Name of the Host Laboratory | Laboratoire d'informatique de l'École polytechnique (LIX, UMR 7161 CNRS)
Website of the Host Laboratory | https://www.lix.polytechnique.fr
Research Group | Algebraic modeling and symbolic computation (MAX)
Internship Supervisor | Grégoire Lecerf
Internship Subject | Reliable numerical integration
Student’s level | ☒ Advanced Undergraduate Students (3rd or 4th year)
               | ☒ Master’s students (1st or 2nd year)
               | ☒ PhD students
Proposed Duration | ☒ 3 months
               | ☒ 4 months
               | ☒ 5 months
               | ☒ 6 months
Prerequisites | Background both in mathematics and computer science: complex analysis, differential equations, algorithms. Skills in computer science will be needed to achieve efficient implementations in C++.
Internship description (max. 15 lines) | Consider a system of differential equations of the form $f'(z) = \Phi(f(z))$, where $f : \mathbb{R} \to \mathbb{R}^n$, $\Phi : \mathbb{R}^n \to \mathbb{R}^n$, and an initial condition is provided at $z_0$ in $\mathbb{R}$. The numeric integration of such a system is a widely studied problem in numeric analysis. The function $\Phi$ can usually be an arbitrary smooth function and the integration is typically done with machine precision. Runge–Kutta schemes are the most common workhorse in this situation. Systems and libraries like Matlab, DifferentialEquations.jl, or Sundials offer extensive suites of solvers adapted to various kinds of problems and accuracy requirements. This internship concerns the certification problem: given $\varepsilon > 0$ and $z_1$ in $\mathbb{R}$ such that the above differential equation can be integrated on $[z_0,z_1]$, how to compute an approximation $g(z)$ of $f(z)$ with $|g(z_1)-f(z_1)| \leq \varepsilon$. There exist several software packages for this task, most of them are restricted to usual double or quadruple precisions. Having appropriate certified counterparts is a major open problem in the area of reliable computing, especially for modern safety-critical applications.
We will restrict ourselves to the frequent case where $\Phi$ is a polynomial or a rational function. The goal
will be to design reliable integration algorithms, to implement them in C++ within the Mathemagix libraries (http://www.mathemagix.org) and to compare them with existing software.

The boxes marked with cross implies eligible