f)/(n+1) - k \end{aligned} \quad (16)$$

By injecting expressions of θ^- , θ^+ into the first order condition (15) it appears that they are solution of an equation independent of n . So equilibrium values of threshold states are independent of n , and:

$$k_n^* = \frac{n}{n+1} 2k^*$$

And finally, the solution of equation (15) and the corresponding productions (16) are equilibrium strategies because individual profit of each firm is concave and first order conditions are satisfied.

Proof of proposition 3

With perfect competition calculations are similar to those with a monopoly. In the short term, for a fixed k , the price is equal to the marginal cost: it is c_h if θ is small and there is excessive capacity, it is c_f if θ is sufficiently high and imports occur, and for intermediary values the production is k and the price is between c_h and c_f . Thresholds states θ^- and θ^+ are:

$$\theta^- = \max \{(bk - a + c_h) / \lambda, -1\} \text{ and } \theta^+ = \min \{(bk - a + c_f) / \lambda, 1\} \quad (17)$$

And the aggregate profit of firms is:

$$\pi = \frac{1}{2} \int_{\theta^-}^{\theta^+} (p - c_h) k d\theta + \frac{1}{2} \int_{\theta^+}^1 (c_f - c_h) k d\theta - c_k k.$$

In the long run this profit is null and

$$\int_{\theta^-}^{\theta^+} (p - c_h) d\theta + \int_{\theta^+}^1 (c_f - c_h) d\theta = 2c_k.$$

Comparing this equation with the first order condition (2) characterizing the monopoly's choice, and the thresholds with perfect competition (17) with those with a monopoly (13) and (14), it appears that equations with perfect competition are similar to those of a monopoly by replacing $2b$ in the latter by b . Consequently the capacity with perfect competition is twice the monopoly's.

D Leakage and passthrough

We will use an expression of $\partial k_n^*/\partial c_h$ as a function of the thresholds θ^- and θ^+ , for proofs of corollary 2 and 3. From the first-order condition (15), $(\theta^+ - \theta^-) b(n + 1)/n \partial k_n^*/\partial c_h = -(1 - \theta^-)$ so

$$\frac{\partial k_n^*}{\partial c_h} = -\frac{1}{b} \frac{n}{n+1} \frac{1 - \theta^-}{\theta^+ - \theta^-} = 0 \quad (18)$$

D.1 proof of corollary 1

The expected quantity produced by a monopoly domestically is

$$Q_h(k, c_h, \lambda) = 0.5 \int_{-1}^{\theta^-} (a + \lambda\theta - c_h)/2bd\theta + 0.5 \int_{\theta^-}^1 kd\theta \quad (19)$$

and the quantity imported:

$$Q_f(k, c_h, \lambda) = 0.5 \int_{\theta^+}^1 [(a + \lambda\theta - c_f)/2b - k] d\theta. \quad (20)$$

The short-term leakage rate is null because Q_f does not depend on c_h , so the numerator of (9) is null.

The long term leakage is independent of n because both the numerator and the denominator are proportional to $n/(n+1)$ (remember that θ^- and θ^+ are independent of n at equilibrium).

The long-term leakage rate is strictly positive if and only if $Q_f(k^*(c_h + \Delta c_h, \lambda), c_h + \Delta c_h, \lambda)$ is strictly positive, i.e. $\theta^+ < 1$. From the proof of lemma 1 (appendix A2, and figure 2 for intuition), it is so if and only if either $c_h + \Delta c_h \leq c_f - 2c_k$ and $\lambda > (c_f - c_k)^2/4(c_f - c_h - c_k)$, or, $c_h + \Delta c_h \geq c_f - 3c_k$ and $\lambda > c_f - c_h - c_k$. And it is equivalent to $c_h + \Delta c_h \geq c_f - c_k - \lambda$ if $\lambda \leq c_k$ and $c_h + \Delta c_h \geq c_f - 2(\lambda c_k)^{1/2}$ otherwise. And both right-hand sides of inequalities are decreasing with respect to λ .

Let us consider a small Δc_h , the leakage rate is $(\partial Q_f/\partial c_h)/(-\partial Q_h/\partial c_h)$. From equation (19) and (20), the two derivatives are:

$$\partial Q_f/\partial c_h = 0.5(1 - \theta^+)(-\partial k^*/\partial c_h) \quad (21)$$

$$-\partial Q_h/\partial c_h = 0.5[(1 + \theta^-)/2b + (1 - \theta^-)(-\partial k^*/\partial c_h)] \quad (22)$$

the inverse of the leakage rate is—injecting (18) with $n = 1$:

$$L_{LT}^{-1} = 1 + \frac{\theta^+ - \theta^-}{1 - \theta^+} + \frac{1 + \theta^-}{1 - \theta^+} \frac{-1/2b}{\partial k^*/\partial c_h} = 1 + \frac{-1/b}{\partial k^*/\partial c_h} \frac{1}{1 - \theta^+}$$

Then, θ^+ is decreasing with respect to λ (cf the proof of lemma 1, points 3. and 4.), and $-\partial k^*/\partial c_h$ is positive and increasing with respect to λ (cf corollary 1). Therefore, the expression above is decreasing and the leakage rate is increasing with respect to λ .

D.2 proof of corollary 2

We first proof the result relative to the long-term pass-through rate. We consider the derivative of the expected price denoted $\mathbb{E}p$ with respect to c_h . The long term derivative is composed of two components a direct one and an indirect one:

$$\frac{d\mathbb{E}p}{dc_h} = \frac{\partial \mathbb{E}p}{\partial c_h} + \frac{\partial \mathbb{E}p}{\partial k} \frac{\partial k^*}{\partial c_h}$$

The first term is the short-term pass through (with a small Δc_h): $\partial \mathbb{E}p/\partial c_h = 0.5(\theta^- + 1)n/(n+1)$ (with the expression (15) for home production). The second term is related to the change of capacity. A marginal change of capacity increases expected price of $\partial \mathbb{E}p/\partial k = (\theta^+ - \theta^-)b/2$, and using (18):

$$\frac{d\mathbb{E}p}{dc_h} = 0.5 \left[(\theta^- + 1) \frac{n}{n+1} + (\theta^+ - \theta^-) b \frac{(1 - \theta^-)}{(\theta^+ - \theta^-)} \frac{n}{b(n+1)} \right] = \frac{n}{n+1}$$

Concerning, the short-term pass-through rate.

- It is smaller than the long-term one, so less than $n/(n+1)$;
- It is positive if and only if $\theta^- > -1$ at c_h . This is true if (cf Lemma 1 or figure 2 for intuition) either $c_h \leq c_f - 2c_k$ and $\lambda > c_k$, or, $c_h \geq c_f - 2c_k$ and $\lambda > (c_f - c_h)^2/4(c_f - c_h - c_k)$. The last inequality is never satisfied if $\lambda \leq c_k$ and is equivalent to $c_f - c_h > 2\lambda(1 + (1 - c_k/\lambda)^{1/2})$ if $\lambda > c_k$.

So, $\theta^- > -1$ if and only if $\lambda > c_k$ and $c_h < c_f - 2\lambda[1 + (1 - c_k/\lambda)^{1/2}]$.

- For small Δc_h the short-term pass-through rate is $\partial \mathbb{E}p / \partial c_h$ i.e. $0.5(1 + \theta^-)n/(n+1)$, and θ^- is increasing with respect to λ (cf proof of lemma 1, points 2. and 4.).