
ECOLE POLYTECHNIQUE
CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

**The Regulator and the Judge: The Optimal Mix in The
Control of Environmental Risk**

Yolande Hiriart
David Martimort
Jerome Pouyet

December 2005

Cahier n° 2005-038

LABORATOIRE D'ECONOMETRIE

1rue Descartes F-75005 Paris

(33) 1 55558215

<http://ceco.polytechnique.fr/>

<mailto:labecox@poly.polytechnique.fr>

The Regulator and the Judge: The Optimal Mix in The Control of Environmental Risk*

Yolande Hiriart¹
David Martimort²
Jerome Pouyet³

Decembre 2005

Cahier n° 2005-038

Résumé: Nous considérons une firme qui génère un risque pour l'environnement via son activité industrielle et qui a une information privée à la fois sur son effort de précaution et sur le montant de ses actifs. Nous étudions l'interaction entre l'audit ex ante de l'effort de précaution par un régulateur et la vérification ex post de la capacité financière par un juge en cas d'accident. Du point de vue des incitations, les deux instruments sont utiles. Le policy-mix optimal dépend de la règle gouvernant l'intervention ex post et de l'efficacité de l'intervention ex ante.

Abstract: A firm engaged in activities which are environmentally risky has private information both on its choice of safety care and on the level of its assets. We study the interaction between the ex ante audit of the firm's precautionary effort by a regulator and the ex post check on the true level of the firm's assets by a judge following an accident. Both instruments are useful from an incentive viewpoint. The optimal policy mix between the regulator and the judge depends on the rule determining the judge's intervention and on the quality of the regulatory enforcement. We discuss the incentives of the judge and the regulator to broaden their mandates and undertake the task of the other branch. Finally, we interpret different scenarios for the regulator's and the judge's interventions in terms of legal principles.

Mots clés : Régulation, Responsabilité, Risque environnemental, Information asymétrique

Key Words : Regulation, Liability, Environmental Risk, Asymmetric Information

Classification JEL: K13, K32, L51

* This paper is part of a research program between IDEI and the French Ministry of Ecology and Sustainable Development on the regulation of risky industrial activity under asymmetric information. Financial and intellectual supports from the Ministry are gratefully acknowledged. We thank Bernard Salanie as well as participants in Bremen EAERE Conference, the Second Toulouse-Montréal Conference on the Law, Economics and Management of Large Scale Risks and the "Economie de l'Environnement et des Ressources Naturelles" Seminar, Maison des Sciences Economiques, Paris, for useful comments. All errors are ours.

¹ University of Toulouse (IDEI, LERNA). Address: IDEI-LERNA, Manufacture des Tabacs, Bat. F, 21 Allée de Brienne, 31000 Toulouse., Phone: +33(0)561128632. Fax: +33(0)561128637. E-mail: yhiriart@cict.fr

² University of Toulouse (IDEI, GREMAQ) & Institut Universitaire de France. Address: IDEI, Manufacture des Tabacs, Bat. F, 21 Allée de Brienne, 31000 Toulouse, France. Phone: +33(0)561128614. Fax: +33(0)561128637. E-mail: martimor@cict.fr

³ Ecole Polytechnique & University of Toulouse (IDEI)

The Regulator and the Judge: The Optimal Mix in The Control of Environmental Risk*

Yolande Hiriart[†]

David Martimort[‡]

Jerome Pouyet[§]

December 21, 2005

Abstract

A firm engaged in activities which are environmentally risky has private information both on its choice of safety care and on the level of its assets. We study the interaction between the ex ante audit of the firm's precautionary effort by a regulator and the ex post check on the true level of the firm's assets by a judge following an accident. Both instruments are useful from an incentive viewpoint. The optimal policy mix between the regulator and the judge depends on the rule determining the judge's intervention and on the quality of the regulatory enforcement. We discuss the incentives of the judge and the regulator to broaden their mandates and undertake the task of the other branch. Finally, we interpret different scenarii for the regulator's and the judge's interventions in terms of legal principles.

JEL Classification: K13, K32, L51.

Keywords: Regulation, Liability, Environmental Risk, Asymmetric Information.

*This paper is part of a research program between IDEI and the French Ministry of Ecology and Sustainable Development on the regulation of risky industrial activity under asymmetric information. Financial and intellectual supports from the Ministry are gratefully acknowledged. We thank Bernard Salanié as well as participants in *Bremen EAERE Conference*, the *Second Toulouse-Montréal Conference on the Law, Economics and Management of Large Scale Risks* and the "Économie de l'Environnement et des Ressources Naturelle" Seminar, Maison des Sciences Économiques, Paris, for useful comments. All errors are ours.

[†]University of Toulouse (IDEI, LERNA). Address: IDEI-LERNA, Manufacture des Tabacs, Bât. F, 21 Allée de Brienne, 31000 Toulouse, FRANCE. Phone: +33(0)561128632. Fax: +33(0)561128637. E-mail: yhiriart@cict.fr.

[‡]University of Toulouse (IDEI, GREMAQ) & Institut Universitaire de France. Address: IDEI, Manufacture des Tabacs, Bât. F, 21 Allée de Brienne, 31000 Toulouse, FRANCE. Phone: +33(0)561128614. Fax: +33(0)561128637. E-mail: martimor@cict.fr.

[§]*Author for correspondence.* École Polytechnique & University of Toulouse (IDEI). Address: Department of Economics, École Polytechnique, 91128 Palaiseau, cedex, FRANCE. Phone: +33(0)169332646. E-mail: jerome.pouyet@polytechnique.edu.

1 Introduction

Whether tort liability or regulation is best suited to cope with accidents is one of the lasting discussions in the law and economics literature. In the field of environmental risk regulation, defining the optimal policy-mix between regulation and liability is an issue of a tantamount importance if one wants that private actors fully internalize the impacts of their decisions on third-parties and the environment. In a world plagued with various informational asymmetries and much uncertainty on the outcomes of production processes that put the environment at risk, relying on either the regulator or the judge are two corrective policies which are well-known to differ both in terms of their effectiveness from an incentive viewpoint but also in terms of their respective administrative costs.

Regulation of environmentally risky ventures requires indeed to enforce standards of care that should be undertaken by private actors and to check compliance. Regulation usually takes the form of routine procedures which take place *ex ante*, i.e. before any harm ever occurs.¹ Tort liability instead is used *ex post* through law suits which are only triggered following an accident. These procedures have a significant incentive role by forcing responsible parties to pay for damages. A proper compensation of the victims or the cleanup of contaminated sites in the case of a disaster require that injuring parties disgorge cash. The judge has a stake in discovering the financial capacity of injuring parties, whereas much of the structure and organization of risky industries precisely aims at escaping those liability payments.²

This brief description of the two kinds of policies available to control environmental risk already stresses a fundamental difference. Whereas risk regulators have expertise to check whether firms shirk on care or not, judges have instead developed the legal expertise to unveil the true assets value of these firms once held liable.³ Those two distinct dimensions of enforcement deal in fact with two incentive problems of a quite different nature. First, it is indeed hard to ascertain whether a firm follows a standard of due care or not and some (random) regulatory inspection is needed: a moral hazard problem. Second, it is often difficult to assess the true value of the firm's assets: an adverse selection problem. The judge's intervention helps unveiling this value by piercing the corporate veil behind which environmentally risky firms may hide.

In this paper, we take as given this functional separation of tasks between the regulator

¹For instance, the *French Directions Régionales de l'Industrie de la Recherche et de l'Environnement* are agencies authorizing agricultural or industrial plants –among which 1148 present a risk a major accident involving hazardous substances– presenting a risk of pollution or nuisance to exert their activity, and are in charge for checking whether firms follow procedures and guidelines for the risk management. In the U.S., such agencies as Occupational Safety and Health Administration (OSHA) or EPA regularly investigate care.

²See Ringleb and Wiggins (1990) for an empirical analysis of these strategies.

³The U.S. 1980 Comprehensive Environmental Response, Compensation and Liability Act (CERCLA) gives an example of the legal arsenal developed by judges to find out the money needed to restore contaminated sites. Under CERCLA, any owner or operator of an environmentally risky venture may be found liable for the potential losses generated by the firm's activity if the latter is itself judgment-proof, i.e., if its assets cannot cover the cleanup costs of the contaminated site.

and the judge and determine the optimal policy mix. Both the judge and the regulator participate to the enforcement of corrective policies and may impose either explicit fines or implicit punishments (such as reputational losses) on firms when they are caught shirking on care or hiding assets. The judiciary and the regulatory branches both contribute to the design of the overall package of incentives and thus, they both improve welfare.

However, the exact interaction between these tasks is far from being obvious. On the one hand, the threat of having the true value of the firm's assets being revealed in a lawsuit might help the regulator to set up the right amount of fines. On the other hand, the threat of being caught shirking by a regulator increases the firm's incentives to exert care. This reduces the likelihood of an accident and thus of an ex post investigation of the firm's assets by the judge.

This paper analyzes this two-way interaction between regulatory and legal interventions and describes the optimal package for enforcement policies in various legal environments. We investigate three different scenarios of increasing complexity.

In the first scenario, the judge only checks the value of the firm's assets if the regulator did not perform any inspection himself. This setting serves as a useful benchmark for the rest of our investigation. When regulatory enforcement is rather efficient, the standard of due care is easily enforced. It is then relatively more tempting for the firm to hide its assets than to shirk on care since the judge is quite unlikely to intervene. Decreasing the probability of regulatory enforcement and, at the same time, increasing the resources that the judge devotes to unveil assets facilitates truth-telling. This increases the possible fines that may be paid if an accident occurs and, by the same token, reduces the regulatory rewards when no accident occurs.⁴ The regulator and the judge are then substitutes.

Quite paradoxically, this is precisely when solving the moral hazard problem (i.e., enforcing the efficient level of care) becomes easier than the adverse selection problem (i.e., finding out the value of the firm's assets) is harder. The optimal policy-mix under this first regime calls for trading off the benefits of ex post and ex ante enforcements.

The scope for this substitutability between the regulator and the judge is then challenged in more complex environments. The judge intervenes now whether regulatory enforcement has taken place or not.

In the second scenario, the judge commits resources to unveil assets whatever the regulatory outcome, i.e., whether a regulatory inspection has taken place or not. Under this second regime, both the regulator and the judge are useful in giving incentives to the firm but they no longer interact.

Finally, in the last scenario, the judge may fine tune the amount of resources devoted to unveil the firm's assets to the regulatory outcome. This scenario reveals a complex web of interaction between the regulator and the judge. When the technology of ex ante audit is efficient, then the likelihood that the firm may encounter both the regulator and the

⁴As we will discuss below, these rewards may either be explicit or implicit, taking the form of reputational gains for instance.

judge is relatively high. When this technology is instead inefficient, then there is a high probability that the firm may just encounter the judge.

Let us now review the relevant literature. Starting with Wittman (1977) and White and Wittman (1983), an earlier trend of the literature has analyzed the performances of ad hoc regulatory and liability mechanisms under either uncertainty or imperfect information, sometimes arguing strongly in favor of liability rules.⁵ Shavell (1984a) discussed and compared the incentive properties of the two policies in a moral hazard environment with also uncertainty on the level of harm.⁶ He showed that liability undermines the level of care for potentially judgment-proof parties or when injuring parties might escape litigation, whereas a regulatory standard performs well when uncertainty on the harm level is sufficiently small. This comparison is somewhat rudimentary both in terms of the incentive mechanisms allowed and because it assumes away the cost of enforcement policies. Still in a framework where the set of mechanisms available is a priori restricted, Kolstad, Ulen and Johnson (1990) did not see liability and regulation necessarily as substitutes and argued that some complementarity may appear between both instruments.⁷ In the present paper, we also start as these latter papers from the presumption that an ex post investigation by the judge certainly generates information whereas regulation helps in enforcing a standard of care. However, information generated on trials does not concern the level of harm as in Shavell (1984a) but instead the level of the firm's sizeable wealth that can be used to compensate the victims. This assumption is more in lines with the motivation behind the legal provisions found in the U.S. in such a law as CERCLA. Working in a model with optimal incentive mechanisms and endogenizing the probability of investigation by either branch, we put on the front line of the analysis the institutional details of the legal environment and the nature of the enforcement costs, showing that those are key elements to delineate the optimal policy-mix. We develop the consequences of the separation of tasks between the regulator and the judge in a framework where no restriction on the set of corrective policies is exogenously imposed, an approach much in the lines advocated in Emons (1994) but with a focus on institutional design.

Boyer and Porrini (2001 and 2004) argued as we do here that regulation and liability (more precisely extended liability towards principal vertically linked with a judgment-proof firm) both involve some kind of monitoring activities. They stress an interesting trade-off coming from the comparison between the cost of a captured regulation and the cost of having the firm's principals (be they lenders or parent firms) with an objective different from the social objective in a context of extended liability. Contrary to these papers, we do not consider regulation and liability as two mutually exclusive alternatives but we are interested in their optimal mix. Also, we do not address the political economy

⁵See also Weitzman (1974) and Yohe (1978).

⁶See also Shavell (1984b) for some informal arguments.

⁷In Hiriart, Martimort and Pouyet (2004), we showed that a joint use of regulation and liability helps implementing the first-best level of care when there are no a priori restriction on incentive mechanisms.

issues and most specifically the reasons why the regulator and the judge should be split as two different entities, a question that we tackle in a companion paper (Hiriart, Martimort and Pouyet (2005)).⁸

Finally, Mookherjee and P'Ng (1992) also stressed the difference between ex ante and ex post monitoring⁹ but, following Shavell (1984a), addressed other sets of issues related to the fact that ex post, the size of a damage is better known than ex ante.

Section 2 presents our model. In Section 3, we provide some useful benchmarks where the overall investigation capacity of the State is limited. In particular, the judge never intervenes. This stresses the difficulty in building a regulatory mechanism able to screen firms according to their assets values without bringing the judge in. In Section 4, we introduce the judge into the picture and stress its interaction with the regulator. Section 5 provides some extensions and discusses further the allocation of tasks between the regulator and the judge. Section 6 concludes. Proofs are relegated to an Appendix.

2 The Model

We consider a relationship between a firm, a regulator and a judge. The cornerstone of our analysis relies on the following ingredients: the environmental risk created by the firm's activities; the presence of asymmetric information in the form of both adverse selection on the value of the firm's assets and moral hazard on safety care; the ex ante intervention of the regulator and ex post intervention of the judge. We now describe these elements.

2.1 The Firm

By its mere activity, a firm may provoke an environmental damage of size D which may harm third-parties. One may think of this damage as an oil spill by oil-carrying vessels during transportation or as chemical leakage from underground storage tanks.

The probability of an accident $\pi(\cdot)$ depends on a firm-specific effort e towards safety care, which, for convenience, is assumed to be binary: $e \in \{0, 1\}$. When the firm exerts the high level of precautionary effort (i.e., when $e = 1$) the probability of an accident is $1 - \pi_1$. By contrast, if the firm undertakes the low effort level (i.e., $e = 0$) then the probability of a damage becomes $1 - \pi_0 > 1 - \pi_1$, with $\Delta\pi \equiv \pi_1 - \pi_0 > 0$. To exert effort $e = 1$ (respectively $e = 0$), the firm must bear a positive non-monetary cost ψ (resp. 0).¹⁰ The privately observed precautionary effort is a moral hazard variable.

To focus on the interesting cases, we shall make the following assumption.

⁸Our point there is to show that splitting ex ante and ex post investigations is the best way to prevent capture of both arms of control.

⁹To make the distinction easier, these authors call the second kind of monitoring 'investigation'. We won't make such semantic difference.

¹⁰Our model could easily be generalized to the case of a monetary cost of maintaining safety care at the cost of a slighter more complex modeling.

Assumption 1. *It is always socially desirable that the firm exerts the high level of precautionary effort.*

This assumption is innocuous since it always holds at equilibrium provided that the damage D in the event of an accident is sufficiently large to offset the costs of inducing such a high level of precautionary effort.

Let us now consider another crucial feature of our model, namely the firm's assets value. The firm owns assets whose total value is denoted by $\theta \in \Theta = \{\underline{\theta}, \bar{\theta}\}$, with $\Delta\theta \equiv \bar{\theta} - \underline{\theta} > 0$. Let $\nu = \text{Prob}(\theta = \bar{\theta}) = 1 - \text{Prob}(\theta = \underline{\theta})$. The firm is privately informed on θ , an adverse selection parameter. As regards the possibility for the firm to conceal the value of its assets, we make the following assumption.

Assumption 2. *Overstatement of the assets' value by the firm is not possible.*

Differently stated, claiming to be of type $\hat{\theta}$ requires the firm to gather hard evidences that the value of its assets is at least as high as $\hat{\theta}$.¹¹

2.2 Ex Ante and Ex Post Interventions

We consider now the 'control' of the firm's activities. This control has to be considered in a broad sense. It includes the many different ways in which the firm's decisions are affected and regulated by the State. Let us introduce the following distinction. Any intervention which occurs ex ante (before the realization of an accident) is undertaken by a regulator \mathcal{R} . By contrast, any intervention which takes place ex post (after the realization of an accident) is undertaken by a judge \mathcal{J} . The view we adopt here is to consider the regulator and the judge as two arms of the same public authority.

Transfers. Transfers to the firm depend on its environmental performances. Let us denote by t_a and t_n the regulatory transfers to the firm following an accident or not.

Although our modeling uses the monetary nature of those rewards and punishments, a broader interpretation of those payments is available.¹² Bad environmental performances sometimes come also with damages to the fixed capital of the firm and to some stakeholders (like workers).¹³ Costs may also be indirect and include tightened future regulations, increases in the number of costly environmental audits undertaken in the future, refusals by the government of authorizations and permits, and new taxes. A good management of environmental risk may also require the training and hiring of experts as permanent employees who improve know-how and affect positively other firm's activities.¹⁴ These

¹¹This is a standard assumption in the literature on contracting with financially constrained agents. See Gale and Hellwig (1985), Townsend (1979), and Lewis and Sappington (2000 and 2001).

¹²This broader interpretation is particularly useful in contexts, like in the U.S., where rewards for good environmental performances may be banned.

¹³Major industrial accidents like Bhopal in India or AZF in France had these features.

¹⁴On the discussion on the indirect costs and benefits of a good management of environmental risks, see Lesourd and Schilizzi (2001).

transfers can also be viewed as a black-box to model the long-term gains for the firm to develop a ‘good reputation’ or the long-term loss if the public authority decides to change the contractor after an accident. Rewards cover the firm’s gains in reputation vis-à-vis its customers, potential contracting partners, the government, its shareholders and more generally the financial community as a whole.^{15,16}

Ex Ante Intervention by the Regulator. Environmental regulators randomly monitor firms under their jurisdiction, trying to ensure that safety standards have been correctly implemented. When such investigation is launched, the regulator is able to discover the precautionary effort level effectively chosen by the firm and can force the firm to implement the standard of care $e = 1$ when it did not initially perform such effort.¹⁷ Importantly, whenever regulatory enforcement has taken place, one knows for sure that the firm ends up exerting a high effort.

The probability that such an ex ante regulatory audit is undertaken is q , interpreted as the intensity of regulatory enforcement. Its social cost is $C_{\mathcal{R}}(q)$ with $C_{\mathcal{R}}(\cdot)$ increasing and convex and satisfying the Inada conditions $C'_{\mathcal{R}}(0) = 0$ and $C'_{\mathcal{R}}(1) = +\infty$ to ensure an interior solution in all conditions below.¹⁸

Two interpretations of this ex ante enforcement stage are possible. In the first one, the regulator always scrutinizes the firm but discovers its effort choice only with some probability. In the second one, the regulator only investigates with some probability but always determines the effort level by doing so.

Ex Post Intervention by the Judge. In the event of an accident, a judge launches a lawsuit against the firm. The purpose of this lawsuit is to find out compensation for harmed third-parties. How much compensation can be taken away from the firm depends on the claimed value of its assets. We assume that the ex post investigation by the judge allows to discover the true value of these assets with some probability p which depends on the amount of resources allocated to the judiciary branch. This ex post investigation has a social cost $C_{\mathcal{J}}(p)$, with $C_{\mathcal{J}}(\cdot)$ being also increasing and convex and satisfying the Inada conditions $C'_{\mathcal{J}}(0) = 0$ and $C'_{\mathcal{J}}(1) = +\infty$ to ensure an interior solution. Again, two possible interpretations of our model are possible. In the first one, the lawsuit is a sure

¹⁵To be completely correct with our modeling which stresses the social costs of those monetary transfers, one should also recognize a social cost of those non-monetary transfers. For instance, reputation gains may also create switching costs in the relationship between the firm and some of its contractual partners. Similarly, tightening future regulations may reduce entry on the market.

¹⁶Another interpretation is that the firm is given a base remuneration for its activities with an additional bonus to be given at the end of the contractual relationship if no accident took place.

¹⁷Of course, this perfect observability upon ex ante investigation is an extreme assumption. In practice regulators observe only how much resources are allocated within the firm to undertake care and whether maintenance, inspections and safety routines are respected. Those observables are related to the exact level of care but might actually be different.

¹⁸Several studies have analyzed the cost of regulatory enforcement from an empirical viewpoint and shown some positive relationships between the frequency of such investigations and its administrative costs. See for instance, Epple and Visscher (1984) and Cohen (1985).

event but is successful in unveiling the true assets of the firm only with probability p . In the second interpretation, the ex post investigation is itself random but always succeeds in unveiling the true level of assets.

Our view of the judge intervention can be given a broader interpretation when one has in mind the insight of the costly state verification literature starting with Townsend (1979) and Gale and Hellwig (1985). Those papers focus on the design of financial contracts by uninformed lenders and sometimes interpret the costly verification of the firm's liabilities that these principals undertake as a costly bankruptcy procedure that would be triggered under strategic default. Viewed in these terms, our model can be viewed as providing insights on the separation of tasks between the regulator and the judge on the incentives of risky ventures to threaten society with bankruptcy.

Contracts. A regulatory contract requests the firm to report the value of its assets before this firm exerts any care. We denote by $\{t_a(\hat{\theta}), t_n(\hat{\theta})\}$ the transfers to the firm if it claims having liability $\hat{\theta} \in \Theta$ depending on whether an accident does occur or not. We denote by $q(\hat{\theta})$ the probability of an ex ante audit of care and by $p(\hat{\theta})$ the probability of an ex post audit of the firm's assets.

By the Revelation Principle, there is no loss of generality in restricting the public authority to offer such direct mechanisms which ensure that the firm truthfully reveals the value of its assets. Incentive compatibility constraints will be studied later on. Note that these mechanisms are in fact characterized a priori by different probabilities of both kinds of audits together with ex post transfers (rewards or fines) that all depend on the firm's claim on its assets. Fines are used in the following cases: first, when the firm is audited ex ante and the regulator figures out that the firm did not comply with the regulatory standard; second, when the firm's assets are verified ex post by the judge and the firm had understated its assets. When the firm is found shirking, either on care or on the value of its assets, the *Maximal Punishment Principle* applies.¹⁹ The firm has to pay fines up to the value of its claimed assets to relax as much as possible incentive constraints. This remark helps us to significantly simplify the exposition of those constraints.

Timing. The sequence of events unfolds as follows:

- At date 0, nature draws the type θ of the firm. The firm is privately informed about the assets' value.
- At date 1, the firm is offered a menu of contracts which, for all possible reports about its assets, stipulate transfers conditional on the occurrence of an accident and investigation policies.

¹⁹See Becker (1968), Baron and Besanko (1984) and Laffont and Martimort (2002, Chapter 3).

- At date 1^+ , the firm announces $\hat{\theta}$ or equivalently picks a contract among the menu offered and decides on the level of precautionary effort e .
- At date 2^- , the regulator audits the firm with probability $q(\hat{\theta})$. If an ex ante audit takes place, the regulator can verify the precautionary effort chosen by the firm. If this effort differs from the socially optimal one, the regulator can both enforce the high level of precautionary effort and impose fines to the firm for non-compliance.
- At date 2, an accident occurs with probability $1 - \pi(e)$.
- At date 2^+ , in the event of an accident, a lawsuit may or may not be initiated depending on the scenario investigated below. The judge discovers the value of the firm's assets θ with probability $p(\hat{\theta})$ and imposes a fine if $\hat{\theta} \neq \theta$. In all cases, transfers are paid according to the contract chosen at date 1^+ .

Different rules determine whether the judge intervenes and when:

- First, the ex ante regulatory investigation can immunize the firm against the threat of ex post legal prosecution. Put differently, the judge only intervenes provided that an accident has occurred and no regulatory investigation has taken place. This setting serves as a useful benchmark for the rest of our investigation. This simple case of immunization is studied in Section 4.1.
- Second, the firm may be prosecuted ex post even if an ex ante audit has taken place and the socially efficient effort is implemented.²⁰ Sections 4.2 and 4.3 propose variations on this second scenario which is a priori more complex but can be easily understood once the first scenario has already been exposed.

At this stage, let us re-emphasize that our setting implicitly depicts some separation of tasks between the regulator and the judge: the regulator cannot verify the firm's claim on its wealth whereas the judge cannot determine ex post the effort chosen by the firm.²¹ Indeed, lack of resources and time may lead the regulator and the judge to specialize in non-overlapping tasks and to develop their own expertise. The regulator might specialize in technical aspects of the regulation while the judge is best suited to assess the financial viability of the firm and the compensation that should be paid to third-parties in the event of an accident. These assumptions will be relaxed in Section 5.

²⁰This level of effort is implemented both on the equilibrium path and off since, when shirking has been detected, the regulator can force the firm to comply with the standard.

²¹Except, of course, when the regulator has intervened ex ante and implemented the standard.

2.3 Incentive Constraints

To understand the nature of the different regulatory regimes that will be considered thereafter, it is useful to write down the firm's incentive constraints depending on its assets. As we already stressed, these constraints capture both the moral hazard and adverse selection sides of the incentive problem.

The $\underline{\theta}$ -firm. First, consider a $\underline{\theta}$ -firm with few assets. From Assumption 2, the only incentive issue is to induce this firm to comply with the standard of care. We can write its moral hazard incentive constraint as:

$$U(\underline{\theta}) = \pi_1 t_n(\underline{\theta}) + (1 - \pi_1) t_a(\underline{\theta}) - \psi \geq (1 - q(\underline{\theta})) [\pi_0 t_n(\underline{\theta}) + (1 - \pi_0) t_a(\underline{\theta})] - q(\underline{\theta})(\psi + \underline{\theta}). \quad (1)$$

The left-hand side depicts the equilibrium payoff of this $\underline{\theta}$ -firm once it complies with the standard. Even if an ex ante investigation takes place, the regulator cannot detect any misconduct and the firm is not fined. By contrast, when this $\underline{\theta}$ -firm shirks on the level of care, it may be detected with probability $q(\underline{\theta})$. In that case, it will be forced by the regulator to adopt the standard of due care and to bear the cost ψ . The firm is also heavily punished whatever the future realization of the environmental risk, i.e., the public authority imposes a net penalty equal to the firm's liability $\underline{\theta}$.²²

Taking into account the limited liability constraint of a $\underline{\theta}$ -firm, namely,

$$t_a(\underline{\theta}) \geq -\underline{\theta}, \quad (2)$$

the moral hazard incentive constraint (1) can be rewritten as:²³

$$U(\underline{\theta}) \geq R(q(\underline{\theta})) - \underline{\theta}, \quad (3)$$

where $R(q) = \left(\frac{\pi_0(1-q) - \pi_1 q}{\pi_1 - \pi_0(1-q)} \right) \psi$. To exert the socially desirable effort level, the $\underline{\theta}$ -firm must be given a liability rent $R(q(\underline{\theta})) - \underline{\theta}$.

To focus on the interesting cases, we will assume throughout the paper that the so-called limited liability rent $R(q) - \theta$ is strictly positive for all (θ, q) in the relevant domain. This condition implies that the $\underline{\theta}$ -firm participation constraint (normalizing outside opportunities at zero) is implied by its limited liability and moral hazard incentive constraints which are compounded into (3). For future references, we highlight that the limited liability rent is decreasing in the firm's assets, i.e., firms which have more assets earn lower

²²Finally, note also that if an accident occurs, the mechanism may require an ex post investigation for this $\underline{\theta}$ -firm. This won't change its payments since, from Assumption 2, this firm cannot be caught lying on its assets. But, doing so may nonetheless help relaxing the incentive constraint of a $\bar{\theta}$ -firm which could be tempted to understate its own assets.

²³If the probability of an ex ante investigation is sufficiently large, constraint (1) is trivially satisfied and the moral hazard problem disappears. In order to get rid of this uninteresting case, we shall assume i) that the ex ante investigation occurs not too frequently ($1 - \pi_1 < (1 - q(\underline{\theta}))(1 - \pi_0)$ in the relevant range); ii) that one wants to increase as much as possible the fine paid by the firm if an accident occurs so that limited liability on the firm's side is a serious impediment to first-best regulation.

limited liability rents since, having to disgorge more cash in the event of an accident, they obtain a lower rent to perform a given level of care.

The $\bar{\theta}$ -firm. Let us now turn to the $\bar{\theta}$ -firm. This firm may not only shirk by not adopting the standard of due care but it may also hide its assets to limit its exposure to liability payments if an accident occurs. This leads us to consider three incentive constraints:

- a pure moral hazard incentive constraint where the $\bar{\theta}$ -firm might shirk only by adopting a low level of care;
- a pure adverse selection incentive constraint where the $\bar{\theta}$ -firm adopts the standard but pretends to have low assets;
- a mixed incentive constraint where both deviations take place simultaneously.

First, note that the pure moral hazard incentive constraint of a $\bar{\theta}$ -firm can be derived exactly as what we did for a $\underline{\theta}$ -firm. Given the $\bar{\theta}$ -firm's liability constraint

$$t_a(\bar{\theta}) \geq -\bar{\theta}, \quad (4)$$

and the definition of the $\bar{\theta}$ -firm's expected utility, we find:

$$U(\bar{\theta}) = \pi_1 t_n(\bar{\theta}) + (1 - \pi_1) t_a(\bar{\theta}) - \psi \geq R(q(\bar{\theta})) - \bar{\theta}. \quad (5)$$

Note that under our assumptions the right-hand side of (5) is positive.

Second, the pure adverse selection incentive constraint prevents the $\bar{\theta}$ -firm from understating its wealth given that it has chosen to comply with the standard of due care. Notice that there is no need to check the assets of a firm claiming being of type $\bar{\theta}$. This would indeed mean incurring the cost of an ex post investigation without relaxing any incentive constraint since, from Assumption 2, only understatement of assets is feasible.²⁴

This leads to write the pure adverse selection incentive constraint as:

$$U(\bar{\theta}) \geq (1 - q(\underline{\theta})) \{ \pi_1 t_n(\underline{\theta}) + (1 - \pi_1) [(1 - p(\underline{\theta})) t_a(\underline{\theta}) + p(\underline{\theta})(-\bar{\theta})] \} \\ + q(\underline{\theta}) \{ \pi_1 t_n(\underline{\theta}) + (1 - \pi_1) t_a(\underline{\theta}) \} - \psi.$$

On the right-hand side we take into account that, once an environmental accident occurs, the firm claiming to have low assets may be audited ex post and caught lying with probability $p(\underline{\theta})$ and have to disgorge its whole wealth. To ease the comparison of the various incentive constraints, the pure adverse selection constraint can be written as:

$$U(\bar{\theta}) \geq U(\underline{\theta}) - (1 - \pi_1)(1 - q(\underline{\theta}))p(\underline{\theta})\Delta\theta. \quad (6)$$

²⁴Indeed, given the focus on direct and truthful contracts, the firm will never lie about its assets level at equilibrium; hence the sole purpose of the investigation policies is to relax the firm's incentive constraints. See Laffont and Martimort (2002, Chapter 3) and the references therein.

This pure adverse selection constraint can be interpreted in the following way.²⁵ If a $\bar{\theta}$ -firm understates its wealth while complying with the standard, it obtains the rent $U(\underline{\theta})$ of a $\underline{\theta}$ -firm but faces a probability $(1 - \pi_1)(1 - q(\underline{\theta}))p(\underline{\theta})$ of being fined up to the value of the hidden assets $\Delta\theta$ if an accident takes place and it is audited ex post. This possibility reduces the rent associated with the understatement of wealth and thus relaxes (6).

Third, considering now the possibility of deviations along both effort and assets, we get the mixed incentive constraint:

$$U(\bar{\theta}) \geq (1 - q(\underline{\theta})) [\pi_0 t_n(\underline{\theta}) + (1 - \pi_0)(1 - p(\underline{\theta}))t_a(\underline{\theta}) - (1 - \pi_0)p(\underline{\theta})\bar{\theta}] - q(\underline{\theta})(\psi + \underline{\theta}),$$

or, after rearranging terms,

$$U(\bar{\theta}) \geq R(q(\underline{\theta})) - \underline{\theta} - (1 - \pi_0)(1 - q(\underline{\theta}))p(\underline{\theta})\Delta\theta. \quad (7)$$

Constraints (6) and (7) show that by pretending having fewer assets, a $\bar{\theta}$ -firm gets at least the utility level of a $\underline{\theta}$ -firm but may lose the hidden assets $\Delta\theta$ if an accident occurs and an ex post intervention unveils its true wealth.

The next lemma turns out to simplify the analysis by reducing the number of relevant constraints for a $\bar{\theta}$ -firm.

Lemma 1. *Consider that (3) and (6) are satisfied; then (7) is satisfied too.*

The intuition is twofold. First, since the probability of an accident is larger with a low effort than with the high one, so is the expected loss if there is a prosecution ex post for a $\bar{\theta}$ -firm claiming a low wealth. Second, given that the adverse selection parameter does not affect the probability of an accident, absent any ex post intervention, the $\bar{\theta}$ -firm which shirks on the effort level earns the same limited liability rent than the $\underline{\theta}$ -firm.

2.4 Social Objectives

Optimal contracts are designed to maximize a social welfare function which incorporates the well-being of victims but also the cost of the incentive program (the cost of regulatory transfers and the administrative costs of the investigations). This objective writes as:

$$\mathcal{W} = \mathbb{E}_\theta \{ -(1 - \pi_1)D - [\pi_1 t_n(\theta) + (1 - \pi_1)t_a(\theta)] - C_{\mathcal{R}}(q(\theta)) - C_{\mathcal{J}}(p(\theta)) \},$$

where $\mathbb{E}_\theta(\cdot)$ is the expectation operator with respect to the wealth level. Expressing this objective as a function of the utility levels left to both types of firms, we get:

$$\mathcal{W} = -(1 - \pi_1)D - \psi - \mathbb{E}_\theta \{ U(\theta) + C_{\mathcal{R}}(q(\theta)) + C_{\mathcal{J}}(p(\theta)) \}.$$

²⁵The constraint is given for the first scenario where the $\underline{\theta}$ -firm can only be suited (with probability $p(\underline{\theta})$) if no ex ante audit took place (with probability $1 - q(\underline{\theta})$). It will then be adapted to each case.

We thus see that the utility levels left to both types of firms should be reduced as much as possible to maximize social welfare. Summarizing, under asymmetric information, the problem becomes:

$$(\mathcal{P}) : \quad \max_{\{U(\cdot), q(\cdot), p(\cdot)\}} \mathcal{W}$$

subject to constraints (3), (5), (6) and (7).

2.5 An Illustration

We now provide one possible application to our model to a real-world situation, which involves highly environmentally risky activities and a strong regulatory oversight.

Envirocare is the only completely privately owned and operated dump in the U.S. that handles the U.S. government's nuclear waste. At stake are lucrative federal contracts, which typically consist in a licence to operate for a given period of time coupled with a possibility of renewal.²⁶

In 1988, Envirocare began disposing of naturally occurring radioactive material. Later, licenses and permits to dispose of mixed waste, uranium mill tailings and Class-A low-level radioactive waste were obtained. Since 1993, more than 14 million cubic feet of lightly radioactive dirt and junk from the Department of Energy has been entombed at this isolated dump. Exposure to radiation is risky and, according to the State of Utah (where the dump is located), clean-up costs for an accident with radioactive wastes are estimated at \$14 to \$313 billion.

The Price Anderson Act limits radioactive waste companies' liability for accidents to \$9,4 billion: Envirocare is a private company that is protected as a corporation from long-term liabilities. According to the Utah Division of Radiation Control, Envirocare has put up a \$14 million bond for closure activities.

Ten federal, state and local agencies regulate Envirocare operations, among which the U.S. Environmental Protection Agency, the Nuclear Regulatory Commission, the U.S. Department of Transportation, the Utah Division of Radiation Control, the Utah Division of Solid and Hazardous Waste. Their tasks are primarily to ensure compliance towards the safety standards and to investigate in the event of an accident. For instance, the Utah Division of Radiation Control has currently more than 10 full-time equivalent employees work on oversight of Envirocare's operations, has daily access to inspect and regulate site operations, performs more than 500 workdays at site for regular inspection.²⁷

²⁶The U.S. Department Of Energy is the successor of the government's Cold-War era Atomic Energy Commission and has inherited the task of cleaning up the radioactive refuse from uranium mines, munitions facilities and other sites around the country.

²⁷Envirocare has repeatedly violated state environmental regulations. In 2000, Envirocare was fined \$44,000 for everything from improperly disposing of radioactive and hazardous wastes to groundwater monitoring violations. In 1999, Envirocare paid approximately \$160,000 in fines.

3 Useful Benchmarks

To better understand some of our results, it is useful to start looking at a few benchmarks in which the ability to audit the firm is limited .

3.1 No Investigation

Let us suppose that the cost of an investigation is infinite, which forces to set $q(\theta) = p(\theta) = 0$ for any θ . Hence constraint (6) can be now be expressed as:

$$U(\bar{\theta}) \geq U(\underline{\theta}), \quad (8)$$

and is hardened by the absence of any ex post investigation. Moreover, the moral hazard incentive constraint of a $\bar{\theta}$ -firm (namely (5)) is directly implied by (6) and the moral hazard incentive constraint of a $\underline{\theta}$ -firm (namely (3)) since the limited liability rent $R(0) - \theta$ is decreasing in the firm's wealth.

Intuitively, the transfers t_a and t_n are the only screening devices. This is not enough to force a firm with large assets to disgorge that cash if an accident occurs. Such a firm can always mimick a $\underline{\theta}$ -one and limit its exposure to liability payments. Therefore, at the optimum of (\mathcal{P}) with no investigation, we necessarily have (6) and (3) binding, or:

$$U(\bar{\theta}) = U(\underline{\theta}) = R(0) - \underline{\theta} = \frac{\pi_0 \psi}{\Delta \pi} - \underline{\theta}. \quad (9)$$

As also shown by Lewis and Sappington (2000) in a similar context, the optimal regulatory policy is easily implemented by a pooling policy which does not depend on the firm's type:

$$t_a(\theta) = -\underline{\theta} \text{ and } t_n(\theta) = -\underline{\theta} + \frac{\psi}{\Delta \pi}, \text{ for any } \theta.$$

Proposition 1. *In the absence of any investigation, fines and rewards do not depend on the firm's assets.*

3.2 The Regulator Only

As in Section 3.1 above, the ability to hide assets strongly structures the optimal regulatory scheme but, now, the ability to fine tune the intensity of the regulatory monitoring gives a tool to screen the different firms according to their assets.

From now on, we will also state an assumption which ensures that (\mathcal{P}) is a concave problem with respect to $q(\bar{\theta})$ and $q(\underline{\theta})$.

Assumption 3. $C_{\mathcal{R}}(R^{-1}(x))$ is convex in x .²⁸

The analysis when only ex ante investigation is feasible is summarized in the next proposition. The solution depends on the extent by which the $\bar{\theta}$ -firm prefers to hide its

²⁸This amounts in fact to having $\frac{C_{\mathcal{R}}''(q)}{C_{\mathcal{R}}'(q)} > R''(q)$, a condition which certainly holds when $C_{\mathcal{R}}(\cdot)$ is convex enough.

assets than to shirk on care. In an *unconstrained regime*, the first effect dominates and only the incentive constraints (3) and (6) are binding. Instead, in a *constrained regime*, all incentives matter and constraints (3), (5) and (6) altogether bind. For simplicity, we present in the text the analysis of an unconstrained regime and leave for the Appendix the analysis of the constrained one.

Proposition 2. *Assume that only an ex ante investigation is feasible, and that uncertainty on assets is large enough, i.e.,*

$$R(q_*^{ea}(\underline{\theta})) + \Delta\theta \geq R(0),$$

where $q_*^{ea}(\underline{\theta})$ is defined below. Then, only the $\underline{\theta}$ -firm is audited with a positive probability $q_*^{ea}(\underline{\theta})$ such that

$$C'_{\mathcal{R}}(q_*^{ea}(\underline{\theta})) = -\frac{1}{1-\nu}R'(q_*^{ea}(\underline{\theta})). \quad (10)$$

At the optimal contract, everything happens as if the firm had the choice within a menu of two regulatory policies. The first one is rather lenient in terms of enforcement of a standard of care but it requires harsh fines if an accident occurs. Only wealthy firms can afford this first scheme. The second contract involves a closer monitoring ex ante but weaker fines following an accident. More cash-constrained firms choose this second scheme.

Henceforth, when regulatory investigation is the sole tool available, all resources are spent on checking ex ante the firms which have little collateral since having them fined ex post does not bring much. This optimal mechanism which discriminates firms according to their financial capacity bears some resemblance with the limited fines that firms incur when they self-report violations of a standard.²⁹ The difference with the self-reporting mechanism comes from the fact that a firm does not report information on its care choice in our model but on its assets.

The intuition behind Proposition 2 can be grasped from first thinking about the case where the size of the firm's assets is perfectly observable. In the absence of the adverse selection problem, the optimal contract is only driven by pure moral hazard. The corresponding moral hazard constraints are necessarily binding and, to reduce agency cost, the firm is audited by the regulator with the same probability whatever its type.³⁰ This probability q^* is such that the marginal investigation cost equals the marginal benefit in terms of rent reduction, or $C'_{\mathcal{R}}(q^*) = -R'(q^*)$. With this ex ante investigation probability, the public authority minimizes the overall cost of dealing with each type where this cost encompasses both the limited liability rent $R(q) - \theta$ withdrawn by each type and the ex ante auditing cost.

²⁹See Innes (2001) for instance.

³⁰Indeed, the limited liability rent of a firm (i.e., the rent obtained when the pure moral hazard constraint is binding) is separable in (q, θ) . Moreover the limited liability rent is decreasing with the firm's wealth.

Of course, this policy can no longer be implemented under adverse selection because, exactly as in Section 3.1, a $\bar{\theta}$ -firm cannot be forced to disgorge its whole assets if an accident occurs and the probability of an ex post investigation is null. In the event of an accident, the $\bar{\theta}$ -firm should thus pay the same amount as a $\underline{\theta}$ -firm. The adverse selection constraint is thus necessarily binding.

As a consequence, two regimes appear depending on whether the moral hazard incentive constraint of the $\bar{\theta}$ -firm is also binding or not. In an unconstrained regime, (5) is slack and both types of firm earn the same level of rent, namely the limited liability rent of the $\underline{\theta}$ -firm $R(q_*^{ea}(\underline{\theta})) - \underline{\theta}$. Hence, only the $\underline{\theta}$ -firm is audited. This regime occurs as long as the gain from shirking on safety care for the $\bar{\theta}$ -firm, i.e., $R(0) - \bar{\theta}$, remains smaller than the rent earned from mimicking the $\underline{\theta}$ -firm, i.e., $R(q_*^{ea}(\underline{\theta})) - \underline{\theta}$.³¹

To relax the adverse selection constraint, the $\underline{\theta}$ -firm is audited more often than the $\bar{\theta}$ -one. This ex ante investigation, whose sole role is a priori to avoid shirking on safety care for a $\underline{\theta}$ -firm, can also be used as a screening device to reduce the $\bar{\theta}$ -firm's incentives to understate its assets. Indeed, auditing the $\underline{\theta}$ -firm more often reduces its limited liability rent. In particular, the reward $t_n(\underline{\theta})$ received for a good environmental performance diminishes. The temptation of a $\bar{\theta}$ -firm to understate its wealth becomes less attractive. At the same time, one way of rewarding the $\bar{\theta}$ -firm for having revealed its assets is to reduce the probability of an ex ante investigation for this firm so that it enjoys more of the liability rent due to moral hazard. Information revelation is obtained by offering a more lenient regulatory policy to those firms which a priori claim being wealthy enough.

4 The Regulator and the Judge

Let us now turn to the full-fledged model where the claim of the firm on its assets can possibly be checked ex post at some cost by the judge. The main lesson of the costly state verification models à la Townsend (1978)-Gale and Hellwig (1985) applies to our framework: the threat of being punished when caught lying reduces the firm's incentives to understate the value of its assets. The main question we address in this section is thus: how the ex ante and ex post investigation policies ought to be optimally combined?

4.1 Immunization

Suppose the judge intervenes only after an accident and the regulator did not intervene ex ante. The two arms intervene thus to solve different incentive problems and in different states of nature.

As in the previous section, the adverse selection constraint (6) is necessarily binding at

³¹In a constrained regime, the moral hazard incentive constraints of both types and the adverse selection constraint bind altogether. Unsurprisingly, such a regime arises when the gain from shirking on safety care for a $\bar{\theta}$ -firm is greater than the gain from pretending to be a $\underline{\theta}$ -firm, i.e., when there is instead little uncertainty on the firm's assets.

the optimum.³² To simplify the analysis, let us focus on an unconstrained regime which arises when the uncertainty on the firm's assets is large enough and only constraints (3) and (6) are binding.³³

Proposition 3. Unconstrained regime with immunization.

Assume that uncertainty on the firm's assets is large enough, i.e.,

$$R(0) \leq R(q_*^{ep}(\underline{\theta})) + \Delta\theta [1 - (1 - \pi_1)(1 - q_*^{ep}(\underline{\theta}))p^{ep}(\underline{\theta})], \quad (11)$$

where $q_*^{ep}(\underline{\theta})$ and $p_*^{ep}(\underline{\theta})$ are defined below. Then, the optimal contract with immunization is such that only the $\underline{\theta}$ -firm is audited ex ante and ex post with positive probabilities such that:

$$C'_{\mathcal{R}}(q_*^{ep}(\underline{\theta})) = -\frac{1}{1-\nu}R'(q_*^{ep}(\underline{\theta})) - \frac{\nu}{1-\nu}(1-\pi_1)p_*^{ep}(\underline{\theta})\Delta\theta, \quad (12)$$

$$C'_{\mathcal{J}}(p_*^{ep}(\underline{\theta})) = \frac{\nu}{1-\nu}(1-\pi_1)(1-q_*^{ep}(\underline{\theta}))\Delta\theta. \quad (13)$$

The analysis of the unconstrained regime with ex post investigation is quite close to the analysis when no such ex post investigation is feasible (see Section 3.2). The gain from shirking on safety care is smaller than the gain from understating assets but, now, this gain has to be discounted by the probability that a judge may discover and take the hidden assets $\Delta\theta$.

Because the pure moral hazard incentive constraint of a $\bar{\theta}$ -firm is slack in this regime, there is no reason to audit ex ante a $\bar{\theta}$ -firm. This would not help relaxing any of the relevant incentive constraints. Instead, an increase in the probability of auditing a ratio $1 - \nu$ of $\underline{\theta}$ -firms above the level $q^*(\underline{\theta})$ had assets been known relaxes constraint (6) since the rent $U(\underline{\theta})$ left to all types of firms decreases with this probability. This positive effect, captured by the first term on the right-hand side of (12), calls for an increase of the ex ante investigation probability beyond $q^*(\underline{\theta})$, since $\frac{1}{1-\nu} > 1$.

However, a second negative effect is at work here (the second term on the right-hand side of (12)). Comparing (10) and (12), we can see that the pure adverse selection constraint (6) is relaxed by decreasing the probability of an ex ante investigation for a $\underline{\theta}$ -firm. Remember that the $\bar{\theta}$ -firm prefers to hide its wealth than to shirk on the level of care because doing so reduces the probability of accident and thus the overall probability of liability exposure. Decreasing the probability of the regulator's monitoring increases the likelihood that the judge intervenes ex post. This increases the threat of being fined if an accident occurs.

Although both kinds of intervention help relaxing incentive constraints, the regulator is relatively inefficient in inducing the $\bar{\theta}$ -firm to disgorge cash as we have already seen

³²Indeed, suppose that it was not the case. Then considering only the two (binding) moral hazard constraints (3) and (5) would again lead to the choice of a common ex ante investigation policy q^* and to undertake no ex post investigation since such investigation is useless in relaxing any of those two pure moral hazard constraints. This would violate (6) since the $\bar{\theta}$ -firm would lie on its assets to minimize its exposure to liability payments.

³³The case of a constrained regime is treated in the Appendix.

in Section 3.2. The judge is instead crucial in doing so. However, the judge intervenes only when an ex ante investigation did not take place, i.e., with probability $(1 - \pi_1)(1 - q(\underline{\theta}))$. Condition (13) reflects the fact that the marginal benefit of an ex post investigation depends on the probability that an ex ante investigation did not take place.

With immunization, everything happens as if, following the regulator's intervention, the firm pays a fine $\underline{\theta}$ that can be raised up to $\bar{\theta}$ if the judge intervenes. To reinforce this intuition, let us describe the optimal transfers. Given that both the limited liability constraint (2) and the moral hazard incentive constraint (3) of a $\underline{\theta}$ -firm are binding, we find:

$$t_{a*}^{ep}(\underline{\theta}) = -\underline{\theta}, \quad (14)$$

$$t_{n*}^{ep}(\underline{\theta}) = -\underline{\theta} + \frac{(1 - q_*^{ep}(\underline{\theta}))\psi}{\pi_1 - \pi_0(1 - q_*^{ep}(\underline{\theta}))}. \quad (15)$$

For a $\bar{\theta}$ -firm, given that both the limited liability constraint (4) and the pure adverse selection constraint (6) are binding, we find:³⁴

$$t_{a*}^{ep}(\bar{\theta}) = -\bar{\theta}, \quad (16)$$

$$t_{n*}^{ep}(\bar{\theta}) = -\bar{\theta} + \underbrace{\frac{(1 - q_*^{ep}(\underline{\theta}))\psi}{\pi_1 - \pi_0(1 - q_*^{ep}(\underline{\theta}))}}_{\text{moral hazard incentive reward for a } \underline{\theta}\text{-firm}} + \underbrace{\frac{\Delta\theta}{\pi_1} [1 - (1 - \pi_1)(1 - q_*^{ep}(\underline{\theta}))p_*^{ep}(\underline{\theta})]}_{\text{adverse selection incentive reward}}. \quad (17)$$

Those transfers can easily be interpreted. Everything happens as if the maximal fine imposed on a $\bar{\theta}$ -firm was equal to its assets but there existed a reward for a good environmental performance incorporating the information rent withdrawn by this firm from private knowledge of its assets. Indeed, the right-hand side of (17) can be decomposed into two pieces: first, the moral hazard incentive reward which induces a high level of care from the $\underline{\theta}$ -firm; second, the adverse selection incentive reward which facilitates truth-telling. Although the first of these terms is reduced with an ex ante investigation, the second one increases with it.

In this setting, some substitutability between the regulator and the judge appears.³⁵

When the regulator benefits from a better supervision technology ($C'_R(\cdot)$ being lower), an ex ante investigation becomes easier, $q^{ep}(\underline{\theta})$ increases and the marginal benefit from an ex post investigation decreases (from (13)). Increasing the frequency of ex ante intervention makes it less valuable to call the judge ex post.³⁶ Reciprocally, when the ex post

³⁴When (11) holds, remember that the pure moral hazard incentive constraint of a $\bar{\theta}$ -firm is strictly satisfied at the optimum.

³⁵Indeed, differentiating (13) leads to:

$$\frac{dp_*^{ep}(\underline{\theta})}{dq_*^{ep}(\underline{\theta})} = -\frac{1}{C''_{\mathcal{J}}(p_*^{ep}(\underline{\theta}))} \frac{\nu}{1 - \nu} (1 - \pi_1)\Delta\theta < 0.$$

³⁶This substitutability can in fact be related to an important property of the $\bar{\theta}$ -firm's rent under immunization. Let us introduce the following notation: $\mathcal{U}(\bar{\theta}/q(\underline{\theta}), p(\underline{\theta}))$ is the high-liability firm's rent when

investigation technology of the judge improves ($C'_{\mathcal{J}}(\cdot)$ being lower), $p^{ep}(\underline{\theta})$ increases and this decreases the marginal benefit from auditing the firm ex ante (from (12)). Altogether, these results show the substitutability between the two arms of enforcement. Although both are jointly used to improve incentives, the better one instrument, the less used is the other, at least as long as the regulatory intervention immunizes the firm against ex post prosecution.

We summarize this discussion as:

Proposition 4. *With immunization, the regulator and the judge are substitutes in solving the incentive problem.*

4.2 No Immunization

Let us now suppose that the judge might always intervene even if the regulator has audited care ex ante.

We first consider a polar situation where the judge unveils the true value of the firm's assets with the same intensity, be the firm audited ex ante by the regulator or not. This would correspond to a scenario where resources are allocated to the judiciary branch beforehand, i.e., before the regulator intervenes.

Straightforward manipulations show several facts. First, the moral hazard constraints are left unchanged. Second, the pure adverse selection constraint of a $\bar{\theta}$ -firm is always more demanding than the mixed constraint. Third, this pure constraint writes now as:

$$U(\bar{\theta}) \geq U(\underline{\theta}) - (1 - \pi_1)p(\underline{\theta})\Delta\theta. \quad (18)$$

Since the judge intervention is now unconditional, the loss for a $\bar{\theta}$ -firm if it understates its assets and is prosecuted ex post no longer depends on the probability of an ex ante investigation: the second-term of the right-hand side of (18) no longer depends on $q(\underline{\theta})$. Two regimes appear depending on whether the pure moral hazard incentive constraint of a $\bar{\theta}$ -firm is binding or not at equilibrium. Still focusing on the unconstrained regime where only constraints (3) and (18) are binding and relegating the analysis of the constrained case to the Appendix, we get:

Proposition 5. *Assume that uncertainty on the firm's assets is large enough, i.e.,*

$$R(0) \leq R(q_*^{ep}(\underline{\theta})) + \Delta\theta [1 - (1 - \pi_1)p_*^{ep}(\underline{\theta})],$$

the public authority audits the $\underline{\theta}$ -firm with probabilities $q(\underline{\theta})$ and $p(\underline{\theta})$. Using our previous analysis, the following inequality is immediately obtained:

$$\mathcal{W}(\bar{\theta}/q, p) + \mathcal{W}(\bar{\theta}/0, 0) \geq \mathcal{W}(\bar{\theta}/q, 0) + \mathcal{W}(\bar{\theta}/0, p).$$

To be more precise, $\mathcal{W}(\bar{\theta}/q(\underline{\theta}), p(\underline{\theta})) = R(q(\underline{\theta})) - \underline{\theta} - (1 - \pi_1)(1 - q(\underline{\theta}))p(\underline{\theta})\Delta\theta$, $\mathcal{W}(\bar{\theta}/0, p(\underline{\theta})) = R(0) - \underline{\theta} - (1 - \pi_1)p(\underline{\theta})\Delta\theta$, $\mathcal{W}(\bar{\theta}/q(\underline{\theta}), 0) = R(q(\underline{\theta})) - \underline{\theta}$, $\mathcal{W}(\bar{\theta}/0, 0) = R(0) - \underline{\theta}$. As a consequence, the rent of the $\bar{\theta}$ -firm is supermodular in $(q(\underline{\theta}), p(\underline{\theta}))$. Equivalently, the social objective is submodular in the two interventions, implying that the regulator and the judge are substitutes.

where $q_*^{ep}(\underline{\theta})$ and $p_*^{ep}(\underline{\theta})$ are defined below. Then, the optimal contract without immunization is such that only the $\underline{\theta}$ -firm is audited ex ante and ex post with positive probabilities such that

$$C'_{\mathcal{R}}(q_*^{ep}(\underline{\theta})) = -\frac{1}{1-\nu}R'(q_*^{ep}(\underline{\theta})), \text{ and}$$

$$C'_{\mathcal{J}}(p_*^{ep}(\underline{\theta})) = \frac{\nu}{1-\nu}(1-\pi_1)\Delta\theta.$$

Without immunization, there is a clear separation between the tasks of the regulator and the judge. Any change in the cost of one kind of investigation has only an impact on the probability of using that particular investigation.

It is again worth describing the optimal transfers for both types. For a $\underline{\theta}$ -firm whose limited liability and moral hazard incentive constraints are binding, we find the same expressions as in (14) and (15). For a $\bar{\theta}$ -firm, whose limited liability and pure adverse selection incentive constraints are binding, we find the following transfers:

$$t_{a*}^{ep}(\bar{\theta}) = -\bar{\theta},$$

$$t_{n*}^{ep}(\bar{\theta}) = -\bar{\theta} + \underbrace{\frac{(1-q_*^{ep}(\underline{\theta}))\psi}{\pi_1-\pi_0(1-q_*^{ep}(\underline{\theta}))}}_{\text{moral hazard incentive reward for a } \underline{\theta}\text{-firm}} + \underbrace{\frac{\Delta\theta}{\pi_1}[1-(1-\pi_1)p_*^{ep}(\underline{\theta})]}_{\text{adverse selection incentive rewards}}.$$

Those transfers decompose the role of the regulator and the judge. Everything happens as if the $\bar{\theta}$ -firm was now always forced to pay fines up to the value of its assets when an accident takes place but then receives an extra reward for a good environmental performance. This extra reward can be decomposed into two pieces: first, the incentive reward offered to a $\underline{\theta}$ -firm to solve its moral hazard problem; second, a pure adverse selection reward to induce truth-telling. The first of these rewards is reduced through an ex ante investigation whereas the second one is reduced by the threat of an ex post prosecution.

In an unconstrained regime, the sum of these two rewards suffices to solve the moral hazard problem of a $\bar{\theta}$ -firm. The difference with Section 4.1 is that each reward is indirectly controlled by either the regulator or the judge.

We summarize this discussion as:

Proposition 6. *Without immunization, the regulator and the judge do not interact in solving the incentive problem.*

4.3 Conditioning

Sections 4.1 and 4.2 have shown that the interaction between the regulator and the judge depends heavily on the rule determining the judge's intervention. We assume that the resources allocated by the judiciary branch to fulfill its role depend on the regulatory

outcome. This conditioning of legal intervention on regulatory outcomes unveils a complex web of interactions between the regulator and the judge.

Let us first introduce some notations: $p_1(\hat{\theta})$ (resp. $p_2(\hat{\theta})$) is the probability that the judge unveils the firm's assets following an accident when the regulator has not audited the firm ex ante (resp. when the regulator has audited the firm ex ante).

One can easily check that the moral hazard constraints are not modified. The pure adverse selection constraint of a $\bar{\theta}$ -firm writes now as:

$$U(\bar{\theta}) \geq U(\underline{\theta}) - (1 - \pi_1) [(1 - q(\underline{\theta}))p_1(\underline{\theta}) + q(\underline{\theta})p_2(\underline{\theta})] \Delta\theta. \quad (19)$$

Again, simple manipulations show that this constraint is more stringent than the modified mixed constraint of a $\bar{\theta}$ -firm.

For the sake of conciseness, we shall focus again only on the unconstrained regime. Considering that only (3) and (19) bind, only a $\underline{\theta}$ -firm will be audited with positive probabilities characterized by the following first-order conditions:

$$C'_{\mathcal{R}}(q(\underline{\theta})) = -\frac{1}{1-\nu}R'(q(\underline{\theta})) + \frac{\nu}{1-\nu}(1-\pi_1)\Delta\theta[p_2(\underline{\theta}) - p_1(\underline{\theta})], \quad (20)$$

$$C'_{\mathcal{J}}(p_1(\underline{\theta})) = \frac{\nu}{1-\nu}(1-\pi_1)(1-q(\underline{\theta}))\Delta\theta, \quad (21)$$

$$C'_{\mathcal{J}}(p_2(\underline{\theta})) = \frac{\nu}{1-\nu}(1-\pi_1)q(\underline{\theta})\Delta\theta. \quad (22)$$

The possibility to fine tune the resources allocated to the judiciary branch to unveil assets as a function of the regulatory outcome has now an ambiguous impact on regulatory intervention.

First, notice that $p_1(\underline{\theta})$ decreases with $q(\underline{\theta})$ (from (21)), and that $p_2(\underline{\theta})$ increases with $q(\underline{\theta})$ (from (22)).

When the ex ante monitoring is rather efficient ($C'_{\mathcal{R}}(\cdot)$ being low), the optimal frequency of ex ante investigation is relatively high, so that $q(\underline{\theta}) \geq 1 - q(\underline{\theta})$. Then, from (21) and (22), we have $p_2(\underline{\theta}) \geq p_1(\underline{\theta})$. There is more ex post investigation by the judge if the regulator did intervene ex ante than if he did not. Bringing back this result into (20), the probability of regulatory inspection is distorted beyond the level that prevails in the absence of ex post intervention.

By contrast, when ex ante investigation is relatively inefficient ($C'_{\mathcal{R}}(\cdot)$ being high), the optimal frequency of ex ante investigation is relatively low, so that $p_2(\underline{\theta}) \leq p_1(\underline{\theta})$ and the second term on the right-hand side of (20) is negative. The probability of a regulatory inspection is thus distorted below the level that prevails in the absence of ex post investigation.

Proposition 7. *With conditioning,*

- i) when regulation is efficient (inefficient), the optimal frequency of inspection is higher (lower) in the presence of ex post investigation than in its absence;*
- ii) the higher the optimal probability of ex ante inspection, the higher (lower) the optimal probability of ex post investigation when the ex ante audit did (not) take place.*

An interpretation of this proposition can be the following. If the marginal cost of ex ante inspection is low (resp. high), the intensity of the regulatory enforcement is high (resp. low) and, conditionally on the fact that an accident occurs, a firm that has been audited ex ante faces a high (resp. low) probability of a lawsuit ex post. The probability for the firm of encountering both the regulator and the judge is relatively high (resp. low).

5 Broadening Mandates and Cross-Checkings

In this section, we briefly comment on how our results should be modified if alternative scenarii regarding the roles of the regulator and the judge were envisioned.

5.1 Cross-Checking Care

So far we assumed that following an accident, the judge only audits the value of the firm's assets. Should the judge be given the possibility to run another expertise to cross-check the regulator's ex ante findings about the precautionary level? Such a duplication of expertise is costly and may increase the legal uncertainty faced by the firms, thereby discouraging firms from entering in the business in the first place. This may also destabilize the regulator and weaken its position with respect to the industry since its credibility might be challenged before court. We shall abstract away from these issues and focus on how this possibility of 'cross-checking of care' affects the firm's incentives.

In this new framework, it is immediate to see that (6) and (7) are unchanged. Indeed, these constraints depict situations where the firm already cheats on its wealth level: when the judge intervenes following an accident, the firm's understatement of its wealth is discovered and the firm is already fined at the highest level consistent with its limited liability.

The possibility for the judge to observe ex post the choice of effort by the firm may only affect the moral hazard constraint of a θ -firm, which writes now as follows:

$$U(\theta) \geq (1 - q(\theta)) [\pi_0 t_n(\theta) + (1 - \pi_0) t_a(\theta)] - q(\theta)(\theta + \psi) - \underbrace{(1 - \pi_0) p(\theta)(1 - q(\theta)) [t_a(\theta) + \theta + \psi]}_{\substack{\text{cross-checking on} \\ \text{the precautionary effort}}}. \quad (23)$$

The new term has a simple explanation: when an accident occurs (with probability $1 - \pi_0$), then with probability $p(\theta)(1 - q(\theta))$ the judge finds evidences of the firm's misconduct while the regulator did not check the firm's care.

However, limited liability on the firm's side implies that this additional possibility for the judge has not impact on the optimal investigation probabilities! Since the regulatory transfer $t_a(\theta)$ already saturates the firm's limited liability constraint in the event of an accident, there is no need to check its precautionary effort ex post. Providing judges with additional expertise capabilities is useless. Judges should not devote resources to check

the regulators' ex ante findings about the precautionary effort implemented by the firms. This justifies the kind of separation of expertise that was actually assumed so far in our analysis.

5.2 Cross-Checking Assets

We now briefly touch upon the reverse question: Should the regulator be given the legal rights to audit ex ante the value of the firm's assets?

To give some preliminary hint to this question, let us denote by $p_{ea}(\hat{\theta})$ (resp. $p_{ep}(\hat{\theta})$) the probability of an ex ante investigation on the firm's assets run by the regulator (respectively, the probability of an ex post investigation run by the judge) when the firm claims to have wealth $\hat{\theta}$. To simplify the analysis, we may even assume that there is no investigation of the firm's choice of precautionary effort.

Clearly, the moral hazard incentive constraints are left unchanged. The remaining incentive constraints for a $\bar{\theta}$ -type of firm write now as follows:

$$U(\bar{\theta}) \geq [1 - p_{ea}(\underline{\theta})]U(\underline{\theta}) - (1 - \pi_1)[1 - p_{ea}(\underline{\theta})]p_{ep}(\underline{\theta})\Delta\theta - p_{ea}(\underline{\theta})(\bar{\theta} + \psi), \quad (24)$$

$$U(\bar{\theta}) \geq [1 - p_{ea}(\underline{\theta})]U(\underline{\theta}) - (1 - \pi_0)[1 - p_{ea}(\underline{\theta})]p_{ep}(\underline{\theta})\Delta\theta - p_{ea}(\underline{\theta})\bar{\theta}. \quad (25)$$

These constraints are easily understood as the pure adverse selection and the mixed ones. Notice that, contrary to the main analysis, different regimes are likely to appear depending on the relevant constraint between (24) and (25).

Providing a full-fledged analysis of this case is beyond the scope of this paper. However, notice that in order to reduce the cost of the incentive constraints of the $\bar{\theta}$ -firm, using both an ex ante and an ex post investigation of the firm's claim about its wealth might be valuable. Indeed, cross-checking the regulator's ex ante expertise about the firm's liability with the judge's ex post investigation in the event of an accident enables to reduce the firm's incentive to understate its wealth. Hence, it might be optimal to broaden the mandate of the regulator by allowing for an ex ante investigation of the firm's liability. This may be quite difficult in practice since environmental regulators generally have not developed this expertise.³⁷

6 Conclusion

We have developed a model to study the respective roles of the regulator and the judge in the control of environmental risk. Our starting point is that there exists a separation of tasks between the regulator who controls safety care ex ante, i.e., before any accident realizes, and the judge who intervenes ex post to find out the true value of the firm's assets

³⁷As an example, Boyd (2001, p.21) notes that "corporate financial auditing is not a traditional strength of environmental regulation". However, one can notice that this might change. Indeed, since the 2003 *Loi Bachelot* in France, the so-called *Seveso* firms are required to inform the regulator about their technical and financial capacities before exerting their activity. In addition, some criminal sanctions can be inflicted to those who fail to report any significant change in these capacities to the regulator.

for compensation. Although different in nature and in timing, both instruments are useful in providing incentives to the firm. However, the precise interaction between the regulator and the judge depends on the rule determining the Courts' intervention.³⁸

The first two scenarii we have explored in this paper could be interpreted in terms of real world legal principles.

In our first scenario, the firm is insulated from the perspective of a lawsuit if it has already been inspected by a regulator, which ensures that the high level of effort has been exerted. Considering the fact that the firm is not prosecuted if an accident occurs and that it must have exerted a high level of care, this first scenario has something in common with the *negligence rule* in the law doctrine where injurers are not held liable for the damage they have caused if they have complied with a standard of due care.

In our second scenario, the judge may intervene if an accident takes place even if the regulator did. This has something in common with the *strict liability rule* in the law doctrine where injurers are held liable for the damage they have caused whatever the care they have exerted.

With this comparison in mind, we can reinterpret our main results in the following way. The immunization of firms from legal investigation that follows a regulatory inspection under the negligence rule creates a substitutability between the regulator and the judge. Instead, under strict liability, the regulator and the judge do not interact.

Our model provides thus strong predictions on the extent of ex ante and ex post enforcements. A negative correlation between expenditures/investigations of both branches is expected under the negligence rule whereas expenditures/investigations in the judiciary branch are not correlated with the regulatory expenditures/investigations under strict liability. We are not aware of any such empirical study but such endeavour would certainly be worth undertaking.

From a theoretical perspective, our model could be extended along several lines. First, risk-aversion on the firms' side may also be an important concern. The presence of the regulator and the judge would affect the standard trade-off between insurance and incentives under moral hazard. We feel confident that the general lessons of our work will carry over to those environments.

Second, the legal procedures by which the judge uncovers assets and pierces the corporate veil have been modeled here as a black-box. Much should be made to understand this stage of the analysis in more details.

We also found that, in the event of an accident, providing the judge with the possibility to run a separate expertise about the firm's choice of precautionary effort is useless. This calls for some sort of separation between the regulatory tasks and the legal intervention: the regulator limits its intervention to the 'technical' aspects of the underlying risk

³⁸We did not rank the policies described through scenarii 1 to 3. Obviously, the greatest social welfare would be obtained when using the highest number of instruments, hence under *conditioning*, as long as interventions do not involve fixed costs ($C_{\mathcal{R}}(0) = 0$ and $C_{\mathcal{J}}(0) = 0$).

(i.e., the care exerted by the firm) and the judge focuses on ‘financial’ aspects, i.e., the firm’s collectable wealth. Clearly, further research is warranted in order to refine these results and to reach a better understanding of the interaction between ex ante and ex post interventions.

For instance, we have taken for granted the cooperation between the regulator and the judge. These restrictions are reasonable approximations of real-world institutions. However, they abstract away from political economy considerations or conflict of interests between regulators and judges. Introducing such considerations should allow us to build a more satisfactory theory of the organization of the control of risky industrial activities and of the separation or integration of regulators and judges.³⁹ We plan to investigate those issues in future research.

References

- Baron, D. and D. Besanko**, (1984), “Regulation, Asymmetric Information and Auditing”, *Rand Journal of Economics*, 15: 447-470.
- Becker, G.S.**, (1968), “Crime and Punishment: An Economic Approach”, *Journal of Political Economy*, 76(2): 169-217.
- Boyd, J.**, (2001), “Financial Responsibility for Environmental Obligations: Are Bonding and Assurance Rules Fulfilling Their Promise?” Discussion Paper 01-42 Resources for the Future.
- Boyer, M. and D. Porrini**, (2001), “Law versus Regulation: A Political Economy Model of Instruments Choice in Environmental Policy”, in A. Heyes (ed.), *Law and Economics of the Environment*, Edward Elgar Publishing Ltd.
- Boyer, M. and D. Porrini**, (2004), “Modelling the Choice Between Regulation and Liability in Terms of Social Welfare”, *Canadian Journal of Economics*, 37: 590-612.
- Cohen, M.**, (1985), “The Costs and Benefits of Oil Spill Prevention and Enforcement”, *Journal of Environmental Economics and Management*, 13: 167-188.
- Emons, W.**, (1994), “The Provision of Environmental Protection Measures under Incomplete Information: An Introduction to the Theory of Mechanism Design”, *International Review of Law and Economics*, 14: 479-491.
- Epple, D. and M. Visscher**, (1984), “Oil Spill Frequency and Size: Modelling Occurrence, Detection and Deterrence”, *Journal of Law and Economics*, 27: 29-61.

³⁹See Hiriart, Martimort and Pouyet (2005) for some analysis along these lines.

- Gale, D. and M. Hellwig**, (1985), “Incentive-Compatible Debt Contracts: The One-Period Model”, *Review of Economic Studies*, 52: 647-663.
- Hiriart, Y., D. Martimort and J. Pouyet**, (2004), “On the Optimal Use of Ex Ante Regulation and Ex Post Liability”, *Economics Letters*, 84: 231-235.
- Hiriart, Y., D. Martimort and J. Pouyet**, (2005), “The Public Management of Environmental Risk: Separating Ex Ante and Ex Post Monitors”, mimeo IDEI Toulouse.
- Innes, R.**, (2001), “Self-Enforcement of Environmental Law”, in A. Heyes (ed.), *Law and Economics of the Environment*, Edward Elgar Publishing Ltd.
- Kolstad, C., T. Ulen and G. Johnson**, (1990), “Ex post Liability for Harm vs. Ex Ante Safety Regulation: Substitutes or Complements”, *American Economic Review*, 80: 888-901.
- Laffont, J.-J. and D. Martimort**, 2002, *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press.
- Lewis, T., and D. Sappington**, (2000), “Contracting with Wealth-Constrained Agents,” *International Economic Review*, 41: 743-767.
- Lewis, T., and D. Sappington**, (2001), “Optimal Contracting with Private Knowledge of Wealth and Ability,” *Review of Economic Studies*, 68: 21-45.
- Lesourd, J.B., and G. Schilizzi**, (2001), *The Environment in Corporate Management*, Edward Elgar.
- Mookherjee, D. and I. P’ng**, (1992), “Monitoring vis-à-vis Investigation in Enforcement of Law?”, *American Economic Review*, 82: 556-565.
- Ringleb, A. and S. Wiggins**, (1990), “Liability and Large Scale, Long-Term Hazards,” *Journal of Political Economy*, 98: 574-595.
- Shavell, S.**, (1984a), “A Model of the Optimal Use of Liability and Safety Regulation”, *Rand Journal of Economics*, 15: 271-280.
- Shavell, S.**, (1984b), “Liability for Harm versus Regulation of Safety”, *Journal of Legal Studies*, 13: 357-374.
- Townsend, R.**, (1979), “Optimal Contracts and Competitive Markets with Costly State Verification”, *Journal of Economic Theory*, 21: 417-425.
- Weitzman, M.**, (1974), “Prices vs. Quantities”, *Review of Economics Studies*, 41: 447-491.
- White, M. and D. Wittman**, (1983), “A Comparison of Taxes, Regulation and Liability Rules Under Imperfect Information”, *Journal of Legal Studies*,

12: 413-425.

Wittman, D., (1977), “Prior Regulation versus Post Liability: The Choice between Input and Output Monitoring”, *Journal of Legal Studies*, 6: 193-212.

Yohe, G., (1978), “Towards a General Comparison of Price Controls and Quantity Controls under Uncertainty”, *Review of Economics Studies*, 45(2): 229-238.

A Appendix

A.1 Proof of Proposition 2

The unconstrained regime. When only (3) and (6) are binding, we have

$$U(\underline{\theta}) = U(\bar{\theta}) = R(q(\underline{\theta})) - \underline{\theta}.$$

Inserting those expressions into the regulator’s objective function and optimizing yields $q_*^{ea}(\bar{\theta}) = 0$ and $q_*^{ea}(\underline{\theta})$ given by (10). This unconstrained regime arises as long as:

$$R(q_*^{ea}(\underline{\theta})) + \Delta\theta \geq R(0),$$

i.e., for $\Delta\theta$ large enough.

The constrained regime. In that case, (\mathcal{P}) can be rewritten as:

$$\min_{\{q^{ea}(\cdot)\}} \mathbb{E}_\theta \{R(q^{ea}(\theta)) + C_{\mathcal{R}}(q^{ea}(\theta))\} \text{ subject to } R(q^{ea}(\bar{\theta})) = R(q^{ea}(\underline{\theta})) + \Delta\theta.$$

This problem can be rewritten using the new variable $\tilde{R}^{ea}(\theta) \equiv R(q^{ea}(\theta))$ as:

$$\min_{\{\tilde{R}^{ea}(\cdot)\}} \mathbb{E}_\theta \left\{ \tilde{R}^{ea}(\theta) + h(\tilde{R}^{ea}(\theta)) \right\} \text{ subject to } \tilde{R}^{ea}(\bar{\theta}) = \tilde{R}^{ea}(\underline{\theta}) + \Delta\theta.$$

This is a convex problem when $h \equiv C_{\mathcal{R}}(R^{-1})$ is convex, which amounts to Assumption 3. The first-order conditions are then sufficient for global optimality. Those conditions write respectively as:

$$1 + h'(\tilde{R}^{ea}(\bar{\theta})) = \frac{\lambda}{\nu}, \tag{A1}$$

$$1 + h'(\tilde{R}^{ea}(\underline{\theta})) = -\frac{\lambda}{1-\nu}, \tag{A2}$$

where λ is the Lagrange multiplier associated to the equality constraint. This yields conditions

$$C'_{\mathcal{R}}(q_{**}^{ea}(\bar{\theta})) = -\left(1 - \frac{\lambda}{\nu}\right) R'(q_{**}^{ea}(\bar{\theta})), \quad (\text{A3})$$

$$C'_{\mathcal{R}}(q_{**}^{ea}(\underline{\theta})) = -\left(1 + \frac{\lambda}{1 - \nu}\right) R'(q_{**}^{ea}(\underline{\theta})), \quad (\text{A4})$$

when one has noticed that $\frac{C'_{\mathcal{R}}(q)}{R'(q)} = h'(R(q))$. Then, the value of the multiplier λ is obtained from solving:

$$\varphi\left(1 - \frac{\lambda}{\nu}\right) - \varphi\left(1 + \frac{\lambda}{1 - \nu}\right) = \Delta\theta, \quad (\text{A5})$$

where $\varphi = (-h')^{-1}$ is strictly decreasing when h is convex. The slackness condition is

$$R(q_{**}^{ea}(\bar{\theta})) = R(q_{**}^{ea}(\underline{\theta})) + \Delta\theta. \quad (\text{A6})$$

The interior solution characterized previously is the optimal solution provided that $\lambda \leq \nu$ since $h' < 0$, or equivalently if and only if $\varphi(0) - \varphi\left(\frac{1}{1-\nu}\right) \geq \Delta\theta$. When $\varphi(0) - \varphi\left(\frac{1}{1-\nu}\right) < \Delta\theta$ then $q_{**}^{ea}(\bar{\theta})$ is set at zero (corner solution) and we have the unconstrained regime that we described before.⁴⁰ Finally, this constrained regime holds as long as

$$R(q_*^{ea}(\underline{\theta})) + \Delta\theta \leq R(0),$$

i.e., for $\Delta\theta$ small enough. This constrained regime holds as long as

$$R(q_*^{ea}(\underline{\theta})) + \Delta\theta \leq R(0),$$

i.e., for $\Delta\theta$ small enough.

A.2 Constrained Regime with Immunization

Let us now describe the optimal contract for a constrained regime which corresponds to the case of a small uncertainty on the firm's assets.

Proposition 8. Constrained Regime with immunization.

Assume that

$$R(0) \geq R(q_*^{ep}(\underline{\theta})) + \Delta\theta [1 - (1 - \pi_1)(1 - q_*^{ep}(\underline{\theta}))p_*^{ep}(\underline{\theta})].$$

Then, the optimal contract entails:

- *Constraints (3), (5) and (6) are binding.*

⁴⁰Note that the $\bar{\theta}$ -firm is audited with positive probability as long as $\Delta\theta$ is small enough (then $\lambda \leq \nu$).

- The ex ante and ex post investigation policies are given by the following first-order conditions:

$$C'_{\mathcal{R}}(q_{**}^{ep}(\bar{\theta})) = - \left(1 - \frac{\lambda}{\nu}\right) R'(q_{**}^{ep}(\bar{\theta})), \quad (\text{A7})$$

$$C'_{\mathcal{R}}(q_{**}^{ep}(\underline{\theta})) = - \left(1 + \frac{\lambda}{1 - \nu}\right) R'(q_{**}^{ep}(\underline{\theta})) - \frac{\lambda}{1 - \nu} (1 - \pi_1) p_{**}^{ep}(\underline{\theta}) \Delta\theta, \quad (\text{A8})$$

$$C'_{\mathcal{J}}(p_{**}^{ep}(\underline{\theta})) = \frac{\lambda}{1 - \nu} (1 - \pi_1) (1 - q_{**}^{ep}(\underline{\theta})) \Delta\theta, \quad (\text{A9})$$

for some λ such that $\nu \geq \lambda > 0$, with the slackness condition

$$R(q_{**}^{ep}(\bar{\theta})) = R(q_{**}^{ep}(\underline{\theta})) + \Delta\theta [1 - (1 - \pi_1)(1 - q_{**}^{ep}(\underline{\theta}))p_{**}^{ep}(\underline{\theta})];$$

thus $q_{**}^{ep}(\underline{\theta}) > q_{**}^{ep}(\bar{\theta})$.

Proof. Similar to the proof of Proposition 3 and is thus omitted. A.4. \square

Roughly speaking, the lessons of Proposition 8 extend those of Proposition 3. One common feature of both the constrained and unconstrained regimes is that the probability of an ex ante investigation decreases with the claimed assets. However, when the uncertainty on the firm's assets is small enough, the gain from shirking on safety care is greater than the gain from lying on assets value. Hence, the pure moral hazard incentive constraint of a $\bar{\theta}$ -firm (5) is necessarily also binding. To relax this constraint, the probability of an ex ante investigation of a $\bar{\theta}$ -firm is now positive although lower than when assets are common knowledge. Because (5) is now binding, the multiplier λ of the pure adverse selection constraint (6) is less than ν . Compared with what happens in an unconstrained regime, this reduces the marginal benefits of increasing the probability of both an ex post and an ex ante investigation of a $\underline{\theta}$ -firm (see the right-hand side of (A8)). Indeed, now the adverse selection constraint (6) can be relaxed by using the probability of auditing ex ante the $\bar{\theta}$ -firm.

A.3 Constrained Regime with No-Immunization

For completeness, we describe the optimal regulatory contract in a constrained regime.

Proposition 9. Constrained regime with no immunization.

Assume that

$$R(0) \geq R(q_*^{ep}(\underline{\theta})) + \Delta\theta [1 - (1 - \pi_1)p_*^{ep}(\underline{\theta})].$$

Then, the optimal contract entails:

- Constraints (3), (5) and (6') are all binding.

- The *ex ante* and *ex post* investigation policies are given by the following first-order conditions:

$$\begin{aligned}
C'_{\mathcal{R}}(q_{**}^{ep}(\bar{\theta})) &= -\left(1 - \frac{\lambda}{\nu}\right) R'(q_{**}^{ep}(\bar{\theta})), \\
C'_{\mathcal{R}}(q_{**}^{ep}(\underline{\theta})) &= -\left(1 + \frac{\lambda}{1 - \nu}\right) R'(q_{**}^{ep}(\underline{\theta})), \\
C'_{\mathcal{J}}(p_{**}^{ep}(\underline{\theta})) &= \frac{\lambda}{1 - \nu}(1 - \pi_1)\Delta\theta,
\end{aligned}$$

for some λ such that $\nu \geq \lambda > 0$, with the slackness condition

$$R(q_{**}^{ep}(\bar{\theta})) = R(q_{**}^{ep}(\underline{\theta})) + \Delta\theta [1 - (1 - \pi_1)p^{ep}(\underline{\theta})].$$

Proof. Similar to the proof of Proposition 3 and is thus omitted. □

A.4 Proofs of Propositions 3 and 5

They are similar to the proof of Proposition 2 and are thus omitted.