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THE HECKSHER-OHLIN-SAMUELSON THEORY REVISITED

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Résumé: Le cadre du papier est le modèle HOS, complété par l'ajout d'un processus de transition progressif vers le libre échange et d'une possible mobilité entre les deux facteurs de production retenus, qui sont le travail qualifié et non qualifié. Deux objectifs sont alors poursuivis. Le premier, dans le contexte traditionnel d'ajustement par les prix, est de regarder le rôle des redistributions de revenus nécessaires pour obtenir à chaque étape une situation Pareto supérieure ou au moins "acceptable", c'est-à-dire préférée par une majorité d'agents. Le deuxième objectif est de considérer une approche "duale", où l'ajustement se fait par l'apparition de chômage et non l'adaptation des prix. On regarde alors ce que deviennent les résultats précédemment obtenus. Les deux objectifs visent à rapprocher le modèle HOS des controverses en cours et de la réalité : le libre échange est en effet un processus plus qu'un résultat et celui-ci dépend fortement du cheminement utilisé ; par ailleurs le monde réel est rempli de rigidités, en particulier salariales. Les conclusions soulignent l'importance des politiques de redistribution de revenus et de formation pour que le libre échange demeure un objectif désirable dans le contexte considéré.

Abstract: The framework of the paper is the HOS model completed by the addition of a progressive transition process to free trade and of a possible mobility between the two factors of production, chosen to be unskilled and skilled labour. Two objectives are then pursued. The first objective, in the traditional approach of price adjustment, is to look at the role of redistribution of incomes to get at each stage a Pareto superior situation or at least, an "acceptable" situation, acceptability implying the support of a majority of agents. The second objective is to consider a "dual" approach, where adjustment takes place through unemployment and not adaptation of prices : then one considers what become the previous results. Both objectives intend to bring the HOS theory nearer to current disputes and reality : indeed free trade is never an achieved goal but a process leading to situations depending heavily on the path followed ; also we live in a world with many rigidities, especialy wage rigidities. The conclusions underline the importance of income redistribution and training policies for free trade to remain a desirable goal in the situations considered.

Mots clés : Libre échange, processus de transition, redistribution de revenus, mobilité du travail

Key Words : Free trade, transition process, income redistribution, mobility of labour

Classification JEL: D5, F1, J6

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Introduction

Whether one believes or not in its adequation to the real world, the Hecksher-Ohlin-Samuelson theory remains a basic reference for all those who try to understand the reasons for and the results from international trade. The predictions of the H.O.S. theory, even in its most elementary presentation with two countries, two commodities and two factors of production, actually serve as guidelines for many economic policy positions and decisions.

This paper then pursues two objectives. The first one is to pay a greater attention than usual to the different distributions of utilities and income generated by different ways of moving to free trade. It thus intends to clarify sentences like "free trade is always better than autarky" by looking at the redistributions of income necessary to obtain Pareto superior situations or, at least, "acceptable" situations, acceptability being defined by the fact that a majority of agents supports a definite change. The second objective is to consider a "dual" approach to the traditional one, where adjustment to free trade takes place through unemployment and not adaptation of prices. Then, one considers what is left of the usual propositions, and also what become the previous conclusions.

Both objectives intend to bring the H.O.S. theory nearer to the current disputes about the implications of free trade. Indeed, on one hand, these disputes rely on the fact that free trade is never an achieved goal but a process, leading to precise situations in fact depending heavily - especially concerning utilities and income distributions - on the path followed. On the other hand, disputes also take place in a world with many rigidities - in particular wage rigidities - which imply that some markets - and in particular the labour markets - may not be balanced.

To achieve our analysis, several changes are then brought to the traditional

$2 \times 2 \times 2$ model :

- a simplification : each commodity is considered to be obtained from a single factor ; this allows to get rid of technical calculations which are unessential for our discussions ;

- a specification : instead of considering as the two basic factors of production labour and capital or land, we choose two types of labour, unskilled and skilled ; this choice is made first because a large part of capital is today internationally fully mobile and land is no more a major input, next because the two related questions of distributions of incomes and unemployment appear to be more and more strongly linked to the level of education ;

- an extension : instead of looking only at the two extreme situations, autarky and free trade, we discuss explicitly intermediary ones, by introducing quota the evolution of which define specific transition processes leading to different final states ; doing so then shows that the question of mobility within the labour force, between unskilled and skilled, is also of an essential importance.

Our results can then be summarized as follows :

- indeed, free trade is an efficient situation and can be made to remain so even in presence of unemployment ;

- but it is not necessarily Pareto superior to autarky or even preferred by a majority ;

- the central question for efficiency, acceptability or Pareto superiority to exist, is the implementation of a policy of strong redistribution of incomes ;

- even with such a redistribution though, conditions on the mobility of labour can appear to be necessary in case of rigidities, for free trade to remain a desirable goal.

The paper is organized as follows. Section I presents the model with the three characteristics mentioned above and a new concept of equilibrium. Section II restates precisely the four traditional core propositions of the H.O.S. theory. Section III discusses the transition to free trade driven by higher and higher quota, in the usual context of wage and price flexibility. Section IV conducts the same analysis in case of wage rigidities and thus explicit unemployment situations. The results obtained are then commented in detail in Conclusion.

1 THE MODEL

1. Presentation

The model presented here has already been introduced in a less elaborate form in Fuchs [1997] and [2003]. It considers exchanges between two zones, North and South, both consuming and producing a good number 1, named textile just for the image, and a good number 2, more sophisticate.

Both goods are produced from labour only, through constant returns to scale technologies with coefficients k_1 and k_2, identical for N and S[1]. An essential feature of the model is then the existence of in fact two types of labour : an unskilled labour available in quantities l_1 and l'_1 in N and S for the production of textile, and a skilled labour available in quantities l_2 and l'_2 in N and S for the production of good 2 (quantities with primes will always be related to S). We add the traditional assumption that there is no geographical mobility of labour between N and S.

[1]This identity is only considered here for the sake of comparison with HOS models. It is not a necessary characteristic.

Productions will then be bounded between 0 and ($i = 1, 2$):

$$
\begin{aligned}
y_i &= k_i l_i \\
y'_i &= k_i l'_i
\end{aligned} \tag{1}
$$

The only differentiation between N and S is due to factor endowments through the inequality :

$$l_1/l_2 < l'_1/l'_2 \tag{2}$$

meaning that unskilled labour is relatively more abundant in S.

Let w_i and w'_i be the unskilled and skilled ($i = 1, 2$) wages in N and S respectively. We shall suppose logically that always :

$$w_1 < w_2 \quad w'_1 < w'_2 \tag{3}$$

Production will take place as soon as the corresponding prices p_i and p'_i will be related to wages through :

$$p_i = w_i/k_i \quad p'_i = w'_i/k_i \tag{4}$$

All along the paper, we shall suppose that wages and prices are expressed in the currency of N. We shall not consider any explicit rate of exchange between N and S. We shall see further on, however, that our assumptions on trade imply constraints on the value and evolution of the implicit underlying rate of exchange.

Then we shall suppose that l_i and l'_i in fact correspond to numbers of workers, each endowed with the same utility function of the form :

$$u(c_i) = (c_1)^{\mu}(c_2)^{1-\mu} \tag{5}$$

where c_i (or c'_i) is the individual consumption of good i and $0 < \mu < 1$. We suppose also that workers are wage earners, trying to sell a unit of labour, i.e they look for the maximal level of u under a budget constraint where the only income is wage and, possibly, positive or negative transfer revenues. Given wage levels w_i and an income w, this program leads to :

$$c_1(w_1, w) = \mu w/p_1 \quad c_2(w_2, w) = (1-\mu)w/p_2 \tag{6}$$

(where all quantities have ' if one deals with S).

Ordinary autarkic equilibrium with full employment (the sum of demands equals maximal supply ; no redistribution) then leads to the simple relation between autarkic wages $w^A_.$(for N, and similarly for S with primes) :

$$(1-\mu)w^A_1 l_1 = \mu w^A_2 l_2 \tag{7}$$

Introducing the function :

$$\beta(x_i, x'_j) = x'_1 x_2 / x'_2 x_1 \tag{8}$$

one can see, from (7), (2) and (4) that :

$$\beta^A \equiv \beta(w^A_i, w'^A_j) = \beta(p^A_i, p'^A_j) < 1 \tag{9}$$

2. Trade relations and the behaviour of agents

We shall now introduce an organization of trade which will allow us to describe progressive transitions from autarky to free trade.

First we shall suppose that the $N - S$ trade is balanced at any stage. This implies that prices of commodities in S, expressed as we said in the currency of N, cannot be both higher or lower than prices in N. Thus, either $p_1' > p_1$ and $p_2' < p_2$; but then $\beta(p_i, p_j') > 1$ and, from (9), one cannot approach autarky ; or :

$$p_1' < p_1 \quad p_2' > p_2 \tag{10}$$

inequalities that we shall suppose to hold in what follows. Introducing :

$$\alpha = w_1'/w_1 = p_1'/p_1 \tag{11}$$

one can easily see from (8) that (10) is equivalent to :

$$\beta(p_i, p_j') < \alpha < 1 \tag{12}$$

Clearly (10) or (12) introduce constraints both on the relative price of commodities in N and S and on the rate of exchange to be chosen for their currencies. As a result, it also implies, if trade exists, that textile is exported by S and good 2 by N.

Next, we shall specify the relation between N and S by the volume $q \geq 0$ of textile that N accepts to import (or S agrees to export). Of course $q = 0$ corresponds to autarky. It will be shown later on that above some value of q the existence of a quota is no more binding and a situation of free trade is achieved.

Within this framework, we can thus precise the behavior of our agents.

In N, given (10) or (12), workers will try to buy imported textile at price $p_1' = \alpha p_1 < p_1$ before local one. For small values of q (the word "small" will be precisely defined later on) demand for imported textile will be greater than supply so that we have to consider a "rationing scheme" (see Benassy [1982]). For the sake of simplicity, we shall consider a scheme proportional to demands i.e., from (6) with p_1' instead of p_1, given q, a worker of type j (with $j = 1$ or 2 according to the fact he is unskilled or skilled) can buy a quantity c_q^j of textile defined by :

$$c_q^j = r(q)\mu w_j/p_1' \tag{13}$$

where $r(.)$ is a continuously increasing function of q with $r(0) = 0$. For the c_q^j to define a rationing scheme one must have of course :

$$\lambda_1 c_q^1 + \lambda_2 c_q^2 = q \tag{14}$$

where λ_j is the number of active workers of type j for a given q (so long there are no transfer revenues, possibly unemployed workers of N can buy nothing ; we shall in what follows forget about the integer character of the λ_j and consider them as percentages of the l_j). Thus (13) and (14) together give :

$$c_q^j = w_j \frac{q}{W(q)} \tag{15}$$

where :

$$W(q) = \lambda_1 w_1 + \lambda_2 w_2 \tag{16}$$

is nothing but the total wages and thus income of N. The share of q that a worker of type j can buy is just his share in total income.

We are then in position to describe completely the behavior of a worker of type j. He will choose a consumption bundle $c_i(w_j, w_1', q)$ which maximizes u given the fact that :

- he can buy textile at price $p_1' = \alpha p_1$ up to the quantity c_q^j ;

- if his income w_j is high enough, he can go on buying textile at price p_1.

According to the values of q, different situations then appear.

Let q_2 be the value of q where rationing stops[2], i.e. where $r(q_2) = 1$ or, from (13) and (15) :

$$q_2/W(q_2) = \mu/p'_1 \tag{17}$$

Then obviously, if $q \geq q_2$ all workers (from (17), q_2 is independant of j) can buy all the textile they wish at price p'_1 so that from (6) :

$$\begin{aligned} c_1(w_j, w'_1, q) &= c_1(w'_1, w_j) \\ c_2(w_j, w'_1, q) &= c_2(w_2, w_j) \end{aligned} \tag{18}$$

and we are in a pure free trade situation.

Next, if $q \leq q_2$, an active worker of type j will solve the program :

$$\max \ u(c_q^j + \gamma_j, c_2)$$

under :

$$\begin{aligned} p_1\gamma_j + p_2c_2 \leq R_j \equiv w_j - p'_1 c_q^j \\ \gamma_j \geq 0 \end{aligned} \tag{19}$$

(note that, from (17), $R_j \geq 0$). Considering only the first constraint, an easy calculation leads to :

$$\begin{aligned} c_1(w_j, w'_1, q) &= c_1(w_1, w_j) + \mu(1-\alpha)c_q^j \\ c_2(w_j, w'_1, q) &= c_2(w_2, w_j) + (1-\mu)\frac{p_1 - p'_1}{p_2}c_q^j \end{aligned} \tag{20}$$

The sum of the values of these two consumptions, at price p_1 and p_2 respectively, is $w_j + (p_1 - p'_1)c_q^j$, the last term of the sum representing the extra purchasing power

[2] q_2 and, further, q_1 may or not exist as we shall see later on.

resulting for j from the access to c_q^j at a cheaper price than p_1.

Considering now in addition the constraint $\gamma_j \geq 0$ i.e. :

$$c_1(w_j, w_1', q) - c_q^j \geq 0$$

one gets from (18) :

$$c_q^j \leq \frac{1}{\rho} c_1(w_1, w_j)$$

with :

$$\rho = 1 - \mu(1 - \alpha) \tag{21}$$

Let then q_1 be the value of q defined by :

$$q_1 / W(q_1) = \mu / \rho p_1 \tag{22}$$

It is the value at which and above which no more local textile is bought in N. Clearly $q_1 < q_2$ (because $\rho p_1 > p_1'$). Thus, if $q \leq q_1$ the demand of a worker of type j is given by (20). If $q_1 \leq q \leq q_2$ we have, using to calculate c_2 the budget constraint (19) :

$$\begin{aligned} c_1(w_j, w_1', q) &= c_q^j \\ c_2(w_j, w_1', q) &= w_j/p_2 - p_1' c_q^j / p_2 \end{aligned} \tag{23}$$

Now simultaneously, from (10) or (12) again, consumers of S will try to buy imported good 2 at price $p_2 = (\beta/\alpha) p_2' < p_2'$ before "local" good 2. Trade balance then implies that the volume q' of good 2 imported by S is related to q by :

$$q' = p'_1 q / p_2 \tag{24}$$

leading consumers of S to possibly ration their demands, in parallel to those of N but now for good 2.

Similarly to above then, we are led to define q'_2 and q'_1 through :

$$q'_2 / W'(q'_2) = (1 - \mu)/p_2 \tag{17'}$$

$$q'_1 / W'(q'_1) = (1 - \mu)/\rho' p'_2 \tag{22'}$$

with

$$\rho' = \mu + (1 - \mu)\beta/\alpha \tag{21'}$$

and one has for the demand function of an agent j in S :

- for $q' \geq q'_2$

$$\begin{aligned} c'_1(w'_j, w_2, q') &= \mu w'_j / p'_1 \\ c'_2(w'_j, w_2, q') &= (1 - \mu) w'_j / p_2 \end{aligned} \tag{18'}$$

- for $0 \leq q' \leq q'_1$

$$\begin{aligned} c'_1(w'_j, w_2, q') &= c'(w'_1, w'_j) + \mu \frac{p'_2 - p_2}{p'_1} c'^j_{q'} \\ c'_2(w'_j, w_2, q') &= c_2(w'_2, w'_1) + (1 - \mu)(1 - \beta/\alpha) c'^j_{q'} \end{aligned} \tag{20'}$$

- for $q'_1 \leq q' \leq q'_2$:

$$c'_1(w'_j, w_2, q') = w'_j/p'_1 - p_2 c'^j_{q'}/p'_1$$
$$c'_2(w'_j, w_2, q') = c'^j_{q'} \tag{23'}$$

3. Definition of q-equilibria

Having specified trade relations and the behaviour of producers and consumers, we can now introduce :

Definition 1

Wages w_j and w'_j, the number of active workers λ_j and λ'_j, and the quota level q, define a q-equilibrium if, given these data, demands equal supply plus imports or minus exports on the four markets for the two goods in N and S.

Analytically this definition reads :

$$\begin{aligned} k_1\lambda_1 + q &= \lambda_1 c_1(w_1, w'_1, q) + \lambda_2 c_1(w_2, w'_1, q) \\ k_2\lambda_2 &= \lambda_1 c_2(w_1, w'_1, q) + \lambda_2 c_2(w_2, w'_1, q) + q' \\ k_1\lambda'_1 &= \lambda'_1 c'_1(w'_1, w_2, q') + \lambda'_2 c'_1(w'_2, w_2, q') + q \\ k_2\lambda'_2 + q' &= \lambda'_1 c'_2(w'_1, w_2, q') + \lambda'_2 c'_2(w'_2, w_2, q') \end{aligned} \tag{25}$$

One then has :

Proposition 1

The set of q-equilibria is defined by the following equations for N and S respectively.

- for N

* if $0 \leq q \leq q_1$

$$(1-\mu)w_1\lambda_1 = \mu w_2\lambda_2 - \rho w_1 q/k_1 \tag{26}$$

* if $q_1 \leq q \leq q_2$:

$$\lambda_1 = 0 \tag{27}$$

- for S

* if $0 \leq q' \leq q'_1$

$$(1-\mu)w'_1\lambda'_1 = \mu w'_2\lambda'_2 + \rho' w'_2 q'/k_2 \tag{26'}$$

* if $q'_1 \leq q \leq q'_2$:

$$\lambda'_2 = 0 \tag{27'}$$

Proof :

According to the values of q and q', the c and c' of (25) are given by (20) and (23) or (20') and (23'). Using (4), (15) and (16) and the analogous for S, one gets easily the expressions above. ∎

The fact that the four equations of (25) reduce to two just reflects that, in each zone, total wages sum up to the value of total production plus trade, and that external trade is balanced.

It is not a surprise that, if no more local good 1 or 2 is bought in N or S, the good is no more produced and the corresponding active populations are zero.

Note, from (26) or (26'), that q-equilibria with $q = 0$, i.e. the autarkic situation, includes the autarkic equilibrium with full employment in N (or S) given by (7).

The continuation of the paper will then mainly consist in solving and discussing the above equations. We shall consider first the "classical" situation, where full employment is preserved $\lambda_j = l_j, \lambda'_j = l'_j$, equilibrium then resulting from adaptation of wages (and, through (4), prices). But we shall consider also the "dual" situation where, due to wage rigidities, equilibrium results from the adaptation of the levels of active population, thus taking into consideration possible unemployment.

Before all that, we shall briefly recall, in the framework of the model, the traditional core propositions of the H.O.S. theory.

2 THE H.O.S. RESULTS

1. Direct free trade and the traditional theorems

Implementing directly free trade between N and S just means lifting any trade restriction and unifying N and S prices for the two goods. A simple calculation then leads, similarly to (7), to the equilibrium condition :

$$(1-\mu)w_1^{FT}(l_1 + l'_1) = \mu w_2^{FT}(l_2 + l'_2) \tag{28}$$

From (2) and immediate algebra, one then gets :

$$\frac{w_1'^A}{w_2'^A} < \frac{w_1^{FT}}{w_2^{FT}} < \frac{w_1^A}{w_2^A} \tag{29}$$

In our specific set up and using the above results, the four "core" theorems of the HOS theory become obvious or simple propositions.

Let us first look at the nature of the trade between N and S. Considering for instance the difference between production and consumption of textile in S one gets, using (6) and (4) :

$$k_1 l_1 - \mu(l'_1 w_1^{FT} + l'_2 w_2^{FT})/p_1^{FT} = (1-\mu)k_1 l'_1 - \mu l'_2 k_1 w_2^{FT}/w_1^{FT}$$

a quantity which, from (7), would be zero if wages had their autarkic value and which is positive from (29). This means that S exports textile and thus imports good 2. Each zone thus exports the commodity the production of which requires the relatively abundant factor, relative abundance being in our set up as well physical (see (2)) as regarded in terms of relative prices (see(9)) : this is the Hecksher-Ohlin theorem.

Next, since each commodity is produced with a single type of labour, equalization of commodity prices directly implies through (4) equalization of the corresponding wages : this is the factor price equalization theorem of Samuelson.

The same reason makes empty the Rybczynski theorem predicting an inverse effect of an increase in one factor endowment on the output level of the commodity using intensively the other.

Last, looking at the effects of protection on wages leads to two levels of results. First it is directly clear from (29) that protection (in fact absolute protection) allows for higher relative wages for unskilled workers of N and for skilled workers of S : this is a weak form of the Stolper-Samuelson theorem. To go further and deal with real wages, we shall introduce a price index. Noting that, from(6), μ is also the share of income used by any agent to buy good 1, $(1-\mu)$ to buy good 2 we define :

$$i^{FT} = \mu p_1^{FT}/p_1^A + (1-\mu)p_2^{FT}/p_2^A \tag{30}$$

and, for any w^{FT} the "real" value :

$$w^{rFT} = w^{FT}/i^{FT} \tag{31}$$

Note that replacing FT by A in (30) and (31) gives gives $i^A = 1$ and $w^{rA} = w^A$:

our index is such that real and nominal coincide at autarky ; i^{FT} is just a specific value of a more general index to be defined later on. Using together (31) and (29) then leads after some easy algebra and an obvious definition of i'^{FT} to :

$$\begin{array}{ll} w_1^A > w_1^{rFT} & w_2'^A > w_2^{rFT} \\ w_2^A < w_2^{rFT} & w_1'^A < w_1^{rFT} \end{array} \tag{32}$$

which corresponds this time to the actual Stolper-Samuelson theorem.

2. Utility evolutions

The utility level of each agent only depends on relative prices or wages and so does not care about "nominal" or "real". Using (5), (6) and (29), one sees easily with obvious notations (the lower index corresponds to the type of worker) that :

$$u_1^A > u_1^{FT} > u_1'^A \qquad u_2^A < u_2^{FT} < u_2'^A \tag{32}$$

This confirms the Stolper-Samuelson theorem : implementing free trade in place of autarky makes worse off the relatively rare workers of a zone and better off the relatively abundant ones. In others terms, implementing free trade directly has no chance to lead to a situation Pareto superior to autarky, even if it may be accepted by majority voting in case $l_1 < l_2$ and $l_1' > l_2'$.

Of course, if one first plays autarky and then lets free trade take place, the final situation will be Pareto superior since exchange is voluntary and improves all utilities. But the final situation obtained in this way is defined by different consumption levels and income distribution than in case of direct implementation. This is the core of the discussion to come.

3. The global income approach

Let us then look at the approach where welfare is measured in term of the global income of N and S. Of course comparisons of their nominal values at autarky and free trade make no sense ; but an interesting result can be obtained for real values. Indeed, the same type of calculation as made for individual incomes, using in addition (7), gives :

$$W^{rFT} = W^{A} \qquad W'^{rFT} = W'^{A} \tag{33}$$

These equalities lead to two conclusions : first, the direct opening of N and S preserves the global real incomes that each zone can get by playing autarky ; but also, and this is of central interest for our analysis, redistributions of individual incomes exist which preserve the real income of each agent.

4. The case of redistribution

It is indeed possible, after a direct implementation of free trade, to redistribute income by giving to each agent the real income w_j^A or $w_j'^A$. This corresponds to the nominal incomes in N :

$$x_j^{FT} = w_j^A i^{FT} \tag{34}$$

(and similarly in S). Defining v_j^{FT} as the utility level corresponding to this new distribution, a direct calculation gives :

$$v_j^{FT} = u_j^A \times X(\frac{w_1^{FT}.w_2^A}{w_2^{FT}.w_1^A})$$

where X can be seen to be a decreasing function with $X(1) = 1$. From (29) thus, one concludes that :

$$v_j^{FT} > u_j^A \qquad \forall j \tag{35}$$

i.e. after a suitable redistribution, directly achieved free trade becomes Pareto superior to autarky.

That the properties of a final free trade situation depends crucially on the way it has been obtained and that redistribution may be necessary to get many of the generally stated welfare achievements will be at the heart of the forthcoming sections.

3 TRANSITION TO FREE TRADE THROUGH WAGE ADJUSTMENT

We shall now study the way in which the economies of N and S converge to free trade when the quota q progressively increases, in case equilibrium is obtained through wage flexibility. We introduce :

Definition 2

For a value q of the quota level, wages w_j and w'_j define a q-equilibrium through wage adjustment if, given these data, equations (26) and (26') are satisfied with $\lambda_j = l_j$ and $\lambda'_j = l'_j$.

1. General results

For North, equation (26) can be rewritten :

$$\frac{w_2(q,\alpha)}{w_1(q,\alpha)} = \frac{1}{\mu l_2}[(1-\mu)\ell_1 + \rho q/k_1] \tag{36}$$

Similarly for South, (26') gives, after a lengthy but straightforward calculation using (21'), (24), (8) (definitions of ρ', q', β) and also (36) :

$$\frac{w'_1(q,\alpha)}{w'_2(q,\alpha)} = \frac{\mu}{1-\mu} \quad \frac{[(1-\mu)l_1 + \rho q/k_1]l'_2 + \alpha\mu l_2 q/k_1}{[l'_1 - q/k_1][(1-\mu)l_1 + \rho q/k_1]} \tag{36'}$$

One can check directly that there is no q_1 solution of (22) or q'_1 solution of (22'), which validates ex post the use of (26) and (26').

Thus (36) and (36') define the equilibrium relative wages in N and S, given q and α. The fact that equilibrium conditions are not sufficient to fix also α cannot be looked at as a surprise since, from our discussion in (II, 2), we have chosen not to explicit a rate of exchange. However, from (12), α cannot be arbitrary but must satisfy :

$$1 > \alpha > \beta(q, \alpha) \equiv \frac{[(1-\mu)l_1 + \rho q/k_1]l_2' + \alpha\mu l_2 q/k_1}{(1-\mu)l_2[l_1' - q/k_1]} \tag{37}$$

inequalities which, using (21) are equivalent to :

$$1 > \alpha > \alpha_L(q) \equiv \frac{l_2'(l_1 + q/k_1)}{l_2 l_1' - (l_2 + \mu l_2')q/k_1(1-\mu)} \tag{38}$$

Note that $q = 0$ (autarky) brings (36) and (36') back to (7). Obviously α_L is an increasing function of q ; $\alpha_L(0) < 1$ from (2) and α_L, and thus α, equals 1 for the value :

$$q^{FT} = \frac{(1-\mu)k_1(l_2 l_1' - l_1 l_2')}{l_2 + l_2'} \tag{39}$$

One can also see that β is an increasing function both in q and α and check that $\beta(q^{FT}, \alpha_L(q^{FT})) = 1$. One can see next that, from (36), w_2/w_1 is increasing both in q and α and check that, from (36'), the same is true for w_1'/w_2'. Last, one can calculate using (28) that :

$$\frac{w_2(q^{FT}, 1)}{w_1(q^{FT}, 1)} = \frac{w_2'(q^{FT}, 1)}{w_1'(q^{FT}, 1)} = \frac{(1-\mu)(l_1 + l_1')}{\mu(l_2 + l_2')} = \frac{w_2^{FT}}{w_1^{FT}} \tag{40}$$

which, ex post, justifies the notation q^{FT} : this value indeed corresponds to a free trade situation where the quota is no more binding.

2. Utility evolutions and democratic acceptability

Let us now examine how the utility level of an agent moves with q. One gets for instance for N, using (5), (20), (6), (15) and (16), and with $A = (\mu k_1)^\mu[(1-\mu)k_2]^{1-\mu}$:

$$u_j(q,\alpha) \equiv u(c_1(w_j, w_1', q), c_2(w_j, w_1', q)) = \frac{Aw_j}{w_1^\mu w_2^{(1-\mu)}}[1 + \frac{(1-\alpha)q/k_1}{l_1 + l_2 w_2/w_1}] \qquad (41)$$

With (36) and (37), one can then study precisely the properties of u_1 and u_2 between 0 and q^{FT}, autarky and free trade. Because of the term between brackets in (41), the value of which is 1 at 0 and q^{FT}, and which is first increasing and finally decreasing in q, the evolution of the u_j is not necessarily monotonous. One gets for instance :

- if $l_2 > l_1$, which from (3) implies $\mu < \frac{1}{2}$, $u_1(u_2)$ is monotonously decreasing (increasing), supposing α increases monotonously with q. This means that no increase in q leads to Pareto superior situations but that quota increase can be supported in N at each stage by majority voting ;

- if $l_1 > l_2$ and $\mu > \frac{1}{2}(1-\frac{l_1 l_2'}{l_1' l_2})$ thus both u_1, and u_2 increase until some $q_{\max} < q^{FT}$ where u_1 begins to decrease ; in other words, increasing q is Pareto superior between autarky and $q_{\max}$ and then blocked by majority voting, which thus does not allow for free trade.

We shall not present here a systematic analysis of all cases, which would be technically very long. But it is interesting to emphasize through the two previous examples that many different situations can appear, with different political supports.

Note that calculations above imply only relative incomes, which can thus be real as well as nominal.

3. The global income approach

We shall now look for a possible extension of (33) for any value of q between 0 and q^{FT}. For that, we shall first introduce an extended price index defined, for the North, by a structure analogous to (30) i.e. :

$$i(q,\alpha) = \frac{W_1}{W} \quad \frac{p_1(q,\alpha)}{p_1^A} + \frac{W_1'}{W} \quad \frac{p_1'(q,\alpha)}{p_1^A} + \frac{W_2}{W} \quad \frac{p_2(q,\alpha)}{p_2^A} \tag{42}$$

where the $W_.$ are just the shares of total income of N devoted to buy local textile, imported textile and good 2 respectively. Using (20), (15), (6) and (36) gives :

$$i(q,\alpha) = \mu \frac{l_1 + \alpha^2 q/k_1}{l_1 + \rho q/k_1} \quad \frac{p_1(q,\alpha)}{p_1^A} + (1-\mu) \quad \frac{l_1 + q/k_1}{l_1 + \rho q/k_1} \quad \frac{p_2(q,\alpha)}{p_2^A} \tag{43}$$

Obviously $i(0,\alpha) = 1$ and, for $q = q^{FT}$ (where $\alpha = 1$) ; $i(q^{FT}, 1) = i^{FT}$.

Lets us then define for any income w :

$$w^r(q,\alpha) = w(q,\alpha)/i(q,\alpha) \tag{44}$$

A tedious but straightforward calculation, using (7) and (35) then gives :

$$W^r(q,\alpha) = W^A \frac{[l_1 + \rho q/k_1]^2}{l_1^2 + l_1 q/k_1[2\rho - \alpha(1-\alpha)\mu] + \rho(q/k_1)^2} \tag{45}$$

For $q = 0$ or $q = q^{FT}(\alpha = \rho = 1)$ total real income of N is, as expected, equal to W^A ; but the equality is no more valid for intermediary values of q. For small q, obviously, $W^r > W^A$ so there is necessarily a value $q^{\max}$ where W^r is maximal. If W^r is the criterium chosen to express welfare in N, this means that $q^{\max}$ and not q^{FT} will be the best possible level of a progressive opening. Tedious calculations lead to similar conclusions for S.

4. The case of redistribution

Similarly to Section II, one can last look for a redistribution of $W(q,\alpha)$ such that, even if the income of N decreases after some $q^{\max}$, thanks to the price diminution for textile, individual utilities remain increasing all along the way to free trade.

Let us for such an exploration consider :

$$x_j(q,\alpha) = w_j^A \; W(q,\alpha)/W^A \tag{46}$$

One has $l_1x_1 + l_2x_2 = W$ so that the x_j define for each q a possible redistribution ; $x_j(0,\alpha) = w_j^A$; last (46) is an extension of (34) : indeed, from (33) and (31) one has $x_j(q^{FT},1) = x_j^{FT}$. Defining $v_j(q,\alpha)$ as the utility level corresponding to the incomes above, lengthy and somewhat subtile calculations allow to show that there exist exchange rate policies (i.e. choices of α satisfying (38)) such that v_1 and v_2 are monotonously increasing in q (see Appendix 1). Similar conclusions can be seen to hold for S.

In other terms, subject to the condition that, at each stage, a suitable redistribution of incomes is performed, progressive move to free trade can be unanimously accepted.

4 TRANSITION TO FREE TRADE THROUGH LABOUR ADJUSTMENT

We shall here study what can be considered as a dual approach to the traditional HOS theory, i.e. the way in which the economies of N and S converge to free trade when still the quota q progressively increases but when, this time, wages are rigid and equilibrium is obtained through adjustments of the level of the labour force employed.

To understand the reasons for the definitions to come, let us then start from autarky equilibrium in N and S and let q (and so q') grow. Wages being kept fixed at their autarkic value, global full employment can be maintained (for N and similarly for S with primes) if :

$$\lambda_1 + \lambda_2 = l_1 + l_2 \equiv l \tag{47}$$

Let us first look at the situation in S. (26') an (47') together with growing values of q imply smaller λ_2' and higher λ_1' or, in other words, that skilled workers accept unskilled position and lower wages (it is unlikely that they prefer unemployment in poor countries where unemployment benefits are inexistant or low) : the new situation is not an improvement but not an impossibility. Situation in N is quite different. Indeed, (26) and (47) together will growing q imply smaller λ_1' and higher λ_2', i.e. that unskilled workers take skilled positions : but this requires training, time, money, and will not necessarily be possible for everyone. Thus a violation of (47) implying unemployment for the unskilled is very likely to occur.

Having in mind this discussion, we then introduce successively :

Definition 3

We call mobility of labour in N a continuous non negative function of q labelled $\wedge$, with $\wedge(0) = 0$ and :

$$\begin{aligned} \wedge(q) \leq \frac{\rho^A \; q l_2}{(1-\mu) k_1 l} \qquad & \text{for q} \in [0, \text{q}_1] \\ \wedge(q) \leq \ell_1 \qquad & \text{for q} \in [\text{q}_1, \text{q}_2,] \end{aligned} \tag{48}$$

(ρ^A is equal to ρ with $\alpha = \alpha^A = w_1'^A / w_1^A$)

To comment this definition, let us consider :

$$\lambda_2(q) = l_2 + \wedge(q) \tag{49}$$

One has $\lambda_2(0) = l_2$ and, from (26) using (7) or (27) :

$$\lambda_1(q) + \lambda_2(q) \leq l \quad \forall q$$

In other words, $\wedge$ allows to describe any admissible evolution of the skilled labour force in N, including perfect rigidity ($\wedge \equiv 0 \Leftrightarrow \lambda_2 = l_2$) and perfect mobility (when $\wedge$ is equal to its upper bounds, (47) is always satisfied.

We then introduce :

Definition 4

For a value q of the quota level, given a mobility of labour $\wedge$, employment levels λ_j and $\lambda_j^{'}$ define a $(q, \wedge)$-equilibrium through labour adjustment if, given these data, equations (26), (27), (49) for N, (26'), (27'), (47') for S are satisfied with $w_j = w_j^A$ and $w_j' = w_j'^A$.

1. General results

For North, using (7), it appears clearly that there is a unique $(q, \wedge)$-equilibrium for $q \in [0, q_1]$, given by :

$$\begin{aligned} \lambda_1(q, \wedge) &= l_1 + \frac{l_1}{l_2} \ \wedge(q) - \frac{\rho^A q}{(1-\mu)k_1} \\ \lambda_2(q, \wedge) &= l_2 + \wedge(q) \end{aligned} \qquad (50)$$

From (48) then, λ_1 satisfies the inequality :

$$l_1 - \frac{\rho^A q}{(1-\mu)k_1} \leq \lambda_1(q, \wedge) \leq l_1 - \frac{\rho^A q l_2}{(1-\mu)k_1 l} \qquad (51)$$

which means that there exists a smaller value $q_1^\wedge$ where $\lambda_1(q_1^\wedge) = 0$ and that :

$$\frac{(1-\mu)k_1 l_1}{\rho^A} \leq q_1^\wedge \leq \frac{(1-\mu)k_1 l_1 l}{\rho^A l_2} \qquad (52)$$

From (50), the value $q_1^\wedge$ can be seen to satisfy (22), hence the notation.

Next, for $q \geq q_1^{\wedge}$, we have $\lambda_1 = 0$, λ_2 being still given by (49). Equation (17) in this situation can then be written :

$$p_1^{'A} q_2 = \mu w_2^A [l_2 + \wedge(q_2)] \tag{53}$$

It clearly has a solution since for $q \geq 0$ the left hand side grows linearly from 0 while the right hand side starts at $\mu w_2^A l_2$ and is bounded. We call $q_2^{\wedge}$ the smaller solution, which, from (53), (48) and (7), satisfies :

$$\frac{(1-\mu)k_1 l_1}{\alpha^A} \leq q_2^{\wedge} \leq \frac{(1-\mu)k_1 l_1 l}{\alpha^A l_2} \tag{54}$$

For South, there is also a unique $(q, \wedge)$-equilibrium, corresponding to perfect mobility of workers. For $q' \in [0, q_1']$ it is given by :

$$\begin{aligned} \lambda_1^{'M}(q, \wedge) &= l_1' + \frac{\rho^{'A} q' l_1'}{\mu k_2 l'} \\ \lambda_2^{'M}(q, \wedge) &= l_2' - \frac{\rho^{'A} q' l_1'}{\mu k_2 l'} \end{aligned} \tag{55}$$

which gives for q_1' the value :

$$q_1^{'M} = \frac{\mu k_2 l_2' l'}{\rho^{'A} l_1'} \tag{56}$$

Next for $q' \geq q_1^{'M}$, we have $\lambda_1^{'M} = l', \lambda_2^{'M} = 0$ and we get from (17) :

$$q_2^{'M} = \frac{\alpha^A \mu k_2 l_2 l'}{l_1} \tag{57}$$

Translated through (24) in terms of quota for N, the two limits given in (56) and (57) become :

$$q_1^M = \frac{(1-\mu)k_1 l_1 l_2' l'}{\alpha^A \rho'^A l_2 l_1'}$$
$$q_2^M = (1-\mu)k_1 l' \tag{58}$$

Let us then discuss the economic meaning of the above mathematics. When q grows from 0, it will first reach $q_1^\wedge$ (or q_1^M) where the production of textile in N (or good 2 in S) disappears : we thus get a situation of "specialization" in N (or S). The fact that it is N or S which specializes first depends on two elements : mobility in N (from (50) the more rigid is the unskilled force, the smaller is $q_1^\wedge$), size of population in S with respect to N (from (58), the larger is l', with fixed proportions of skilled and unskilled, the greater is q_1^M). Then, two scenarios may occur. Either q_1^M (or $q_1^\wedge$) is reached next, which means that both zones become specialized in their domain of relative advantage ; then, when $q_2^\wedge$ or q_2^M is reached, a free trade situation prevails since further increase in q does not change consumptions or productions. Or $q_2^\wedge$ (or q_2^M) is reached before the other q_1, which means again that a free trade situation prevails but, this time, with only one zone specialized. From (54) and (58), the size of $q_2^\wedge$ is very sensitive to the relative cost of unskilled labour between S and N, the size of q_2^M to the size of the population of S.

2. Utility evolutions and democratic acceptability

From our assumptions, the evolution of utility of agents of S when q grows is simple to analyze : unskilled workers, which are likely to be a larger majority, see their utility level increase since, with an unchanged income, they have greater access to cheaper good 2 ; the same is true for skilled workers who keep their job ; only skilled workers who take unskilled positions are dissatisfied, at least for small values of q, because of their lower income. Support for progressive opening thus can never gather unanimity but always begins with a majority.

The situation in N is quite different since, except in case of perfect mobility, appear unemployed workers. In the absence of redistribution, since they have a zero income and thus a zero level of utility, they will never support any sort of opening.

Using (50) the number of these workers is :

$$U^{\wedge}(q) = l - \lambda_1 - \lambda_2 = \frac{\rho^A q}{(1-\mu)k_1} - \frac{l}{l_2} \wedge (q) \tag{59}$$

where $U^{\wedge}$ cannot be larger than l_1. Of course, if $l_2 > l_1$ moving to free trade is always supported by a majority. In case $l_1 > l_2$, let $q^{\wedge}_{\max}$ be the first value[3] of q where $U^{\wedge}$ become larger than $\frac{1}{2}l$. Then, in N, not only support for opening will never gather unanimity but, above the threshold $q^{\wedge}_{\max}$, it may even lose majority support (for a more detailed discussion of this assertion, see Fuchs [1997]).

3. The global income approach

Wages being kept fixed, we can this time look first at nominal incomes.

For N we have, from (50) :

$$W(q, \wedge) = W^A + \frac{W^A}{l_2} \wedge (q) - \frac{\rho^A q}{(1-\mu)k_1} w_1^A \tag{60}$$

Clearly, perfect rigidity implies that W decreases with q from W^A, perfect mobility implies that W increases from W^A. Clearly also, there exists a limit $\wedge^m$, with $\wedge^m(q) = \rho^A p_1^A q / w_2^A$, for which W remains constant to W^A (for a more detailed discussion, see Fuchs [2003]).

For S we have, from (55) :

$$W'(q') = W'^A - \frac{\rho'^A q' l'_1}{\mu k_2 l'} (w'^A_2 - w'^A_1) \tag{61}$$

where, without surprise, W' is decreasing with q' from W'^A.

But, of course, real incomes are the interesting objects. They are still defined

[3] $q^{\wedge}_{\max}$ may or not exist according to $\wedge$. For $\wedge = 0$ (perfect rigidity), one has for instance $q^{\wedge}_{\max} = (1-\mu)k_1 l / 2\rho^A$

through (44) where $i(q, \wedge)$ is given by (42) and (50) for N, (42') and (55) for S .

For N one then gets :

$$i(q, \wedge) = 1 - (1 - \alpha)\frac{p_1'^A q}{W(q, \wedge)}$$

so that one has :

$$W^r(q, \wedge) = \frac{W(q, \wedge)}{W(q, \wedge) - (1 - \alpha)p_1'^A q} . W(q, \wedge) \geq W(q, \wedge) \tag{62}$$

Again one can calculate a limit $\wedge_r^m$ such that $W^r(q, \wedge_r^m) = W^A$; then a mobility higher than $\wedge_r^m$ guaranties that $W^r > W^A$. One can also look for maximal values of q until which W^r is increasing or remains greater than W^A. One can last look for conditions such that for $q = q_2^\wedge$ or q_2^M (achieved free trade) again $W^r \geq W^A$.

Similar discussions can take place for S with :

$$i'(q) = 1 - (1 - p_2^A / p_2'^A)\frac{p_2^A q'}{W'(q, \wedge)}$$

4. Redistribution and training

At the difference of Section II and III, it is now clear that redistribution will not necessarily be sufficient to have an opening process supported by unanimity or even by majority : obviously, from 2., unemployed people with no income can block many decisions in N; but also, intuitively, from 3., if the loss in global income is too important, redistribution itself may appear not to be sufficient to keep utility level of many.

So the question we are going to deal with now is : which conditions on mobility (and thus training) are necessary for one to find a redistribution giving for instance unanimity support to opening ?

Let us for that consider first, in N, the following four types of agents $h = 1, 2, 3, 4$ where :

. $h = 1$ corresponds to the λ_1 active unskilled agents ;

. $h = 2$ corresponds to the l_2 initially active skilled agents ;

. $h = 3$ corresponds to the $l - \lambda_1 - \lambda_2$ unemployed unskilled agents ;

. $h = 4$ corresponds to the $\lambda_2 - l_2$ active skilled but initially unskilled agents.

Given any $(q, \wedge)$-equilibrium and the associated income $W(q, \wedge)$ of N, a redistribution of income is defined by a set $(x_h(q, \wedge))$ such that :

$$\lambda_1 x_1(q, \wedge) + l_2 x_2(q, \wedge) + (l - \lambda_1 - \lambda_2) x_3(q, \wedge) + (\lambda_2 - l_2) x_4(q, \wedge) = W(q, \wedge) \quad (63)$$

Since demand functions are homogeneous of degree 1 in income, redistribution does not change total demands and a $(q, \wedge)$-equilibrium is transformed in an other $(q, \wedge)$-equilibrium. Now a straightforward calculation shows that, between 0 and $q_1^\wedge$, the utility level associated with the income distribution above is given by :

$$v_h(q, \wedge) = B x_h(q, \wedge)[1 + (p_1^A - p_1'^A) q / W(q, \wedge)] \quad (64)$$

(where $B = \mu^\mu (1 - \mu)^{1-\mu} / (p_1^A)^\mu (p_2^A)^{1-\mu}))$. It will remain equal to the autarky level if $\wedge$ is such that, for each q

$$x_h(q, \wedge)[1 - (p_1^A - p' A_1) q / W(q, \wedge)] = w_j \quad (65)$$

where $j = 1$ for $h = 1, 3$ or 4 and $j = 2$ for $h = 2$. If this is true, multiplying each equality (65) by the number of corresponding agents and adding gives, thanks to (63), the necessary condition :

$$W(q,\wedge) + (p_1^A - p_1'^A)q = W^A$$

which, using (60), corresponds to the specific $\wedge^P$ given by :

$$\wedge^P(q) = p_1'^A q / w_2^A \tag{66}$$

A similar but more lengthy calculation allows to define $\wedge^P$ between $q_1^\wedge$ and $q_2^\wedge$. Then, obviously, defining $\wedge_1 > \wedge_2$, by $\wedge_1(q) > \wedge_2(q) \forall q$, the simultaneous implementations in N of a mobility of labour $\wedge > \wedge^P$ and of a redistribution policy satisfying (65) will generate $(q,\wedge)$-equilibria unanimously preferred to autarky.

The situation in S is more simple since we have supposed a perfect mobility of labour. We then only have to consider three type of agents $h' = 1, 2, 3$, where :

. $h' = 1$ corresponds to the l_1' originally unskilled workers ;

. $h' = 2$ corresponds to the λ_2' remaining skilled workers ;

. $h' = 3$ corresponds to the $l_2' - \lambda_2'$ unskilled but previously skilled workers.

Let then $(x_{h'}'(q'))$ be a redistribution of income in S. Between 0 and $q_1'^M$, the associated utility levels will be :

$$v_{h'}(q') = Ax_{h'}'(q')[1 + (p_2'^A - p_2^A)q'/W'(q')] \tag{64'}$$

They would remain equal to the autarky levels, if, for each q' :

$$x_{h'}'(q')[1 + (p_2'^A - p_2^A)q'/W'(q')] = w_j' \tag{65'}$$

where $j = 1$ for $h' = 1$ and $j = 2$ for $h' = 2$ or 3. Using (55) one then easily sees that this condition is independent of q'. Some lengthy but straightforward calculation then shows that the left hand side of (65') is always smaller than the right hand side: the negative income effect is always larger than the positive price effect coming from imports of good 2. The same can be shown to be true between $q_1'^M$ and $q_2'^M$. Thus, due to the evolution of skilled workers, even with a strong redistribution process, the support for opening may only come from unskilled workers.

Conclusion

In a pure exchange economy, any lift of barriers to trade between zones obviously leads to final situations which are Pareto superior to initial ones, since opportunities are increased and exchange only takes place if the involved agents obtain a higher level of utility. If, in addition, markets are perfect, free trade then appears to be Pareto optimal.

The case of production economies is more subtle and one has to be very precise about the situations compared. In our approach, a vocabulary of game theory is more appropriate. The agents start with their available labour force and (in the model) a zero utility level : this is situation S_0. Then, from S_0, the following new situations can be obtained and compared :

- S_1 when the agents "play" autarky of North and South ;

- S_2 (or $S_2(q)$) when they play direct free trade (or direct opening with a level q of quota) ;

- S_3(or $S_3(q)$) when they first play S_1 and then totally (or up to q) lift barriers.

Of course, all these situations are Pareto superior to S_0 ; S_3 (or $S_3(q)$) is Pareto superior to S_1 ; S_2 is distinct from S_3(or $S_2(q)$ from $S_3(q)$) ; in case of perfect markets S_2 and S_3 are both Pareto optimal.

The questions discussed in the paper are then the following :

- Section II recalls the Stolper-Samuelson theorem ; playing S_2 rather than S_1 makes some people better off but others worse off ; S_2 is not Pareto superior to S_1

but, in some cases, can be decided by majority voting. It is also shown, however, that an income redistribution exists, changing S_2 in S_2^r, such that S_2^r is Pareto superior to S_1.

- Section III deals with wage adjustments generated by quota increase. Then in some circumstances, things go as above with $S_2(q)$ instead of S_2 ; but, in other cases, with $q' > q''$, $S_2(q')$ is Pareto superior to $S_2(q'')$ only so long q' is smaller than some $q_{\max} < q_{FT}$. In any case however, there always exist for each q an income redistribution such that $S(q')$ is Pareto superior to $S(q'')$ until free trade.

- Section IV deals with labour adjustment generated by quota increases. Things then are naturally more difficult for the agents. In South, only unskilled workers support opening. In North a process of opening cannot lead to Pareto superior situations due to the existence of unemployed ; and even majority support may fail after some $q^{\wedge}_{\max}$. The new conclusion is that, to preserve global real income - and thus possibility for a suitable redistribution to lead to Pareto superior situations - either one cannot go beyond some quota level for a given mobility of labour, or mobility of labour has to be itself superior to some definite function : if not, even redistribution of incomes cannot allow to improve the situation of all.

It would make no sense of course to discuss how far from the real world stands the model presented in this paper. It helps in any case to shed a crude light on two questions :

- even in case of perfect markets, income redistributions are essential for many of the currently stated advantages of free trade to be undoubtedly true ;

- if in addition exist wage rigidities - which do correspond to many actual situations - then mobility between the existing types of labours is an essential feature to counteract the effects of rigidities, that redistribution is not in any case sufficient to compensate.

Appendix 1

Straightforward calculations first lead to :

$$v_j(q,\alpha) = u_j^A \times \frac{1+q/k_1l_1}{[1+\rho q/(1-\mu)k_1l_1]^{1-\mu}}$$

The sign of $\frac{dv_j}{dq}$ is then easily seen to be the sign of

$$[1-\rho] + \frac{\mu\rho q}{(1-\mu)k_1l_1} - \frac{d\rho}{dq} \times q[1+q/k_1l_1] \qquad (A,1)$$

Let us thus choose as an example of exchange rate adaptation :

$$\rho = \rho_0 + (1-\rho_0)(\frac{q}{q^{FT}})^{\frac{1}{n}}$$

where q^{FT} is given by (39) and $n \in N$. ρ is an increasing function of q with $\rho(0) = \rho_0, \rho(q^{FT}) = 1$. Clearly, the first of the three terms in (A,1) is decreasing, the second and the third are increasing.

A short calculation and use of the fact that $\lim_{n\to+\infty}(1+\frac{1}{n})^n = e$ allows thus on one hand to see that the difference between the first and third term in (A,1) is positive if :

$$q < Q = q^{FT}e^{-(1+q^{FT}/k_1l_1)}$$

On the other hand, the difference between the second and third term remain positive if :

$$q > Q(n) = \frac{(1-\rho_0)(1-\mu)(q^{FT}+k_1l_1)}{n\rho_0\mu}$$

For n large enough thus $Q(n) < Q$ and the corresponding ρ is such that (A,1) is positive between 0 and q^{FT} so that v_j is an increasing function of q for both j.

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